PHYSICAL REVIEW D **78,** 071501(R) (2008)

Suppression of the shear viscosity in a "semi"-quark-gluon plasma

Yoshimasa Hidaka[1] and Robert D. Pisarski[2]

[1]*RIKEN BNL Research Center, Brookhaven National Laboratory, Upton, New York 11973, USA*
[2]*Department of Physics, Brookhaven National Laboratory, Upton, New York 11973, USA*

(Received 26 March 2008; published 27 October 2008)

We consider QCD at temperatures T near T_c, where the theory deconfines. We distinguish between a "complete" quark-gluon plasma (QGP), where the vacuum expectation value of the renormalized Polyakov loop is near unity, essentially constant with T, and the "semi"-QGP, where the loop changes strongly with T. Lattice simulations indicate that in QCD, there is a semi-QGP from below T_c to a few times that. Using a semiclassical model, we compute the shear viscosity, η, to leading order in perturbation theory. We find that near T_c, where the expectation value of the Polyakov loop is small, that η/T^3 is suppressed by two powers of the loop. For heavy ions, this suggests that during the initial stages of the collision, hydrodynamic behavior at the LHC will be characterized by a shear viscosity which is significantly larger than that at RHIC.

DOI: 10.1103/PhysRevD.78.071501 PACS numbers: 25.75.Nq, 12.38.Mh

The collisions of heavy ions at RHIC have demonstrated clear signals for a novel regime. Much interest has focused on collective properties, especially elliptical flow, which appear to be well described by a system in which the (dimensionless) ratio of the shear viscosity, η, to the entropy, s, is small [1–10]. These results do not agree with the expectations of a weakly coupled plasma, and have been described as a "strong" quark-gluon plasma (QGP) [3,10].

It is reasonable to expect that the running coupling in QCD, $\alpha_s(T) = g^2(T)/(4\pi)$, might be large near the critical temperature, T_c. Typically, one expects the coupling to be in a nonperturbative regime for momenta less than $\sim$1 GeV. Since the transition temperature is $\sim$200 MeV [11–15], the coupling could well be large at several (and maybe many) times T_c.

Indeed, perhaps the coupling is so large that the relevant limit is of infinite coupling. It is possible to compute in a $\mathcal{N} = 4$ supersymmetric SU(N_c) theory when the number of colors, N_c, and $\alpha_s N_c$, are both infinite [16–18]. The $\mathcal{N} = 4$ supersymmetric theory is conformally invariant, so η/s is independent of temperature, and is small, $= 1/4\pi$ [16,17]. If QCD is analogous to the $\mathcal{N} = 4$ theory [18], then for some region above T_c, η/s should remain small, and not change markedly with temperature.

To have a coupling which is relatively moderate in strength at T_c would be exceptional. At least for the pressure, this might occur because of the ubiquitous factors of 2π which accompany the temperature T in the imaginary time formalism [19]. Using an effective theory in three dimensions [20], a two loop computation shows that this does, in fact, occur: at T_c, the effective coupling is only $\alpha_s^{\rm eff} \approx 0.3$ [21].

The challenge is then to understand how the confining transition, with a large decrease in pressure, occurs for moderate coupling [22]. We find it useful to view deconfinement as the ionization of color charge. Without dynamical quarks, in the confined phase there is no ionization of color. Conversely, far into the deconfined phase, color is completely ionized, either with or without quarks. In a non-Abelian gauge theory, the expectation value of the (renormalized) Polyakov loop characterizes the degree to which color charge is ionized. Lattice simulations [11] find that the Polyakov loop is small near T_c, and near one at a few times T_c [12–14]. This regime, which we view as one of partial ionization, coincides with the drop in the pressure (relative to the ideal gas term). For want of a better term, we refer to this as the "semi"-QGP. Above a few times T_c, there is a "complete" QGP, where both the renormalized Polyakov loop, and the pressure/T^4, are essentially constant.

