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Determination of neutrino mass hierarchy and θ_{13} with a remote detector of reactor antineutrinos

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We describe a method for determining the hierarchy of the neutrino mass spectrum and θ_{13} through remote detection of electron antineutrinos from a nuclear reactor. This method utilizing a single, 10-kiloton scintillating liquid detector at a distance of 49–63 kilometers from the reactor complex measures mass-squared differences involving ν_3 with a one(ten)-year exposure provided $\sin^2(2\theta_{13}) > 0.05(0.02)$. Our technique applies the Fourier transform to the event rate as a function of neutrino flight distance over neutrino energy. Sweeping a relevant range of δm^2 resolves separate spectral peaks for $\delta m^2{}_{31}$ and $\delta m^2{}_{32}$. For normal (inverted) hierarchy $|\delta m^2{}_{31}|$ is greater (lesser) than $|\delta m^2{}_{32}|$. This robust determination requires a detector energy resolution of $3.5\%/\sqrt{E}$.

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I. INTRODUCTION

Neutrinos have different masses as evidenced by their well established mixing and oscillations [1]. Knowledge of the spectrum of neutrino masses is currently incomplete. We know ν_2 to be more massive than ν_1 ($m_2 > m_1$) [2] with $\delta m^2{}_{21} = (7.9 \pm 0.7) \times 10^{-5}$ eV^2 [3]. Although we know $|\delta m^2{}_{31}| \approx |\delta m^2{}_{32}| = (2.5 \pm 0.5) \times 10^{-3}$ eV^2 [4,5], we do not know if the hierarchy is normal ($m_3 > m_2$) or inverted ($m_3 < m_1$). The hierarchy can be determined by measuring both $|\delta m^2{}_{31}|$ and $|\delta m^2{}_{32}|$ with a precision better than $\delta m^2{}_{21}/|\delta m^2{}_{31}| \approx 0.03$. For normal (inverted) hierarchy $|\delta m^2{}_{31}|$ is greater (lesser) than $|\delta m^2{}_{32}|$. Determination of neutrino mass hierarchy is fundamental to the development of models of particle physics [6] with significant implications for cosmology and astrophysics.

The expression for the survival probability of electron neutrinos involving 3-neutrino mixing is given by [7,8]

$$\begin{aligned} P_{ee} = 1 - \{&\cos^4(\theta_{13})\sin^2(2\theta_{12})\sin^2(\Delta_{21}) \\ &+ \cos^2(\theta_{12})\sin^2(2\theta_{13})\sin^2(\Delta_{31}) \\ &+ \sin^2(\theta_{12})\sin^2(2\theta_{13})\sin^2(\Delta_{32})\}, \end{aligned}$$

where θ_{12} and θ_{13} are mixing angles, $\Delta_{ij} = 1.27(|\delta m^2{}_{ji}|L)/E_\nu$ control the oscillations with $\delta m^2{}_{ji} \equiv m_j^2 - m_i^2$ the neutrino mass-squared difference of ν_j and ν_i in eV^2, L is the neutrino flight distance in meters, and E_ν is the neutrino energy in MeV. Three terms, each oscillating with a "frequency" in L/E space specified by $\delta m^2{}_{ji}$, suppress the survival probability an amount determined by the mixing angles. At present we knowθ_{13} is small [9] and θ_{12} is large and less than $\pi/4$ [2]. The first term with the lowest frequency dominates the suppression. It is responsible for the deficit of solar neutrinos and the conspicuous spectral distortion of reactor antineutrinos [3]. For nonzero θ_{13}, the second term provides greater suppression than the third term. Clearly the ability to measure oscillations influenced by mass-squared differences involving ν_3 requires $\theta_{13} \neq 0$. Sensitivity to these oscillations is greatest when $\Delta_{21} = \pi/2$, which provides maximum suppression by the dominant term and thereby the highest signal to noise ratio. For the normal hierarchy of neutrino masses ($m_3 > m_2 > m_1$), Δ_{31} is slightly greater than Δ_{32} giving the second term a slightly higher frequency than the third term. Whereas for the inverted hierarchy of neutrino masses ($m_2 > m_1 > m_3$), Δ_{31} is slightly smaller than Δ_{32} giving the second term a slightly lower frequency than the third term. It is thus possible to determine neutrino mass hierarchy by resolving the small ($\sim 3\%$) difference in the frequency of the second and third terms.

