

Tests of flavor universality for neutrino-Z couplings in future neutrino experiments

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We investigate the physics potential of the NuSOnG experiment to probe new physics contributions to $Z\nu\nu$ couplings in muon-neutrino-electron elastic and neutral-current deep-inelastic scattering processes. We employ an effective Lagrangian approach and do not *a priori* assume universality of the coupling of neutrinos to Z . We obtain 95% C.L. limits on possible universality violating couplings.

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I. INTRODUCTION

The standard model (SM) of particle physics has been extensively tested by experiments at CERN, Fermilab Tevatron, and elsewhere. These experimental results confirm the $SU_L(2) \otimes U_Y(1)$ gauge structure of the SM. Measurement of gauge boson couplings to fermions provides us with important data for the determination of $SU_L(2)$ and $U_Y(1)$ couplings. The charged lepton couplings to the Z boson have been measured with a sensitivity of $O(10^{-4})$ [1]. However Z boson couplings to individual neutrinos have not been tested with comparably good accuracy. For example, the experimental limits on ν_e and ν_μ couplings to Z are approximately 100 times worse than e and μ couplings to Z [1]. Universality of the coupling of neutrinos to Z is another assumption of the SM which has not been tested with a good accuracy. This assumption simply states that ν_e , ν_μ , and ν_τ couple with the same strength to Z at tree level.

Many parameters of the SM have been very precisely tested at, for example, CERN e^+e^- collider LEP. At LEP, couplings of neutrinos to Z are constrained by the invisible Z width which receives contributions from all neutrino flavors. Therefore, it is impossible to discern possible universality violating neutrino Z couplings from the LEP data. It is possible to constrain new physics contributions to $Z\nu\nu$ that respect universality. Recent limits on these contributions are [1,2]

$$|\Delta_e + \Delta_\mu + \Delta_\tau| \leq 0.009, \quad (1)$$

where the parameters Δ_e , Δ_μ , and Δ_τ describe possible deviations from the SM coming from new physics. They modify neutrino neutral current as [2]

$$J_\mu^{NC} = \frac{1}{2} \sum_i [1 + \Delta_i] \bar{\nu}_i \gamma_\mu \nu_i. \quad (2)$$

These new physics contributions respect universality if the equality $\Delta_e = \Delta_\mu = \Delta_\tau$ holds.

The CHARM II Collaboration obtained data on $\nu_\mu e \rightarrow \nu_\mu e$ scattering. These data together with LEP results place the limit [2,3]

$$|\Delta_\mu| \leq 0.037. \quad (3)$$

Using the limits given in Eqs. (1) and (3) we equivalently have the limit

$$|\Delta_e + \Delta_\tau| \leq 0.046. \quad (4)$$

The universality of ν_e and ν_μ coupling to the neutral weak current has also been tested experimentally by the CHARM Collaboration [4]. The ratio of the coupling constants is given by $g_{\nu_e \bar{\nu}_e} / g_{\nu_\mu \bar{\nu}_\mu} = 1.05_{-0.18}^{+0.15}$. From this ratio and previous limits, the following bounds can be obtained:

$$\begin{aligned} -0.13 &\leq -\Delta_\mu + \Delta_e \leq 0.20, \\ -0.167 &\leq \Delta_e \leq 0.237. \end{aligned} \quad (5)$$

The processes impacting only a single neutrino flavor could violate neutrino flavor universality and therefore provide more information about new physics probes on $Z\nu\nu$ couplings compared to invisible decay width experiments of the Z boson. Recently, a new, high energy, high statistics neutrino scattering experiment, called NuSOnG (neutrino scattering on glass), has been proposed [5]. Such a ‘‘terascale’’ (with energies of 1 TeV and beyond) experiment could offer unprecedented physics opportunities. NuSOnG experiment uses a Tevatron-based neutrino beam to study $\nu_\mu e^- \rightarrow \nu_\mu e^-$ and $\nu_\mu e^- \rightarrow \nu_e \mu^-$ reactions as well as neutral- and charged-current deep-inelastic scattering with high statistics.