In this paper we consider how the shear viscosity changes in the semi-QGP. We make numerous drastic assumptions. If the coupling is moderate even down to T_c, perhaps we can treat the semi-QGP by means of a semiclassical approximation. Our ansatz is extremely simple, just a background field with constant A_0, Eq. (1). We then compute to leading order in α_s and $\log(1/\alpha_s)$ [23,24], for an infinite number of colors and flavors [25]. We ignore the change of η with α_s, to concentrate on how it changes as the expectation value of the Polyakov loop decreases. We find that when the loop is small, that the shear viscosity is suppressed by two powers of the Polyakov loop. This implies that η/T^3 decreases significantly in the semi-QGP as the theory cools, from a few times T_c down to T_c. Since $\eta \sim 1/\alpha_s^2$ at small α_s, Eq. (3), including the running of the QCD coupling could reinforce this trend. Such a large decrease near T_c is very different from the strong QGP, where η/s changes little [3,10], if at all [18], with temperature.

There are numerous examples of nonrelativistic systems which exhibit a minimum in the shear viscosity near T_c [8,16]. The present analysis suggests how this might arise dynamically in QCD; see, e.g. [3]. Our result is also reminiscent of the anomalous viscosity in a turbulent plasma [5].

The Polyakov loop represents the propagator of an infinitely massive test quark, which we assume is in the fundamental representation [12–14,22,26–30]. Its magnitude can be viewed as the probability for the test quark to propagate. This is near one at asymptotically high temperature, where the plasma is completely ionized.

In a SU(N_c) gauge theory without dynamical quarks, in the confined phase the propagator for a test quark vanishes identically. This is because the Polyakov loop carries $Z(N_c)$ charge, and the confined phase is $Z(N_c)$ symmetric. Thus without dynamical quarks, there is absolutely no ionization of $Z(N_c)$ charge below T_c, only above.

The expectation value of the renormalized Polyakov loop, $\langle \ell \rangle$, is extracted from that of the bare loop after a type of mass renormalization [12–14,30]. For a SU(3) gauge theory without quarks, we rely upon the data of Ref. [13]. From their Fig. 1, $\langle \ell \rangle = 0$ below T_c, and jumps to ~ 0.5 at T_c^+; it rises rapidly, with $\langle \ell \rangle \sim 0.9$ by $2.0T_c$. It then rises slowly, and reaches ~ 1.1 by $\sim 4T_c$. Its value is then constant from $\sim 4T_c$ up to the highest temperature measured, $\sim 12T_c$. (In perturbation theory, $\langle \ell \rangle$ exceeds unity.) Thus in a SU(3) gauge theory without quarks, there is a confined phase below T_c, a semi-QGP from exactly T_c^+ to somewhere between ~ 2–$4T_c$, and a complete QGP above that. As a phase with partial ionization, by its nature it is not possible to precisely define the boundary between the semi-QGP and the complete QGP.

In QCD the Polyakov loop is no longer a strict order parameter, since dynamical quarks also carry $Z(N_c)$ charge. Thus even below T_c, the $Z(N_c)$ charge of a test quark is shielded by the thermal ionization of dynamical quark antiquark pairs. With sufficiently many flavors of quarks, this could happen even at a rather low temperature.

At present, however, numerical simulations indicate only a modest ionization of color below T_c, at least for three colors and 2 + 1 flavors of dynamical quarks. From Fig. 11 of Ref. [14], the expectation value of the Polyakov loop is very small below $0.8T_c$; it then rises to ~ 0.3 at T_c, and is near one at $\sim 2T_c$. We take this to show that in QCD that there is a semi-QGP between $\sim 0.8T_c$ to perhaps ~ 2–$3T_c$. Thus with dynamical quarks, there is a semi-QGP in both the hadronic phase, from $\sim 0.8T_c$ to T_c, and in the deconfined phase, from T_c to ~ 2–$3T_c$; again, there is a complete QGP above that.

