Isospin mass splittings of heavy baryons in heavy quark symmetry

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In this paper, the electromagnetic mass differences of heavy hadrons are discussed, while the relevant hyperfine interactions are ignored. The effects of one-photon exchange interaction and up-down quark mass difference are parametrized. Two mass difference equations $2\Sigma_c^+ - (\Sigma_c^{++} + \Sigma_c^0) = 2\Sigma_b^0 - (\Sigma_b^+ + \Sigma_b^-)$ and $(\Xi_{cc}^+ - \Xi_{cc}^{++}) + (\Xi_{bb}^- - \Xi_{bb}^0) = 2(\Xi_{bc}^0 - \Xi_{bc}^+)$ for the heavy baryons are obtained. In addition, the masses of Σ_b^0, Ξ_b^0, and Ξ_{cc}^{++} are predicted based on the known experimental data.

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I. INTRODUCTION

For the heavy baryons which contain one heavy quark, all of the s-wave charmed sector has been found at present. However, except for particle Λ_b^0, which was found in the early 1980s, there has been no significant progress in searching s-wave bottomed sector until last year. Recently, some bottomed baryons were discovered at Fermilab. They are the exotic relatives of the proton and neutron $\Sigma_b^{(*)+}$ and $\Sigma_b^{(*)-}$ found by the CDF Collaboration [1] and the triple-scoop baryon Ξ_b^- found by the D0 and CDF collaborations [2,3]. In addition, for the heavy baryons which contain two heavy quarks, only the doubly charmed baryon Ξ_{cc}^+ has been observed by the SELEX Collaboration [4,5] (in fact, the *BABAR* [6] and BELLE [7] experiments failed to observe the SELEX states). It is reasonable that the remainder of s-wave heavy baryons, which include (i) the double strange baryons Ω_b and doubly heavy baryons Ξ_{bc} and Ξ_{bb}, (ii) the excited states of, for example, the triple-scoop baryon Ξ_b' and Ξ_b^*, and (iii) the isospin partners of the known baryons (namely, Σ_b^0, Ξ_b^0, and Ξ_{cc}^{++}), will be observed in the foreseeable future. The particles (i) and (ii) have been studied in some research. This paper focuses on the type (iii) particles, which are based on the heavy quark symmetry (HQS).

Isospin or $SU(2)$ symmetry originates from treating the up and down quarks as an isospin doublet. This symmetry is broken by the up-down quark mass difference, and also by electromagnetic interactions, which distinguish the different charges carried by the up and down quarks. For the former contribution, the u and d quarks are intrinsically light, and their bare mass difference is about several MeV [8]. However, within the limit of a hadron, the u and d quark masses are suitably described by the constituent values which are about 350 MeV greater than the intrinsic ones. In fact, the precise values depend not only on the binding energies of various quarks but also on the context. Therefore, the effective mass difference of u and d quarks is quite uncertain. In this study, the detailed dynamics were not included, but were parametrized following evaluation. For the latter contribution, it is widely accepted that quantum electrodynamics (QED) is the correct theory for electromagnetic (EM) interactions. In QED, the photons mediate EM forces among charged particles. Therefore, this paper aims to discuss the one-photon exchange interaction between the different quarks. As mentioned in Ref. [9], the EM interaction between i and j quarks leads to two kinds of energy contribution. One is the Coulomb energy

$$\Delta E_{\rm coul} = \alpha e_i e_j \left\langle \frac{1}{r_{ij}} \right\rangle, \tag{1}$$

where α is the fine structure constant, e_i is the charge of quark i, and $\langle 1/r_{ij} \rangle$ is the expectation value of the inverse distance between i and j quarks. In the flavor $SU(3)$ limit, $\langle 1/r_{ij} \rangle$ is universal throughout a multiplet. Another energy contribution is the EM hyperfine splitting

$$\Delta E_{hf}^e = {\rm const} \times \alpha e_i e_j |\Psi_{ij}(0)|^2 \frac{\langle \sigma_i \cdot \sigma_j \rangle}{m_i m_j}, \tag{2}$$

where $|\Psi_{ij}(0)|^2$ is the square of the s-wave function of two quarks at zero relative separation and $\sigma_i(m_i)$ is the spin (mass) of quark i. According to the conclusion of Ref. [9] and the experimental data [1,8], this EM hyperfine splitting contributes to systematic uncertainty of the experimental results and can be ignored if one of the quarks is heavy.

