PHYSICAL REVIEW D **78,** 066020 (2008)

Cosmology of the closed string tachyon

Ian Swanson
School of Natural Sciences, Institute for Advanced Study, Princeton, New Jersey 08540, USA
(Received 12 June 2008; published 29 September 2008)

The spacetime physics of bulk closed string tachyon condensation is studied at the level of a two-derivative effective action. We derive the unique perturbative tachyon potential consistent with a full class of linearized tachyonic deformations of supercritical string theory. The solutions of interest deform a general linear dilaton background by the insertion of purely exponential tachyon vertex operators. In spacetime, the evolution of the tachyon drives an accelerated contraction of the universe and, absent higher-order corrections, the theory collapses to a cosmological singularity in finite time, at arbitrarily weak string coupling. When the tachyon exhibits a null symmetry, the worldsheet dynamics is known to be exact and well defined at tree level. We prove that if the two-derivative effective action is free of *nongravitational* singularities, higher-order corrections always resolve the spacetime curvature singularity of the null tachyon. The resulting theory provides an explicit mechanism by which tachyon condensation can generate or terminate the flow of cosmological time in string theory. Additional particular solutions can resolve an initial singularity with a tachyonic phase at weak coupling, or yield solitonic configurations that localize the universe along spatial directions.

DOI: 10.1103/PhysRevD.78.066020 PACS numbers: 11.25.−w, 11.25.Pm, 11.25.Sq, 04.20.Dw

I. INTRODUCTION

The study of tachyon condensation in open string theory has led to a number of important insights into the nature of instabilities in quantum gravity. Direct calculations in open string field theory have provided detailed evidence in support of Sen's conjecture (for useful reviews of this subject, see, e.g., [1–4], and [5] for recent developments). Namely, the open string tachyon represents an unstable mode of a space-filling D-brane. The process of tachyon condensation drives the decay of the D-brane, and the endpoint is an excited state of the closed string vacuum that carries the energy of the original D-brane. Solitonic configurations can also arise, represented as D-branes filling a lower number of spatial dimensions.

Attempts to understand closed string tachyon condensation initially focused on localized tachyons (see, e.g., [6–14]). A well-known example is the theory of winding tachyons localized on the conical orbifold $\mathbb{C}/\mathbb{Z}_N$ [6]. Studies using brane probes, renormalization group flow, and string field theory have provided evidence that tachyon condensation in these systems drives a reduction in the orbifold rank N, and a resolution of the conical singularity.

Similar to the open string tachyon of a space-filling D-brane, the *bulk* closed string tachyon fills spacetime completely, and presents a tantalizing analogy. Adopting the lessons of the open string problem, it is natural to guess that the bosonic bulk closed string tachyon signals an instability of spacetime itself, and the transition to a stable endpoint of tachyon condensation represents the decay of spacetime altogether. According to this analogy, solitons of the tachyon condensate appear as lower-dimensional spacetime, allowing for a dynamical transition between physical theories with different numbers of spatial dimensions. A number of recent studies have provided indirect evidence in support of this picture in bosonic supercritical string theory [15–17].[1]

Overall, the problem of understanding bulk closed string tachyon condensation in detail has been approached in roughly three regimes: (1) At the level of string field theory, (2) from the perspective of the worldsheet conformal field theory (CFT), and (3) within the framework of spacetime effective theories.

String field theory.—Relative to the corresponding problem in open string theory, attempts to study closed string tachyon condensation using field theoretic techniques are hindered by the relative intractability of closed string field theory [19–24]. The action itself is nonpolynomial, and closed string tachyons couple to the dilaton and the metric, making it necessary to carefully account for the backreaction of the tachyon condensate on the background. Progress has been made in computing the bulk closed string tachyon potential perturbatively in the strength of the tachyon [25–30] (see also [14,31] for analogous work on localized tachyons), and some evidence has emerged that a critical point of the potential may exist [32,33].

Worldsheet CFT.—A number of results have demonstrated that substantial progress can be made in understanding bulk closed string tachyon condensation directly as renormalization group flow in the worldsheet CFT (e.g., [15–18,34–38]). Concrete conclusions can be reached by focusing on a class of exact solutions of the string theory in which quantum corrections are tightly constrained and calculable to all orders in perturbation theory. The simplest

[1]Corresponding systems in superstring theory exhibit a more baroque landscape of stable and semistable endpoints of tachyon condensation [18].

examples arise in bosonic string theory when the tachyon condenses with a purely exponential profile varying in a null direction:

$$T \sim \exp(\beta X^+), \tag{1.1}$$

where β is constant. These theories are particularly straightforward, since the tachyon vertex operator is nonsingular in the vicinity of itself, and the diagrammatic structure of the CFT indicates that the theory is exact at tree level [16,17].

To linearized order in the deformation, the tachyon couples to the $2D$ CFT as a potential, and the onset of null tachyon condensation can be modeled on the worldsheet as the nucleation of a bubblelike region of a nonzero tachyon. Inside this region, string states see a potential wall that rapidly increases into the future, and at late times the bubble itself expands outward from the nucleation point at the speed of light [16]. The resulting picture is an expanding region of the tachyon condensate from which all string states are expelled. Since no physical degrees of freedom persist inside the bubble at late times, this configuration has been called the "bubble of nothing," similar in spirit to the Witten instanton solution described in [39]. The expectation is that dynamical spacetime ceases to exist deep inside the bubble.

A related process was found to drive dynamical dimensional reduction (or "dimension quenching"), wherein high potential walls from the tachyon condensate localize one or more (but not greater than $D - 2$, where D is the dimension of spacetime) spatial coordinates on the worldsheet [15,17]. By again adopting a null tachyon profile, quantum corrections can be computed exactly at finite loop order in perturbation theory. In essence, the worldsheet fields that feel the potential become infinitely massive and decouple from the theory. Classically, this amounts to a deficit in central charge contribution from the worldsheet degrees of freedom. This discrepancy is resolved at the quantum level by the simultaneous one-loop renormalization of the dilaton gradient and the string-frame metric.

Stated succinctly, both of these processes confirm qualitatively the expectation that a natural consequence of bosonic closed string tachyon condensation is the spontaneous decay of spacetime. On the worldsheet this is manifested as the creation of regions of "nothing," where exponentially growing potential walls prevent the presence of string states, and several examples of this process are now well understood in the language of worldsheet conformal field theory.

Effective actions.—If the intuition coming from the worldsheet CFT is correct, the two-dimensional theory should give rise to very interesting phenomena in spacetime (see, e.g., [40,41]). However, the spacetime dynamics in the presence of the tachyon configurations discussed above are not well understood. Such solutions fall into a class of exponential tachyon profiles evolving in the background of a linear dilaton, and it is unclear whether the spacetime effective actions typically studied in the literature consistently support these systems.

In this paper we aim to examine bulk closed string tachyon condensation directly as a dynamical cosmological process in spacetime. While this question is most easily and directly studied at the level of an effective action, it is difficult to assess the reliability of this approach without having a unique expression for the potential in which the fields of interest evolve. Furthermore, even if a potential is known, it is still unclear whether a low-derivative truncation, for example, is sufficient to capture the worldsheet physics of tachyonic fields.

To address these questions, we adopt a simple strategy. Since it seems unlikely that a single two-derivative effective action will consistently support all known tachyonic solutions of bosonic string theory, we aim to focus on a specific but nontrivial class of solutions that contains controllable models of bulk tachyon condensation. Namely, the theories of interest are linearized deformations of the exact supercritical ($D > 26$) linear dilaton background of bosonic string theory, characterized by the insertion of purely exponential tachyon vertex operators. We use these solutions to guide the formulation of a consistent two-derivative effective action, including a specific form for the tachyon potential.

The tachyon perturbations of interest are formulated at linearized order in conformal perturbation theory. In other words, the tachyon profiles under consideration obey a linearized equation of motion. In general, the solutions capture the dominant behavior of the string theory when the system is perturbative in the strength of the tachyon. However, we show that one can rely on exact solutions of the worldsheet CFT to study the effects of higher-order corrections on the spacetime dynamics outside this perturbative regime. For the null tachyon system in particular, higher-order contributions to the action can be captured in the spacetime solution in a single undetermined function of the tachyon. We demonstrate that when this function is chosen such that nongravitational singularities are forbidden to appear in the action, and the gravity sector is constrained to be unitary, all possible curvature singularities are either removed or placed at $T = \infty$.

In the next section we review the tachyonic solutions of interest, and demonstrate that a unique two-derivative effective action can be computed that supports the aforementioned class of solutions perturbatively in the strength of the tachyon. In Sec. III we study the process of tachyon condensation as cosmological evolution in a Friedmann Robertson Walker (FRW) target space. Focusing on a timelike tachyon profile [$T \sim \exp(-\beta^0 X^0)$], we show that, modulo higher-order corrections, the evolution of the tachyon is realized in spacetime as a big crunch occurring in finite time, at arbitrarily weak string coupling. (Numerical integration of the spacetime equations of motion for the dilaton, tachyon, and FRW scale factor verifies that the effective action reproduces the timelike tachyon solution quantitatively in the region of classical validity.) We also analyze the exact null tachyon solution in the background of a timelike linear dilaton rolling to weak coupling. In this case, the worldsheet bubble of nothing is realized in spacetime as a logarithmically expanding region in which the scale factor collapses to a singularity. In Sec. IV, we consider the null tachyon solution on general grounds, and demonstrate that the naive gravitational singularity can be resolved classically. The resulting system provides a very simple mechanism by which the flow of cosmological time is either initiated or halted by string theory. This suggests a class of toy models of the big bang, which can be studied at weak coupling. We extend this analysis in Sec. V to study solitonic configurations that localize the universe along a spatial direction.

II. THE EFFECTIVE ACTION

Our initial goal is to derive an effective action that perturbatively supports a full class of tachyonic solutions of bosonic string theory. The solutions of interest are linearized tachyonic deformations of the linear dilaton CFT, characterized by the insertion of purely exponential tachyon vertex operators. The linear dilaton background is taken to be that of supercritical bosonic string theory defined in $D > 26$ spacetime embedding dimensions, labeled by X^μ, $\mu \in 0, 1, \ldots, D-1$.

In obtaining a consistent effective action, we will allow nontrivial functions of the matter fields to appear multiplying the Einstein-Hilbert term. It will therefore be useful to introduce the following nomenclature when referring to different reference frames of the effective action:

(i) In the *sigma-model frame*, the metric $G^\sigma_{\mu\nu}$ is that which appears naturally in the $2D$ worldsheet CFT. In all of the solutions of interest, the sigma-model metric will be that of flat, D-dimensional Minkowski space. In this frame, the Einstein-Hilbert term does not appear canonically, though the dilaton dependence of the effective action appears as an overall factor of $\exp(-2\Phi)$.

(ii) The *string frame* will refer to the frame in which the Einstein-Hilbert term in the effective action appears with just a factor of $\exp(-2\Phi)$ [while the collective dilaton dependence of the action remains as an overall factor of $\exp(-2\Phi)$].

(iii) In the *Einstein frame*, the Einstein-Hilbert term appears canonically, and the prefactor $\exp(-2\Phi)$ is removed by Weyl transformation.

When possible ambiguity arises, the frame will be specified by the subscript or superscript labels σ, S, and E. At the classical level, none of these frames is preferred, in principle, over the others (since there is no equivalence principle). To analyze the cosmological aspects of the solutions at hand, however, it is easiest and most intuitive to work either in the string frame or Einstein frame.

When the sigma-model metric is flat ($G^\sigma_{\mu\nu} = \eta_{\mu\nu}$), the linear dilaton background alone comprises an exact solution with a vanishing B field, in which the worldsheet path integral is precisely Gaussian, and the (constant) dilaton gradient $v_\mu \equiv \partial_\mu \Phi$ satisfies

$$v \cdot v = -\frac{D-26}{6\alpha'}. \tag{2.1}$$

In other words, the worldsheet beta functions for this background vanish to all orders in α':

$$\beta^{G^\sigma} = \beta^\Phi = \beta^B = 0. \tag{2.2}$$

The $2D$ energy-momentum tensor, in worldsheet light-cone coordinates $\sigma^\pm = -\sigma^0 \pm \sigma^1$, is

$$\begin{aligned} \mathcal{T}_{++} &= -\frac{1}{\alpha'}{:}\partial_{\sigma^+}X^\mu \partial_{\sigma^+}X_\mu{:} + \partial^2_{\sigma^+}(v_\mu X^\mu), \\ \mathcal{T}_{--} &= -\frac{1}{\alpha'}{:}\partial_{\sigma^-}X^\mu \partial_{\sigma^-}X_\mu{:} + \partial^2_{\sigma^-}(v_\mu X^\mu). \end{aligned} \tag{2.3}$$

We now wish to introduce bulk tachyonic deformations of the linear dilaton CFT. To linear order in the field strength, the deformation is implemented by the insertion of a single tachyon vertex operator into all correlation functions. The perturbation is marginal as long as the matter part of the vertex operator is constrained to appear as a conformal primary of weight (1, 1). (In other words, the tachyon vertex operator can be made Weyl invariant, with the above energy-momentum tensor.) This is achieved at linear order by satisfying the on-shell condition

$$\partial^2 T(X) - 2v^\mu \partial_\mu T(X) + \frac{4}{\alpha'}T(X) = 0. \tag{2.4}$$

The strategy is to focus on the general class of solutions to this equation given by

$$T(X) = \mu \exp(\beta_\mu X^\mu), \qquad v_\mu = \text{const}, \qquad G^\sigma_{\mu\nu} = \eta_{\mu\nu}, \tag{2.5}$$

where μ is a free, real parameter (not to be confused with the spacetime index), and both v_μ and β_μ are constant, D-dimensional vectors. In general, the solutions in this class are neither exact in the α' expansion, nor in conformal perturbation theory. Even in the $\alpha' \to 0$ limit, we expect that the all-orders dynamics of the effective theory can become strongly corrected relative to the linearized approximation when the tachyon is of order 1.