There is discussion in the literature of various methods to determine neutrino mass hierarchy using reactor antineutrinos. These explore the potential for measuring distortions of the energy spectrum due to nonzero θ_{13} [10,11]. We describe below a unique and robust method.

II. PRECISION MEASUREMENT OF MASS-SQUARED DIFFERENCES INVOLVING ν_3

Neutrino oscillation experiments using reactor antineutrinos are well established. These traditionally involve electron antineutrino disappearance as described by the survival probability equation given above. Using the standard reactor antineutrino event rate spectrum, we generate data samples in a scintillating liquid detector with an energy resolution of $3.5\%/\sqrt{E}$. The neutrino event spectrum peaks at about 3.6 MeV. This suggests an optimum baseline distance of $L = \pi(3.6\text{ MeV})/\{2.54(7.9 \pm 0.7) \times 10^{-5}\text{ eV}^2\} = 56 \pm 7$ km for measuring oscillations involving ν_3. The effect of neutrino mixing on the reactor antineutrino event spectrum at a distance of 50 km is exhibited by a broad modulation of Δ_{21} producing a local minimum of event rate at neutrino energy just above 3 MeV. Superposed, for nonzero θ_{13}, is the narrow modulation of Δ_{31}. There is a broadening of the Δ_{21} and Δ_{31} modulations with increasing neutrino energy. Plotting the event rate as a function of neutrino flight distance divided by neutrino energy (L/E) makes the modulations uniform as we show in Fig. 1 for a 1000 kT-y exposure of a detector fixed at 50 km.

The new approach we describe in this paper utilizes the power of transform methods to extract the signal due to nonzero θ_{13}. We show in Fig. 2 the Fourier transform of the data expected for an exposure of 1000 kT-y at a distance of 50 km from an 8 $\mathrm{GW_{th}}$ nuclear reactor complex. The transform samples 1000 bins in L/E space, while sweeping over values of δm^2. At small δm^2 the spectrum is dominated by the broad Δ_{21} modulation. It is not possible using this technique to measure the δm^2 value associated with this feature because only about one cycle of the Δ_{21} modulation is present in reactor neutrinos at a distance of 50 km. The prominent peak in the spectrum is due to the many cycles of Δ_{31} modulation, allowing measurement of θ_{13}. This peak measures $\delta m^2{}_{31}$ with a precision of about 1% for $\sin^2(2\theta_{13}) = 0.1$.

Using this technique it is possible to determine the neutrino mass hierarchy by resolving a shoulder on the main peak due to Δ_{32}. This shoulder, which has a power reduced by a factor of $\cot^4(\theta_{12})$, appears at smaller δm^2 for normal hierarchy and at larger δm^2 for inverted hierarchy. The displacement of this shoulder from the main peak is $\delta m^2{}_{21}$. We show in Fig. 3 just the top of the peak for the two possible hierarchies, where $\delta m^2{}_{31}$ and $\delta m^2{}_{21}$ are fixed at experimental values given above.

