PHYSICAL REVIEW D **78,** 074001 (2008)

$\Upsilon(4S,5S) \to \Upsilon(1S)\eta$ transitions in the rescattering model and the new *BABAR* measurement

Ce Meng[1] and Kuang-Ta Chao[1,2]

[1]*Department of Physics and State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, China*
[2]*Center for High Energy Physics, Peking University, Beijing 100871, China*

(Received 22 July 2008; published 2 October 2008)

The η transitions of $\Upsilon(4S,5S)$ into $\Upsilon(1S,2S)$ are studied in the rescattering model by considering the final state interactions above the $B\bar{B}$ threshold. The width of the η transition of $\Upsilon(4S)$ into $\Upsilon(1S)$ is found to be larger than that of the dipion transition, and the ratio of $\Gamma(\Upsilon(4S)\to\Upsilon(1S)\eta)$ to $\Gamma(\Upsilon(4S)\to\Upsilon(1S)\pi^+\pi^-)$ is predicted to be $R_4 = 1.8 - 4.5$, which is about 2 orders of magnitude larger than the expectation of the conventional hadronic transition theory, and is supported by the new *BABAR* measurement. The widths of the η transitions of $\Upsilon(5S)$ are found to be sensitive to the coupling constants $g_{\Upsilon(5S)B^{(*)}B^{(*)}}$ due to a large cancellation between contributions from the $B\bar{B}$, $B^*\bar{B}$ + c.c., and $B^*\bar{B}^*$ channels, and only a rough estimate $\Gamma(\Upsilon(5S)\to\Upsilon(1S,2S)\eta) = 10$–$200$ KeV can be given. The widths of the η' transitions of $\Upsilon(4S,5S)$ are also discussed, and they could be much smaller than that of the corresponding η transitions mainly due to the tiny phase space.

DOI: 10.1103/PhysRevD.78.074001 PACS numbers: 14.40.Gx, 13.25.Gv, 13.75.Lb

I. INTRODUCTION

Hadronic transitions of heavy quarkonia are important for understanding both the heavy quarkonium dynamics and the formation of light hadrons. Because heavy quarkonium is expected to be compact and nonrelativistic, at least for the lower-lying states, QCD multiple expansion (QCDME) approach (for recent reviews see, e.g. [1,2]) can be used in the analysis of these transitions. However, the justification of QCDME scenario becomes problematic for higher-exited heavy quarkonia. Particularly, when the excited state lies above the open flavor thresholds, the coupled-channel effects may change the QCDME scenario markedly and add new mechanisms to the analysis of its hadronic transitions.

In a previous paper [3], we use the final state rescattering model [4] to study the dipion transitions of $\Upsilon(5S)$ and $\Upsilon(4S)$. In this model, the $\Upsilon(5S/4S)$ first decays to $B^{(*)}\bar{B}^{(*)}$, and then the B meson pair turns into a lower Υ state and two pions through exchange of another $B^{(*)}$ meson. We find that there is a huge difference, which is about a factor of 200–600 in magnitude [3], between the partial widths of dipion transitions of $\Upsilon(5S)$ and $\Upsilon(4S)$. This result is consistent with the measurement of the Belle Collaboration [5]. The coupled-channel effects (or the meson-loop effects) in the transitions $\Upsilon(nS)\to\Upsilon(mS)\pi\pi$, where $n = 2$, 3, 4, 5 and $m < n$, are also studied by Simonov *et al.* [6,7]. In a recent calculation for the dipion transitions of $\Upsilon(5S)$ [7], their result confirms ours [3] at the quantitative level.

The above results indicate that it might be unnecessary to introduce exotic interpretations for the $\Upsilon(5S)$ resonance such as the Y_b state [8] to account for the experimental data [5] if the rescattering model [3] can be proved efficient enough. Therefore, it is very useful to study other features of the final state rescattering mechanism. One evident feature of the rescattering mechanism is the strong energy-dependence of the decay rates, which can induce significant shifts of 10–20 MeV of the observed resonance peaks in $\Upsilon(5S)\to\Upsilon(mS)\pi^+\pi^-$ relative to that of $\Upsilon(5S)\to B^{(*)}\bar{B}^{(*)}$ [9].