A. Particular solutions

Two particular solutions in the class described above will play an important role in the analysis that follows. The *timelike tachyon* is defined by the profile

$$T(X^0) = \mu \exp(-\beta^0 X^0). \tag{2.6}$$

We will study this solution in the background of a timelike dilaton

$$\Phi = -v^0 X^0 \equiv -qX^0. \tag{2.7}$$

Since the timelike dilaton profile appears in several places, it is convenient to assign $v^0 \equiv q$. The dilaton component of the worldsheet beta function equations demands that

$$q = \pm\sqrt{\frac{D-26}{6\alpha'}}. \tag{2.8}$$

By choosing the branch of this equation that renders q positive definite in $D > 26$, the string coupling decreases toward the future. In turn, the tachyon marginality condition requires

$$\beta^0 = q \pm \sqrt{\frac{4}{\alpha'} + q^2}. \tag{2.9}$$

Arranging the tachyon to increase toward the future (i.e. requiring β^0 to be negative definite), and substituting the solution for q from above, one obtains

$$\beta^0 = \frac{\sqrt{D-26} - \sqrt{D-2}}{6\alpha'}. \tag{2.10}$$

The parameter μ amounts to a shift of X^0, so we can study the condensation of the timelike tachyon at arbitrarily weak string coupling by taking $\mu \ll 1$.

The second solution that will be examined below is the *null tachyon*, characterized by the profile

$$T(X^+) = \mu \exp(\beta_+ X^+) = \mu \exp\left(\frac{\beta_+}{\sqrt{2}}(X^0 + X^j)\right), \tag{2.11}$$

where X^j is an arbitrary spatial direction ($j \in 1, 2, \ldots, D-1$). In the presence of the timelike linear dilaton background, the tachyon equation of motion is satisfied when

$$\beta_+ = \frac{2\sqrt{2}}{q\alpha'}. \tag{2.12}$$

Once again we can choose the strength of the tachyon to increase into the future, in the direction of weak string coupling, which amounts to selecting the $q > 0$ branch of Eq. (2.8). In this case, however, the tachyon is constant along light fronts for which $X^+ = \text{const}$. We can think of this solution as an approximate description of a bubble of the tachyon condensate, which nucleates on the worldsheet and expands outward at the speed of light (see Fig. 1). Since the tachyon vertex operator couples as a potential in the worldsheet sigma model, the growth of the tachyon is manifested as the appearance of a diagonal Liouville wall. String states are prevented from passing deep into the potential wall, so at late times the theory exhibits an expanding region that is completely devoid of physical degrees of freedom.

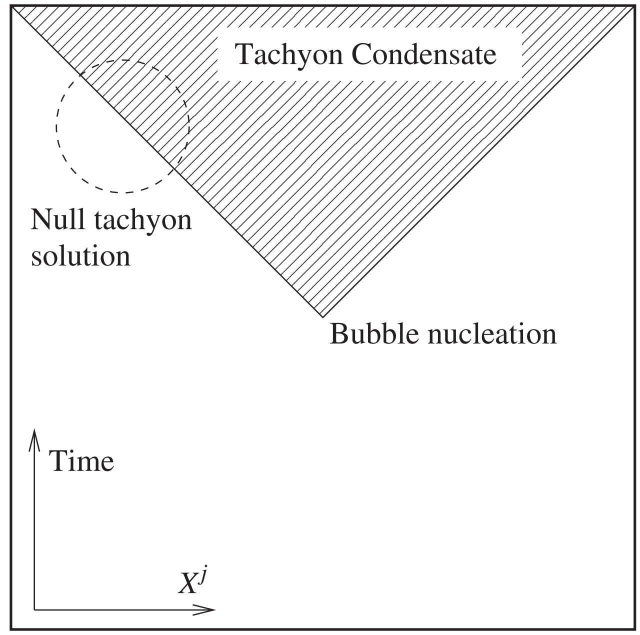

FIG. 1. Schematic diagram of the worldsheet bubble of nothing. The null tachyon solution can be thought of as focusing on a region to the left of the origin of the X^j coordinate. Physical degrees of freedom do not persist deep inside the tachyon condensate.

B. Higher-order corrections

Since the goal of this study is to analyze closed string tachyon condensation in regimes where the classical description is reliable, it is useful to briefly review the various sources of higher-order corrections that can arise in the effective theory. First, higher genus corrections become strong when the string coupling g_S is of order 1. The effective string coupling is defined by $g_S = \exp(\Phi)$, so the classical limit corresponds to a dilaton expectation value of $\Phi = -\infty$. All of the solutions of interest exhibit a tachyon profile that increases in the direction of decreasing string coupling, and the entire region of noninfinitesimal tachyon can be placed at arbitrarily small g_S. For the purposes of the present analysis, therefore, we will focus strictly on the weakly coupled regime.

Working to linearized order in conformal perturbation theory, the exact linear dilaton CFT is deformed by the insertion of a single tachyon vertex operator into all correlation functions. To this order, the conformal invariance of the tachyon deformation is encoded by the tachyon equation of motion (2.4). In general, the insertion of multiple vertex operators will lead to singularities, and conformal invariance at higher order will be restored by corrections to the linearized equation of motion. At the level of the effective theory, conformal perturbation theory corresponds to an expansion of the action in nonlinearities of fields.

Finally, the contribution of higher-dimension operators to the worldsheet beta equations translates in spacetime to the appearance of higher-dimension operators in the effective action. These operators are suppressed by corresponding powers of α', and the classical limit is reached by taking $\alpha' \to 0$, which is the limit of infinite string tension.

With respect to the above corrections, the null tachyon solution turns out to be remarkably simple at the level of the worldsheet CFT. Consider the $2D$ worldsheet theory with general tachyon profiles of the form $T(X) = \mu \exp(\beta_\mu X^\mu)$. For a general constant β^μ, the insertion of two tachyon vertex operators will lead to singularities when the positions of the operators become coincident. To be precise, singularities of normal-ordered operators arise in a free theory when propagators contract free fields in one operator with those of a second operator. In the case of the tachyon vertex operators considered here, operators depend only on $\beta_\mu X^\mu$, so all field contractions, and all higher-order corrections to the linearized tachyon equation of motion, will be proportional to $\beta_\mu \beta^\mu$. For the null tachyon, however, $\beta_\mu \beta^\mu$ vanishes identically.

Furthermore, when the null tachyon solution is expressed in light-cone coordinates, it is straightforward to demonstrate [16,17] that (1) the propagator for the fields $X^\pm$ is oriented, directed from X^+ to X^-, and (2) all interaction vertices in the theory depend only on X^+. As such, there are no Feynman diagrams beyond tree level, and hence no quantum loop corrections whatsoever.[2] Taken together, the above facts indicate that the linearized tree-level solution (2.11) is exactly conformally invariant. We will rely on this fact below to study the general spacetime dynamics of the null tachyon.

It turns out that, in large spacetime dimensions, the timelike tachyon solution exhibits properties similar to the null tachyon. From Eq. (2.10), one obtains

$$\beta_\mu \beta^\mu = -\frac{4}{\alpha'^2 D} + O(1/D^2) \qquad \text{(timelike tachyon)}, \tag{2.13}$$

which vanishes in the $D \to \infty$ limit. Furthermore, it is straightforward to show that worldsheet loops are also suppressed at large D (see, e.g., [15]). Near $D = \infty$ and $g_S \ll 1$, therefore, higher-order corrections to the timelike tachyon solution (2.7) are strongly suppressed.

C. String-frame effective action

The basic approach to constructing a spacetime effective action of the worldsheet CFT of string theory is to find an action whose equations of motion encode the condition that all Weyl anomalies in the $2D$ theory vanish. Here we will restrict the analysis to an action containing terms with at most two spacetime derivatives; higher-dimension terms are suppressed by higher powers of α'. Since our initial goal is to study the spacetime physics of the tachyonic solutions in Eq. (2.5), which are tree-level solutions of the worldsheet beta function equations, the hope is to capture the classical physical content of these solutions within the framework of a two-derivative effective action.

The simple class of tachyonic solutions described above (2.5) is not directly supported by the form of the spacetime effective action most often studied in the literature. The essential additional ingredient is that the Einstein-Hilbert term must appear with a nontrivial tachyon-dependent prefactor if the theory is to simultaneously support the solutions of interest and admit a nonvanishing tachyon potential. Without the latter, there would of course be no hope of reproducing tachyon scattering amplitudes, for example, at the level of the effective action.

Our starting point will therefore be the most general two-derivative effective action for the dilaton, metric, and tachyon [16]. We begin in the sigma-model frame with the following generic form:

$$S = \frac{1}{2\kappa^2}\int d^D x \sqrt{-\det G^\sigma}[\mathcal{F}_1 R^\sigma - \mathcal{F}_2 (\nabla\Phi)^2 - \mathcal{F}_3(\nabla T)^2 - \mathcal{F}_4 - \mathcal{F}_5 \nabla T \cdot \nabla \Phi]. \tag{2.14}$$

The coupling κ is related to the Newton constant by $G_N = \kappa^2/8\pi$. Tree-level string amplitudes are defined such that the dilaton dependence of the tree-level action appears as an overall factor of $\exp(-2\Phi)$, and we have absorbed this prefactor into each of the functions $\mathcal{F}_i$. Apart from this factor, the $\mathcal{F}_i$ are understood to be completely arbitrary functions of the tachyon. For convenience, and to align conventions with the literature, we encode the explicit tachyon dependence of these functions via the following definitions:

$$\begin{aligned} \mathcal{F}_1 &\equiv e^{-2\Phi} f_1(T), \qquad \mathcal{F}_2 \equiv -4 e^{-2\Phi} f_2(T), \\ \mathcal{F}_3 &\equiv e^{-2\Phi} f_3(T), \qquad \mathcal{F}_4 \equiv 2 e^{-2\Phi} V_\sigma(T), \\ \mathcal{F}_5 &\equiv e^{-2\Phi} f_5(T). \end{aligned} \tag{2.15}$$

The functions $f_i(T)$ on the right-hand side of Eq. (2.15) depend only on the tachyon, including the potential $V_\sigma(T)$.

In terms of these functions, the Einstein equation appears as

$$\begin{aligned} 0 = {}& (\nabla^\mu\nabla^\nu - G^{\mu\nu}_\sigma \nabla^2 + \tfrac{1}{2} G^{\mu\nu}_\sigma G^{\rho\sigma}_\sigma R_{\rho\sigma} - R^{\mu\nu})\mathcal{F}_1 \\ & - \tfrac{1}{2} G^{\mu\nu}_\sigma \mathcal{F}_4 + \mathcal{F}_2 \partial^\mu\Phi\partial^\nu\Phi - \tfrac{1}{2} G^{\mu\nu}_\sigma \mathcal{F}_2 (\partial\Phi)^2 \\ & + \mathcal{F}_3 \partial^\mu T \partial^\nu T - \tfrac{1}{2} G^{\mu\nu}_\sigma \mathcal{F}_3 (\partial T)^2 - \tfrac{1}{2}\mathcal{F}_5 G^{\mu\nu}_\sigma \partial_\rho T \partial^\rho \Phi \\ & + \tfrac{1}{2}\mathcal{F}_5 \partial^\mu T \partial^\nu \Phi + \tfrac{1}{2}\mathcal{F}_5 \partial^\nu T \partial^\mu \Phi, \end{aligned} \tag{2.16}$$

where ∇^μ is the usual covariant derivative. The dilaton and tachyon equations of motion read, respectively,

$$\begin{aligned} 0 = {}& -2Rf_1 + 8f_2(\partial\Phi)^2 - 8f_2'\partial T \cdot \partial\Phi - 8 f_2 \nabla^2\Phi \\ & + (2f_3 + f_5')(\partial T)^2 + f_5 \nabla^2 T + 4V_\sigma, \end{aligned} \tag{2.17}$$

[2]This is possible because the theory is not unitary prior to enforcing conformal gauge constraints.

$$0 = f_1' R + (4f_2' - 2f_5)(\partial\Phi)^2 + f_3'(\partial T)^2 - 4f_3\partial\Phi\cdot\partial T - 2V_\sigma' + f_5\nabla^2\Phi + 2f_3\nabla^2 T, \tag{2.18}$$

where $f_i'(T) \equiv \partial_T f_i(T)$.