The remainder of this paper is organized as follows. Section II presents a brief review on the heavy quark effective theory. Section III is the analysis of the heavy mesons and the heavy baryons which contain one or two heavy quarks. Finally, the conclusions are given in Sec. IV.

II. HEAVY QUARK EFFECTIVE THEORY

It was found in 1989 that, within the limit $m_Q \to \infty$, quark-gluon dynamics are independent of the heavy quark flavor and spin [10]. This is called HQS, which is not present in the full QCD Lagrangian. Thus, HQS is valid only when the typical gluon momenta are much less than the heavy quark mass m_Q.

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The full QCD Lagrangian for a heavy quark (c, b, or t) is given by

$$\mathcal{L}_Q = \bar{Q}(i\gamma_\mu D^\mu - m_Q)Q, \tag{3}$$

where $D^\mu \equiv \partial^\mu - ig_s T^a A^{a\mu}$ with $T^a = \lambda^a/2$. Inside a hadronic bound state containing a heavy quark, the heavy quark Q interacts with the light degrees of freedom by exchanging gluons with the momenta of order $\Lambda_{\rm QCD}$, which is much smaller than its mass m_Q. Consequently, the heavy quark is close to its mass shell, and its velocity does not deviate much from the hadron's four-velocity v. In other words, the heavy quark's momentum p_Q is close to the "kinetic" momentum $m_Q v$ resulting from the hadron's motion

$$p_Q^\mu = m_Q v^\mu + k^\mu, \tag{4}$$

where k^μ is the so-called "residual" momentum and is of order $\Lambda_{\rm QCD}$ and the corresponding change in the heavy quark velocity vanishes as $\Lambda_{\rm QCD}/m_Q \to 0$. Thus it is appropriate to introduce the "large" and "small" component fields h_v and H_v by

$$h_v(x) = e^{im_Q v\cdot x}\mathcal{P}_+ Q(x), \qquad H_v(x) = e^{im_Q v\cdot x}\mathcal{P}_- Q(x), \tag{5}$$

where $\mathcal{P}_\pm$ are the positive and negative energy projection operators

$$\mathcal{P}_\pm = \frac{1 \pm \not{v}}{2}, \tag{6}$$

with $\mathcal{P}_\pm^2 = \mathcal{P}_\pm$ and $\mathcal{P}_\pm \mathcal{P}_\mp = 0$, and $\mathcal{P}_+$ satisfies the useful identity

$$\mathcal{P}_+ \gamma^\mu \mathcal{P}_+ = \mathcal{P}_+ v^\mu \mathcal{P}_+. \tag{7}$$

$h_v(x)$ and $H_v(x)$ are related to the original field $Q(x)$ by

$$Q(x) = e^{-im_Q v\cdot x}[h_v(x) + H_v(x)]. \tag{8}$$

It is clear that h_v annihilates a heavy quark with velocity v, while H_v creates a heavy antiquark with velocity v. In the heavy meson's rest frame $v = (1, \vec{0})$, $h_v(H_v)$ correspond to the upper (lower) two components of $Q(x)$. In terms of the new fields, the QCD Lagrangian for a heavy quark given by (3) takes the following form:

$$\mathcal{L}_Q = \bar{h}_v iv\cdot D h_v - \bar{H}_v(iv\cdot D + 2m_Q)H_v + \bar{h}_v i\not{D}_\perp H_v + \bar{H}_v i\not{D}_\perp h_v, \tag{9}$$

where $D_\perp^\mu = D^\mu - v^\mu v\cdot D$ is orthogonal to the heavy quark velocity, $v\cdot D_\perp = 0$. In (9), h_v describes the massless degrees of freedom, whereas H_v corresponds to fluctuations with twice the heavy quark mass. The heavy degrees of freedom represented by H_v can be eliminated using the equations of motion of QCD. By substituting (8) into $(i\not{D} - m_Q)Q(x) = 0$ and multiplying it by $\mathcal{P}_\pm$, we can obtain