Another important feature is that some of the power counting rules in the QCDME approach may fail in the rescattering model. For example, in the QCDME approach, the dipion transition of heavy quarkonium can be achieved through the E1-E1 (electric-dipole) transition, whereas the η transition is dominated by the E1-M2 (magnetic quadrupole) transition, which is associated with the spin-flip effects of the heavy quarks, due to the η quantum number being $J^{PC} = 0^{-+}$ [1,2]. Therefore, in the QCDME approach the η transition is expected to be strongly suppressed relative to the corresponding dipion transition, whereas there is no such suppression in the rescattering model. In fact, within the framework of the QCDME approach, Kuang [1] predicted the ratios

$$R_n = \frac{\Gamma(\Upsilon(nS)\to\Upsilon(1S)\eta)}{\Gamma(\Upsilon(nS)\to\Upsilon(1S)\pi^+\pi^-)} \sim 10^{-2}\text{–}10^{-3} \quad (1)$$

for $n = 2$, 3, which are roughly in agreement with the new measurements by the CLEO Collaboration [10]:

$$R_2 = 1.1^{+0.5}_{-0.4}\times 10^{-3}, \qquad R_3 < 7\times 10^{-3}. \quad (2)$$

However, recently the preliminary result reported by the *BABAR* Collaboration [11] indicates that for the $\Upsilon(4S)$ the ratio

$$R_4 = \frac{\Gamma(\Upsilon(4S)\to\Upsilon(1S)\eta)}{\Gamma(\Upsilon(4S)\to\Upsilon(1S)\pi^+\pi^-)} = 2.41 \pm 0.40 \pm 0.12, \quad (3)$$

which is larger than that for the $\Upsilon(2S,3S)$ in (2) by 2 orders of magnitude or more. This is another puzzling problem for hadronic transitions of heavy quarkonium aside from the $\Upsilon(5S)$ dipion transitions [5]. Here, again, a possible and natural interpretation for this anomalously large difference between R_4 and R_n ($n < 4$) is that the rescattering mechanism is dominant in the hadronic transitions of $\Upsilon(4S)$, since it lies above the $B\bar{B}$ threshold, whereas $\Upsilon(nS)$ ($n < 4$) are below the open bottom threshold and hence described by the conventional hadronic transition theory.

In this paper, we will clarify whether the ratio R_4 can be as large as (3) in the rescattering model, and we will give some predictions for the η transitions of the $\Upsilon(5S)$ state. We will first introduce the rescattering model and the notation of $\eta - \eta'$ mixing in Sec. II. Then, we will numerically analyze the η as well as η' transitions of $\Upsilon(4S, 5S)$ in turn in Sec. III. A summary will be given in the last section.

II. THE MODEL

In the rescattering model, the transitions $\Upsilon(4S, 5S) \to \Upsilon(1S)\eta$ can arise from scattering of the intermediate state $B^{(*)}_{(s)}\bar{B}^{(*)}_{(s)}$ by exchange of another $B^{(*)}_{(s)}$ meson. The typical diagrams for the $B^{(*)}\bar{B}^{(*)}$ channels are shown in Fig. 1, and the other ones can be related to those in Fig. 1 by the charge conjugation transformation $B \leftrightarrow \bar{B}$ and isospin transformations $B^0 \leftrightarrow B^+$ and $\bar{B}^0 \leftrightarrow B^-$. Therefore, the amplitudes of