In this paper we investigate the physics potential of this future experiment to probe possible new physics contributions to $Z\nu\nu$ couplings. To carry out a more general treat-

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ment we do not assume universality of the coupling of neutrinos to Z .

II. EFFECTIVE LAGRANGIAN APPROACH TO $Z\nu\nu$ COUPLINGS

There is extensive literature on nonstandard interactions of neutrinos [6–11]. New physics contributions to neutrino- Z couplings can be investigated in a model-independent way by means of the effective Lagrangian approach. In this approach, new interactions appearing at a higher energy scale Λ influence physics at lower energies as small deviations from the standard model. These corrections are described by nonrenormalizable effective Lagrangians which respect the symmetries of the standard model. Specifically we consider the $SU(2)_L \otimes U(1)_Y$ invariant effective Lagrangian introduced in Ref. [12]. Possible deviations from the SM that may violate neutrino- Z coupling universality are described by the following dimension-6 effective operators:

$$O_j = i(\phi^\dagger D_\mu \phi)(\bar{\ell}_j \gamma^\mu \ell_j), \quad (6)$$

$$O'_j = i(\phi^\dagger D_\mu \tilde{\tau} \phi) \cdot (\bar{\ell}_j \gamma^\mu \tilde{\tau} \ell_j), \quad (7)$$

where ℓ_j is the left-handed lepton doublet for flavor $j = e, \mu$ or τ ; ϕ is the scalar doublet; and D_μ is the covariant derivative, defined by

$$D_\mu = \partial_\mu + i\frac{g}{2}\tilde{\tau} \cdot \vec{W}_\mu + i\frac{g'}{2}YB_\mu. \quad (8)$$

Here g and g' are the $SU(2)_L$ and $U(1)_Y$ gauge couplings, Y is the hypercharge and the gauge fields $W_\mu^{(i)}$ and B_μ sit in the $SU(2)_L$ triplet and $U(1)_Y$ singlet representations, respectively.

The most general $SU(2)_L \otimes U(1)_Y$ invariant Lagrangian up to dimension-6 operators, containing new physics contributions that may violate universality of the neutrino Z coupling, is then given by

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \sum_{j=e,\mu,\tau} \frac{1}{\Lambda^2} (\alpha_j O_j + \alpha'_j O'_j), \quad (9)$$

where, \mathcal{L}_{SM} is the SM Lagrangian, Λ is the energy scale of new physics and α_j, α'_j are the anomalous couplings. After symmetry breaking, Lagrangian in Eq. (9) reduces to [12]

$$\mathcal{L}' = \frac{g}{\cos\theta_W} J_\mu^{NC} Z^\mu, \quad (10)$$

with

$$J_\mu^{NC} = \left[\frac{1}{2} + \frac{v^2}{2\Lambda^2} (-\alpha_j + \alpha'_j) \right] \bar{\nu}_{jL} \gamma_\mu \nu_{jL} + \left[-\frac{1}{2} + \sin^2\theta_W - \frac{v^2}{2\Lambda^2} (\alpha_j + \alpha'_j) \right] \bar{\ell}_{jL} \gamma_\mu \ell_{jL}. \quad (11)$$

In this effective current subscript ‘‘L’’ represents the left-

handed leptons and v represents the vacuum expectation value of the scalar field. (For definiteness, we take $v = 246$ GeV and $\Lambda = 1$ TeV in the calculations presented in this paper).

As can be seen from the current in Eq. (11), the operators of Eq. (6) and (7) modify not only the neutrino currents, but also the left-handed charged lepton currents. On the other hand right-handed charged lepton currents are not modified. We show in the next section that this fact has important consequences in $\nu_\mu e^- \rightarrow \nu_\mu e^-$ scattering.

