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Bjorken sum rule and perturbative QCD frontier on the move

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The reasonableness of the use of perturbative QCD notions in the region close to the scale of hadronization, i.e., below $\lesssim 1$ GeV is under study. First, the interplay between higher orders of pQCD expansion and higher-twist contributions in the analysis of recent Jefferson Lab (JLab) data on the generalized Bjorken sum rule function $\Gamma_1^{p-n}(Q^2)$ at $0.1 < Q^2 < 3\ \mathrm{GeV}^2$ is studied. It is shown that the inclusion of the higher-order pQCD corrections could be absorbed, with good numerical accuracy, by change of the normalization of the higher-twist terms. Second, to avoid the issue of unphysical singularity (Landau pole at $Q = \Lambda \sim 400$ MeV), we deal with the ghost-free analytic perturbation theory (APT) that recently proved to be an intriguing candidate for a quantitative description of light quarkonia spectra within the Bethe-Salpeter approach. The values of the twist coefficients μ_{2k} extracted from the mentioned data by using the APT approach provide a better convergence of the higher-twist series than with the common pQCD. As the main result, a good quantitative description of the JLab data down to $Q \simeq$ 350 MeV is achieved.

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I. INTRODUCTION

The analysis of deep inelastic scattering (DIS) data by combination of perturbative quantum chromodynamics (pQCD) and operator product expansion (OPE) provides us with a test site for combining both the perturbative and nonperturbative (NP) QCD contributions in the low energy domain. In particular, the generalized (Q^2-dependent) Bjorken sum rule (BSR) [1] is a renowned target ground for testing different possibilities [2,3]. Fortunately, fresh Jefferson Lab data [4] give information on the spin-dependent BSR amplitude $\Gamma_1^{p-n}(Q^2)$ behavior close to the confinement/hadronization scale. Meanwhile, in this region the common theoretical pQCD analysis is spoiled by the unphysical singularities in the infrared (IR) region at a scale $\sim\Lambda \sim 400$ MeV.

To cure this disease (known as the Landau pole trouble) of the pQCD expansion, we use the analytic perturbation theory (APT) approach [5] based on the causality principle implemented as analyticity imperative in the complex Q^2-plane for the QCD coupling $\alpha_s(Q^2)$ in the form of the Källen-Lehmann spectral representation (for a review on APT concepts and algorithm see Ref. [6]).

In principle, the shift of the pQCD frontier (i.e., the boundary above which pQCD is applicable) and the re-arrangement of the total contribution between perturbative and nonperturbative terms is possible by an appropriate modification of perturbative series. Examples are provided, say, by IR renormalons [7–9] and the extractions of higher twists (HT) using various approximations of pQCD [10]. In the present paper, we systematically explore the possibility of such a shift by extracting the values of HT terms using various approximations and modifications of PT to analyze recent JLab data [11–13] on the BSR.

We found that a particular form of solution to the renormalization group (RG) equation, namely, the "denominator" form [14] [see Eq. (2.4) below] is much more suitable for the use in the low-Q^2 region than the most popular ones based on the "multistory" Eq. (9.5) of the Particle Data Group [15] compendium [Eq. (2.3) below]. The inclusion of a higher-order (HO) of PT demonstrates a "duality" between HO and HT, in the sense that HT terms are absorbed, with good numerical accuracy, into HO terms. As a result, HT coefficients decrease. At the same time, we observed that the description of the data is improved only up to the two-loop order of PT, which may be a signal of the asymptotic character of PT series in the region close to the Landau pole.

A further shift of the pQCD frontier is achieved by using the analytic APT modifications of pQCD in the analysis of BSR below 1 GeV making possible investigation of the interplay of PT and HT contributions. As a result, we find that while the next-to-leading twist (μ_4) term is larger in APT than in the usual pQCD, the HT ($\mu_{6,8}$) coefficients in APT are smaller (making the impression of HT series being convergent) and allowing for a reasonable description of the data down to $Q \sim 350$ MeV.