We now require that the effective action in Eq. (2.14) support the class of tachyonic solutions of interest, given in Eq. (2.5) above. (See, e.g., [42] for a similar technique, employed in open string theory.) Projecting onto these solutions, the Einstein equation (2.16) stipulates the following constraint:

$$\begin{aligned}0 = {}& \eta_{\mu\nu}[2V_\sigma + 4(2f_1 - f_2)v\cdot v \\ &+ T[f_5 v\cdot\beta + 2f_1'(\beta\cdot\beta - 4v\cdot\beta) \\ &+ (2f_1'' + f_3)\beta\cdot\beta T]] + 8(f_2 - f_1)v_\mu v_\nu \\ &+ T[(v_\mu\beta_\nu + \beta_\mu v_\nu)(4f_1' - f_5) \\ &- 2\beta_\mu\beta_\nu(f_1' + (f_1'' + f_3)T)].\end{aligned} \tag{2.19}$$

The tachyon equation of motion becomes

$$0 = (2f_2' - f_5)v\cdot v - \partial_T V_\sigma - 2f_3 T v\cdot\beta + \beta\cdot\beta T(f_3 + \tfrac{1}{2}f_3' T), \tag{2.20}$$

while the dilaton equation of motion gives

$$0 = 8f_2 v\cdot v + 4V_\sigma - 8f_2' T v\cdot\beta + f_5 T\beta\cdot\beta + (2f_3 + f_5')T^2\beta\cdot\beta. \tag{2.21}$$

The above equations (2.19), (2.20), and (2.21) are uniquely satisfied in terms of the lone function $f_1(T)$ by the following solution:

$$\begin{aligned}f_2(T) &= f_1(T), \\ f_3(T) &= -f_1''(T) - \frac{f_1'(T)}{T}, \\ f_5(T) &= 4f_1'(T), \\ V_\sigma(T) &= -\frac{1}{2}(4f_1(T)v\cdot v + Tf_1'(T)(\beta\cdot\beta - 4v\cdot\beta) \\ &\quad + T^2 f_1''(T)\beta\cdot\beta).\end{aligned} \tag{2.22}$$

From the solution for $f_3(T)$, we see that for the tachyon kinetic term to be finite at $T = 0$, we must have that

$$f_1'(T)|_{T=0} = 0. \tag{2.23}$$

In addition, we also require the conformal invariance of the exponential tachyon perturbation to linearized order in conformal perturbation theory [i.e., that it satisfies the on-shell condition in Eq. (2.4) above]:

$$\beta\cdot\beta - 2v\cdot\beta + \frac{4}{\alpha'} = 0. \tag{2.24}$$

Imposing this, along with the condition that the dilaton component of the beta function equations vanish (2.1), we can eliminate v^μ from the potential (2.22). One recovers the following form:

$$V_\sigma(T) = \frac{1}{2}\Big[4f_1(T)\Big(\frac{D-26}{6\alpha'}\Big) + Tf_1'(T)\Big(\beta\cdot\beta + \frac{8}{\alpha'}\Big) - T^2 f_1''(T)\beta\cdot\beta\Big]. \tag{2.25}$$

At this stage, we could demand that the action be independent of the form of any particular solution. The condition that the potential be completely independent of β^μ translates to a condition on the function $f_1(T)$ of the form

$$\partial_{\beta^\mu} V(T) = \beta_\mu(Tf_1'(T) - T^2 f_1''(T)) \equiv 0. \tag{2.26}$$

So for nonzero tachyon and nonzero β_μ, we recover the following form for the function $f_1(T)$:

$$f_1(T) = c_0 + c_1 T^2, \tag{2.27}$$

where c_1 and c_2 are free constant parameters.

There is another route to this result. In general, we should impose that *any* solution to the equations of motion must preserve conformal invariance to leading order in perturbation theory. For a general background, this means that the system should satisfy

$$\nabla^2 T - 2\partial_\mu\Phi\partial^\mu T + \frac{4}{\alpha'}T = 0. \tag{2.28}$$

In other words, for solutions of the effective theory, the conditions of conformal invariance imposed by the $2D$ worldsheet theory should emerge as a prediction, rather than an input, of the effective action. In essence, we demand that the spacetime effective action correctly reproduce the known worldsheet tachyon scattering amplitudes to leading order in conformal perturbation theory and α'. To be sure, higher-order corrections can become important for completely general backgrounds. For the purposes of this calculation we can consider imposing (2.28) in the presence of small deviations from solutions that are known to exist in a perturbative regime.

For general T and Φ, the dilaton and tachyon equations of motion take the form [dropping the explicit T dependence of $f_1(T)$]

$$\begin{aligned}0 = {}& f_1'\nabla^2 T - 2f_1'\partial\Phi\cdot\partial T + \frac{f_1''}{2}\partial T\cdot\partial T \\ &- \frac{f_1}{2}(4\nabla^2\Phi - 4\partial\Phi\cdot\partial\Phi + R) - \frac{f_1'}{2T}\partial T\cdot\partial T \\ &+ V_\sigma(T), \\ 0 = {}& [f_1' - T(f_1'' + f_1'''T)]\partial T\cdot\partial T \\ &+ T[f_1' T(4\nabla^2\Phi - 4\partial\Phi\cdot\partial\Phi + R) \\ &+ 2(f_1' + f_1''T)(2\partial\Phi\cdot\partial T - \nabla^2 T) - 2T\partial_T V_\sigma(T)].\end{aligned} \tag{2.29}$$

Note that the Einstein equation simplifies significantly when the dilaton equation is enforced (specifically, terms proportional to $G^\sigma_{\mu\nu}$ drop out entirely):

$$2f_1\nabla_\mu\nabla_\nu\Phi - f_1'\nabla_\mu\nabla_\nu T + \frac{f_1'}{T}\partial_\mu T\partial_\nu T + f_1 R_{\mu\nu} = 0. \tag{2.30}$$

Using the trace of this equation, the dilaton equation of motion also simplifies,

$$4f_1\partial\Phi\cdot\partial\Phi - 4f_1'\partial\Phi\cdot\partial T + f_1 R + \left(f_1'' + \frac{f_1'}{T}\right)\partial T\cdot\partial T + 2V_\sigma = 0. \tag{2.31}$$

Employing the dilaton equation of motion (2.29) to eliminate $\nabla^2\Phi$, and imposing the condition (2.28), the tachyon equation of motion yields the following condition on $f_1(T)$:

$$T^2 f_1 f_1''' - (f_1 - Tf_1')(f_1' - Tf_1'') = 0. \tag{2.32}$$

At the level of the effective theory, conformal perturbation theory corresponds to an expansion in the strength of the tachyon field. Since the condition is that conformal invariance is imposed at leading order, we can solve this equation perturbatively in T:

$$f_1(T) = \sum_{n=0}^{\infty} c_n T^n. \tag{2.33}$$

The function $f_1(T)$ appears as a coefficient of the Einstein-Hilbert term in the effective action

$$S_{\rm EH} \sim \int d^D X e^{-2\Phi} f_1(T) R^\sigma, \tag{2.34}$$

so we require $c_0 = 1$ for the theory to properly reduce to the unperturbed linear dilaton background when the tachyon vanishes. Furthermore, the leading-order contribution from (2.32) establishes that $c_1 = 0$. It is then clear, working order by order, that Eq. (2.32) is satisfied perturbatively to all orders in T for any c_2, and

$$c_k = 0, \quad \forall\ k > 2. \tag{2.35}$$

Furthermore, for the tachyon potential to be tachyonic, the constant c_2 must be negative definite. It turns out that the magnitude of c_2 drops out of the entire action under a constant rescaling of the tachyon. Setting $c_2 = -1$, we recover the solution

$$f_1(T) = 1 - T^2. \tag{2.36}$$

We pause to emphasize an important aspect of this result. As noted above, the solutions under consideration (2.5) are, in general, not exact. For the tachyon perturbation to remain conformally invariant to higher orders in conformal perturbation theory, the linearized tachyon equation (2.28) will inevitably acquire nonlinear corrections. These corrections will ultimately alter the condition on the function $f_1(T)$ in Eq. (2.32). We should therefore not regard the solution in Eq. (2.36) as exact to all orders in the strength of the tachyon:

$$f_1(T) = 1 - T^2 + O(T^3). \tag{2.37}$$

The resulting tachyon potential in the sigma-model frame takes the following form:

$$V_\sigma(T) = \frac{1}{3\alpha'}((D-26) - (D-2)T^2) + O(T^3). \tag{2.38}$$

Happily, the potential is now completely independent of the vector β^μ to the order of interest, rendering the effective theory independent of any particular solution.

For the moment, we wish to study the leading-order dynamics of this theory in certain special cases, and we will momentarily drop all reference to higher-order corrections in the small-T expansion (though we will return to this issue in Sec. IV). At this stage, the spacetime action takes the form

$$S = \frac{1}{2\kappa^2}\int d^D X\sqrt{-\det G^\sigma}e^{-2\Phi} \times \bigg[(1-T^2)R^\sigma + 4(1-T^2)\partial_\mu\Phi\partial^\mu\Phi + 8T\partial_\mu T\partial^\mu\Phi - 4\partial_\mu T\partial^\mu T + \frac{2}{3\alpha'}(26 - D + (D-2)T^2)\bigg]. \tag{2.39}$$

To canonicalize the gravity sector, we can invoke a spacetime Weyl transformation

$$G^S_{\mu\nu} = e^{2\omega(T)}G^\sigma_{\mu\nu}, \tag{2.40}$$

where $G^S_{\mu\nu}$ is the string-frame metric, and

$$\omega(T) = \frac{\log(1-T^2)}{D-2}. \tag{2.41}$$

In the next section, however, we will move completely over to the Einstein frame. We will therefore combine the above field redefinition with an additional Weyl transformation that renders the Einstein-frame action in canonical form.

III. SPACETIME COSMOLOGY

In this section we focus on the dynamics of the timelike and null tachyon solutions, as encoded by the spacetime effective action. As noted above, we expect the semiclassical analysis to be reliable when the strength of the tachyon is small compared to 1. The goal is to study the general behavior of these solutions in regions of classical validity, but we also wish to establish that, in the absence of higher-order corrections, singularities eventually arise as a consequence of tachyon condensation.

A. Translation to the Einstein frame

Because the Einstein-Hilbert term in the sigma-model action appears with the prefactor $f_1(T)$, the Weyl rescaling that renders this term canonical in the Einstein frame depends on both the dilaton and the tachyon:

$$G^E_{\mu\nu} \equiv e^{2\omega(\Phi,T)} G^\sigma_{\mu\nu}. \tag{3.1}$$

Under this field redefinition, the Ricci term in the Einstein frame takes the form

$$S_{\rm EH} \sim \int d^D X \sqrt{-\det G^E} f_1(T) \times \exp((2-D)\omega(\Phi,T) - 2\Phi) R^E. \tag{3.2}$$

We therefore find the following expression for $\omega(\Phi, T)$:

$$\omega(\Phi, T) = \frac{-2\Phi + \log(1-T^2)}{D-2}. \tag{3.3}$$

At this stage, it is easy to see that as the tachyon magnitude evolves from some small initial value at early times to $T = \pm 1$, the Einstein metric inevitably encounters a big crunch in finite time. However, if the tachyon evolves slowly from zero, the metric can reach this singularity deep within a region where the string theory is weakly coupled. When T^2 increases beyond unity, the Einstein term in the sigma-model frame changes sign, and the conformal transformation (3.1) becomes imaginary. This can be interpreted as a signal that the gravitational theory becomes nonunitary for $T^2 > 1$.

It is interesting that the theory encounters a singularity at the point where we expect the dynamics to acquire strong corrections relative to the linearized approximation (2.5). Returning to this issue in Sec. IV, we will consider the ability of higher-order corrections to resolve the singularity. If such corrections contribute to higher-order terms in $f_1(T)$ (2.37), the tachyon dependence in the metric will obviously change. For now, however, we aim to study the action as it stands, leaving open the question of higher-order corrections. Our goals in this section are (1) to establish that this singular region in fact arises in the classical analysis, and (2) to test whether the effective action reliably reproduces the classical solutions of interest (2.5) away from singular points.