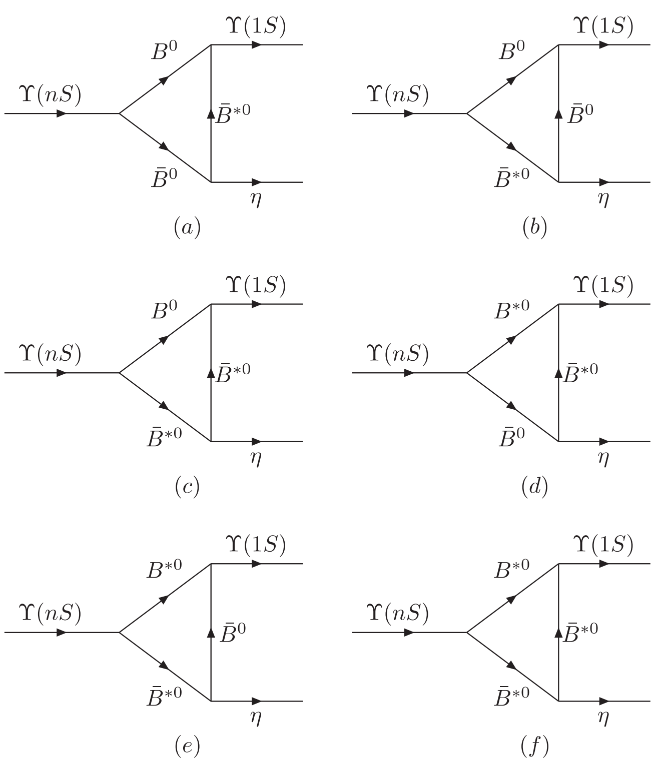

FIG. 1. The diagrams for $\Upsilon(nS) \to B^{(*)0}\bar{B}^{(*)0} \to \Upsilon(1S)\eta$. Other diagrams can be obtained by charge conjugation transformation $B \leftrightarrow \bar{B}$ and isospin transformations $B^0 \leftrightarrow B^+$ and $\bar{B}^0 \leftrightarrow B^-$.

Figs. 1(a)–1(f) should be multiplied by a factor of 4, respectively. As for $B_s^{(*)}\bar{B}_s^{(*)}$ channels, the typical diagrams are the same as those in Fig. 1, but the multiplied factor should be 2.

To evaluate the amplitudes, we need the following effective Lagrangians [3,12]:

$$\mathcal{L}_{\Upsilon BB} = g_{\Upsilon BB}\Upsilon_\mu(\partial^\mu B B^\dagger - B\partial^\mu B^\dagger), \tag{4a}$$

$$\mathcal{L}_{\Upsilon B^* B} = \frac{g_{\Upsilon B^* B}}{m_\Upsilon}\varepsilon^{\mu\nu\alpha\beta}\partial_\mu\Upsilon_\nu(B^*_\alpha \overleftrightarrow{\partial}_\beta B^\dagger - B\overleftrightarrow{\partial}_\beta B^{*\dagger}_\alpha), \tag{4b}$$

$$\mathcal{L}_{\Upsilon B^* B^*} = g_{\Upsilon B^* B^*}(-\Upsilon^\mu B^{*\nu}\overleftrightarrow{\partial}_\mu B^{*\dagger}_\nu + \Upsilon^\mu B^{*\nu}\partial_\nu B^{*\dagger}_\mu - \Upsilon_\mu \partial_\nu B^{*\mu} B^{*\nu\dagger}), \tag{4c}$$

$$\mathcal{L}_{B^* B\eta} = i g_{B^* B\eta} B^*_\mu \partial^\mu \eta B^\dagger, \tag{4d}$$

$$\mathcal{L}_{B^* B^*\eta} = i\frac{g_{B^* B^*\eta}}{m_{B^*}}\varepsilon^{\mu\nu\alpha\beta}\partial_\mu B^*_\nu B^{*\dagger}_\alpha \partial_\beta \eta, \tag{4e}$$

where $\overleftrightarrow{\partial} = \overrightarrow{\partial} - \overleftarrow{\partial}$. In the heavy quark limit, the coupling constants in (4) can be related to each other by heavy quark spin symmetry as

$$g_{\Upsilon BB} = g_{\Upsilon B^* B} = g_{\Upsilon B^* B^*}, \tag{5}$$

$$g_{B^* B\eta} = g_{B^* B^*\eta}. \tag{6}$$

Particularly, the coupling constants for $\Upsilon(4S)$ and $\Upsilon(5S)$ can be determined by the observed values of their partial decay widths to the bottom meson pairs.

All the coupling constants will be determined below. However, it is necessary to emphasize here that the determinations do not account for the off-shell effects of the exchanged $B^{(*)}$ mesons, of which the virtualities can not be ignored. Such effects can be compensated by introducing, e.g., the monopole [4] form factors for off-shell vertices. Let q denote the momentum transferred and m_i the mass of the exchanged meson; the form factor can be written as

$$\mathcal{F}(m_i, q^2) = \frac{(\Lambda + m_i)^2 - m_i^2}{(\Lambda + m_i)^2 - q^2}. \tag{7}$$

For comparison, we will use the same cutoff $\Lambda = 660$ MeV as the one used in the numerical analysis of $\Upsilon(4S, 5S) \to \Upsilon(1S)\pi^+\pi^-$ [3].