We characterize the semi-QGP by the following approximation [12,22,26–29]. The Polyakov loop is the trace of a straight Wilson line in imaginary time. An expectation value for the Polyakov loop which is not near one implies a nontrivial distribution for the eigenvalues of this Wilson line. We thus expand about a constant, background field for the timelike component of the vector potential,

$$A_0^{\rm cl} = Q\frac{T}{g}, \tag{1}$$

where g is the coupling constant for an SU(N_c) gauge theory. The matrix Q is diagonal in color space, with components Q^a, $a = 1 \dots N_c$. The Wilson line is $L = \exp(iQ/T)$, and the bare Polyakov loop is the first moment, $\ell = {\rm tr}L/N_c$. This and higher moments, ${\rm tr}L^j/N_c$ for $j = 2 \dots (N_c - 1)$, are all gauge invariant. Although we expand about a given Q, only integrals over the complete Q distribution are physically relevant. Expansion about $Q \neq 0$ is familiar from semiclassical calculations of the 't Hooft loop in weak coupling [26].

When the spatial volume is a sphere so small that the coupling g^2 runs nearly to zero, at $N_c = \infty$ deconfinement is characterized precisely by the changes in the Q distribution [27]. We conjecture that this remains true in the infinite volume limit, for any N_c [12,22,28,29]. At present, the detailed form of the Q distribution in infinite volume is unknown; notably, this would determine the pressure in terms of the Q's. (Determining the Q distribution from numerical simulations of effective theories appears promising [29].) In lieu of this, at a given temperature we take both the entropy and the expectation value of the (renormalized) Polyakov loop from numerical simulations on the lattice [12–14]. For higher moments of the Wilson line, we choose two different forms for the Q distribution. Unexpectedly, we find the shear viscosity is most insensitive to which distribution we take.

To compute the shear viscosity, we use a Boltzmann equation [23,24] in the presence of a background field; $Q \neq 0$ acts like an imaginary chemical potential for the color charge, in the subspace of diagonal generators [26]. We find it useful to use the double line notation of 't Hooft, taking both N_c and N_f to be infinite [25,27] (although this is not essential [31]). How the Q's enter depends upon the representation of the gauge group [26]. Quarks in the fundamental representation have one color line, so the propagator in a background Q field has a momenta with one color index, $P_\mu^a = (p_0 + Q^a, \boldsymbol{p})$. (We also define $P_0^{-a} = p_0 - Q^a$.) Adjoint gluons have two color lines, so the propagator in a background Q field has two color indices, $P_\mu^{ab} = (p_0 + Q^{ab}, \boldsymbol{p})$, where $Q^{ab} = Q^a - Q^b$. In the imaginary time formalism, the Euclidean energies p_0 are even (odd) multiples of πT for bosons (fermions), while each component Q^a is typically a nonintegral multiple of $2\pi T$ [26,27].

To compute amplitudes in real time, one takes the background field to be nonzero only for the part of the contour in imaginary time, and not in real time [32]. The energies $p_0 + Q^a \to -i\omega$, where the mass shells remain on the light cone, $\omega = \pm E$, $E = \sqrt{\boldsymbol{p}^2}$.

For quarks, what typically enters are distribution functions $\tilde{n}(E - iQ^a)$, where $\tilde{n}(E)$ is the usual Fermi-Dirac statistical distribution function. To compute, expand this as [27]

$$\frac{1}{e^{(E-iQ^a)/T} + 1} = \sum_{j=1}^{\infty} (-)^{j+1} e^{-j(E-iQ^a)/T}. \tag{2}$$

The first term, $\sim \exp(-E/T)$, represents the Boltzmann approximation to the quantum distribution function, and is accompanied by $\exp(iQ^a/T)$. Expansion to jth order brings in a factor of $\sim\exp(-jE/T)$, and is accompanied by $\exp(ijQ^a/T)$. For a given process, the moments of the Wilson line which enter depend upon the detailed routing of the color indices and the like. As an example, consider the trace of the quark propagator: then the first, Boltzmann term involves the trace of $\exp(iQ^a/T)$, which is the Polyakov loop, ℓ; terms to jth order involve the jth moment of the Wilson line, $\mathrm{tr}L^j$.