It is straightforward to carry out the transformation in Eq. (3.1) on the remaining terms in the action. To canonicalize the dilaton kinetic term, we perform an additional rescaling:

$$\Phi = \tfrac{1}{2}\sqrt{D-2}\,\phi. \tag{3.4}$$

We recover the following two-derivative effective action in the Einstein frame:

$$\begin{aligned} S = \frac{1}{2\kappa^2} \int d^D X \sqrt{-\det G^E} \times \Bigg[& R^E - \partial_\mu \phi \partial^\mu \phi - \frac{4}{\sqrt{D-2}} \frac{T}{(1-T^2)} \partial_\mu \phi \partial^\mu T \\ & - \frac{4(D-2+T^2)}{(D-2)(1-T^2)^2} \partial_\mu T \partial^\mu T \\ & - \frac{2}{3\alpha'} (1-T^2)^{-(D/(D-2))} (D - 26 - (D-2)T^2) \\ & \times e^{2\phi/\sqrt{D-2}} \Bigg]. \end{aligned} \tag{3.5}$$

One can see that the singularity in the metric at $T = \pm 1$ is also translated to the kinetic terms.

We have thus found a specific form of the dilaton-tachyon potential $V_E(\Phi, T)$ in the Einstein frame:

$$V_E(\Phi, T) = \frac{1}{3\alpha'} (1-T^2)^{-(D/(D-2))} (D - 26 - (D-2)T^2) \times e^{2\phi/\sqrt{D-2}}. \tag{3.6}$$

The potential is depicted for $D = 30$ in Fig. 2. From this picture it is easy to understand the generic behavior of the system evolving from a state with zero tachyon. At the outset, the theory evolves toward weak coupling as the dilaton rolls down its potential toward decreasing negative values (to the right in Fig. 2). Small fluctuations of the tachyon eventually cause it to roll toward positive or negative magnitude, reaching $T = \pm 1$ asymptotically.

At zero tachyon, the potential increases exponentially in the direction of increasing positive ϕ (to the left in Fig. 2, which is the direction of strong string coupling). There is a critical magnitude of the tachyon,

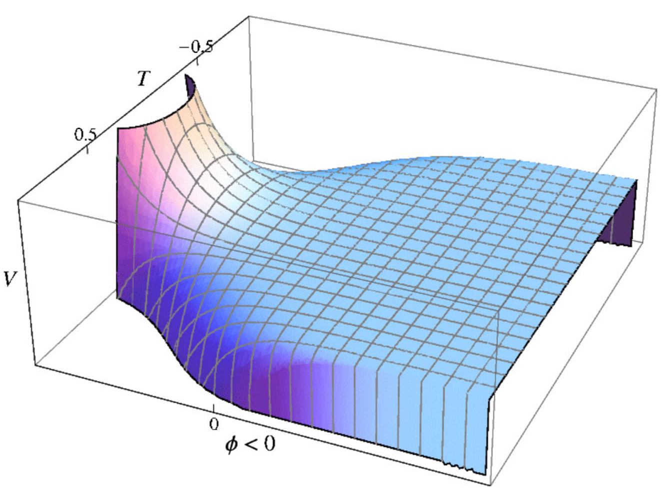

FIG. 2 (color online). The dilaton-tachyon potential $V_E(\phi, T)$ in the Einstein frame (depicted at $D = 30$, $\alpha' = 1$). When the tachyon passes the critical value $\pm T^*$, the potential decreases as a function of ϕ in the direction of strong coupling.

$$T^* = \sqrt{1 + \frac{24}{2 - D}}, \quad (3.7)$$

at which the potential vanishes identically. When the tachyon is above this magnitude, the potential is negative and decreases exponentially as a function of ϕ in the direction of strong coupling. At weak coupling the potential is essentially flat, falling off steeply at $T = \pm 1$.

B. Timelike tachyon

We now want to focus, in particular, on the timelike tachyon:

$$T = \mu \exp(-\beta^0 t_\sigma), \qquad \Phi = \tfrac{1}{2}\sqrt{D-2}\phi = -q t_\sigma, \qquad G^\sigma_{\mu\nu} = \eta_{\mu\nu}. \quad (3.8)$$

We have labeled the sigma-model time coordinate as $X^0_\sigma \equiv t_\sigma$. Again, by choosing the dilaton gradient q to be positive definite in $D > 26$, the string coupling decreases toward the future.

To study the cosmology associated with these solutions directly, we will proceed by moving to a coordinate system in which the Einstein metric is of FRW form:

$$ds^2_E = -dt^2_E + a(t_E)^2 d\vec{X}^2_E. \quad (3.9)$$

We wish to study the timelike tachyon on shell, and the conformal rescaling in Eq. (3.1) provides a precise translation between FRW coordinates X^μ_E and sigma-model coordinates X^μ_σ:

$$dt^2_E = e^{2\omega(\Phi,T)} dt^2_\sigma, \qquad a(t_E)^2 d\vec{X}^2_E = e^{2\omega(\Phi,T)} d\vec{X}^2_\sigma. \quad (3.10)$$

(For simplicity, we are keeping the time dependence of the dilaton and tachyon implicit.)

We can keep the translation among spatial coordinates trivial (i.e., $X^i_E = X^i_\sigma$, $i \in 1, 2, \ldots, D-1$) by assigning

$$a(t_E) = \exp(\omega(\Phi, T)) = \exp\left(\frac{-2\Phi + \log(1 - T^2)}{D-2}\right). \quad (3.11)$$

This leaves the translation between timelike coordinates explicit. When the tachyon is zero, we recover

$$t_E(t_\sigma) = \frac{D-2}{2q} \exp\left(\frac{2q}{D-2} t_\sigma\right) + \text{const} \qquad (T = 0). \quad (3.12)$$

After the tachyon acquires a vacuum expectation value, the translation for general dilaton and tachyon profiles takes the form

$$t_E(t_\sigma) = \int_1^{t_\sigma} d\xi e^{-2\Phi(\xi)/(D-2)} (1 - T(\xi)^2)^{1/(D-2)} + \text{const}. \quad (3.13)$$

For the solution at hand (3.8), we obtain

$$t_E(t_\sigma) = \frac{D-2}{2(q-\beta^0)} \exp\left(\frac{2q t_\sigma}{D-2}\right)(1 - \Lambda(t_\sigma))^{1/(2-D)}(1 - \Lambda(t_\sigma)^{-1})^{1/(D-2)} {}_2F_1\left(\frac{1}{2-D}, \frac{q-\beta^0}{\beta^0(D-2)}, \frac{q + \beta^0(D-3)}{\beta^0(D-2)}, \Lambda(t_\sigma)\right) + \text{const}, \quad (3.14)$$

where $\Lambda(t_\sigma)$ is defined by

$$\Lambda(t_\sigma) \equiv \frac{1}{\mu^2} e^{2\beta^0 t_\sigma}. \quad (3.15)$$

${}_2F_1(a, b, c, z)$ is the hypergeometric function, which exhibits a branch cut in the complex z plane along the real z axis from $z = 1$ to ∞.

It turns out that we can reexpress the solution $t_E(t_\sigma)$ as

$$t_E(t_\sigma) = \mu^{2q/(\beta^0(D-2))} \frac{e^{-(i\pi/(D-2))}}{2\beta^0} B_{\Lambda(t_\sigma)}\left(\frac{q-\beta^0}{\beta^0(D-2)}, \frac{D-1}{D-2}\right) + \text{const}, \quad (3.16)$$

where $B_z(a, b)$ is the incomplete Euler beta function $B_z(a, b) = \int_0^z d\xi (1-\xi)^{b-1} \xi^{a-1}$. The beta function $B_z(a, b)$ also exhibits a branch cut, though in this case it runs along the negative real z axis from $z = -\infty$ to $z = 0$. As it stands, $t_E(t_\sigma)$ (modulo the integration constant) contributes a constant imaginary piece when t_σ lies in the region prior to the final singularity. In the analysis that follows it will be understood that this contribution is subtracted by absorbing it into the integration constant.

The timelike tachyon solution can thus be expressed as a function of the time coordinate in the Einstein frame by inverting Eq. (3.16) to generate t_σ as a function of t_E. There is not a convenient closed-form expression for $t_\sigma(t_E)$, however. Keeping this relationship implicit, one obtains

$$T(t_E) = \mu e^{-\beta^0 t_\sigma(t_E)}. \quad (3.17)$$

Likewise, the dilaton evolves according to

$$\Phi(t_E) = \frac{1}{2}\sqrt{D-2}\phi = -\sqrt{\frac{D-26}{6\alpha'}}t_\sigma(t_E). \tag{3.18}$$

Substituting into the general form for the scale factor in Eq. (3.11), we find

$$a(t_E) = \exp\left(\frac{2qt_\sigma(t_E) + \log(1-\mu^2 e^{-2\beta^0 t_\sigma(t_E)})}{D-2}\right). \tag{3.19}$$

By construction, this is an exact (albeit particular) classical solution to the equations of motion of the spacetime effective action in the Einstein frame.

It is straightforward to plot the behavior of the scale factor numerically. This is done in Fig. 3, along with the corresponding evolution of the string coupling. (Here, and in the analysis that follows, α' can be set to any convenient value without affecting the qualitative behavior of the solutions.) When the tachyon is small, the scale factor evolves linearly as a function of t_E. As the tachyon evolves away from zero, the universe enters a phase of accelerated contraction. Eventually, as the tachyon strength approaches 1, the scale factor collapses to zero. The region of a non-negligible tachyon condensate can exist entirely within a region of weak string coupling $g_S \approx 0$.

In sigma-model coordinates, the tachyon magnitude reaches 1 at

$$t_\sigma^{\text{crunch}} = \frac{1}{\beta^0}\log\mu = \frac{6\alpha' \log\mu}{\sqrt{D-26} - \sqrt{D-2}}. \tag{3.20}$$

In the Einstein frame, this translates to the statement that the scale factor formally collapses to a singularity at the time

$$t_E^{\text{crunch}} = \frac{\mu^{2q/(\beta^0(D-2))}}{2\beta^0}\cos\left(\frac{\pi}{D-2}\right)B\left(\frac{q-\beta^0}{\beta^0(D-2)}, \frac{D-1}{D-2}\right), \tag{3.21}$$

where the expression on the right-hand side is given in terms of the complete Euler beta function

$$B(a, b) = \Gamma(a)\Gamma(b)/\Gamma(a+b). \tag{3.22}$$

For fixed spacetime dimensions, we have a one-parameter family of solutions of the timelike tachyon system, parametrized by the constant μ. In Fig. 4 we plot four such solutions for $\mu = \{0.38, 0.42, 0.46, 0.5\}$ in a fixed dimension. One can see explicitly that by decreasing μ, the tachyon reaches $T \approx 1$ more gradually, placing the final singularity of the scale factor farther out in the direction of decreasing string coupling.

The relationship between μ and t_E^{crunch} is depicted for various D in Fig. 5. From this plot we can also see that in higher spacetime dimensions, the crunch is placed at weaker coupling for fixed μ. To this is added the additional effect that, for a fixed time t_E, increasing the dimension D alone reduces the string coupling (at large D, the dilaton gradient scales as $q \approx -\sqrt{D}$).

It is straightforward to compute the spacetime equations of motion in the Einstein frame. If, for example, we are interested in studying the behavior of the timelike tachyon in the background of a timelike linear dilaton, we can

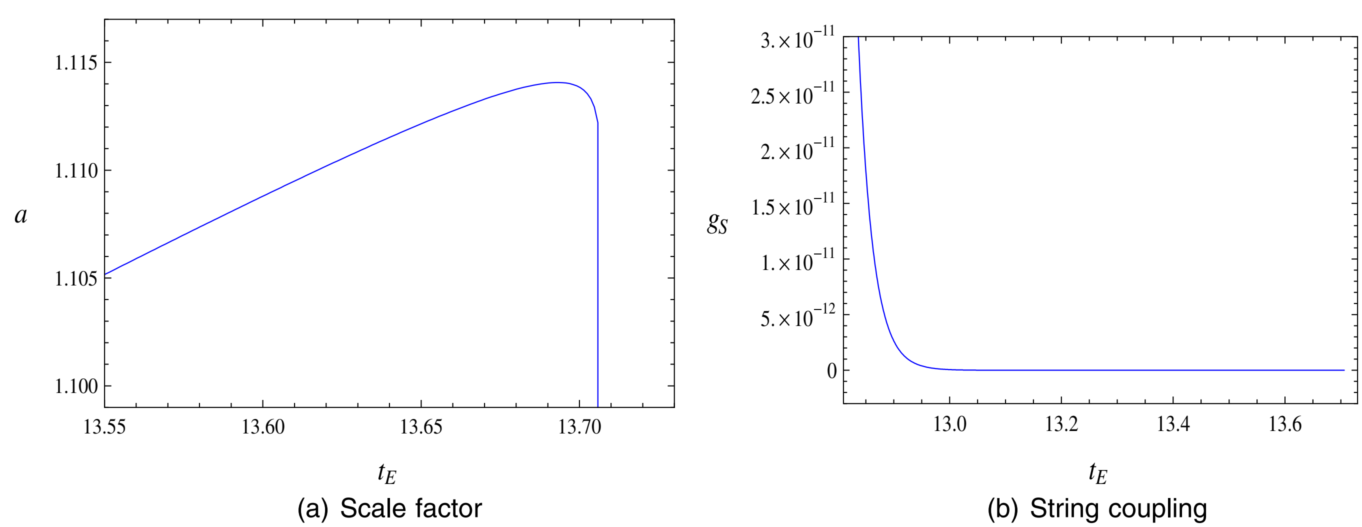

FIG. 3 (color online). The evolution of the timelike tachyon ($\mu = 1/2$, $D = 1000$). The scale factor [panel (a)] evolves linearly as a function of time in the Einstein frame while the tachyon is small (i.e., it evolves at a critical equation of state). As the magnitude of the tachyon increases, the scale factor enters a phase of accelerated contraction, and eventually reaches a singularity. The string coupling [panel (b)] decreases throughout this process.