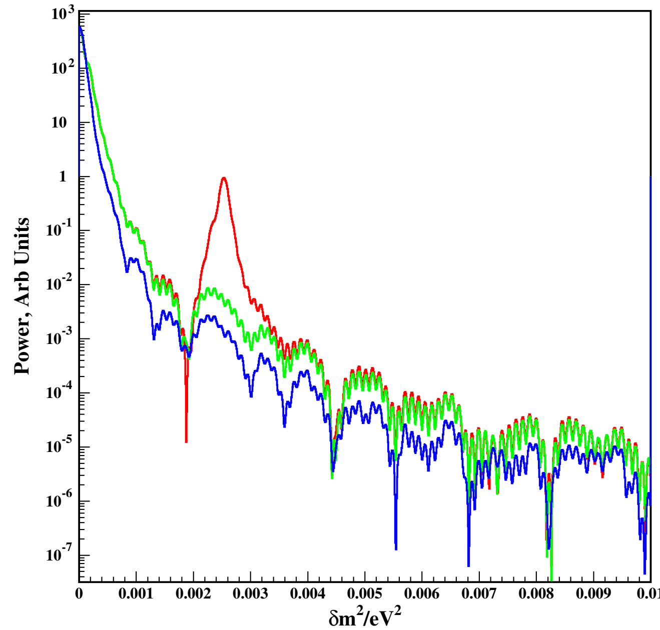

FIG. 2 (color online). Fourier power spectrum with modulation in units of eV^2 and power in arbitrary units on the logarithmic scale. The peak due to Δ_{31} with $\sin^2(2\theta_{13}) = 0.1$ is prominent.

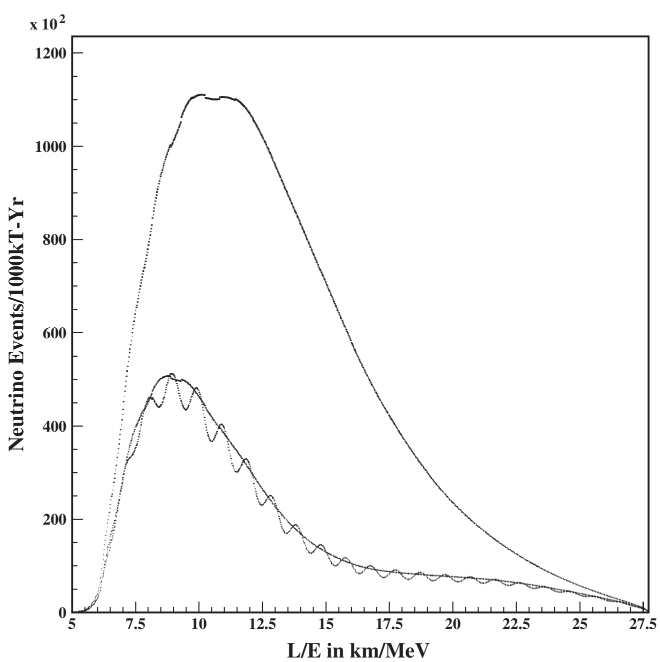

FIG. 1. Event rate versus L/E in units of km/MeV for: no oscillations (top curve), oscillations with $\theta_{13} = 0$ (lower smooth curve), and oscillations with $\sin^2(2\theta_{13}) = 0.1$.

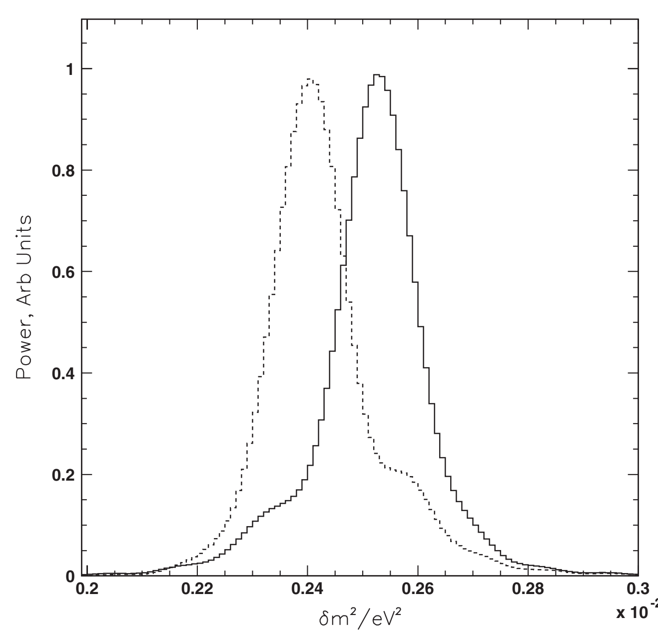

FIG. 3. Neutrino mass hierarchy (normal = solid; inverted = dashed) is determined by the position of the small shoulder on the main peak.