Comparing currents (2) and (11) we express the parameters Δ_j in terms of couplings α_j and α'_j :

$$\Delta_j = \frac{v^2}{\Lambda^2} (-\alpha_j + \alpha'_j). \quad (12)$$

We see that the parameters Δ_j actually consist of two independent couplings which need to be constrained by the experiments.

III. $\nu_\mu - e$ ELASTIC AND NEUTRAL-CURRENT DEEP-INELASTIC SCATTERING

Muon-neutrino-electron elastic scattering is described by a t -channel Z exchange diagram. As we discussed in the previous section, not only the $\nu_\mu \nu_\mu Z$ vertex but also the $e^- e^- Z$ vertex is modified by the effective Lagrangian. The differential cross section is given by

$$\frac{d\sigma(\nu_\mu e^- \rightarrow \nu_\mu e^-)}{dy} = \frac{2G_F^2 m_e E_\nu}{\pi} \left(1 + \frac{v^2}{\Lambda^2} (-\alpha_\mu + \alpha'_\mu) \right)^2 \times \left[\eta^2 + \epsilon_+^2 (1-y)^2 - \eta \epsilon_+ \frac{m_e}{E_\nu} y \right], \quad (13)$$

$$y = \frac{E'_e - m_e}{E_\nu}, \quad 0 \leq y \leq \frac{1}{1 + \frac{m_e}{2E_\nu}}, \quad (14)$$

where E_ν and E'_e are the initial neutrino and final electron energies, m_e is the mass of the electron, G_F is the Fermi constant, Λ is the energy scale of new physics, and v is the vacuum expectation value of the scalar Higgs field. The parameters η and ϵ_+ appearing above are defined as

$$\eta = -\frac{1}{2} + \sin^2\theta_W - \frac{v^2}{2\Lambda^2} (\alpha_e + \alpha'_e), \quad (15)$$

$$\epsilon_+ = \sin^2\theta_W. \quad (16)$$

We see from Eqs. (13) and (15) that the contribution of α_e to the cross section is equal to the contribution of α'_e . It is then impossible to distinguish α_e from α'_e and therefore we only consider the coupling α_e in our numerical calculations. The couplings α_μ and α'_μ can be distinguished from α_e and α'_e with the help of polarization. For left-handed final state electrons, only the term proportional to η^2 contributes to the differential cross section. On the other

hand, for right-handed final state electrons only the term proportional to ϵ_+^2 contributes. Therefore, the right-handed cross section isolates the couplings $(-\alpha_\mu + \alpha'_\mu)$. The interference term proportional to $\eta\epsilon_+$ does not contribute if we neglect the mass of the final electron.

The neutrino magnetic dipole moment is very small in the SM, but it may receive contributions from new physics. With the neutrino magnetic moment there is a t -channel photon exchange diagram which contributes to the process $\nu_\mu e^- \rightarrow \nu_\mu e^-$. This contribution increases the cross section by [13–17]

$$\frac{\Delta d\sigma(\nu_\mu e^- \rightarrow \nu_\mu e^-)}{dy} = \mu^2 \frac{\pi\alpha^2}{m_e^2} \left(\frac{1}{y} - 1\right), \quad (17)$$

where μ is the neutrino magnetic moment measured in units of Bohr magnetons. Consistency of $\nu_\mu e$ cross sections with SM expectations tightly constrains the neutrino magnetic moment [14], $\mu < 10^{-9}\mu_B$. Therefore, the contribution (17) is very little especially for high energy neutrinos due to y dependence [13]. For this reason we will neglect this photon exchange contribution.

In Fig. 1 we plot the differential cross section as a function of y for various values of the anomalous couplings. We see from this figure that deviation of the differential cross section from its SM value is larger for $\alpha_e = 1$ as compared with $\alpha_\mu = 1$ and $\alpha'_\mu = 1$ cases. The shape of the curves for $\alpha_\mu = 1$ and $\alpha'_\mu = 1$ are exactly the same with the SM curve. But the behavior of the $\alpha_e = 1$ curve is slightly different from the SM one. Its deviation from the SM increases as the parameter y increases. Hence a terascale neutrino facility could in principle probe physics that yields $\alpha_e \neq 0$.