II. THE BJORKEN SUM RULE IN CONVENTIONAL PT

The Bjorken integral is defined

$$\Gamma_1^{p-n}(Q^2) = \int_0^1 dx(g_1^p(x, Q^2) - g_1^n(x, Q^2)), \qquad (2.1)$$

via the spin-dependent proton and neutron structure functions g_1^p, g_1^n with $x = Q^2/2M\nu$, the energy transfer ν, and the nucleon mass M. At large Q^2, the BSR comes to its renowned form $\Gamma_1^{p-n} = g_A/6$, where $g_A = 1.267 \pm 0.004$

is the nucleon axial charge defined from the neutron β-decay. At finite $Q \gg \Lambda$, the BSR is given by the OPE series in $1/Q^{i-2}$ with even $i = 2, 4\ldots$ being the number of a twist and the pQCD series in α_s^n. The expression for the perturbative part of $\Gamma_1^{p-n}(Q^2)$ including the HT contribution is (see e.g. Ref. [8])

$$\Gamma_{1,PT}^{p-n}(Q^2) = \frac{g_A}{6}\left[1 - \frac{\alpha_s}{\pi} - 3.558\left(\frac{\alpha_s}{\pi}\right)^2 - 20.215\left(\frac{\alpha_s}{\pi}\right)^3 - O(\alpha_s^4)\right] + \sum_{i=2}^{\infty}\frac{\mu_{2i}}{Q^{2i-2}} \tag{2.2}$$

with numerical values given at $n_f = 3$ and weak dependence of μ_{2i} on $\log Q^2$ neglected. The first nonleading-twist term [16] can be expressed [13] as

$$\mu_4^{p-n} \approx \tfrac{4M^2}{9} f_2^{p-n},$$

in terms of the color polarizability f_2.

Within the pQCD, the α_s coupling is usually taken in the form (Eq. (7) in [17], Eq. (9.5) in PDG [15]) expanded over $\ln L/L$ with ($L = \ln(Q^2/\Lambda^2)$, $b_k = \beta_k/\beta_0$)

$$\bar{\alpha}_s^{(4)}(L) = \tfrac{1}{\beta_0 L} - \frac{b_1}{\beta_0^2}\frac{\ln L}{L^2} + \frac{1}{\beta_0^3 L^3}[b_1^2(\ln^2 L - \ln L - 1) + b_2] - \frac{1}{\beta_0^4 L^4}\left[b_1^3\left(\ln^3 L - \frac{5}{2}\ln^2 L - 2\ln L + \frac{1}{2}\right) + 3b_1 b_2 \ln L - \frac{b_3}{2}\right]. \tag{2.3}$$

Here, the second term in the first line is the 2-loop contribution and the third term usually is referred to as "the 3-loop one," while all contents of the second and third lines are treated on an equal footing with the 4-loop term $\sim b_3$.

However, it is evident that pieces of genuine 2-loop contribution proportional to b_1 are entangled with the higher-loop ones. This defect is absent in the more compact *denominator representation* [14],

$$\tfrac{1}{\bar{\alpha}_s^{(3)D}(L)} = \beta_0 L + b_1\left[\ln L + \ln\left(1 + \frac{b_1 \ln L}{\beta_0 L}\right) + \frac{b_1^2 - b_2}{\beta_0 L}\right], \tag{2.4}$$

which, being generic for the PDG expression, is closer to the iterative RG solution and, hence, more precise. Below, we shall refer to it as "denom."

A detailed higher-twist analysis based on the total set of low energy SLAC and JLab data was performed in Ref. [12]. The result of the combined fit done in the Q^2 range 0.66–10.0 GeV^2 is $f_2(Q^2 = 1\ \text{GeV}^2) = -0.101 \pm 0.027$ and $\mu_6/M^4 = 0.084 \pm 0.011$ (elastic contribution included). The fitting procedure of Refs. [12,13] taking the pQCD leading-twist term calculated at NLO α_s and using the 2-loop denom coupling $\bar{\alpha}_s^{(2),D}$ was repeated. We succeeded in obtaining the central values of Refs. [12,13] in the two-parametric fit with the output

$$f_2 = -0.096 \pm 0.012, \qquad \mu_6/M^4 = 0.087 \pm 0.004,$$

where the errors are statistical only and $\chi^2 = 0.48$. These results are compatible with HT extraction performed in Ref. [18].

It is of special interest to study the BSR data with the elastic contribution (necessarily present in the OPE framework [19]) excluded, since the low-Q^2 behavior of such an "inelastic" BSR integral (coinciding with the usual BSR for $Q^2 \to \infty$) is constrained by the Gerasimov-Drell-Hearn sum rule [20], and one may investigate its continuation to low energies [3]. To this goal, doing the same NLO combined fit (elastic contribution excluded), one gets

$$f_2^{\text{inel}} = -0.080 \pm 0.016, \qquad \mu_6/M^4 = 0.022 \pm 0.005,$$

with $\chi^2 = 0.91$. The difference is noticeable starting from μ_6 which is natural due to a decrease of an elastic contribution with growing Q^2.