Gluons are similar, except that distribution functions $n(E - iQ^{ab})$ enter, where $n(E)$ is the Bose-Einstein statistical distribution function. Again, as an example consider summing over the indices of the gluon propagator. To avoid taking the trace, which is part of the correction in $1/N_c$, one sums separately over a and b. The first, Boltzmann term involves the traces of $\exp(iQ^{ab}/T)$, which becomes $|\mathrm{tr}L|^2$; terms to jth order become $|\mathrm{tr}L^j|^2$.

In computing perturbatively about a trivial vacuum, one naturally divides the momenta into hard momenta, whose components are $\sim T$, or soft momenta, where both the energy ω and spatial momenta p are $\sim gT$ [23]. This remains valid at $Q \neq 0$ for the energies ω; the Q's themselves are $\sim T$, and so hard.

We have computed the hard thermal loops (HTLs) in the quark and gluon self-energies for $Q \neq 0$ [31]. HTLs are the dominant contributions when the external momenta are soft. To leading order, $\sim g^2$, the dominant term involves an integral over hard momenta, times an angular integral over soft momenta [23]. The dependence on the Q's only affect the integral over the hard momenta, through the change in the statistical distribution functions; the angular integral is unchanged.

For the quark self-energy, the only change in the HTL is the change in the Debye mass, which now depends upon the direction in color space. For the gluon self-energy, there is the usual HTL, again modified by the change in the Debye mass. Besides the HTL, which is $\sim g^2T^2$, there are also terms $\sim g^2T^3Q/\omega$. These new terms are directly proportional to tadpole terms, and arise because $Q \neq 0$ induces a net color charge. These terms are physical for a 't Hooft loop [26], and are corrections to the color electric field of a $Z(N_c)$ interface [31]. In vacuum, however, there is not net color charge, and so we assume that the only change in the soft gluon propagator is the change of the Debye mass.

We now outline the calculation of the shear viscosity [31]. To leading order, both in g^2 and in $\log(1/g)$, we find that the result can be written as [23,24,31]

$$\frac{\eta}{T^3} = \frac{c_\eta}{g^4\log(1/g)}\mathcal{R}(L). \tag{3}$$

The constant c_η depends upon the number of colors, N_c, and flavors, N_f [24]. As noted before, for calculational reasons we compute for $N_c = \infty$, with N_f/N_c fixed, implicitly assuming that $\mathcal{R}(L)$ is relatively insensitive to N_c.

At this order, the viscosity is determined by the scattering of $2 \to 2$ particles, where all particles have hard momenta. They interact through the exchange of a single, soft field in the t channel. For the soft field, we use the HTL approximation. The Debye mass changes when $L \neq 1$, but this does not enter at leading logarithmic order.

For the pure glue theory at infinite N_c, to leading order in g^2 and $\log(1/g)$ [23,24], two scattering processes contribute to the collision term: $P_1^{ab} + P_3^{bc} \to P_2^{ad} + P_4^{dc}$ and $P_1^{ab} + P_3^{cd} \to P_2^{ad} + P_4^{cb}$. This is nearly forward scattering, with the spatial momenta $\boldsymbol{p}_1 \approx \boldsymbol{p}_2$, and $\boldsymbol{p}_3 \approx \boldsymbol{p}_4$. The momenta of the exchanged gluon, $\boldsymbol{p}_1 - \boldsymbol{p}_2 = \boldsymbol{p}_4 - \boldsymbol{p}_3$, must be soft to give $\log(1/g)$.