Neglecting terms of order $\frac{m_e}{E_\nu}$, we obtain the total cross section:

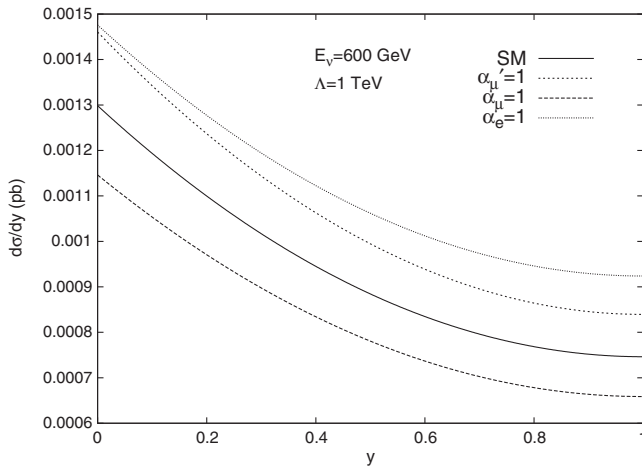


FIG. 1. Differential cross section as a function of y for various values of the anomalous couplings. Only one of the anomalous couplings is kept different from their SM value.

$$\begin{aligned} \sigma(\nu_\mu e^- \rightarrow \nu_\mu e^-) &= \frac{2G_F^2 m_e E_\nu}{\pi} \left(1 + \frac{v^2}{\Lambda^2} (-\alpha_\mu + \alpha'_\mu)\right)^2 \\ &\times \left[\frac{\sin^4 \theta_W}{3} + \left(\frac{1}{2} - \sin^2 \theta_W \right. \right. \\ &\left. \left. + \frac{v^2}{2\Lambda^2} (\alpha_e + \alpha'_e)\right)^2 \right]. \end{aligned} \quad (18)$$

Since the details of the experiment are yet to be worked out, it is not possible to assess all systematic errors. We assume that systematic errors are of the same order as the statistical ones. We studied 95% C.L. bounds using two-parameter χ^2 analysis. The χ^2 function is given by

$$\chi^2 = \left(\frac{\sigma_{\text{SM}} - \sigma_{\text{AN}}}{\sigma_{\text{SM}} \delta_{\text{exp}}}\right)^2, \quad (19)$$

where σ_{AN} is the cross section containing new physics effects and $\delta_{\text{exp}} = \sqrt{\delta_{\text{stat}}^2 + \delta_{\text{syst}}^2}$, $\delta_{\text{stat}} = \frac{1}{\sqrt{N}}$ is the statistical error and δ_{syst} is the systematic error. The number of events is taken to be $N = 75\,000$ which is compatible with the number of events studied in Ref. [5]. We reparametrize the couplings as

$$\alpha_e = \alpha_\mu + \delta_1 = \alpha'_\mu + \delta_2. \quad (20)$$

Thus, possible nonzero values of the couplings δ_1 or δ_2 implies universality violation between interactions $\nu_\mu \nu_\mu Z$ and $\nu_e \nu_e Z$: Any modification of the SM $\nu_\mu \nu_\mu Z$ and $\nu_e \nu_e Z$ couplings that respect universality is described by $\delta_1 = \delta_2 = 0$ (or equivalently by $\alpha_\mu = \alpha'_\mu = \alpha_e$). In Figs. 2–4 we show 95% C.L. allowed regions for the parameter spaces $\alpha_\mu - \delta_1$, $\alpha'_\mu - \delta_2$, and $\alpha_\mu - \alpha'_\mu$. In Fig. 4 we also show the limit (area bounded by dotted lines) obtained from inequality (3). We see from this figure that the limit obtained from the CHARM II data is approximately 6 times weaker than our limits.