In order to assess the quantitative ability of an experiment to discriminate between normal and inverted hierarchy, we have written a simulation program which generates and analyzes data sets from an idealized detector with 8.5×10^{32} free proton targets (10 kT) and an 8 $\mathrm{GW_{th}}$ nuclear reactor complex. We have varied the distance, $\sin^2(2\theta_{13})$ and exposure time typically for 1000 simulated experiments at each set of parameters.

At this stage we have not included detector specific background sources such as those due to cosmic ray muons traversing the detector, radio impurities, geological antineutrinos, or antineutrinos from other (more distant) reactors. The cosmic ray induced background depends upon depth of water or rock overburden, so must be assessed for the individual proposed location. We know, however, that this is of no concern at depths greater than 3000 mwe, though lesser depths may be acceptable. Other reactors will make a small contribution, if sites are chosen on the basis of not having significant additional flux (though to a certain extent these can be included in the analysis). In general, we do not expect background to compromise the proposed method, since the added antineutrinos start at random distances relative to the detector, so make no coherent contribution to the Fourier transform on L/E at the frequency of interest. One may think of such background, if uniformly distributed in L/E as simply contributing to the zeroth term in the transform, the total rate. Of course, the more random events in a finite sample, the more background across the δm^2 spectrum. In any case, at this stage we neglect background, reserving the study for more specific applications.

We have studied several algorithms for determining the mass hierarchy, noting that the periodicity (δm^2), if evident, is measured to 0.1% precision. In practice this is limited by systematic uncertainties in terms of interpretation as a particular mass difference, probably the energy scale uncertainty (of order 1%). However, in the data set the peak is known to whatever we fit it to, and we can analyze the data employing that knowledge. Hence, knowing the primary peak ($\delta m^2{}_{31}$), we need to determine if the secondary peak ($\delta m^2{}_{32}$) is at greater or lesser periodicity (δm^2). Although we do not know the displacement ($\delta m^2{}_{21}$) exceedingly accurately ($\sim 10\%$), we know the uncertainty in the displacement relative to the primary peak is small ($\sim 0.3\%$) and about an order of magnitude less than the displacement relative to the primary peak ($\delta m^2{}_{21}/\delta m^2{}_{31} \approx 3\%$). Hence, we can examine how well the data fit each hierarchy hypothesis. For presentation here, we use a ‘‘matched filter’’ approach, which one can think of as the Fourier transform of the correlation function, producing a numerical value for each hypothesis.

In Fig. 4 we show in a scatter plot the distribution of ‘‘experimental’’ results for one year exposures at distances of 30 and 50 km with normal and inverted hierarchy. Each of the 1000 experiments at each distance yields two numerical values corresponding to the output of the matched filter for each hierarchy hypothesis. The plot displays the normal hierarchy test value N on the horizontal axis and the inverted hierarchy test value I on the vertical axis. One