In the lightest pseudoscalar-meson nonet of the $SU(3)$ quark model, there are two isoscalar components, which can be written in the octet-singlet basis as

$$\eta_8 = \sqrt{\frac{1}{6}}(u\bar{u} + d\bar{d} - 2s\bar{s}), \qquad \eta_0 = \sqrt{\frac{1}{3}}(u\bar{u} + d\bar{d} + s\bar{s}),$$

where η_8 is one of the Goldstone bosons in the octet representation of the chiral symmetry. If the intrinsic glue component is negligible, the physical wave functions of η and η' can then be written as

$$|\eta\rangle = \cos\theta_P|\eta_8\rangle - \sin\theta_P|\eta_0\rangle,$$
$$|\eta'\rangle = \sin\theta_P|\eta_8\rangle + \cos\theta_P|\eta_0\rangle, \tag{8}$$

where the mixing angle θ_P has been determined in many places in the literature and the value is in the range from $-13°$ to $-22°$ (see, e.g., Ref. [13]). We will choose a moderate value

$$\theta_P = -17°,$$

in our numerical analysis.

As the first step, we will treat the η as a pure η_8 state, and leave the mixing effect to be considered in the following section. An evident advantage of this treatment is that one can relate the coupling constant $g_{B^*B\eta}$ to $g_{D^*D\pi}$ using heavy quark flavor symmetry and chiral symmetry [12]:

$$g_{B^*B\eta} = -\frac{1}{2}g_{B_s^*B_s\eta} = \frac{1}{\sqrt{6}}\frac{\sqrt{m_B m_{B^*}}}{\sqrt{m_D m_{D^*}}}g_{D^*D\pi} = \frac{\sqrt{2}}{\sqrt{3}}\frac{\sqrt{m_B m_{B^*}}}{f_\pi}g, \tag{9}$$

where $f_\pi = 131$ MeV is the π decay constant, and the coupling constant $g \approx 0.6$ is determined by the measurement of the decay width of $D^{*+} \to D^0\pi^+$ [14].

In the rescattering $\Upsilon(4S, 5S) \to B\bar{B} \to \Upsilon(1S)\eta$, the intermediate process $\Upsilon(4S, 5S) \to B\bar{B}$ can take place in a real or virtual way, which corresponds to the imaginary part or the real part of the amplitude, respectively. If the $\Upsilon(nS)$ state lies above the $B^{(*)}\bar{B}^{(*)}$ threshold, the absorptive part (imaginary part) of the amplitude arising from Fig. 1 can be evaluated by the Cutkosky rule. For the process $\Upsilon(nS) \to B^{(*)}(p_1) + \bar{B}^{(*)} \to \Upsilon(mS) + \eta$, the absorptive part of the amplitude reads

$$\mathbf{Abs}_{\,i} = \frac{|\vec{p}_1|}{32\pi^2 m_{\Upsilon(nS)}}\int d\Omega\mathcal{A}_i(\Upsilon(nS) \to B^{(*)}\bar{B}^{(*)})\mathcal{C}_i(B^{(*)}\bar{B}^{(*)} \to \Upsilon(mS)\eta), \tag{10}$$

where $i = (a, b, c, d, e, f)$, and $d\Omega$ and $\vec{p}_1$ denote the solid angle of the on-shell $B^{(*)}\bar{B}^{(*)}$ system and the 3-momentum of the on-shell $B^{(*)}$ meson in the rest frame of $\Upsilon(nS)$, respectively.

The evaluation of the real part of the amplitude is difficult to be achieved and will bring large uncertainties inevitably. Fortunately, for the transitions $\Upsilon(4S, 5S) \to \Upsilon(1S)\eta$, the contributions from the real part are expected to be small, because the masses of $\Upsilon(4S, 5S)$ are not very close to the open flavor thresholds as those of $X(3872)$ [15] and $Z(4430)$ [16]. In the previous paper [3], we have roughly estimated the contributions to the dipion transitions of $\Upsilon(4S, 5S)$ from the real parts through the dispersion relation and found that these contributions are negligible for $\Upsilon(5S)$ and somewhat comparable to those from the imaginary parts for $\Upsilon(4S)$. The same argument should be also valid here for the η transitions of $\Upsilon(4S, 5S)$. As in [3], we will neglect the contribution from the real part and use (10) to determine the full amplitude in the calculations. This scheme is efficient enough to serve our aims.