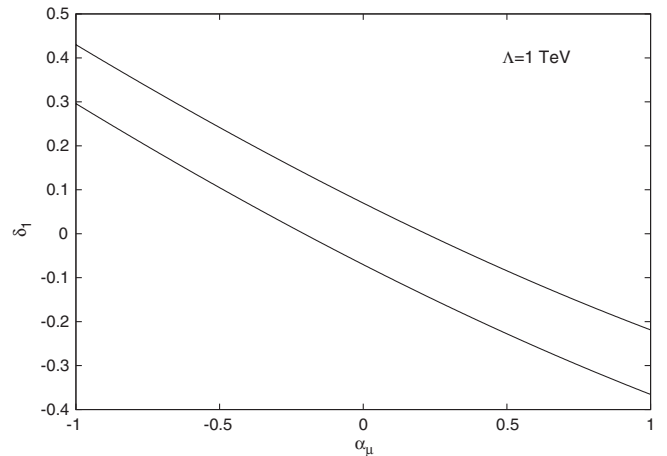


FIG. 2. 95% C.L. sensitivity bound on the parameter space $\alpha_\mu - \delta_1$. Sensitivity bound is the area restricted by the lines. α'_μ is taken to be zero.

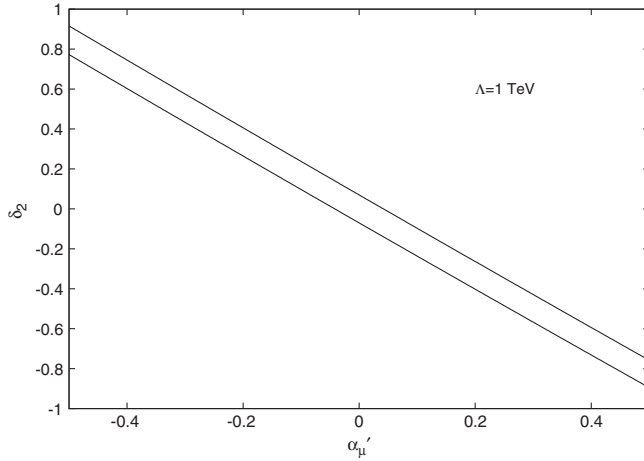


FIG. 3. 95% C.L. sensitivity bound on the parameter space $\alpha'_\mu - \delta_2$. Sensitivity bound is the area restricted by the lines. α_μ is taken to be zero.

The NuSONG experiment will also provide high statistics ν_μ deep-inelastic scattering from the nuclei in glass. The expected number of events for ν_μ neutral-current deep-inelastic scattering is 190×10^6 [5]. In comparison NuTeV had 1.62×10^6 deep-inelastic scattering events in neutrino mode [18]. Therefore, NuSONG will provide 2 orders of magnitude more events. Since quark couplings to Z are not modified by operators (6) and (7), the hadron tensor does not receive any contribution. It is defined by the standard form [19,20]

$$W_{\mu\nu} = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} \right) F_1(x, Q^2) + \frac{\hat{p}_\mu \hat{p}_\nu}{p \cdot q} F_2(x, Q^2) - i \epsilon_{\mu\nu\alpha\beta} \frac{q^\alpha p^\beta}{2p \cdot q} F_3(x, Q^2), \quad (21)$$

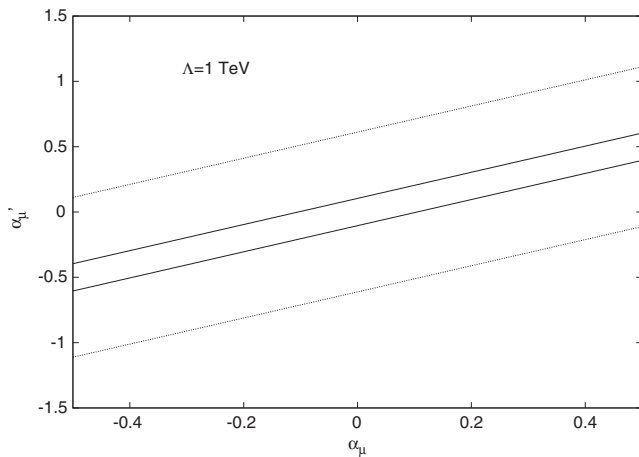


FIG. 4. The area bounded by the solid lines is 95% C.L. sensitivity bound on the parameter space $\alpha_\mu - \alpha'_\mu$. Dotted lines show the limits obtained from CHARM II data. α_e is taken to be zero.