To explore the fit results' sensitivity to the PT order and to the form of α_s below 1 GeV, we have also performed fits at the 1-, 2-, and 3-loop levels. The minimal borders of fitting domains in Q^2 were settled from the *ad hoc* restriction $\chi^2 \le 1$.

From Fig. 1, one sees that the results obtained with 2- (blue online) and 4-loop (green online) expressions for the denom coupling are better consistent with the BSR data at $Q < 1$ GeV than those based on the PDG expression (2.3), though 3-loop results do not differ significantly.

Indeed, by using Eq. (2.4) one may extend the applicability domain of Eq. (2.2) down to $Q^2 \sim 0.27$ GeV^2. At the same time, Eq. (2.3) works well only down to $\sim$0.47 GeV^2 due to extra $\ln^n L/L^{n+1}$ singularities.

The fitting of BSR data in 2, 3, 4 loops over the fixed range 0.6 GeV $< Q <$ 2.0 GeV yields a "swap" between the higher orders of PT and HT terms. In Fig. 2, we show one-parametric fits with 2-, 3-loop α_s pQCD to the BSR. One can see there that the higher-loop contributions are effectively "absorbed" into the value of μ_4 of which the magnitude decreases with increasing of the loop order while all the fitting curves are very close to each other. This observation reveals a kind of duality between perturbative α_s series and nonperturbative $1/Q^2$ series.

This also means the appearance of a new aspect of quark hadron duality, the latter being the necessary ingredient of

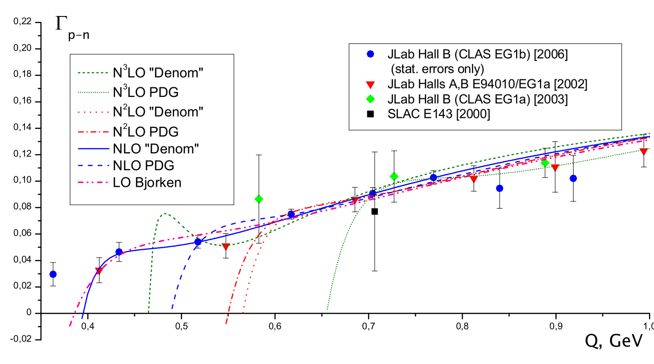

FIG. 1 (color online). Best 3-parametric fits of JLab and SLAC data on Bjorken SR calculated within α_s in PDG form (2.3) and in the denom one (2.4) at various loop orders.

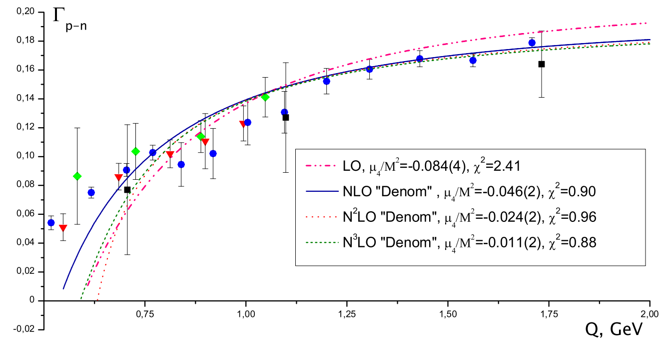

FIG. 2 (color online). One-parametric fits of the JLab and SLAC data on Bjorken SR calculated in the denom form in different loop orders.

all the QCD applications in the low energy domain. Usually, it is assumed [21] that the perturbative effects are less important there than the power ones due to a nontrivial structure in a QCD vacuum.

In our case, the PT corrections essentially enter into the game, so that the pQCD HO terms are relevant in the domain where the concepts of traditional hadronic physics are usually applied.

The interplay between partonic and hadronic degrees of freedom in the description of Gerasimov-Drell-Hearn SR and BSR may be also observed in the surprising similarity between the results of "resonance" [22] and "parton" [3] approaches.

At the same time, from Fig. 1 it follows that the higher (3- and 4-loop) PT orders yield a worse description of the BSR data, probably implying the asymptotic character of the series in powers of α_s.