$$-iv\cdot D h_v = i\not{D}_\perp H_v, \tag{10}$$

$$(iv\cdot D + 2m_Q)H_v = i\not{D}_\perp h_v. \tag{11}$$

$H_v(x)$ can be eliminated to obtain the equation of motion for h_v. It is easy to check that the resulting equation follows from the effective Lagrangian

$$\mathcal{L}_{Q,\rm eff} = \bar{h}_v iv\cdot D h_v + \bar{h}_v i\not{D}_\perp \frac{1}{(iv\cdot D + 2m_Q - i\epsilon)} i\not{D}_\perp h_v. \tag{12}$$

$\mathcal{L}_{Q,\rm eff}$ is the Lagrangian of the heavy quark effective theory, and the second term of (12) allows for a systematic expansion in terms of iD/m_Q. Taking into account that $\mathcal{P}_+ h_v = h_v$, and using the identity

$$\mathcal{P}_+ i\not{D}_\perp i\not{D}_\perp \mathcal{P}_+ = \mathcal{P}_+\left[(iD_\perp)^2 + \frac{g_s}{2}\sigma_{\alpha\beta}G^{\alpha\beta}\right]\mathcal{P}_+, \tag{13}$$

where

$$G^{\alpha\beta} = T_a G_a^{\alpha\beta} = \frac{i}{g_s}[D^\alpha, D^\beta] \tag{14}$$

is the gluon field strength tensor, thus

$$\mathcal{L}_{Q,\rm eff} = \bar{h}_v iv\cdot D h_v + \frac{1}{2m_Q}\bar{h}_v(iD_\perp)^2 h_v + \frac{g}{4m_Q}\bar{h}_v \sigma_{\alpha\beta}G^{\alpha\beta}h_v + \mathcal{O}\left(\frac{1}{m_Q^2}\right). \tag{15}$$

The new operators at order $1/m_Q$ are

$$\mathcal{O}_1 = \frac{1}{2m_Q}\bar{h}_v(iD_\perp)^2 h_v, \tag{16}$$

$$\mathcal{O}_2 = \frac{g_s}{4m_Q}\bar{h}_v \sigma^{\mu\nu}G_{\mu\nu}h_v, \tag{17}$$

where $\mathcal{O}_1$ is the gauge invariant extension of the kinetic energy arising from the off-shell residual motion of the heavy quark, and $\mathcal{O}_2$ describes the color magnetic interaction of the heavy quark spin with the gluon field. It is clear that both $\mathcal{O}_1$ and $\mathcal{O}_2$ break the flavor symmetry, while $\mathcal{O}_2$ breaks the spin symmetry as well. For instance, $\mathcal{O}_1$ would introduce a common shift to the masses of pseudoscalar and vector heavy mesons, and $\mathcal{O}_2$ is responsible for the color hyperfine mass splittings $\delta m_{\rm HF}$.

This work did not concern the effects of strong $1/m_Q$ corrections because they vanished when the mass difference of two ground-state hadrons, which are the same heavy flavor but variant charge, is taken into consideration. The full QCD Lagrangian, as $m_Q \to \infty$, can be reduced to

$$\mathcal{L} = \mathcal{L}_Q + \mathcal{L}_q + \mathcal{L}_g \to \bar{h}_v iv \cdot D h_v + \bar{q}(i\gamma_\mu D^\mu - m_q)q - \tfrac{1}{4}F_a^{\mu\nu}F_{a\mu\nu}. \quad (18)$$