where p_μ is the nucleon momentum, q_μ is the momentum of the Z propagator, $Q^2 = -q^2$, $x = \frac{Q^2}{2p \cdot q}$, and

$$\hat{p}_\mu \equiv p_\mu - \frac{p \cdot q}{q^2} q_\mu.$$

The structure functions are defined as follows [21,22]:

$$F_1 = \frac{1}{2} \sum_i (g_V^2 + g_A^2)_i (q_i + \bar{q}_i), \quad (22)$$

$$F_2 = 2xF_1, \quad (23)$$

$$F_3 = 2 \sum_i (g_V g_A)_i (q_i + \bar{q}_i), \quad (24)$$

where $(g_V)_i$, $(g_A)_i$, and q_i are the weak charges and quark distribution functions of the i th quark flavor. In our calculations, parton distribution functions of Martin, Roberts, Stirling and Thorne [23] have been used. We assume an isoscalar nucleus $N = (p + n)/2$. This would be a good assumption if the glass target was pure SiO_2 . Natural silicon is 92.2% ^{28}Si , 4.7% ^{29}Si , and 3.1% ^{30}Si , where only ^{29}Si is not isoscalar [24]. Naturally occurring oxygen is 99.8% ^{16}O . Hence the error incurred by assuming an isoscalar target would be not more than a few percent.

Possible new physics contributions coming from the operators in (6) and (7) only modify the lepton tensor:

$$L_{\mu\nu} = \frac{1}{2} \left(1 + \frac{v^2}{\Lambda^2} (-\alpha_\mu + \alpha'_\mu) \right)^2 (k_\mu k'_\nu + k'_\mu k_\nu - k \cdot k' g_{\mu\nu} + i \epsilon_{\mu\nu\alpha\beta} k^\alpha k'^\beta), \quad (25)$$

where k_μ and k'_μ are the momenta of initial and final state neutrinos. Therefore, ν_μ neutral-current deep-inelastic scattering isolates the couplings α_μ and α'_μ . It does not

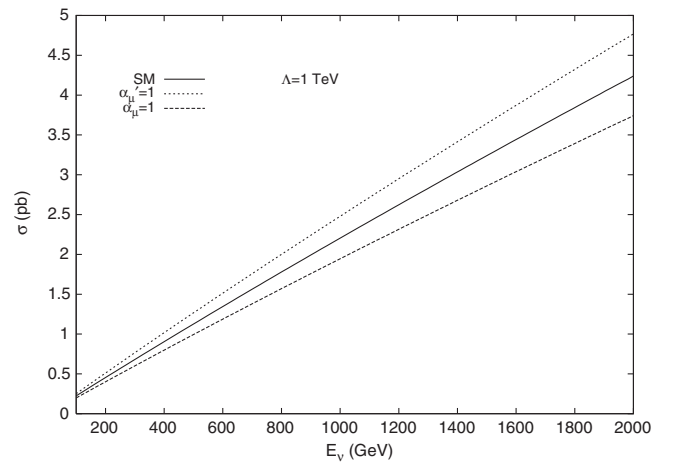


FIG. 5. Integrated total cross section of $\nu_\mu N \rightarrow \nu_\mu X$ as a function of initial neutrino energy for various values of the anomalous couplings. Only one of the anomalous couplings is kept different from their SM value.