In the absorptive part, which corresponds to the real rescattering process, the intermediate states $B^{(*)}\bar{B}^{(*)}$ are on shell. Similar to the case of the dipion transitions of $\Upsilon(4S, 5S)$ [3], the amplitude in (10) is proportional to $|\vec{p}_1|^3$. This very fact results in both the large difference between the dipion transition rates of $\Upsilon(5S)$ and $\Upsilon(4S)$ [3] and the markedly peak shift effect in $\Upsilon(5S) \to \Upsilon(mS)\pi^+\pi^-$ [9]. One can generally expect the similar effects to emerge in the corresponding η transitions.

III. NUMERICAL RESULTS AND DISCUSSIONS

Since the contribution from the imaginary part of the rescattering amplitude corresponds to the real decay process $\Upsilon(nS) \to B^{(*)}\bar{B}^{(*)}$, the coupling constants $g_{\Upsilon(nS)B^{(*)}B^{(*)}}$ should be determined by the measured values of the decay widths of $\Upsilon(4S, 5S) \to B^{(*)}\bar{B}^{(*)}$ [14], and the results are given by [3]

$$g_{\Upsilon(4S)BB} = 24, \tag{11}$$

$$g_{\Upsilon(5S)BB} = 2.5, \tag{12}$$

$$g_{\Upsilon(5S)B^*B} = 1.4 \pm 0.3, \tag{13}$$

$$g_{\Upsilon(5S)B^*B^*} = 2.5 \pm 0.4. \tag{14}$$

The value of $g_{\Upsilon(4S)BB}$ in (11) is typical and is comparable to the estimation using the vector meson dominance model [15] for $g_{\Upsilon(1S)BB}$:

$$g_{\Upsilon(1S)BB} \approx \frac{m_{\Upsilon(1S)}}{f_{\Upsilon(1S)}} \sim 15, \tag{15}$$

where the decay constant $f_{\Upsilon(1S)}$ can be determined by the leptonic width of $\Upsilon(1S)$. However, the values determined from the $\Upsilon(5S)$ data in (12)–(14) are small. This may be partly due to the fact that as a higher-excited $b\bar{b}$ state, the wave function of $\Upsilon(5S)$ has a complicated node structure (with four nodes), and the coupling constants will be small if the p values $|\vec{p}_1|$ of $B^{(*)}\bar{B}^{(*)}$ channels (1060–1270 MeV) are close to those corresponding to the zeros in the decay amplitude. The symmetry relation in (5) can also be violated by the same reason. This fact has been confirmed by a specific calculation [17] recently.

Following Ref. [3], for the other coupling constants $g_{\Upsilon(mS)B^{(*)}B^{(*)}}$ $(m < 5)$, we assume that the symmetry relations in (5) hold, and they are equal to each other, which is implied by comparison between (11) and (15).

Since the rescattering amplitude is proportional to $|\vec{p}_1|^3$, one can expect that the contributions from $B_s\bar{B}_s$ channels are generally much smaller than those from $B\bar{B}$ channels, although there is an enhancement factor of 2 in the coupling constant $g_{B_s^*B_s\eta}$ in (9). Therefore, here we only use the central value of the decay widths of $\Upsilon(5S) \to B_s^{(*)}\bar{B}_s^{(*)}$ [18] to determine the coupling constant $g_{\Upsilon(5S)B_s^{(*)}B_s^{(*)}}$:

$$g_{\Upsilon(5S)B_sB_s} = 1.4, \qquad g_{\Upsilon(5S)B_s^*B_s} = 2.0, \qquad g_{\Upsilon(5S)B_s^*B_s^*} = 7.5. \tag{16}$$

Here, the coupling constant $g_{\Upsilon(5S)B_s^*B_s^*}$ is larger than the others and those in (12)–(14). This can be understood by the fact that the p value of $B_s^*\bar{B}_s^*$ channel $|\vec{p}_1| \approx 0.48$ GeV is small, which makes the amplitude away from the zero points sufficiently [17]. This can also serve as evidence in favor of the usual bottomonium interpretation of $\Upsilon(5S)$ in addition to its usual leptonic decay width [14].