One may ask to what extent the troubles mentioned above are due to the unphysical singularities at $Q \sim \Lambda$ in PT series for $\Gamma_{1,PT}^{p-n}$. Their influence is essential just at $Q <$ 1 GeV where the HT terms play an important role.

The APT is free of such problems, thus providing a tool to investigate the behavior of HT terms extracted directly from the low energy data. This provides a motivation for the analysis performed in the next section.

III. THE BJORKEN SUM RULE IN APT

According to the approach developed by Solovtsov *et al.* [23], the APT modification of BSR with HT power corrections looks like

$$\begin{aligned}\Gamma_1^{p-n}(Q^2) &= \Gamma_{1,\mathrm{APT}}^{p-n}(Q^2) + \sum_{i=2}^{\infty} \frac{\mu_{2i}^{\mathrm{APT}}}{Q^{2i-2}},\\ \Gamma_{1,APT}^{p-n}(Q^2) &= \frac{g_A}{6}[1 - \Delta_{1,\mathrm{APT}}^{p-n}(Q^2)],\\ \Delta_{1,\mathrm{APT}}^{p-n} &= 0.318\mathcal{A}_1^{(3)}(Q^2) + 0.361\mathcal{A}_2^{(3)}(Q^2)\\ &\quad + 0.652\mathcal{A}_3^{(3)}(Q^2) + \ldots\end{aligned} \tag{3.1}$$

It should be noted that the APT Euclidean functions in the 1-loop case are simple enough [5]

$$\begin{aligned}\mathcal{A}_1^{(1)}(Q^2) &= \frac{1}{\beta_0}\left[\frac{1}{L} + \frac{\Lambda^2}{\Lambda^2 - Q^2}\right],\\ \mathcal{A}_2^{(1)}(l) &= \frac{1}{\beta_0^2}\left(\frac{1}{L^2} - \frac{Q^2\Lambda^2}{(Q^2-\Lambda^2)^2}\right),\\ \mathcal{A}_{k+1}^{(1)} &= -\frac{1}{k\beta_0}\frac{d\mathcal{A}_k^{(1)}}{dL}.\end{aligned} \tag{3.2}$$

where $L = \ln\left(\frac{Q^2}{\Lambda^2}\right)$. The higher $\mathcal{A}_k$ are related to the lower ones recursively by differentiation. Analogous 2- and 3-loop level expressions involve a little-known special Lambert function and are more intricate [24,25].

Meanwhile, even for the 3-loop APT case, there exists a possibility to employ *the effective log approach* proposed by Solovtsov and one of the authors [26] and extended recently (see Eqs. (12), (14) in Ref. [27]) to higher APT Euclidean and Minkowskian functions. In the present context, one may use simple model one-loop expressions (3.2) with some *effective 2-loop log* L^* accumulating the 2-loop log-of-log

$$\begin{aligned}\mathcal{A}_{1,2,3}^{(3)}(L) \to \mathcal{A}_{1,2,3}^{\mathrm{mod}} &= \mathcal{A}_{1,2,3}^{(1)}(L^*);\\ L^* = L + B(n_f)\ln\sqrt{L^2 + 2\pi^2}; &\qquad B = \beta_1/\beta_0^2.\end{aligned} \tag{3.3}$$

Happily enough, the second term does not undergo a significant variation in the intermediate few GeV region. Indeed, as $B(n_f = 3) = 0.79$ and $B(4) = 0.74$, in the region $Q < 5$ GeV involved in the BSR data analysis, a simple approximation

$$\begin{aligned}L^* &\simeq L + B(n_f)\ln\sqrt{2\pi^2} = 2\ln(Q/\Lambda_{\mathrm{eff}}^{(1)}),\\ \Lambda_{\mathrm{eff}}^{(1)} &= e^{-(1/2)B(n_f)\ln\sqrt{2\pi^2}}\Lambda^{(3)} \sim 0.50\Lambda^{(3)}\end{aligned} \tag{3.4}$$

happens to be accurate enough. That is, instead of the cumbersome 3-loop expressions for the APT functions, in Eq. (3.1) one can use the 1-loop expressions (3.2) with $\Lambda_{\mathrm{mod}} = \Lambda_{\mathrm{eff}}^{(1)}$ value given by the second relation (3.4).