Under these approximations, the dependence upon the background field Q only enters through the integrals over the statistical distribution functions for the hard fields, p_1 and p_3. While the statistical distribution functions are complex when $Q \neq 0$, after summing over both emission and absorption processes, all contributions to the stress energy tensor are real. For the shear viscosity, the integrals which enter are

$$\begin{gathered}\int_0^\infty dp\,p^4 n(p - iQ^{ab}),\\ \int_0^\infty dp\,p^j n(p - iQ^{ab})(1 + n(p - iQ^{ad})),\end{gathered} \tag{4}$$

where $j = 2$ and 4. One then expands the distribution functions as in Eq. (2), to obtain a power series in moments of the Wilson line. With quarks, there are more diagrams, as the distribution functions between quarks and gluons mix [24]. The integrals are similar, but also involve $\tilde{n}(E - iQ^a)$. [We also take χ, the function which parametrizes the deviation from equilibrium, as $\chi(p) \sim p^2$ [24]: this is valid to $<1.0\%$ for $Q = 0$, and is exact at small ℓ, where Boltzmann statistics applies.]

As written in Eq. (3), the result is a function of the Wilson line, $\mathcal{R}(L)$, times the usual perturbative result; thus $\mathcal{R} = 1$ when $L = 1$. This simple form is not valid beyond leading logarithmic order. If $\ell = \mathrm{tr}L/N_c$ is small, and dominates higher moments, we find the result vanishes like the square of the loop:

$$\mathcal{R}(L) \approx a_2(\lambda)\left(\frac{\ell + 4\lambda}{\ell + \lambda}\right)\ell^2, \qquad \ell \ll 1, \tag{5}$$

where $\lambda = N_f/N_c$: $a_2(0) \approx 3.31$, $a_2(1) \approx 1.01$.

The behavior at small ℓ can be understood as follows. In the pure glue theory, when gluons with momenta P_1^{ab} and P_3^{bc} scatter, the Q^b charges cancel, so summation over a and c gives a collision term $\sim g^4\ell^2$ [times $\log(1/g)$, which we suppress for brevity]. There is no such cancellation for the scattering of gluons with momenta P_1^{ab} and P_3^{cd}, which is only $\sim g^4\ell^4$. The source term [24] is like the trace of the gluon propagator, $\sim\ell^2$, and so without quarks, at small ℓ the gluon contribution to the shear viscosity is $\eta_{\mathrm{gluon}} \sim (\ell^2)^2/(g^4\ell^2) \sim \ell^2/g^4$.

With quarks, the collision term is dominated by a quark scattering off of an antiquark, $P_1^a + P_3^{-a} \to P_2^c + P_4^{-c}$. The Q^a charges cancel, so this is $\sim g^4\ell^0$ at small ℓ. If a quark scatters off of a quark, $P_1^a + P_3^b \to P_2^b + P_4^a$, the Q charges do not cancel, and the scattering is $\sim g^4\ell^2$. Gluons also scatter off of quarks, by exchanging a gluon: $P_1^{ab} + P_3^b \to P_2^{ac} + P_4^c$; the Q^b charges cancel, so this is $\sim g^4\ell$. The mixing of the gluon and quark distribution functions [24] can be neglected at small ℓ: quark antiquark annihilation, $P_1^a + P_3^{-b} \to P_2^{ac} + P_4^{c(-b)}$, is $\sim g^4\ell^2$, while Compton scattering, $P_1^a + P_3^{bc} \to P_2^{ab} + P_4^c$, is $\sim g^4\ell^3$. For the source term, the quark contribution is like summing over the quark propagator, $\sim\ell$, so the quarks contribute to the viscosity as $\eta_{\text{quark}} \sim (\ell)^2/(g^4\ell^0) \sim \ell^2/g^4$. In the presence of quarks, gluons contribute to the viscosity as $\eta_{\text{gluon}} \sim (\ell^2)^2/(g^4\ell) \sim \ell^3/g^4$, down by $\sim\ell$ to the quark contribution.