A. $\Upsilon(4S) \to \Upsilon(1S)\eta$

Only Fig. 1(a) is allowed for the real rescattering process $\Upsilon(4S) \to B\bar{B} \to \Upsilon(1S)\eta$. For a pure η_8 component, the result reads

$$\Gamma(\Upsilon(4S) \to \Upsilon(1S)\eta) = 2.92 \text{ KeV}. \tag{17}$$

Together with the prediction for the width $\Gamma(\Upsilon(4S) \to \Upsilon(1S)\pi^+\pi^-) = (1.47 \pm 0.03)$ KeV in the same model [3], we can get the ratio

$$R_4 \simeq 2.0, \tag{18}$$

which is consistent with the experimental measurement [11] in (3).

Although the absolute value of the width in (17) suffers from large uncertainties due to the cutoff Λ, the real part contamination, and the coupling constants $g_{\Upsilon(1S)B^*B}$, the situation for the ratio R_4 in (18) should be better, since many of the uncertainties canceled out in the ratio. On the other hand, the ratio is indeed sensitive to the description of the production of $\pi^+\pi^-$. In Ref. [3], we assume that the scalar resonance ($\sigma, f_0(980)\ldots$) contributions are dominant in the dipion production and estimate the coupling constant $g_{\sigma BB}$ through symmetry and rescaling analysis. The value of $g_{\sigma BB}$ used by us is lager than the one [19] deduced from linear representation of chiral symmetry [20] by 20% in magnitude. The later value [19] will enhance the ratio in (18) by a factor of 1.5.

Another large uncertainty of R_4 comes from the mixing between η_8 and η_0 in (8). While the mixing angle θ_P is rather well determined, there is no reliable information for the coupling constant $g_{B^*B\eta_0}$. As a tentative assumption, we choose the value of $g_{B^*B\eta_0}$ in the range from zero to the value of $g_{B^*B\eta_8}$ determined in (9), and then the results are given by

$$R_4 = 1.8\text{–}3.1, \tag{19}$$

$$\Gamma(\Upsilon(4S) \to \Upsilon(1S)\eta') = 0.1\text{–}0.3 \text{ KeV}. \tag{20}$$

Here, the width $\Gamma(\Upsilon(4S) \to \Upsilon(1S)\eta')$ is very small mainly due to the tiny phase space.

To sum up, we find the ratio to be $R_4 = 1.8$–4.5 in the rescattering model, which agrees with the experimental measurement [11] in (3). This can serve as another evidence for the dominant role of the rescattering mechanism in the hadronic transitions of $\Upsilon(4S, 5S)$, which lie above the open bottom threshold.

B. $\Upsilon(5S) \to \Upsilon(1S/2S)\eta$

To study the real rescattering effects in the transitions $\Upsilon(5S) \to \Upsilon(1S/2S)\eta$, one needs to evaluate the imaginary part of the amplitudes for all the diagrams in Fig. 1 and for both $B^{(*)}\bar{B}^{(*)}$ channels and $B_s^{(*)}\bar{B}_s^{(*)}$ channels. The contributions from the $B_s^{(*)}\bar{B}_s^{(*)}$ channels are very small as one can see later. Thus, in Table I, we only list the contributions from the $B\bar{B}$, $B^*\bar{B}$ + c.c., and $B^*\bar{B}^*$ channels, respectively, and totally.

We use the central values in (12)–(14) to evaluate the transition width obtained from each single channel. The results shown in Table I are all about 100 KeV, and are much greater than the width of $\Upsilon(4S) \to \Upsilon(1S)\eta$ in (17). The reason is just the same as the large difference between the dipion transition widths of $\Upsilon(5S)$ and $\Upsilon(4S)$, namely, the $\Upsilon(5S)$ has much larger $|\vec{p}|$ values than $\Upsilon(4S)$.

However, there is a large cancellation between these three channels. As a result, the total widths of these transitions, which are listed in the last line of Table I, are very small. This cancellation makes the widths very sensitive to the coupling constants determined in (12)–(14), which can be seen through the large error bars in Table I. If we choose all the coupling constants $g_{\Upsilon(5S)B^{(*)}B^{(*)}} = 2.5$, the calculated widths for the $5 \to 1$ and $5 \to 2$ transitions will be 145 and 83 KeV, respectively. So, these two widths cannot be determined accurately and we can only give loose estimates for them:

$$\Gamma(\Upsilon(5S) \to \Upsilon(1S)\eta) = 20\text{–}150 \text{ KeV}, \tag{21}$$

$$\Gamma(\Upsilon(5S) \to \Upsilon(2S)\eta) = 10\text{–}100 \text{ KeV}. \tag{22}$$