This Lagrangian can be responsible for binding, such as a heavy quark and a light quark in the heavy quark limit. Since an exact solution to the QCD bound state problem does not exist, a phenomenological approach is taken by assuming that, after summing all the two-particle irreducible diagrams for a heavy-light system, the effective coupling between a heavy quark (ψ_Q) and a light quark (ψ_q) can be written as

$$\mathcal{L}_I^{Qq} = g_0 \bar{h}_v i\gamma_5[\mathcal{F}(-iv\cdot\partial)\psi_q]\cdot[\mathcal{F}(iv\cdot\partial)\bar{\psi}_q]i\gamma_5 h_v \quad (19)$$

in the pseudoscalar channel, where g_0 is a coupling constant, and $\mathcal{F}$ is a form factor whose presence is expected for an effective interaction resulting from nonperturbative QCD dynamics. $\mathcal{L}_I^{Qq}$ can be considered as a generalized four-fermion coupling model [11,12] inspired by QCD in the heavy quark limit. If indeed the above assumption is reasonable, $\mathcal{L}_I^{Qq}$ should produce a bound state of pseudoscalar heavy meson with physical mass m_M. Consequently, the sum of all iterations of diagrams should have a pole at the reduced mass

$$\bar{\Lambda}_{\bar{q}} \equiv m_M - m_Q, \quad (20)$$

which is independent of the heavy flavor. However, if considering the EM interaction, there are other contributions to m_M. This will be discussed in the following section. As to the heavy baryons which contain one ($\mathcal{B}$) and two heavy quarks ($\mathcal{B}'$), we can also define the corresponding reduced masses as

$$\bar{\Lambda}_{qq} \equiv m_{\mathcal{B}} - m_Q, \quad (21)$$

$$\bar{\Lambda}_q \equiv m_{\mathcal{B}'} - m_Q - m_{Q'}, \quad (22)$$

which are independent of the heavy flavor, too.

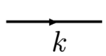
All the above derivations concerning the Lagrangian can be suited to the EM interaction based on the following replacements:

$$g_s \to e_Q e, \qquad T_a \to 1, \qquad A^{a\mu} \to A^\mu. \quad (23)$$

In addition, as mentioned in Sec. I, the EM hyperfine splitting contributes to systematic uncertainty of the experimental results, and can be ignored when the mass difference of heavy hadrons is considered. We may assume that the contributions of $\mathcal{O}_1$ and $\mathcal{O}_2$ are the same order, and both are neglected here. Then the full QED Lagrangian can be reduced to

$$\mathcal{L} = \mathcal{L}_Q + \mathcal{L}_q + \mathcal{L}_\gamma \to \bar{h}_v iv \cdot D h_v + \bar{q}(i\gamma_\mu D^\mu - m_q)q - \tfrac{1}{4}F^{\mu\nu}F_{\mu\nu}. \quad (24)$$

From (24), we can easily derive the Feynman rules for this Lagrangian:


$$: \quad \frac{i}{v\cdot k}\frac{1+\not{v}}{2} \quad \text{(for heavy quark propagator)}, \quad (25)$$

$$: \quad ie_Q ev^\mu \quad \text{(for heavy quark–photon coupling)}. \quad (26)$$

Therefore, it can be inferred that the Coulomb energy between heavy (Q) and light (q) quarks

$$\Delta E_{\rm coul} = \alpha e_Q e_q \left\langle \frac{1}{r_{Qq}} \right\rangle \quad (27)$$

is independent of the heavy flavors.

III. ANALYSES OF HEAVY MESONS AND HEAVY BARYONS

The simplest case is discussed first. As mention in Sec. I, $SU(2)$ symmetry breaking comes from the up-down quark mass difference and the EM interactions which distinguish the different charges carried by the up and down quarks. The mass of a $(Q\bar{q})$ meson with EM Coulomb energy $e_Q e_{\bar{q}} \delta m_{Q\bar{q}}$ can be written as

$$M(Q\bar{q}) = m_Q + \bar{\Lambda}_{\bar{q}} + e_Q e_{\bar{q}} \delta m_{Q\bar{q}}, \quad (28)$$

where q is the u or d quark and $\delta m_{Q\bar{q}}$ is proportional to $\langle 1/r_{Q\bar{q}}\rangle$. Here, $SU(2)$ breaking of the $1/m_Q$ contributions is also ignored since they are higher order effects. Thus,