To obtain results valid for all values of ℓ, some assumption about higher moments must be made. We used two forms. The first is to take a simple step function, of width β, about the origin, so that $\mathrm{tr}L^n/N_c = \sin(n\beta)/(n\beta)$. The other is to take a Q distribution as in the Gross-Witten matrix model [12,27,28]. By the methods described above, it is straightforward to obtain results, although their analytic form is unwieldy. These forms can be easily evaluated numerically, though, as shown in Fig. 1. We find that $\mathcal{R}(L)$ is insensitive to the assumption about higher moments, changing by at most a few percent over the entire range of ℓ. We find that with both eigenvalue distributions, that the collision term generates a cusp in $\mathcal{R}(L)$ near $\ell = 1$. We expect that this nonanalytic behavior will be washed out by corrections to higher order, which enter for $Q \sim gT$.

We conclude with some general comments. The Polyakov loop is proportional to the propagator of an infinitely massive test quark, and as such, has no direct relation to the propagation of dynamical fields. In our ansatz, however, the quasiparticles are fluctuations about the background field in Eq. (1). When the expectation value of the Polyakov loop is small, this background field universally suppresses the propagation of any colored field. For heavy fields, this is reasonable, but for light fields, it is nontrivial. If the light fields have hard momenta $p \geq T$, then suppression by the Polyakov loop is the dominant effect, with other corrections down by powers of g. At soft momenta, $p \sim gT$, light fields are not only suppressed by a (small) Polyakov loop, but altered by the change in their hard thermal loops.

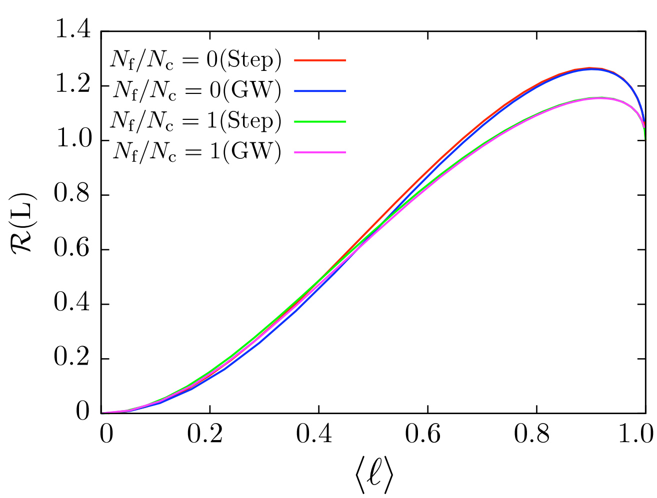

FIG. 1 (color online). The function $\mathcal{R}(L)$ of Eq. (3), versus ℓ. "Step" and "GW" denote Q distributions with a simple step function and that in the Gross-Witten matrix model [12,27,28], respectively.

In general, the shear viscosity is proportional to the ratio of a source and a collision term [23,24]. In the absence of a background field, the source term is of order one, and so the shear viscosity can only be small if the coupling constant, and so the collision term, is large. This is the central idea which motivates the strong QGP [3,10]. In contrast, in our analysis of the semi-QGP, the quasiparticles are weakly coupled, but are fluctuations about a nontrivial background field. It is this background field which suppresses *both* the source and collision terms, to give $\eta/T^3 \sim \ell^2$ at small ℓ.

We suggest that this is not an artifact of our approximations. If deconfinement truly represents the ionization of color charge, then it is reasonable to expect that the propagation of all colored fields are suppressed at small ℓ. In particular, while we have computed only to leading order in g^2 and $1/\log(1/g)$, one can show that within our ansatz, that $\eta/T^3 \sim \ell^2$ as $\ell \to 0$, order by order in perturbation theory at $N_c = \infty$ [31].

Taking the result for η/T^3 to leading logarithmic order, Eq. (3) gives suppression near T_c. Whether high order corrections in perturbation theory modify this can only be answered within a detailed program of resummation. If η/T^3 is suppressed near T_c, it can be verified through numerical simulations on the lattice [15].