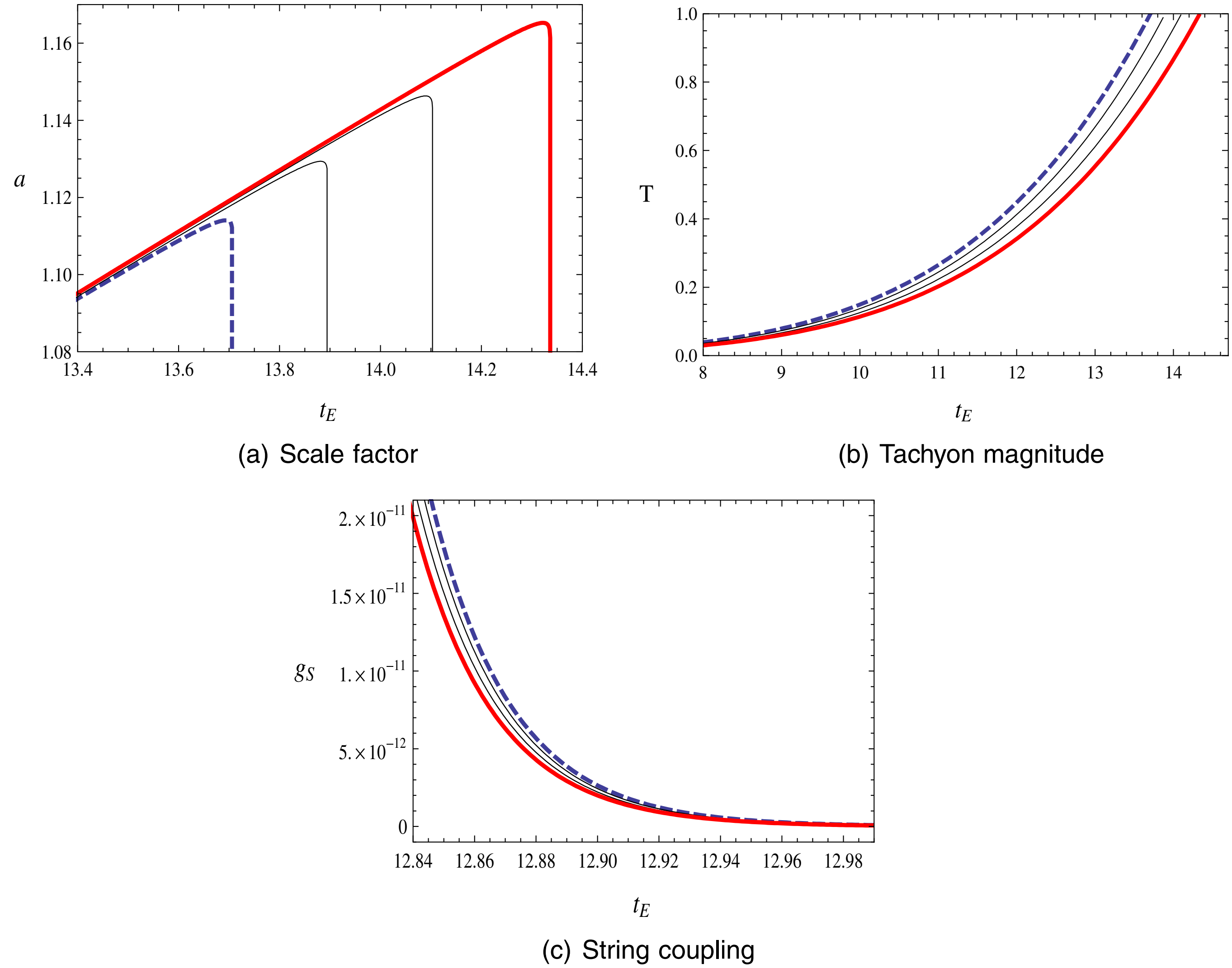

FIG. 4 (color online). The timelike tachyon system from $\mu = 0.38$ (red-solid line) to $\mu = 0.5$ (blue-dashed line), with $D = 1000$. The intermediate solid curves are generated at $\mu = 0.42$ and $\mu = 0.46$. The effect of decreasing μ is to place the final curvature singularity deeper in the direction of decreasing string coupling. For each value of μ the scale factor [panel (a)] collapses in a big crunch precisely when the tachyon [panel (b)] reaches 1. This process can occur throughout a region in which the string coupling [panel (c)] is small.

proceed with the ansatz that both the tachyon and dilaton fields depend only on the Einstein-frame time coordinate t_E. Furthermore, we can work in the coordinate system given in Eq. (3.9), in which the metric $G^E_{\mu\nu}$ takes the form of a spatially flat ($k = 0$) FRW metric:

$$ds_E^2 = -dt_E^2 + a(t_E)^2 d\vec{X}^2. \tag{3.23}$$

With these assumptions, variation of the dilaton in the action yields

$$0 = \ddot{\phi} + (D-1)H\dot{\phi} + \partial_\phi V(\phi, T) + \frac{2}{\sqrt{D-2}} \frac{1}{(1-T^2)} \times \left[(D-1)HT\dot{T} + T\ddot{T} + \frac{1+T^2}{1-T^2}\dot{T}^2\right], \tag{3.24}$$

where H is the Hubble parameter $H \equiv \dot{a}/a$. Varying the tachyon gives

$$0 = \frac{1}{2}\sqrt{D-2}(\ddot{\phi} + (D-1)H\dot{\phi})T + \frac{D-2+T^2}{(1-T^2)}(\ddot{T} + (D-1)H\dot{T}) + \frac{2D-3+T^2}{(1-T^2)^2}T\dot{T}^2 + \frac{1}{4}(D-2)(1-T^2)\partial_T V(\phi, T). \tag{3.25}$$

The pressure p and energy density ρ of the background take the form

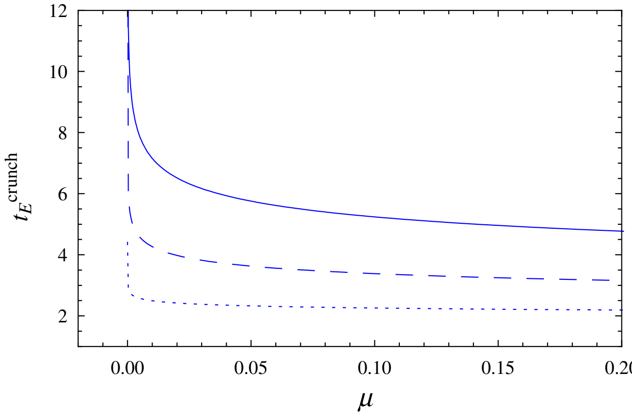

FIG. 5 (color online). The time at which the universe reaches a crunch in the Einstein frame is prolonged by decreasing μ, or by increasing D. The curves above are depicted with $D = 30$ (dotted curve), $D = 50$ (dashed curve), and $D = 100$ (solid curve).

$$p = \frac{1}{2\kappa^2}\left[\dot{\phi}^2 + \frac{4}{\sqrt{D-2}}\frac{T}{(1-T^2)}\dot{\phi}\dot{T} + \frac{4(D-2+T^2)}{(D-2)(1-T^2)^2}\dot{T}^2 - 2V(\phi, T)\right],$$
$$\rho = p + \frac{2}{\kappa^2}V(\phi, T). \quad (3.26)$$

The Einstein equations then reduce to

$$\frac{\ddot{a}}{a} = -\kappa^2\rho\frac{(D-3)+(D-1)w}{(D-2)(D-1)}, \quad (3.27)$$

where w is the usual equation of state $w \equiv p/\rho$. We note that the critical equation of state, defining the boundary between an accelerating and a decelerating cosmology, is [16]

$$w_{\text{crit}} = -\frac{D-3}{D-1}. \quad (3.28)$$

In other words, the scale factor accelerates as a function of t_E if the equation of state lies in the range $-1 \leq w < w_{\text{crit}}$.

At this point we can integrate Eqs. (3.24), (3.25), and (3.27) numerically, given a set of initial conditions. As noted above, solutions in the worldsheet theory fall into a family parametrized by μ (for fixed D and α'). While this parameter is absent from the point of view of the effective theory alone, the information contained in μ can be translated to the effective dynamics in the form of integration constants.

For the particular solutions under consideration, it is instructive to plot the equation of state as the system evolves toward the final singularity. This is displayed in Fig. 6. It is easy to see that as the tachyon remains small, each system evolves at the critical equation of state (3.28) (marked by the green horizontal line). As the tachyon evolves, each system acquires an equation of state lying above the critical value, indicating a phase of decelerating scale factor.

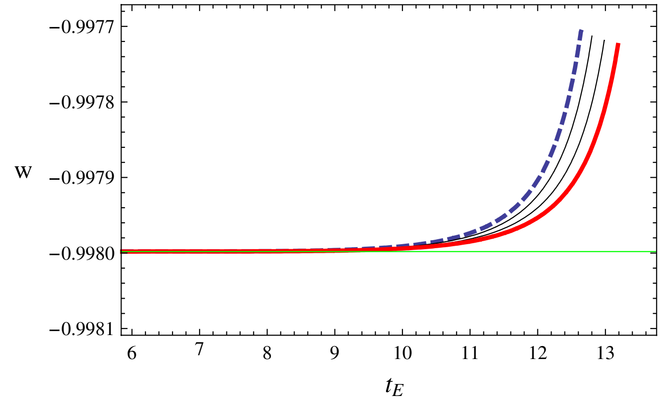

FIG. 6 (color online). The evolution of the equation of state for the timelike tachyon solutions under consideration (at $D = 1000$). The individual curves correspond to values of μ from $\mu = 0.38$ (red-solid line) to $\mu = 0.5$ (blue-dashed line). The intermediate solid curves are generated at $\mu = 0.42$ and $\mu = 0.46$. The critical equation of state, given by $w_{\text{crit}} = -(D-3)/(D-1)$, is depicted by the horizontal green line.

C. The null tachyon

We now turn to the null tachyon solution in the same setting. Our goal for the moment is to characterize the singular region exhibited by the null tachyon that arises in the absence of higher-order corrections to the action. (We will consider higher-order effects in the next section.) In sigma-model coordinates, the null tachyon solution is specified by

$$T(t_\sigma, X_\sigma^1) = \mu\exp\left(\frac{\beta_+}{\sqrt{2}}(t_\sigma + X_\sigma^1)\right) \equiv \mu\exp(\beta X^+),$$
$$\Phi(t_\sigma) = -qt_\sigma, \qquad G_{\mu\nu}^S = \eta_{\mu\nu}, \quad (3.29)$$

where X_σ^1 is a transverse embedding coordinate in the sigma-model frame. To keep the notation concise, we have relabeled the constant light-cone vector as $\beta_+ \equiv \beta$. Consistency of the string theory requires

$$q = \pm\sqrt{\frac{D-26}{6\alpha'}}, \qquad q\beta = \frac{2\sqrt{2}}{\alpha'}. \quad (3.30)$$

Again, choosing the positive branch of the dilaton gradient yields a system in which the dilaton rolls to weak coupling in the future while the tachyon grows exponentially at fixed X_σ^1.

In the Einstein frame, we will adopt a coordinate system in which the metric is again of FRW form:

$$ds_E^2 = -dt_E^2 + a(t_E, X_E^1)^2 dX_E^i dX_E^i;$$
$$i \in 1, 2, \ldots, D-1. \quad (3.31)$$

The scale factor $a(t_E, X_E^1)$ now depends both on the timelike direction t_E and the single spatial coordinate X^1_E. As before, we can choose the mapping between Einstein-frame and sigma-model coordinates to be trivial for the spatial directions X^i:

$$X^i_E = X^i_\sigma \equiv X^i, \qquad i \in 1, 2, \ldots, D-1. \tag{3.32}$$

These relations imply the following dependence of the scale factor on the dilaton and tachyon:

$$a(t_E, X^1) = \exp\left(\frac{-2\Phi + \log(1 - T^2)}{D-2}\right), \tag{3.33}$$

where T is now a function of both t_σ and X^1 (3.29). The relation

$$dt_E^2 = e^{2\omega(\Phi,T)} dt_\sigma^2 \tag{3.34}$$

gives the following mapping for the Einstein coordinate t_E:

$$t_E(t_\sigma, X^1) = -\frac{\mu^{-(2\sqrt{2}q/(\beta(D-2)))}}{\sqrt{2}\beta} \exp\left(\frac{i\pi + 2qX^1}{2-D}\right) B_{\Lambda(t_\sigma, X^1)} \times \left(\frac{q\sqrt{2}+\beta}{\beta(2-D)}, \frac{D-1}{D-2}\right) + \text{const}, \tag{3.35}$$

where

$$\Lambda(t_\sigma, X^1) \equiv \frac{1}{\mu^2} \exp(-\sqrt{2}\beta(t_\sigma + X^1)). \tag{3.36}$$

As with the timelike tachyon, we need to absorb a constant imaginary contribution into the integration constant on the right-hand side of Eq. (3.35).