If we take $\Lambda^{(3)} = 380$ MeV [28], then $\Lambda_{\mathrm{eff}}^{(1)} \simeq 190$ MeV. The corresponding maximal errors of the model (3.3) for first and second functions are [27] $\delta\mathcal{A}_1^{\mathrm{mod}}/\mathcal{A}_1^{\mathrm{mod}} \simeq 4\%$ and $\delta\mathcal{A}_2^{\mathrm{mod}}/\mathcal{A}_2^{\mathrm{mod}} \simeq 8\%$ at $Q \sim \Lambda^{(3)}$, which seems to be sufficiently accurate. Indeed, as far as $\mathcal{A}_1(Q = 400\ \mathrm{MeV}) = 0.532$ and $\mathcal{A}_2(400\ \mathrm{MeV}) = 0.118$, the total error in $\Gamma_{1,\mathrm{APT}}^{p-n}$ is mainly determined by the first term, being of the order $\delta\Gamma^{p-n}/\Gamma^{p-n} \simeq \delta\mathcal{A}_1^{\mathrm{mod}}/\pi \sim 1\%$, i.e., less than the data uncertainty.

Turn now to the 3-loop APT part of the Bjorken integral $\Gamma_{1,\mathrm{APT}}^{p-n}(Q^2)$. Its value is quite stable with respect to small variations of Λ (in contrast with a huge instability of $\Gamma_{1,PT}^{p-n}$): it changes now by about 2%–3% within the interval $\Lambda^{(3)} = 300$–400 MeV.[1]

[1]In particular, this means that the low-Q BSR data cannot be used for a determination of Λ in the APT approach.

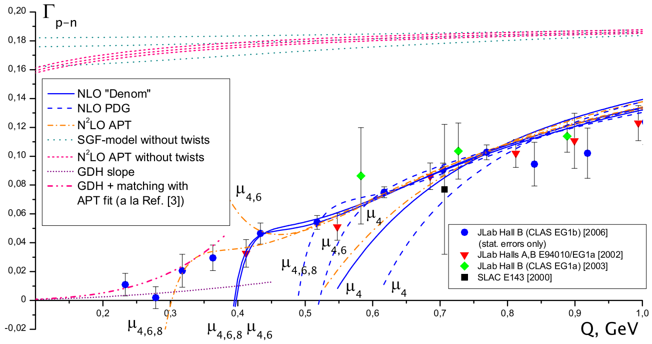

FIG. 3 (color online). Best 1-, 2-, 3-parametric fits of the JLab and SLAC data on Bjorken SR calculated with NLO denom (solid lines) and PDG (dashed lines) couplings and N^{2}LO APT (dashed-dotted lines) at fixed $\Lambda_{\rm QCD}$ value corresponding to the world average. We also show the pQCD part of the BSR at different values of $\Lambda^{(3)} = 300$, 350, 400 MeV calculated within APT (short-dashed lines) and SGF model [29] at different values of the glueball mass $M_0 = 1.2$, 1.0, 0.8 GeV (with $\Lambda = 380$ MeV) (dotted lines).

We also performed the comparison with Simonov's "glueball-freezing model" (SGF model) [29]—see Fig. 3, with similar to PDG $1/L$-type loop expansion for freezed coupling

$$\alpha_B(Q^2) = \alpha_s^{(2)}(\bar{L}), \qquad \bar{L} = \ln\left(\frac{Q^2+M_0^2}{\Lambda^2}\right) \quad \text{(SGF)},$$

where 2-loop $\alpha_s^{(2)}$ is taken in the form of two terms from the first line in Eq. (2.3) with the logarithm modified by a "glueball mass" $M_0 \sim 1$ GeV and the usual PT expansion in powers of α_B in Γ^{p-n} is adopted.

Extending the analysis of Ref. [23] to lower Q values, we estimated the relative size of APT contributions to the BSR. It turned out that the third term $\sim\mathcal{A}_3$ contributes no more than 5% to the sum, thus supporting the practical convergence of the APT series.