If color is only partially ionized in the semi-QGP, then some color singlet states persist for a range of temperatures above T_c, before they eventually disassociate into colored constituents. Similarly, with dynamical quarks colored fields contribute below T_c, in the hadronic part of the semi-QGP. Including both effects is clearly challenging; here we just speculate about the complete result. Although hydrodynamics depends upon η/s, without a complete theory we can only discuss the behavior of η/T^3. Working up from low temperatures in the hadronic phase, η/T^3 appears to decrease with increasing T, as in a liquid [3,9,10]. Working down from high temperatures, to leading logarithmic order we find a suppression in the semi-QGP near T_c. Combining these two effects gives a minimum for η/T^3 near T_c, as in nonrelativistic systems [8,16]. It seems implausible that the minimum is precisely at T_c; certainly in QCD, where there is no true phase transition [11,14].

It is natural to suspect that heavy ion collisions at RHIC have probed some region in the semi-QGP. Since one needs a small value of the shear viscosity to fit the experimental data [1–10], perhaps one is near T_c. Heavy ion collisions at the LHC may probe temperatures which are significantly higher, possibly well into the complete QGP. If so, then at small times collisions at the LHC create a system with large shear viscosity; as the system cools through T_c, the shear viscosity then drops. Thus the semi-QGP predicts that at short times, the hydrodynamic behavior of heavy ion collisions at the LHC is qualitatively different from that at RHIC [31].

In contrast, models of a strong QGP predict that hydrodynamics at the LHC will be similar to that at RHIC, characterized by a small value of η/s [3,7,10]. In particular, while in $\mathcal{N} = 4$ supersymmetry the pressure/T^4 is constant [17], several models have been proposed to fit the QCD pressure right down to T_c [18]. Even so, in *all* of these models η/s is independent of temperature, and so remains small, $= 1/4\pi$ [18].

We eagerly await the experimental results for heavy ions from the LHC, which may be as unexpected and exciting as those from RHIC first were.

This research was supported in part by the RIKEN BNL Research Center and by the U.S. Department of Energy under Cooperative Research Agreement No. DE-AC02-98CH10886. R. D. P. also thanks the Alexander von Humboldt Foundation for their support.