At this point we can see that the theory reaches a curvature singularity at

$$t_\sigma^{\text{crunch}} = -X^1 - \frac{\sqrt{2}}{\beta} \log\mu. \tag{3.37}$$

In FRW coordinates, this equates to

$$t_E^{\text{crunch}} = -\frac{\mu^{-(2\sqrt{2}q/(\beta(D-2)))}}{\sqrt{2}\beta} \exp\left(\frac{2qX^1}{2-D}\right) \times \cos\left(\frac{\pi}{2-D}\right) B\left(\frac{q\sqrt{2}+\beta}{\beta(2-D)}, \frac{D-1}{D-2}\right). \tag{3.38}$$

To an observer at fixed negative X^1, the singular region appears to approach from the positive X^1 direction at a speed given by

$$v_{\text{bubble}} = (D-2)\sqrt{\frac{3\alpha'}{2(D-26)}}\frac{1}{t_E}. \tag{3.39}$$

This picture is intuitively consistent with what we know from the 2D CFT. On the worldsheet, the growth of the tachyon appears as a potential wall that increases exponentially with X^+ (in sigma-model coordinates), preventing the presence of string states deep inside the region of the tachyon condensate. To a rough approximation, we would expect spacetime to become nondynamical in a region that

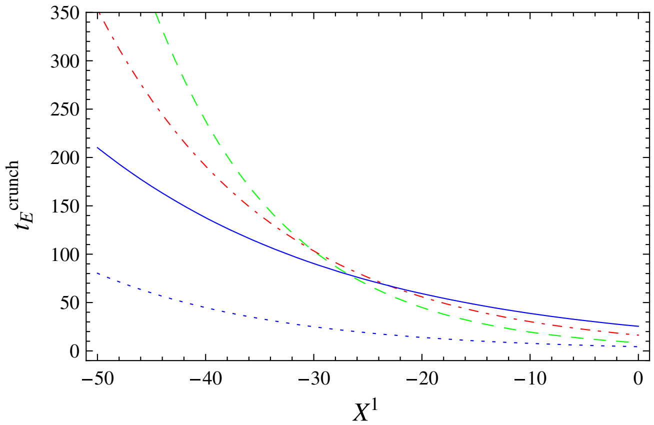

FIG. 7 (color online). The time at which the universe reaches the final crunch in the presence of the null tachyon, as a function of X^1. The curves shown above are depicted for $\mu = 0.5$, $D = 30$ (dotted blue curve), $D = 50$ (dashed green curve), $D = 150$ (dot-dashed red curve), and $D = 350$ (solid blue curve).

grows outward from the origin of the X^1 coordinate. The nontrivial translation between sigma-model and Einstein-frame coordinates modifies the picture somewhat, but the general outcome is as expected. In Fig. 7 we plot t_E^{crunch} as a function of X^1 for various D. In spacetime, the bubble of nothing indeed emerges as a surface of zero metric that expands outward in the direction of negative X^1.

IV. RESOLVED SINGULARITIES

The action under consideration thus far was derived under the constraint that it support the complete class of tachyonic solutions defined in Eq. (2.5). Terms in the effective action suppressed by higher powers of α' have been systematically dropped, and the form of $f_1(T)$ was derived at linearized order (in conformal perturbation theory). In this section, we wish to consider the possible effects of higher-order corrections.

A. The null tachyon

By focusing strictly on the null tachyon solution in the background of a timelike linear dilaton rolling to weak coupling (3.29), we can study various higher-order corrections in isolation. As noted above, the tree-level worldsheet solution is well defined and exactly conformally invariant to all orders in perturbation theory, and nonperturbatively in α'. Furthermore, the effects of finite string coupling can be made arbitrarily small by placing the strongly coupled region deep in the past.

Since the null tachyon is exact in α', including higher-dimension operators in the effective action consistently cannot lead to corrections to the spacetime equations of motion that are not automatically satisfied by the tree-level solution. Similarly, while corrections associated with conformal perturbation theory will inevitably alter the tachyon marginality condition (2.4), such corrections should be satisfied trivially by the null tachyon solution. However, the constraint equation (2.32) for $f_1(T)$ was derived for the nonexact tachyon profiles (plus fluctuations) appearing in Eq. (2.5).[3] Corrections to the marginality condition will ultimately contribute nontrivial corrections to Eq. (2.32). The conclusion is that the only higher-order effects that can appear directly in the spacetime null tachyon system arise as corrections to the function $f_1(T)$.[4] This raises the possibility that the curvature singularity that naively appears in the above analysis is resolved by higher-order effects.

B. Generalized constraints on $f_1(T)$

Without directly computing higher-order corrections to the effective action (a problem that lies beyond the scope of this study), we would like to understand on general grounds the terms that can arise as corrections to $f_1(T)$, subject to the condition that the theory always supports the null tachyon as an exact solution.

Consider the effective action for general $f_1(T)$. As it stands, this action is consistent with the condition that it supports only the null tachyon as an exact solution. Because of the null symmetry, the β dependence in the potential [see, e.g., Eq. (2.25)] is automatically absent:

$$S = \frac{1}{2\kappa^2}\int d^D X\sqrt{-\det G^S}e^{-2\Phi} \times\bigg[R^S - \frac{4}{D-2}\bigg(\partial_\mu\Phi\partial^\mu\Phi + \frac{f_1'}{f_1}\partial_\mu\Phi\partial^\mu T\bigg) + \frac{1}{f_1}\bigg[f_1'' + f_1'\bigg(\frac{1}{T} - \frac{(D-1)}{(D-2)}\frac{f_1'}{f_1}\bigg)\bigg]\partial_\mu T\partial^\mu T - \frac{1}{3\alpha'}f_1^{-(D/(D-2))}(24Tf_1' + 2(D-26)f_1)\bigg]. \tag{4.1}$$

Here the action appears in the string frame, with the Einstein-Hilbert term expressed in canonical form. For general $f_1(T)$, the string-frame metric is related to the sigma-model metric by

$$G^S_{\mu\nu} = e^{2\omega(T)}G^\sigma_{\mu\nu}, \qquad \omega(T) = \frac{\log f_1(T)}{D-2}. \tag{4.2}$$

Of course, we can also move to the Einstein frame using the Weyl transformation

$$G^E_{\mu\nu} = e^{2\omega(\Phi,T)}G^\sigma_{\mu\nu}, \qquad \omega(\Phi,T) = \frac{-2\Phi + \log f_1(T)}{D-2}. \tag{4.3}$$

For the finite dilaton, singularities of the metric arise whenever $f_1(T)$ vanishes.[5]

We now want to study constraints on higher-order corrections to $f_1(T)$ under the following general conditions:

(1) The theory encodes the standard, nontachyonic linear dilaton background in the limit $T = 0$.
(2) The gravity sector remains unitary for all values of T.
(3) Nonmetric prefactors of the matter sector kinetic terms remain finite for all values of T.
(4) Expanding in the strength of the tachyon, $f_1(T)$ is defined to quadratic order by $f_1(T) \approx 1 - T^2$.

The last condition guarantees that the potential in a canonical gravity frame is tachyonic. If there is any hope that the effective action can reliably reproduce the physics of the null tachyon for all finite values of T, each of these conditions must be met. For the moment, we leave open the possibility that the metric encounters a singularity as a consequence of tachyon condensation.

As noted in Sec. II, the first condition imposes

$$f_1(0) = 1. \tag{4.4}$$

Furthermore, the condition that the gravity sector remain unitary can be satisfied by demanding that $f_1(T)$ be nonnegative for all T. This is intuitive from the perspective of the action in the sigma-model frame, where the Ricci term appears multiplied by $f_1(T)$. In addition, we see that for the tachyon kinetic term to remain finite at $T = 0$,

$$f_1'(T)|_{T=0} = 0. \tag{4.5}$$

This indicates that $T = 0$ is a critical point of $f_1(T)$.

The tachyon kinetic term (as well as the $\partial_\mu T\partial^\mu\Phi$ term) can also become singular at points where $f_1(T)$ vanishes. The relevant factors are $f_1''(T)/f_1(T)$ and $f_1'(T)/f_1(T)$, so we demand that $f_1'(T)$ and $f_1''(T)$ vanish whenever $f_1(T) = 0$. This implies that points where $f_1(T)$ vanishes must either be inflection points or points where $f_1(T)$ vanishes identically over a finite region.[6] Since $f_1(T)$ is everywhere nonnegative, however, there can be no inflection points coinciding with points where $f_1(T)$ vanishes. Furthermore, if $f_1(T)$ vanishes identically over a finite region, the *entire* action becomes identically zero in this region. If we hope to reliably encode the dynamics of the complete string theory for all values of the tachyon, we are forced to reject this scenario. We conclude that, under the second and third constraints above, $f_1(T)$ can never vanish at finite values of T. At finite T, therefore, the null tachyon avoids *all* cosmological singularities.

C. A candidate effective action

We would now like to understand the relation between the above constraints on $f_1(T)$ and the perturbative form

[3]Recall that the null tachyon is supported as a classical solution for *any* $f_1(T)$.

[4]To be sure, analogous statements do not hold for nonexact solutions.

[5]In the sigma-model frame, this corresponds to a vanishing Einstein term in the action.

[6]An example of a C^∞ function that is nontrivial in some region but vanishing in another is $f_1(T) = \exp(-1/(a^2 - x^2))\Theta(a^2 - x^2)$, where $\Theta(x)$ is the step function.

that was computed in Sec. II. The constraint imposed there was that the effective action should support all linearized tachyonic perturbations to the linear dilaton CFT of the form

$$T = \mu \exp(\beta_\mu X^\mu), \tag{4.6}$$

subject to the condition that the tachyon profile satisfies the (linearized) tachyon equation of motion in Eq. (2.4). For solutions other than the null tachyon, and in the absence of an additional regulator (like large D, for example), corrections to the linearized marginality condition can generate higher-order corrections to the general constraint equation for $f_1(T)$, (2.32). As noted above, we should therefore regard the solution for $f_1(T)$ used in Sec. II as an approximation valid at small T:

$$f_1(T) \approx 1 - T^2, \qquad T \ll 1. \tag{4.7}$$

A natural additional constraint on the general function $f_1(T)$ is that it reproduce the perturbative expansion around small T [i.e., condition (4) above]. This guarantees that the effective action supports the full class of solutions in Eq. (2.5) in regions where the solutions themselves are not strongly corrected by higher-order effects. A simple exact form for $f_1(T)$ that meets all of the above criteria and reproduces the known solution at small T is

$$f_1(T) = \exp(-T^2) = 1 - T^2 + O(T^4). \tag{4.8}$$

The resulting action in the string frame takes the form (using the rescaled dilaton)

$$S = \frac{1}{2\kappa^2}\int d^D X \sqrt{-\det G^S}\, e^{-\sqrt{D-2}\phi}\Big[R^S - \partial_\mu\phi\partial^\mu\phi - \frac{4}{\sqrt{D-2}} T\partial_\mu\phi\partial^\mu T - \frac{4}{D-2}(T^2 + D - 2)\partial_\mu T\partial^\mu T - \frac{2}{3\alpha'}\exp\Big(\frac{2T^2}{D-2}\Big)(D - 26 - 24T^2)\Big]. \tag{4.9}$$

As desired, the potential

$$V_S(T) = \frac{1}{3\alpha'}\exp\Big(\frac{2T^2}{D-2}\Big)(D - 26 - 24T^2) \tag{4.10}$$

is tachyonic. In this case, however, it is well defined at $T = 1$. It can easily be verified that this potential agrees with the tachyon potential computed above in Eq. (3.6), up to and including cubic order in an expansion around small T.