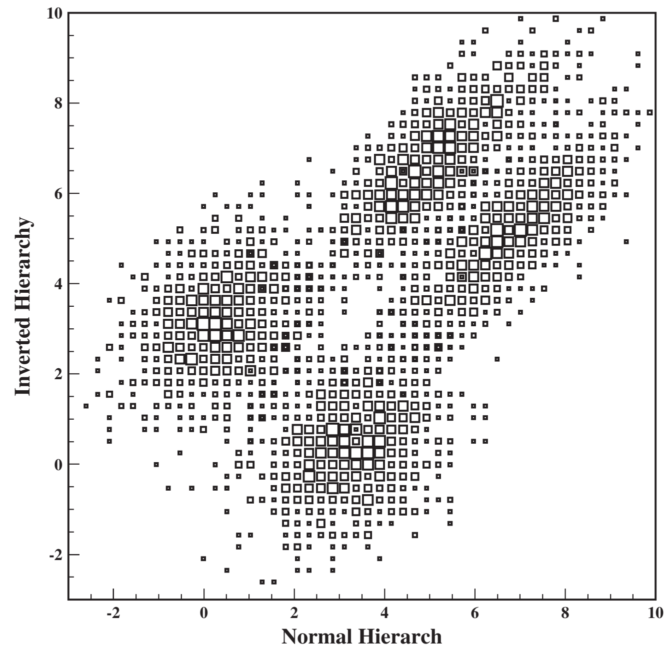

FIG. 4. Scatter plot for the dependence of the hierarchy test on distance. The points below the positively sloped diagonal line (lower right) are sets of 1000 experiments at 30 km (larger values) and 50 km (smaller values) with normal hierarchy. Points above the positively sloped diagonal line (upper left) are experiments with inverted hierarchy.

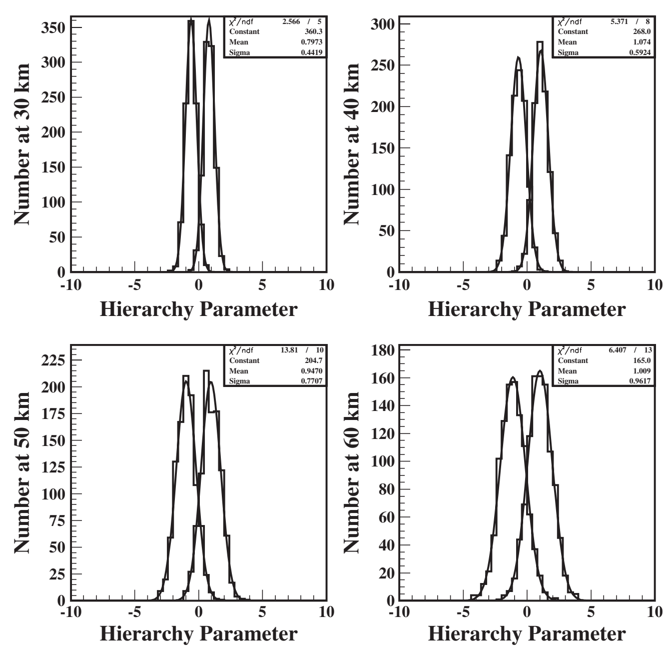

FIG. 5. Hierarchy parameter distributions for 30, 40, 50, and 60 km. Solid histograms are with normal hierarchy, dashed with inverted. Distributions fit well to a Gaussian.

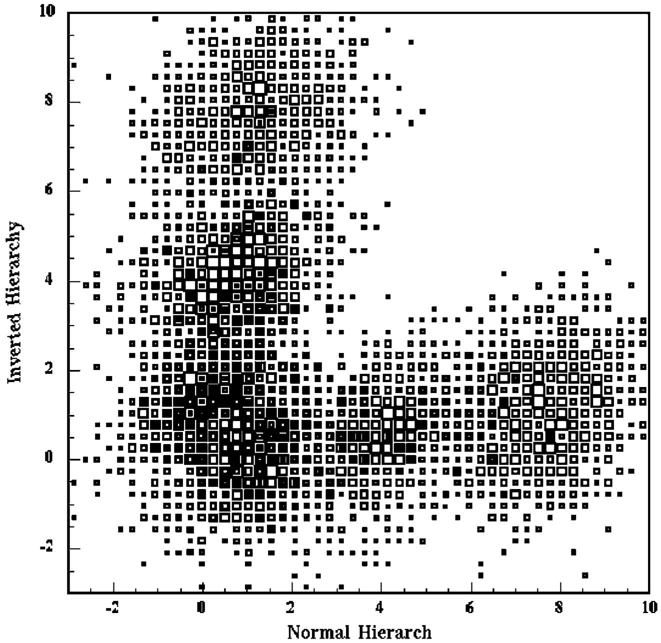

FIG. 6. Scatter plot for the dependence of the hierarchy test on $\sin^2(2\theta_{13})$. The points below the positively sloped diagonal line (lower right) are sets of 1000 experiments at $\sin^2(2\theta_{13}) = 0.04$ (smallest values), 0.12 (intermediate values), and 0.20 (largest values) with normal hierarchy. Points above the positively sloped diagonal line (upper left) are experiments with inverted hierarchy. Note that the separation increases with larger values of $\sin^2(2\theta_{13})$.