As for the contributions from the $B_s^{(*)}\bar{B}_s^{(*)}$ channels, they are very small as we have mentioned. Choosing the values of the coupling constants in (16), the total decay widths from these channels are only about 1 KeV. Needless to say, there is also a large cancellation between these channels. Moreover, the width for an individual channel is only about

TABLE I. Transition widths of $\Upsilon(5S) \to B^{(*)}\bar{B}^{(*)} \to \Upsilon(mS)\eta$ in units of KeV. The error bars come from those of $g_{\Upsilon(5S)B^*B}$ and $g_{\Upsilon(5S)B^*B^*}$.

Channel	$\Upsilon(5S) \to \Upsilon(1S)\eta$	$\Upsilon(5S) \to \Upsilon(2S)\eta$
$B\bar{B}$	80	78
$B^*\bar{B}$ + c.c.	70	59
$B^*\bar{B}^*$	141	172
Total	30^{+22+24}_{-17-17}	9^{+13+17}_{-7-8}

10 KeV, which is much smaller than those from $B^{(*)}\bar{B}^{(*)}$ channels.

Similar to the case of $\Upsilon(4S) \to \Upsilon(1S)\eta$, the mixing between η and η' can cause additional uncertainties of 50% in magnitude to the decay widths in (21) and (22). The width of the transition $\Upsilon(5S) \to \Upsilon(1S)\eta'$ is about 10–40 KeV due to the mixing.

IV. SUMMARY AND DISCUSSION

In summary, we study the effects of long-distance final state interactions on the η transitions of $\Upsilon(4S, 5S)$ in the rescattering model. We find that the width of the η transition of $\Upsilon(4S)$ to $\Upsilon(1S)$ is larger than that of the dipion transition, and the ratio of the former to the latter is predicted to be $R_4 = 1.8$–4.5, which is consistent with the experimental data [11]. This result, together with those in Ref. [3], indicate that the real rescattering mechanism can be dominant in the hadronic transitions of the higher Υ-states that lie above the $B\bar{B}$ threshold.

Estimations for the virtue rescattering effects can be roughly made through the same procedure by using the dispersion relation as that suggested in Ref. [3]. The contributions are 3–6 KeV and 1.1–1.5 KeV to the widths, respectively, of the η and dipion transitions of $\Upsilon(4S)$ to $\Upsilon(1S)$, while the ratio R_4 in (18) will be enhanced by a factor of 1.2–1.5. In addition, the virtual rescattering contributions are found to be about $(1$–$2) \times 10^{-2}$ KeV and $(5$–$6) \times 10^{-4}$ KeV to the widths $\Gamma(\Upsilon(nS) \to \Upsilon(1S)\eta)$ for $n = 3$ and 2, respectively. The former is larger than but roughly consistent with the upper limit of the width measured by the CLEO Collaboration [10], while the latter is smaller than the measurement [10] by an order of magnitude, which indicates that the QCDME mechanism may be dominant in the transition $\Upsilon(2S) \to \Upsilon(1S)\eta$ since $\Upsilon(2S)$ is far below the $B\bar{B}$ threshold. Note that the absolute values of these transitions are sensitive to the values for the coupling constants, e.g., $g_{\Upsilon(1S)B^*B}$. If the value in (15) is used instead of that in (11) for the $g_{\Upsilon(1S)B^*B}$, the absolute transition widths of $\Upsilon(4S) \to \Upsilon(1S)\eta$ and $\Upsilon(4S) \to \Upsilon(1S)\pi\pi$ as well as $\Upsilon(3S) \to \Upsilon(1S)\eta$ and $\Upsilon(2S) \to \Upsilon(1S)\eta$ will decrease by almost a factor of 3, while the ratio R_4 remain unchanged, and thus all predictions for $\Upsilon(nS) \to \Upsilon(1S)\eta$ ($n = 4$, 3, 2) can become consistent with observed data. Another point is that the above estimations for the virtue rescattering effects depend on the cutoff parameter Δ (see Ref. [3] for details), and we have chosen $\Delta = m_{B^*} - m_B$ in our calculations as in Ref. [3].