Of course, there are other possible completions of $f_1(T)$ that meet all of the conditions described above. Another example is

$$f_1(T) = \frac{1}{\cosh(\sqrt{2}T)} = 1 - T^2 + O(T^4), \tag{4.11}$$

which also resolves the curvature singularity in the null tachyon solution at finite T. The string-frame action takes the form

$$S = \frac{1}{2\kappa^2}\int d^D X \sqrt{-\det G^S}\, e^{-\sqrt{D-2}\phi}\Big[R^S - \partial_\mu\phi\partial^\mu\phi + \frac{2\sqrt{2}}{\sqrt{D-2}}\tanh(\sqrt{2}T)\partial_\mu\phi\partial^\mu T - \Big(\frac{2}{D-2} + \frac{2(D-3)}{(D-2)}\mathrm{sech}^2(\sqrt{2}T) + \frac{\sqrt{2}}{T}\tanh(\sqrt{2}T)\Big) \times \partial_\mu T\partial^\mu T - \frac{2}{3\alpha'}\mathrm{sech}^{-(2/(D-2))}(\sqrt{2}T) \times (D - 26 - 12\sqrt{2}T\tanh(\sqrt{2}T))\Big]. \tag{4.12}$$

The qualitative properties of the null tachyon solution are equivalent for both forms of $f_1(T)$ given above. For the purposes of the present study, therefore, we will employ the exponential form in Eq. (4.8) in the analysis that follows. While it would be interesting to study a wider class of solutions, we emphasize that the resolution of the crunch is universal to all $f_1(T)$ satisfying the above conditions.

D. Resolution of the singularity

On general grounds, we expect that the dynamics of the effective action in Eq. (4.9) resolve the cosmological singularity that appears naively in the null tachyon solution. We now examine in detail how the singularity is avoided.

The string-frame metric is given in Eq. (4.2) above. We again choose the Weyl transformation to act trivially on the transverse spatial coordinate X^1_σ, so it is convenient to drop the subscript label. The string-frame time coordinate thus evolves as a function of t_σ and X^1 according to

$$t_S(t_\sigma, X^1) = \frac{1}{\sqrt{2}\beta}\mathrm{Ei}\Big(-\frac{\mu^2}{D-2}e^{\sqrt{2}\beta(t_\sigma + X^1)}\Big) + \mathrm{const}, \tag{4.13}$$

where $\mathrm{Ei}(x)$ is the exponential integral function $\mathrm{Ei}(x) = -\int_{-x}^{\infty}(e^{-\xi}/\xi)d\xi$ [$\mathrm{Ei}(x)$ exhibits a branch cut in the complex x plane running from $x = -\infty$ to $x = 0$, though for real x we take the principal value of the integral]. For convenience we set the constant term to zero.

The (string-frame) FRW scale factor now takes the form

$$a_S(t_\sigma, X^1) = \exp\Big(\frac{\log f_1(T)}{D-2}\Big) = \exp\Big(-\frac{T(t_\sigma, X^1)^2}{D-2}\Big). \tag{4.14}$$

For the null tachyon, this becomes

$$a_S(t_\sigma, X^1) = \exp\Big(-\frac{\mu^2}{D-2}e^{\sqrt{2}\beta(t_\sigma + X^1)}\Big). \tag{4.15}$$

Expressed in terms of sigma-model coordinates, the outcome is clear. For fixed X^1, the evolution of t_σ drives an accelerated contraction of the scale factor [in the positive branch of (2.8), $\beta > 0$]. However, $a_S(t_\sigma, X^1)$ can never reach a true singularity in finite t_σ, as the singular point has been moved to $t_\sigma = \infty$.

Interestingly, the dependence on X^1 drops out when the scale factor is expressed as a function of string-frame coordinates. To see this, first note that Eq. (4.13) can be inverted to give the sigma-model time coordinate as a function of t_S and X^1:

$$t_\sigma(t_S, X^1) = -X^1 + \frac{1}{\sqrt{2}\beta}\log\left(-\frac{D-2}{\mu^2}\mathrm{Ei}^{-1}(\sqrt{2}\beta t_S)\right). \tag{4.16}$$

In the string frame, the scale factor thus takes the explicit form

$$a_S(t_S) = \exp(\mathrm{Ei}^{-1}(\sqrt{2}\beta t_S)). \tag{4.17}$$

Naively, the system reaches a curvature singularity at $t_S = 0$. Relative to the sigma-model frame, however, the physics in the string frame is dramatically redshifted as the scale factor collapses. We can see this directly by plotting the string-frame time coordinate t_S as a function of t_σ (see Fig. 8). As the system collapses, the coordinate t_S steadily ceases to evolve with increasing t_σ, and reaches $t_S = 0$ only asymptotically at $t_\sigma = \infty$. So, from the point of view of the string frame, the system avoids reaching the singularity because the dynamics are severely redshifted; the tachyon generates a smooth cutoff of cosmological time t_S. To be certain, there is a cosmological big crunch at finite time, but from the point of view of the underlying fundamental string, the physics is completely smooth.

In the Einstein frame, the action takes the form

$$S = \frac{1}{2\kappa^2}\int d^DX\sqrt{-\det G^E}\Big[R^E - \partial_\mu\phi\partial^\mu\phi - \frac{4}{\sqrt{D-2}}T\partial_\mu\phi\partial^\mu T - \frac{4}{D-2}(T^2+D-2)\partial_\mu T\partial^\mu T - \frac{2}{3\alpha'}e^{2\phi/\sqrt{D-2}}e^{2T^2/(D-2)}(D-26-24T^2)\Big]. \tag{4.18}$$

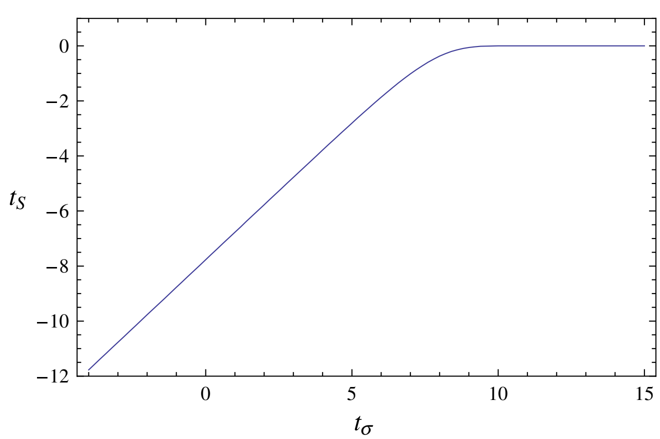

FIG. 8 (color online). The "flow of time" in the string frame, as a function of t_σ. Deep inside the tachyon condensate, the spacetime dynamics becomes infinitely redshifted relative to the sigma model. (The plot depicted is generated at $D = 1000$, $\mu = 0.5$, $X^1 = 0$.)

Expressed in this frame, the scale factor acquires a dependence on the dilaton gradient $q = -\partial_{t_\sigma}\Phi$:

$$a_E(t_\sigma, X^1) = \exp\left(-\frac{1}{D-2}(\mu^2 e^{\sqrt{2}\beta(t_\sigma+X^1)} - 2qt_\sigma)\right). \tag{4.19}$$

The FRW time coordinate evolves with t_σ according to

$$t_E(t_\sigma) = \int_1^{t_\sigma} d\xi \exp\left(-\frac{1}{D-2}(2q\xi - e^{\sqrt{2}\beta(X^1+\xi)})\right) + \text{const.} \tag{4.20}$$

Once again, this expression can be formally inverted to express t_σ as a function of t_E and X^1. By construction, the scale factor in Eq. (4.19) and the tachyon and dilaton profiles in Eq. (3.29) are implicit particular solutions of the equations of motion.

Figure 9 depicts the evolution of the scale factor in the Einstein frame. When the tachyon is small, the universe evolves approximately linearly in t_E for all X^1. This is just a reflection of the fact that at zero tachyon the theory exhibits an equation of state that is precisely critical [$w = -(D-3)/(D-1)$]. As the tachyon increases in strength, the scale factor collapses, approaching a singularity asymptotically at $t_\sigma = \infty$. To an observer at some fixed negative X^1, the region of collapse appears to expand in the negative X^1 direction outward from the origin. As the scale factor approaches zero, the cosmological time t_E ceases to evolve as a function of t_σ, and spacetime becomes frozen in a near singularity.

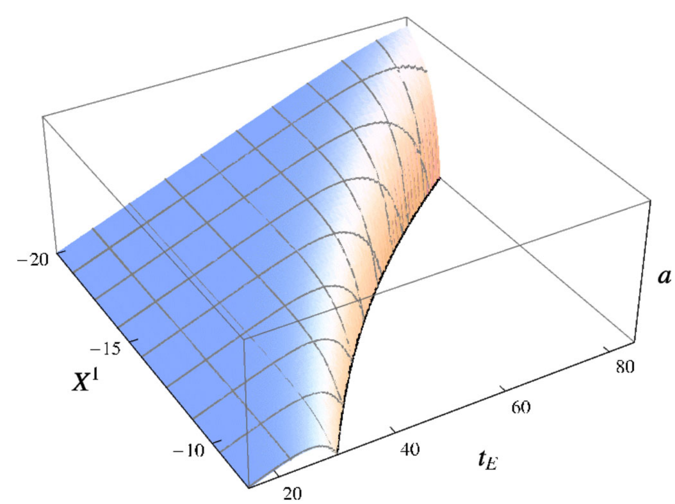

FIG. 9 (color online). The bubble of nothing in the Einstein frame is a surface of an asymptotically vanishing metric. The scale factor initially grows linearly as a function of t_E, then collapses to a near-singular region that expands outward in the negative X^1 direction ($D = 1000$, $\mu = 0.5$).

E. Toy models of the big bang

As noted above, the beta function equations of the $2D$ worldsheet CFT stipulate that the dilaton gradient $q = -\partial_{t_\sigma}\Phi$ satisfy

$$q^2 = \left(\frac{D-26}{6\alpha'}\right)^2. \tag{4.21}$$

We can also study the second branch of this solution, where q is negative definite. In this case, the dilaton evolves from weak coupling in the far past and rolls in the direction of strong coupling toward the future. The linearized tachyon equation of motion (2.4) is satisfied under the condition $\beta = \frac{2\sqrt{2}}{q\alpha'}$, so β is also negative definite. Choosing this branch is therefore equivalent to invoking an overall time reversal on the previous solution. The strength of the tachyon thus decreases with evolving time, reaching zero in the infinite future. (This general setup of a tachyonic big bang was studied in detail in [43], with a particular focus on the importance of fluctuations.) Although this is essentially an extremely fine-tuned solution, let us briefly consider the picture that emerges in its own right.

Deep in the weakly coupled regime, the tachyon is large, and the FRW scale factor is correspondingly small. As the tachyon decreases in strength, the scale factor rapidly accelerates from a near singularity. (In the Einstein frame, the evolution of the dilaton eventually takes over, and the scale factor contracts linearly as a function of t_E for fixed X^1.) This situation presents an interesting toy model of the big bang that can be studied at arbitrarily weak string coupling. Figure 10 depicts the scale factor as a function of t_S. Deep in the past the scale factor is near zero and, as the tachyon passes below a critical value, the universe rapidly expands outward.

When viewed as a function of t_σ, the temporal coordinate in the string frame is also frozen deep in the past. This is depicted in Fig. 11, which displays a plot of $t_S(t_\sigma)$. The "flow of time" in the string frame is thus generated as the tachyon evolves toward zero.

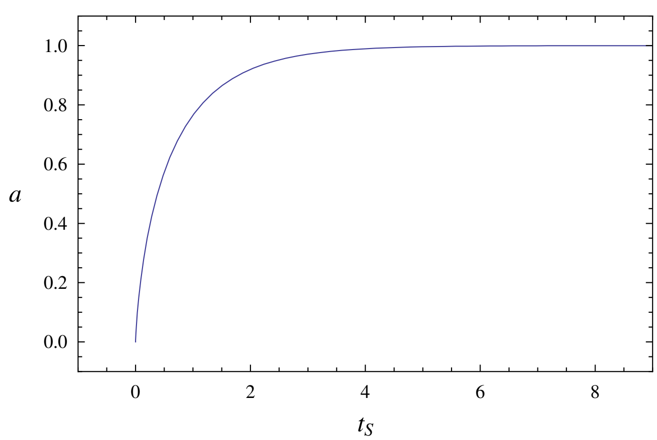

FIG. 10 (color online). A toy model of the big bang ($D = 1000$, $\mu = 0.5$, $X^1 = 0$). The second branch of the null tachyon solution causes the scale factor to emerge from a near singularity and rapidly accelerate, reaching a constant value (in the string frame) when the tachyon shrinks to zero size.

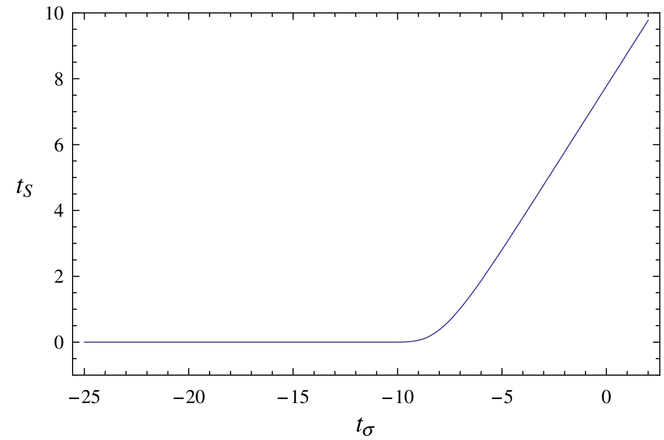

FIG. 11 (color online). In a tachyonic big bang, cosmological time in the string frame is frozen in the deep past, as a function of sigma-model time t_σ. As the tachyon evolves toward zero, time begins to flow monotonically in the positive direction ($D = 1000$, $\mu = 0.5$).