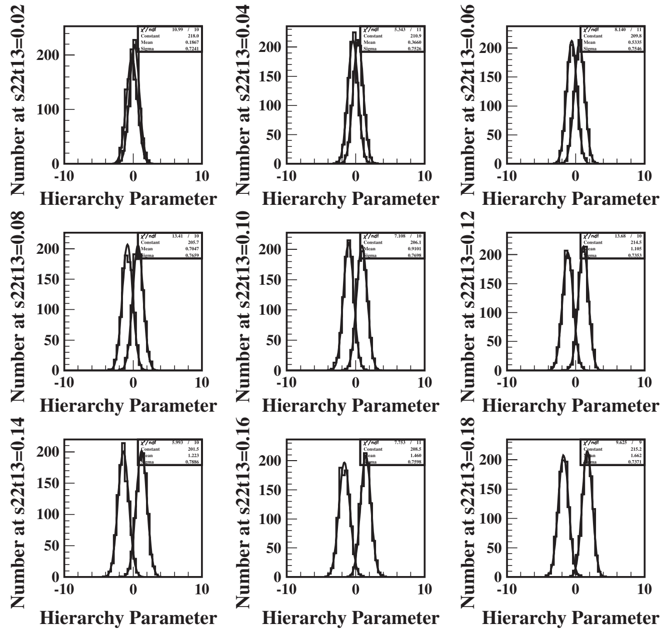

FIG. 7. Hierarchy parameter distributions for 1000 experiments each with $\sin^2(2\theta_{13})$ values of 0.02, 0.04, 0.06, 0.08, 0.10, 0.12, 0.14, 0.16, and 0.18. Solid histograms are with normal hierarchy, dashed with inverted.

sees a nice separation of the hierarchy hypotheses along the diagonal with positive slope (symmetry line).

We construct a hierarchy parameter by projecting the distributions onto the diagonal line with negative slope (perpendicular to the symmetry line) and equivalent to $(N - I)/(N + I)$. This is illustrated in Fig. 5 in four panels each corresponding to a different distance. Each projection fits well to a Gaussian distribution. Separation of the projections is quite good ($>95\%$) over the entire range examined, from 30–75 km, but degrades at distances less than 40 km and greater than 65 km.

Next we examine the sensitivity of the hierarchy determination to $\sin^2(2\theta_{13})$. In Fig. 6 we present a scatter plot of hierarchy tests for 1000 experiments of one year exposure at each of $\sin^2(2\theta_{13}) = 0.04$, 0.12, and 0.20, all at 50 km range. One sees that the distributions are well separated at $\sin^2(2\theta_{13})$ values more than about 0.04. The values of the hierarchy parameter are plotted, in the same projection as above for the distance study, in Fig. 7. It thus appears as though such an experiment can probe the hierarchy down to $\sin^2(2\theta_{13})$ values of 0.02 with an exposure of 100 kT-y (with the caveats about site specific background). This sensitivity is the subject of a subsequent study [12].

III. CONCLUSIONS

We demonstrate a robust method using a single remote detector of reactor antineutrinos that measures θ_{13} by employing a Fourier transform, and determines neutrino mass hierarchy by resolving mass-squared differences involving ν_3. This determination is provided with an exposure of 10 (100) kT-y and $\sin^2(2\theta_{13}) > 0.05(0.02)$. This method does not depend on precise measuring or modeling of the reactor flux spectrum nor observation of matter effects.

Note added.—The hierarchy determination is sensitive to the actual values of $\delta m^2{}_{31}$ and $\delta m^2{}_{32}$. This is explored in a subsequent paper [12]. After completion of this work, a related study also employing Fourier transform techniques was reported, which supports the results presented here [13].

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