$$M(Q\bar{d}) - M(Q\bar{u}) = \delta\bar{\Lambda}_{\bar{d}-\bar{u}} + e_Q \delta m_{Q\bar{q}}, \quad (29)$$

where $\delta\bar{\Lambda}_{\bar{d}-\bar{u}} = \bar{\Lambda}_{\bar{d}} - \bar{\Lambda}_{\bar{u}}$. From the experimental values [8], we can obtain

$$D^+ - D^0 = \delta\bar{\Lambda}_{\bar{d}-\bar{u}} + \tfrac{2}{3}\delta m_{Q\bar{q}} = 4.78 \pm 0.1 \text{ MeV}, \quad (30)$$

$$B^0 - B^- = \delta\bar{\Lambda}_{\bar{d}-\bar{u}} - \tfrac{1}{3}\delta m_{Q\bar{q}} = 0.37 \pm 0.24 \text{ MeV} \quad (31)$$

(the particle names stand for their masses), and consequently

$$\delta\bar{\Lambda}_{\bar{d}-\bar{u}} = 1.84 \pm 0.16 \text{ MeV}, \quad (32)$$

$$\delta m_{Q\bar{q}} = 4.41 \pm 0.26 \text{ MeV}. \quad (33)$$

As shown, $\delta\bar{\Lambda}_{\bar{d}-\bar{u}}$ is only 1.84 MeV and smaller than the value of $m_d - m_u$ [8]. The reason is that since the d quark is heavier, it is also more tightly bound, so that part of the mass difference $m_d - m_u$ is canceled by the larger binding energy of the d quark. As to the value of $\delta m_{Q\bar{q}}$, it provides a crucial test for the phenomenological models within the heavy quark effective theory to the Coulomb interaction of QED. In addition, there are two kinds of $1/m_Q$ correction one may consider in (29). One is the strong hyperfine interaction energy [9]

$$\Delta E^s_{hf} = \text{const} \times |\Psi_{Q\bar{q}}(0)|^2 \frac{\langle \sigma_Q \cdot \sigma_{\bar{q}} \rangle}{m_Q m_{\bar{q}}}. \tag{34}$$

Then $\delta\bar{\Lambda}_{\bar{d}-\bar{u}}$ will be replaced as

$$\delta\bar{\Lambda}_{\bar{d}-\bar{u}} \to \delta\bar{\Lambda}_{\bar{d}-\bar{u}} + \text{const} \times |\Psi_{Q\bar{q}}(0)|^2 \frac{\langle \sigma_Q \cdot \sigma_{\bar{q}} \rangle}{m_Q m_{\bar{d}}} \times \frac{m_{\bar{u}} - m_{\bar{d}}}{m_{\bar{u}}}, \tag{35}$$

where we assume that $\Psi_{Q\bar{d}}(0) \simeq \Psi_{Q\bar{u}}(0)$. The additional term is suppressed not only by $1/m_Q$ but also by $m_{\bar{u}} - m_{\bar{d}}/m_{\bar{u}}$. The other is the EM hyperfine $1/m_c$ corrections because the heavy quark limit for the charm quark is not as good as the bottom one, so then the terms such as (2) must be added to (28). The additional parameters from the above two corrections will complicate (29), so that a phenomenological model needs to be used to handle the corrections.

Next, for a (Qqq) baryon, its relevant mass can be written as

$$M(Qqq) = m_Q + \bar{\Lambda}_{qq} + \sum_{i\neq j} e_i e_j \delta m_{ij}, \tag{36}$$

where i, j are heavy or light quarks. Here the parametrized factor $e_q e_q \delta m_{qq}$ contains not only the Coulomb energy but also the hyperfine contribution. Then the mass differences of the isospin multiplet are

$$M(Qdd) - M(Quu) = \delta\bar{\Lambda}_{dd-uu} - 2e_Q \delta m_{Qq} - \tfrac{1}{3}\delta m_{qq}, \tag{37}$$

$$M(Qdd) - M(Q\{ud\}) = \delta\bar{\Lambda}_{dd-\{ud\}} - e_Q \delta m_{Qq} + \tfrac{1}{3}\delta m_{qq}, \tag{38}$$