Note that the APT functions $\mathcal{A}_k$ contain the $(Q^2)^{-k}$ power contributions which effectively change the fitted values of μ terms. In particular, subtracting the extra $(Q^2)^{-1}$ term induced by the APT series

$$\Gamma_{1,\rm APT}^{p-n}(Q^2) \simeq \frac{g_A}{6} + f\left(\frac{1}{\ln(Q^2/\Lambda_{\rm eff}^{(1)^2})}\right) + \varkappa\frac{\Lambda_{\rm eff}^{(1)^2}}{Q^2} + \mathcal{O}\left(\frac{1}{Q^4}\right)$$

with $\varkappa = 0.43$, we get

$$\frac{\mu_4^{\rm APT} + \varkappa\Lambda_{\rm eff}^{(1)^2}}{M^2} \simeq \frac{\mu_4}{M^2} \simeq -0.048, \qquad \Lambda_{\rm eff}^{(1)} \sim 0.2 \text{ GeV}, \tag{3.5}$$

that nicely correlates with the result in Ref. [12]: $\mu_4/M^2 \simeq -0.045$. This demonstrates the concert of the APT analysis with the usual PT one for the BSR data at $Q^2 \geq 1$ GeV^2.

In Fig. 3, we show best fits of the combined data set for the function $\Gamma_1^{p-n}(Q^2)$ in the PT and the APT approaches. The corresponding numerical results are given in Table I. Our fit gives the HT values indicating a better convergence of the OPE series due to decreasing magnitudes and alternating signs of consecutive terms, in contrast to the usual PT fit results.

TABLE I. Combined fit results for the HT terms in APT and conventional PT in PDG and denominator forms.

Method	$Q^2_{\rm min}$, GeV^2	μ_4/M^2	μ_6/M^4	μ_8/M^6
NLO PDG	0.50	−0.043(2)	0	0
$\Lambda = 380$ MeV	0.30	−0.074(4)	0.025(2)	0
	0.27	−0.049(5)	−0.007(5)	0.009(1)
NLO denom	0.47	−0.046(2)	0	0
$\Lambda = 340$ MeV	0.17	−0.066(2)	0.013(4)	0
	0.17	−0.061(4)	0.009(3)	0.0005(3)
N^{2}LO APT	0.47	−0.054(1)	0	0
$\Lambda = 380$ MeV	0.17	−0.065(2)	0.0081(5)	0
	0.10	−0.069(2)	0.0114(9)	−0.0006(1)

It is worth noting that the best APT fit allows one to describe well all the BSR data at scales down to $Q \sim 350$ MeV with only the first three terms of the OPE series, unlike the usual PT case, where such fits happened to be impossible (due to the ghost issue) even for an increasing number of HT terms. This means that the lower bound of the pQCD applicability (supported by power HT terms) now may be shifted down to $Q \sim \Lambda_{\rm QCD} \simeq 350$ MeV.

However, it seems to be difficult to get a description in the region $Q < \Lambda_{\rm QCD}$. This is not surprising, because the expansion in positive powers of Q^2 and its matching [3] with the HT expansion are relevant here. In this respect, the $\Lambda_{\rm QCD}$ scale appears as a natural border between "higher-twist" and "chiral" nonperturbative physics.

IV. CONCLUSION AND OUTLOOK

The separation of perturbative and NP physics may be different if some modification of perturbation theory is adopted. To test such a separation, we performed a systematic comparison of HT terms extracted from the very accurate JLab data on BSR in the framework of both the common PT and the APT in QCD and came to the following results.

(i) The evidence that the denominator form (2.4) of the QCD coupling α_s is more suitable in the low Q region is given (see Fig. 1).

(ii) A kind of duality between HO of PT and HT is observed so that HO terms absorb part of the HT contributions moving the pQCD frontier between the PT and HT contribution to lower Q values (see Fig. 2).

(iii) This situation is more pronounced in the APT where convergence of both the HO and HT series is much better. While the twist-4 term happened to be larger in magnitude in the APT than in the PT, the subsequent terms are essentially smaller and quickly decreasing (as the APT absorbs some part of NP dynamics described by HT). This is the second reason of the shift of pQCD frontier to lower Q values.

As the main result, a satisfactory description of the data down to $Q \sim \Lambda_{\mathrm{QCD}} \simeq 350$ MeV is achieved by taking the analytic HO and HT contributions into account simultaneously (see Fig. 3).

In a sense, this could be natural if the main reason of such a success was the disappearance of unphysical singularities. We have in mind that the singularity-free APT and SGF QCD couplings are very close in the domain $Q \gtrsim 400$ MeV. Moreover, various lattice data [30] (see also reviews [31] and references therein) yield similar α_s curves there.

It will be very interesting to explore the interplay between PT and NP physics against other low energy experimental data.

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