[1] D. Teaney, J. Lauret, and E. V. Shuryak, Phys. Rev. Lett. **86**, 4783 (2001); K. Dusling and D. Teaney, Phys. Rev. C **77**, 034905 (2008).
[2] P. F. Kolb and U. W. Heinz, arXiv:nucl-th/0305084; H. Song and U. W. Heinz, Phys. Rev. C **77**, 064901 (2008); Phys. Rev. C **78**, 024902 (2008); G. Kestin and U. W. Heinz, arXiv:0806.4539.
[3] T. Hirano and M. Gyulassy, Nucl. Phys. **A769**, 71 (2006).
[4] P. Romatschke and U. Romatschke, Phys. Rev. Lett. **99**, 172301 (2007); M. Luzum and P. Romatschke, Phys. Rev. C **78**, 034915 (2008).
[5] M. Asakawa, S. A. Bass, and B. Muller, Phys. Rev. Lett. **96**, 252301 (2006); P. Romatschke, arXiv:0710.0016.
[6] Z. Xu and C. Greiner, Phys. Rev. Lett. **100**, 172301 (2008); Z. Xu, C. Greiner, and H. Stocker, Phys. Rev. Lett. **101**, 082302 (2008).
[7] H. Niemi, K. J. Eskola, and P. V. Ruuskanen, arXiv:0806.1116.
[8] L. P. Csernai, J. I. Kapusta, and L. D. McLerran, Phys. Rev. Lett. **97**, 152303 (2006); R. A. Lacey *et al.*, *ibid.* **98**, 092301 (2007).
[9] A. Dobado and F. J. Llanes-Estrada, Phys. Rev. D **69**, 116004 (2004); K. Itakura, O. Morimatsu, and H. Otomo, Phys. Rev. D **77**, 014014 (2008).
[10] M. Gyulassy and L. McLerran, Nucl. Phys. A **750**, 30 (2005); B. Muller and J. L. Nagle, Annu. Rev. Nucl. Part. Sci. **56**, 93 (2006); S. Mrowczynski and M. H. Thoma, *ibid.* **57**, 61 (2007); E. V. Shuryak, arXiv:0807.3033.
[11] Z. Fodor, Proc. Sci., LATTICE2007 (2007) 011.
[12] A. Dumitru, Y. Hatta, J. Lenaghan, K. Orginos, and R. D. Pisarski, Phys. Rev. D **70**, 034511 (2004).
[13] S. Gupta, K. Hübner, and O. Kaczmarek, Phys. Rev. D **77**, 034503 (2008).
[14] M. Cheng *et al.*, Phys. Rev. D **77**, 014511 (2008).
[15] H. B. Meyer, Phys. Rev. D **76**, 101701 (2007).
[16] P. Kovtun, D. T. Son, and A. O. Starinets, Phys. Rev. Lett. **94**, 111601 (2005).
[17] D. T. Son and A. O. Starinets, Annu. Rev. Nucl. Part. Sci. **57**, 95 (2007).
[18] N. Evans and E. Threlfall, arXiv:0805.0956; S. S. Gubser and A. Nellore, arXiv:0804.0434 [Phys. Rev. D (to be published)]; U. Gursoy, E. Kiritsis, L. Mazzanti, and F. Nitti, arXiv:0804.0899 [Phys. Rev. Lett. (to be published)].
[19] E. Braaten and A. Nieto, Phys. Rev. D **53**, 3421 (1996).
[20] F. Di Renzo, M. Laine, Y. Schröder, and C. Torrero, J. High Energy Phys. 09 (2008) 061.
[21] M. Laine and Y. Schröder, J. High Energy Phys. 03 (2005) 067; Phys. Rev. D **73**, 085009 (2006).
[22] R. D. Pisarski, Phys. Rev. D **74**, 121703(R) (2006).
[23] M. Le Bellac, *Thermal Field Theory* (Cambridge University Press, Cambridge, 2000).
[24] P. Arnold, G. D. Moore, and L. G. Yaffe, J. High Energy Phys. 11 (2000) 001; 05 (2003) 051.
[25] G. 't Hooft, Nucl. Phys. **B72**, 461 (1974).
[26] T. Bhattacharya, A. Gocksch, C. Korthals Altes, and R. D. Pisarski, Phys. Rev. Lett. **66**, 998 (1991); Nucl. Phys. **B383**, 497 (1992); A. Gocksch and R. D. Pisarski, *ibid.* **B402**, 657 (1993); C. Korthals Altes, A. Kovner, and M. A. Stephanov, Phys. Lett. B **469**, 205 (1999); P. Giovannangeli and C. P. Korthals Altes, Nucl. Phys. **B608**, 203 (2001); **B721**, 1 (2005); **B721**, 25 (2005).
[27] B. Sundborg, Nucl. Phys. **B573**, 349 (2000); O. Aharony, J. Marsano, S. Minwalla, K. Papadodimas, and M. Van Raamsdonk, Adv. Theor. Math. Phys. **8**, 603 (2004); Phys. Rev. D **71**, 125018 (2005).
[28] A. Dumitru, J. Lenaghan, and R. D. Pisarski, Phys. Rev. D **71**, 074004 (2005).
[29] A. Dumitru and D. Smith, Phys. Rev. D **77**, 094022 (2008).
[30] As an ultraviolet divergent quantity, the overall normalization of the Polyakov loop is arbitrary. All values quoted are normalized so that $\langle \ell \rangle \to 1$ as $T \to \infty$, with vanishing zero point energy [12–14]; see, especially, Sec. II A and Appendix C of Ref. [13].
[31] Y. Hidaka and R. D. Pisarski (unpublished).
[32] K. Furuuchi, Phys. Rev. D **73**, 046004 (2006).