where $\{ud\}$ is the symmetry form $(ud + du)/\sqrt{2}$. As mentioned in the case of heavy meson, the heavier the light degree of freedom, the larger the binding energy ε_{qq}. If assuming that there are three types of ε_{qq}, they are proportional to the mass of light degree of freedom m_{qq}, $m_{qq}^{-1/3}$, and independent of m_{qq}, which correspond to the Coulombic, linear, and a square well potential of either finite or infinite height, respectively. For the first and third types, the reduced mass $\bar{\Lambda}_{qq} \sim m_{qq} - \varepsilon_{qq}$ is easily checked that it is proportional to m_{qq}. For the second type, the mass differences $\delta\bar{\Lambda}_{dd-\{ud\}}$ and $\delta\bar{\Lambda}_{\{ud\}-uu}$ can be rewritten as

$$\begin{aligned} \delta\bar{\Lambda}_{dd-\{ud\}} &\sim m_{dd} - m_{\{ud\}} \\ &\quad + c\frac{m_{dd} - m_{\{ud\}}}{m_{dd}^{1/3} m_{\{ud\}}^{1/3}(m_{dd}^{2/3} + m_{dd}^{1/3} m_{\{ud\}}^{1/3} + m_{\{ud\}}^{2/3})}, \\ \delta\bar{\Lambda}_{\{ud\}-uu} &\sim m_{\{ud\}} - m_{uu} \\ &\quad + c\frac{m_{\{ud\}} - m_{uu}}{m_{uu}^{1/3} m_{\{ud\}}^{1/3}(m_{uu}^{2/3} + m_{uu}^{1/3} m_{\{ud\}}^{1/3} + m_{\{ud\}}^{2/3})}, \end{aligned} \tag{39}$$

where c is a dimensional constant. For the typical values of $m_{dd,\{ud\},uu}$, the equation

$$\frac{m_{uu}^{1/3}(m_{uu}^{2/3} + m_{uu}^{1/3} m_{\{ud\}}^{1/3} + m_{\{ud\}}^{2/3})}{m_{dd}^{1/3}(m_{dd}^{2/3} + m_{dd}^{1/3} m_{\{ud\}}^{1/3} + m_{\{ud\}}^{2/3})} = 1 \tag{40}$$

is satisfied to $\sim 2\%$. Then, for the above three types of ε_{qq}, $\delta\bar{\Lambda}_{qq-qq'}$ is almost proportional to $m_{qq} - m_{qq'}$. In addition, following the similar derivations, the above conclusion is also suitable to the cases that ε_{qq} is proportional to $m_{qq}^{n/n'}$ (n and n' are the nonzero integers). Therefore, we can obtain a relation $\delta\bar{\Lambda}_{dd-uu} \cong 2\delta\bar{\Lambda}_{dd-\{ud\}}$ by using the equation $m_{dd} + m_{uu} = 2m_{\{ud\}}$. Then (37) and (38) give the mass difference relation

$$2\Sigma_c^+ - (\Sigma_c^{++} + \Sigma_c^0) = 2\Sigma_b^0 - (\Sigma_b^+ + \Sigma_b^-). \tag{41}$$

From the experimental values [1,8], we have

$$\begin{aligned} \Sigma_c^0 - \Sigma_c^{++} &= -0.27 \pm 0.11 \text{ MeV}, \\ \Sigma_c^0 - \Sigma_c^+ &= 0.9 \pm 0.4 \text{ MeV}, \\ \Sigma_b^- - \Sigma_b^+ &= 7.4 \pm 2.3 \text{ MeV}, \\ \Sigma_b^- &= 5815.2 \pm 2.0 \text{ MeV}, \end{aligned}$$

and predict

$$\Sigma_b^- - \Sigma_b^0 = 4.7 \pm 1.0 \text{ MeV}, \tag{42}$$

$$\Sigma_b^0 = 5810.5 \pm 2.2 \text{ MeV}. \tag{43}$$

In addition, the relevant parameters in (37) and (38) are obtained:

$$\delta\bar{\Lambda}_{dd-\{ud\}} = 2.8 \pm 0.8 \text{ MeV}, \tag{44}$$

$$\delta m_{Qq} = 3.8 \pm 1.2 \text{ MeV}, \tag{45}$$

$$\delta m_{qq} = 2.1 \pm 0.8 \text{ MeV}. \tag{46}$$

Comparing (44) and (45) with (32) and (33), it is found that, for the central values, $\delta\bar{\Lambda}_{dd-\{ud\}} > \delta\bar{\Lambda}_{\bar{d}-\bar{u}}$ and $\delta m_{Qq} < \delta m_{Q\bar{q}}$. The reason is that since the strength of the strong coupling between two quarks is smaller than that between a quark and an antiquark, not only the canceled part of mass difference $m_d - m_u$ in the baryon is smaller than that in the meson, but also the expectation value of the inverse distance. Therefore, the results lead to the above inequalities. As to a (Qsq) heavy baryon which contains one strange quark, the corresponding mass equation is

$$M(Qsq) = m_Q + \bar{\Lambda}_{sq} + \sum_{i \neq j} e_i e_j \delta m_{ij}. \quad (47)$$

Following a similar procedure, we can obtain

$$\Xi_b^- - \Xi_b^0 = \Xi_c^0 - \Xi_c^+ + \delta m_{Qq}. \quad (48)$$

From the experimental data $\Xi_c^0 - \Xi_c^+ = 3.1 \pm 0.5$ MeV [8], $\Xi_b^- = 5792.9 \pm 3.0$ MeV [3], and (45), we obtain the predictions

$$\Xi_b^- - \Xi_b^0 = 6.9 \pm 1.1 \text{ MeV}, \quad (49)$$

$$\Xi_b^0 = 5786.0 \pm 3.2 \text{ MeV}. \quad (50)$$

Finally, we consider a $(QQ'q)$ doubly heavy baryon, and write its mass as

$$M(QQ'q) = m_Q + m_{Q'} + \bar{\Lambda}_q + \sum_{i \neq j} e_i e_j \delta m_{ij}. \quad (51)$$

For the two heavy quarks (Q, Q') that are (c, c), (b, c), and (b, b), we have the following results:

$$\Xi_{cc}^+ - \Xi_{cc}^{++} = \delta\bar{\Lambda}_{d-u} - \tfrac{4}{3}\delta m_{Qq}, \quad (52)$$

$$\Xi_{bc}^0 - \Xi_{bc}^+ = \delta\bar{\Lambda}_{d-u} - \tfrac{1}{3}\delta m_{Qq}, \quad (53)$$

$$\Xi_{bb}^- - \Xi_{bb}^0 = (\Xi_{cc}^+ - \Xi_{cc}^{++}) + 2\delta m_{Qq}, \quad (54)$$

and the mass difference relation

$$(\Xi_{cc}^+ - \Xi_{cc}^{++}) + (\Xi_{bb}^- - \Xi_{bb}^0) = 2(\Xi_{bc}^0 - \Xi_{bc}^+). \quad (55)$$

The assumption $\delta\bar{\Lambda}_{d-u} = \delta\bar{\Lambda}_{dd-\{ud\}}$ can be used because these situations are in the baryons. From the experimental data $\Xi_{cc}^+ = 3518.7 \pm 1.7$ MeV [5] and the values of (44) and (45), we can predict