As for the η transitions of $\Upsilon(5S)$, the widths are very sensitive to the coupling constants $g_{\Upsilon(5S)B^{(*)}B^{(*)}}$ because there is a large cancellation between the contributions from $B\bar{B}$, $B^*\bar{B}$ + c.c., and $B^*\bar{B}^*$ channels. Thus we can only give very loose estimations $\Gamma(\Upsilon(5S) \to \Upsilon(1S, 2S)\eta) = 10$–$200$ KeV.

Besides, the widths of the η' transitions of $\Upsilon(4S, 5S)$ are generally expected to be much smaller than those of the corresponding η transitions due to the tiny phase space, but we need to have a better understanding for the couplings of the flavor-singlet η_0 to $B^{(*)}B^{(*)}$ meson pairs before we can draw a definite conclusion on the η' transitions.

In conclusion, the observed anomalously large η transition rate of $\Upsilon(4S)$ to $\Upsilon(1S)$ might be explained in the rescattering model above the open bottom threshold, despite large uncertainties in chosen parameters.

ACKNOWLEDGMENTS

We wish to thank H. Q. Zheng for helpful discussions. This work was supported in part by the National Natural Science Foundation of China (No. 10675003 and No. 10721063).

Note added.—When this manuscript was completed, a paper on the transitions of $\Upsilon(nS) \to \Upsilon(1S)\eta$ ($n = 2$, 3, 4, 5) appeared [21]. The predicted width of $\Upsilon(4S) \to \Upsilon(1S)\eta$ is in agreement with ours and the predicted width of $\Upsilon(5S) \to \Upsilon(1S)\eta$ is close to the lower limit given by us, but without large error bars as ours. However, the predicted width of $\Upsilon(3S) \to \Upsilon(1S)\eta$ [21] is greater than the experimental upper limit in (2) by a factor of 200–500.

[1] Y.-P. Kuang, Front. Phys. China **1**, 19 (2006).
[2] M. B. Voloshin, Prog. Part. Nucl. Phys. **61**, 455 (2008); E. Eichten, S. Godfrey, H. Mahlke, and J. L. Rosner, arXiv:hep-ph/0701208 [Rev. Mod. Phys. (to be published)].
[3] C. Meng and K. T. Chao, Phys. Rev. D **77**, 074003 (2008).
[4] H. Y. Cheng, C. K. Chua, and A. Soni, Phys. Rev. D **71**, 014030 (2005); P. Colangelo, F. De Fazio, and T. N. Pham, Phys. Lett. B **542**, 71 (2002); Phys. Rev. D **69**, 054023 (2004); X. Liu, B. Zhang, and S. L. Zhu, Phys. Lett. B **645**, 185 (2007).
[5] K. F. Chen *et al.* (Belle Collaboraion), Phys. Rev. Lett. **100**, 112001 (2008).
[6] Yu. A. Simonov, Phys. At. Nucl. **71**, 1048 (2008); JETP Lett. **87**, 147 (2008); arXiv:0804.4635.
[7] Yu. A. Simonov and A. I. Veselov, arXiv:0805.4499.
[8] W. S. Hou, Phys. Rev. D **74**, 017504 (2006).
[9] C. Meng, K. T. Chao, Phys. Rev. D **78**, 034022 (2008).
[10] Q. He *et al.* (CLEO Collaboration), arXiv:0806.3027.
[11] N. Arnaud, arXiv:0805.2591.
[12] R. Casalbuoni *et al.*, Phys. Rep. **281**, 145 (1997).

[13] C. E. Thomas, J. High Energy Phys. 10 (2007) 026, and references therein.
[14] W. M. Yao *et al.*, J. Phys. G **33**, 1 (2006); see also the Web site http://pdg.lbl.gov/.
[15] C. Meng and K. T. Chao, Phys. Rev. D **75**, 114002 (2007).
[16] C. Meng and K. T. Chao, arXiv:0708.4222.
[17] Yu. A. Simonov, A. I. Veselov, arXiv:0805.4518.
[18] K. Kinoshita, arXiv:0805.3866.
[19] Y. R. Liu, X. Liu, W. Z. Deng, and S. L. Zhu, Eur. Phys. J. C **56**, 63 (2008).
[20] W. A. Bardeen, E. J. Eichten, and C. T. Hill, Phys. Rev. D **68**, 054024 (2003).
[21] Yu. A. Simonov and A. I. Veselov, arXiv:0806.2919.