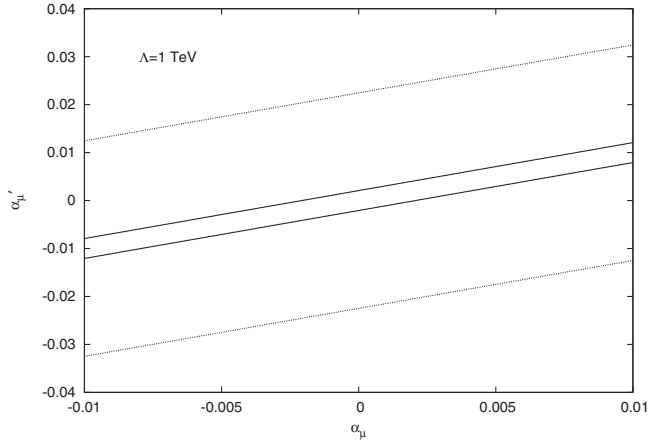


FIG. 6. The area bounded by the solid (dotted) lines is 95% C.L. sensitivity bound on the parameter space $\alpha_\mu - \alpha'_\mu$ for NuSONG (NuTeV) statistics of $\nu_\mu N \rightarrow \nu_\mu X$ scattering.

receive any contribution from α_e and α'_e . As we have discussed this is not the case in $\nu_\mu e^- \rightarrow \nu_\mu e^-$.

The behavior of the integrated total cross section as a function of initial neutrino energy is plotted for various values of anomalous couplings in Fig. 5. We see from the figure that the cross section has a linear energy dependence in the energy interval 100–2000 GeV. Deviation of the anomalous cross sections from their SM value increase as the energy increases. Therefore, high energy neutrino experiments are expected to reach a high sensitivity to probe these anomalous couplings.

In Fig. 6 we show 95% C.L. sensitivity bounds on the parameter space $\alpha_\mu - \alpha'_\mu$ for NuSONG and NuTeV statistics. We observe from the figure that NuSONG has approximately 10 times more sensitive bounds than NuTeV for $\nu_\mu N \rightarrow \nu_\mu X$ scattering. Neutral-current deep-inelastic scattering limits can be combined with $\nu_\mu e^- \rightarrow \nu_\mu e^-$ limits to place bounds on universality violating parameters δ_1 and δ_2 . Combining results of Figs. 3 and 6 we obtain the bound $-0.074 \leq \delta_2 \leq 0.074$ ($\alpha_\mu = 0$). Similarly combining Figs. 2 and 6 we obtain the bound $-0.071 \leq \delta_1 \leq$

0.071 ($\alpha'_\mu = 0$). These bounds can be compared with CHARM limits. From the first inequality of (5) we obtain $-2.2 \leq \delta_2 \leq 3.3$ for $\delta_1 = 0$ and $-3.3 \leq \delta_1 \leq 2.2$ for $\delta_2 = 0$. Therefore, $\nu_\mu e^- \rightarrow \nu_\mu e^-$ and $\nu_\mu N \rightarrow \nu_\mu X$ scattering processes at NuSONG provide approximately 40 times more restricted limits for δ_2 and δ_1 compared with CHARM limits.

IV. CONCLUSIONS

In some schemes new physics neutrinos participation may be observable at lower energies, such as neutrino scattering through an unparticle exchange [25]. However, to probe most of the neutrino interactions beyond the standard model would require energetic neutrino beams such as those employed in the NuSONG proposal or beta beam proposals [26,27]. In this paper we explored signatures for deviation from flavor universality in neutrino-Z boson couplings. We found that the proposed NuSONG experiment can place almost an order of magnitude better limits than the CHARM experiment in the muon-neutrino-electron scattering mode. We have also shown that deep-inelastic scattering measurements with NuSONG can place significantly better than 1 order of magnitude improvement on limits of universality breaking than previous measurements. Thus, coupled with possible complementary measurements of electron neutrino-electron scattering cross section at beta beam experiments [28,29], the NuSONG experiment can be a powerful probe of new neutrino physics.

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