$$\Xi_{cc}^+ - \Xi_{cc}^{++} = -2.3 \pm 1.7 \text{ MeV}, \quad (56)$$

$$\Xi_{cc}^{++} = 3521.0 \pm 2.4 \text{ MeV}, \quad (57)$$

$$\Xi_{bc}^0 - \Xi_{bc}^+ = 1.5 \pm 0.9 \text{ MeV}, \quad (58)$$

$$\Xi_{bb}^- - \Xi_{bb}^0 = 5.3 \pm 1.1 \text{ MeV}. \quad (59)$$

It is worth noting that the SELEX Collaboration seeks the particle Ξ_{cc}^{++} in the corresponding decay modes [13]. It is expected that the oncoming data can confirm our calculations. In addition, although $m_d > m_u$, the mass of $\Xi_{cc}^+(ccd)$ is smaller than that of $\Xi_{cc}^{++}(ccu)$ from (56). The reason is similar to the case of mass difference between $\Sigma_c^+(cud)$ and $\Sigma_c^{++}(cuu)$; namely, since the charge of d quark is negative, the Coulomb energies between $c(u)$ and d quarks reduce the masses of $\Xi_{cc}^+(ccd)$ and $\Sigma_c^+(cud)$. The situations are opposite in the particles $\Xi_{cc}^{++}(ccu)$ and $\Sigma_c^{++}(cuu)$. Therefore, the mass inequalities are reversed. The predictions of this work are summarized and the other theoretical calculations and the experimental data are listed in Table I. In previous literature, [14] parametrized the intrinsic quark mass difference and the Coulomb and magnetic-moment interactions, [15] used the MIT bag model, [16] studied the relativized quark model, and [17,18] used the potential models.

IV. CONCLUSIONS

This study calculated the isospin mass splittings of heavy baryons by ignoring the EM hyperfine interactions. Both the light degrees of freedom and Coulomb energies of the heavy baryons are parametrized. In addition to deriving two mass difference equations, $2\Sigma_c^+ - (\Sigma_c^{++} + \Sigma_c^0) = 2\Sigma_b^0 - (\Sigma_b^+ + \Sigma_b^-)$ and $(\Xi_{cc}^+ - \Xi_{cc}^{++}) + (\Xi_{bb}^- - \Xi_{bb}^0) = 2(\Xi_{bc}^0 - \Xi_{bc}^+)$, we also obtained the numerical values of

TABLE I. Experimental data, the predictions of this work, and the other theoretical calculations (in units of MeV).

	Experiment	This work	[14]	[15]	[16]	[17]	[18]
$\Sigma_c^0 - \Sigma_c^{++}$	-0.27 ± 0.11	input	-3.4	0.01	-1.4	-0.12	-1.20
$\Sigma_c^0 - \Sigma_c^+$	0.9 ± 0.4	input	-0.8	0.83	0.2	0.96	0.36
$\Sigma_b^- - \Sigma_b^+$	7.4 ± 2.3	input			5.6	3.58	3.57
$\Sigma_b^- - \Sigma_b^0$		4.7 ± 1.0			3.7	2.85	2.51
$\Xi_c^0 - \Xi_c^+$	3.1 ± 0.5	input	-0.6	1.72		4.67	2.83
$\Xi_b^- - \Xi_b^0$		6.9 ± 1.1				7.25	5.39
$\Xi_{cc}^+ - \Xi_{cc}^{++}$		-2.3 ± 1.7	-4.7	-1.11		-1.87	-2.96
$\Xi_{bc}^0 - \Xi_{bc}^+$		1.5 ± 0.9					
$\Xi_{bb}^- - \Xi_{bb}^0$		5.3 ± 1.1					
$2\Sigma_c^+ - (\Sigma_c^{++} + \Sigma_c^0)$	-2.0 ± 0.8	input				-2.04	-1.92
$2\Sigma_b^0 - (\Sigma_b^+ + \Sigma_b^-)$		-2.0 ± 0.8				-1.12	-0.45
Σ_b^0		5810.5 ± 2.2					
Ξ_b^0		5786.0 ± 3.2					
Ξ_{cc}^{++}		3521.0 ± 2.4					

some isospin mass differences. Moreover, the masses of particles Σ_b^0, Ξ_b^0, and Ξ_{cc}^{++} are predicted based on the known experimental data. According to the estimations, the decay modes $\Xi_{cc}^{++} \rightarrow \Lambda_c^+ K^- \pi^+ \pi^+$ and $\Xi_{cc}^{++} \rightarrow pD^+ K^- \pi^+$ which are mentioned by the experimentalists [13] are allowed. However, the phase space of the former is obviously larger than that of the latter.

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