V. SOLITONIC SOLUTIONS

Part of the benefit of having a specific effective action in hand is the ability to study new solutions that are not used as input in deriving the action itself. In this section we look for static solutions of the effective action that localize the universe in spatial, rather than temporal, directions.

The general problem of using closed string tachyon condensation to localize or remove a spatial dimension has been studied in detail in [15,17,33,40,44]. In [17], the tractability of the null tachyon was used to derive *exact* solutions that dynamically remove spatial dimensions from the theory. The central charge deficit that arises in the classical worldsheet CFT from the removal of these dimensions is made up of one-loop quantum corrections. Because these solutions lie outside of the class of tachyon perturbations considered here (2.5), and because they are inherently quantum mechanical on the worldsheet, we will not attempt to study them directly in the present context.

Instead, we adopt the strategy of [40], which studied this problem directly at the level of effective actions. The authors of [40] were able to find toy models that localize spatial directions in the presence of general tachyon potentials. We will follow this analysis in searching for analogous solutions of the effective action computed above.

A simple approach is to introduce as an ansatz a spatially varying sigma-model metric of the form

$$ds^2_\sigma = (dX^1)^2 + \tilde{a}(X^1)^2 \eta_{\mu'\nu'} dX^{\mu'} dX^{\nu'},$$
$$\mu', \nu' \in 0, 2, 3, \ldots, D-1, \tag{5.1}$$

with a warp factor $\tilde{a}(X^1)$ that depends only on X^1. Following [40], we observe that, in general, the dilaton will depend on all of the embedding coordinates X^μ, though by the $(D-1)$-dimensional Lorentz invariance we can move to a frame in which the dilaton depends only on X^1 and X^2:

$$\Phi = \Phi(X^1, X^2). \tag{5.2}$$

The aim is to find codimension one solitons, in which the tachyon depends solely on the spatial direction X^1. The nontrivial components of the Einstein equation (2.30) in the sigma-model frame yield the conditions [for general $f_1(T)$]

$$0 = f_1[(D-2)(\tilde{a}')^2 + \tilde{a}\tilde{a}'' - 2\tilde{a}\tilde{a}'\partial_1\Phi] + f_1'T'\tilde{a}\tilde{a}', \tag{5.3}$$

$$0 = f_1\left(2\partial_1^2\Phi - (D-1)\frac{\tilde{a}''}{\tilde{a}}\right) + f_1'\left(\frac{T'^2}{T} - T''\right), \tag{5.4}$$

$$0 = \frac{2f_1}{\tilde{a}}(\tilde{a}\partial_1\partial_2\Phi - \tilde{a}'\partial_2\Phi), \tag{5.5}$$

$$0 = -f_1[(D-2)(\tilde{a}')^2 + \tilde{a}\tilde{a}'' - 2(\partial_2^2\Phi + \tilde{a}\tilde{a}'\partial_1\Phi)] - f_1'T'\tilde{a}\tilde{a}'. \tag{5.6}$$

Equations (5.3) and (5.6) combine to give the condition

$$f_1\partial_2^2\Phi = 0. \tag{5.7}$$

Equation (5.3) alone implies that $\partial_1\Phi$ can only depend on the X^1 coordinate (f_1 is a function of T only, which we assume depends only on X^1), so

$$\partial_2\partial_1\Phi = 0. \tag{5.8}$$

With the general condition that $f_1(T)$ is nonzero at finite T, we obtain the following generic form for the dilaton:

$$\Phi(X^1, X^2) = F(X^1) + QX^2, \tag{5.9}$$

where $F(X^1)$ is some function of X^1 only, and Q is a constant. This same condition was derived for the codimension one soliton configurations studied in [40]. Equation (5.5) then reduces to

$$Q\tilde{a}' = 0, \tag{5.10}$$

implying that either the sigma-model metric is precisely flat, or the dilaton is independent of X^2.

Let us first consider solutions with a flat sigma-model metric:

$$ds_\sigma^2 \equiv \eta_{\mu\nu}dX^\mu dX^\nu. \tag{5.11}$$

In this case, the remaining nontrivial component of the Einstein equation appears as

$$2f_1F'' - f_1'T'' + \frac{f_1'}{T}T'^2 = 0. \tag{5.12}$$

The dilaton and tachyon equations of motion take the form

$$0 = f_1(F'^2 + Q^2) - f_1'F'T' + \frac{1}{4}\left(f_1'' + \frac{f_1'}{T}\right)T'^2 + \frac{1}{2}V_\sigma,$$
$$0 = \left(\frac{f_1'}{T} - f_1'' - f_1'''T\right)T'^2 + 4f_1'T(F'' - F'^2 - Q^2) + 2(f_1' + f_1''T)(2F'T' - T'') - 2T\partial_T V_\sigma. \tag{5.13}$$

Employing the perturbative solution $f_1(T) \approx 1 - T^2$ and substituting the tachyon potential into these equations yields the following conditions on the transverse component of the dilaton:

$$F' = \frac{1}{2\alpha'T'}(4T + \alpha'T''), \qquad F'' = \frac{1}{(T^2-1)}(TT'' - T'^2). \tag{5.14}$$

We also recover an explicit expression for the longitudinal dilaton gradient in terms of the tachyon,

$$Q^2 = -\frac{1}{6(T^2-1)}\left[6T'^2 - 6TT'' + \frac{1}{\alpha'}(T^2-1)(D-26) + \frac{3}{2\alpha'T'^2}(T^2-1)(4T+\alpha'T'')^2\right]. \tag{5.15}$$

The conditions on the transverse dilaton $F(X^1)$ in Eq. (5.14) impose the following differential equation for the tachyon profile:

$$\frac{1}{(T^2-1)}(T'^2 - TT'') + \frac{1}{2\alpha'T'^2}[4T'^2 - T''(4T + \alpha'T'') + \alpha'T'T'''] = 0. \tag{5.16}$$

This equation is satisfied exactly by the exponential profile

$$T = \mu\exp(\beta_1 X^1). \tag{5.17}$$

With this solution, the transverse dilaton is linear in X^1, with a gradient given by

$$F' = \frac{2}{\alpha'\beta_1} + \frac{\beta_1}{2}. \tag{5.18}$$

The longitudinal dilaton gradient takes the form[7]

$$Q^2 = -\frac{\beta_1^2}{4} - \frac{4}{\alpha'^2\beta_1^2} - \frac{D-14}{6\alpha'}. \tag{5.19}$$

For real β_1, Q^2 can only be nonnegative for $D \le 2$. [In fact, this conclusion can be reached using the full exponential form $f_1(T) = \exp(-T^2)$, or the cosh form $f_1(T) = 1/\cosh(\sqrt{2}T)$, in the above equations.] If a timelike direction is present at all, the *only* consistent solution exists in $D = 2$, with $Q = 0$. The resulting system is described by an exponential tachyon profile with a spacelike linear

[7]One can check that the tachyon profile satisfies the marginality condition, and $Q^2 + F'^2 = -\frac{D-26}{6\alpha'}$.

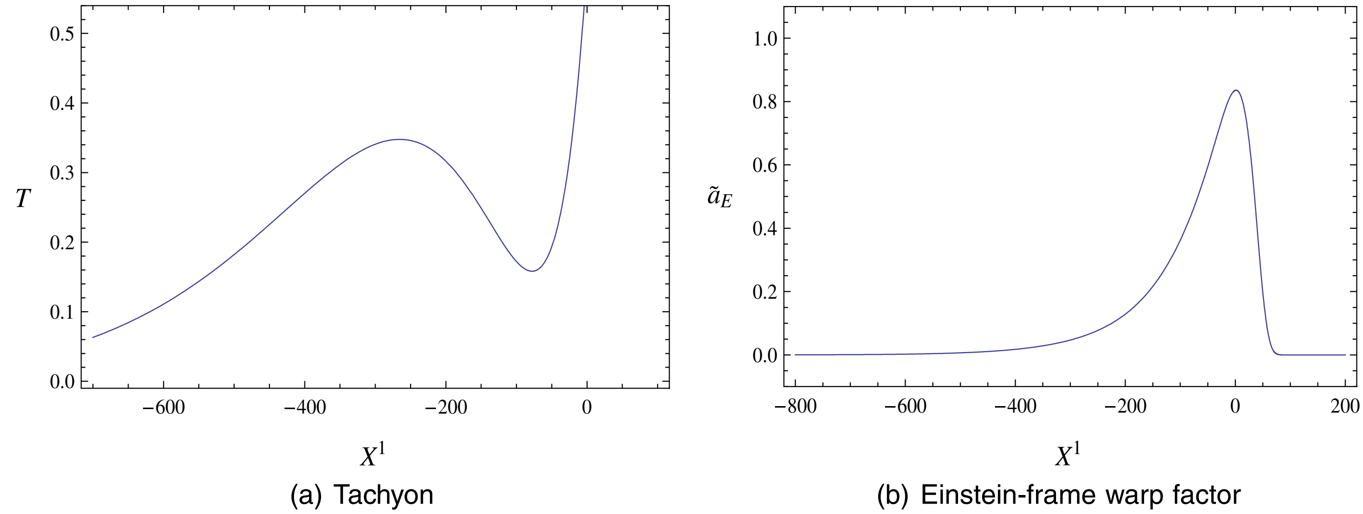

FIG. 12 (color online). A lump configuration of the spatially varying tachyon [panel (a)] with a flat sigma-model metric. The asymptotic regions are dominated by an overall exponential prefactor in the solution that is present to all orders in α'. The Einstein-frame metric [panel (b)] exhibits a solitonic configuration of localized spacetime.

dilaton, both varying in the X^1 direction. The dilaton gradient and tachyon profile are determined by

$$F' = \beta_1 = \pm \frac{2}{\sqrt{\alpha'}}. \tag{5.20}$$

Therefore, the only static solution consistent with a single exponential tachyon and an exactly flat sigma-model metric lives strictly in $D = 2$.

Of course, this restriction is lifted if the dilaton is allowed to vary in the timelike direction (so the configuration is no longer static). For example, consider the form

$$\Phi(X^0, X^1) = F(X^1) - qX^0. \tag{5.21}$$

In this case, with a tachyon profile of the form in Eq. (5.17), the dilaton is again linear along the X^1 direction, with gradient given by Eq. (5.18). However, the gradient in the timelike direction is given by (5.19), with an overall sign change: $q^2 = -Q^2$. This is positive definite for all $D \geq 2$. The special case in which $F' = q$ corresponds to spacelike Liouville theory with a null linear dilaton in the critical dimension $D = 26$.

It is instructive to look for other static tachyon solutions. One method is to solve (5.16) order by order in α'. To $O(\alpha'^3)$, one obtains

$$\begin{aligned} T(X^1) = {} & \mu e^{\beta_1 X^1} + \alpha' e^{\beta_1 X^1}(c_1 + c_2 X^1) \\ & + \alpha'^2 e^{\beta_1 X^1}\left(c_3 + c_4 X^1 + \frac{1}{2\mu} c_2^2 (X^1)^2\right) \\ & + \alpha'^3 e^{\beta_1 X^1}\left(c_5 + c_6 X^1 + \frac{1}{\mu}\left(c_2 c_4 - \frac{1}{2\mu} c_1 c_2^2\right)\right. \\ & \left. \times (X^1)^2 + \frac{1}{6\mu^2} c_2^3 (X^1)^3\right) + O(\alpha'^4), \end{aligned} \tag{5.22}$$

where c_n are free constants. It is easy to find a set of constants c_n such that the tachyon exhibits a "lump" configuration over some intermediate range in X^1 (and the square of the longitudinal dilaton gradient is positive definite).

The exponential prefactor $\exp(\beta_1 X^1)$ is present in the solution (5.22) to all orders in α', multiplying terms that are universally polynomial in X^1. The prefactor therefore dominates in the asymptotic regions, and the tachyon is forced to vanish at $X^1 = \pm\infty$, depending on the sign of β_1. An example lump profile is depicted in Fig. 12(a). In the string frame, the metric exhibits a configuration that is finite over a semi-infinite region, with a localized "pseudosoliton" in some separate region. The resulting picture is a semi-infinite universe in D dimensions, with an effectively lower-dimensional neighboring parallel universe. In the Einstein frame, the semi-infinite region of the finite metric can be removed by arranging the transverse dilaton to increase in the appropriate direction. This is displayed in Fig. 12(b), with $f_1(T) = \exp(-T^2)$.

Of course, one should keep in mind that the effective action itself is not an exact description of string theory, and higher-order effects are likely to become important in the absence of some special mechanism (as with the null tachyon). Even so, the leading-order behavior of nonexact solutions can often serve as a useful qualitative guide in determining the types of solutions that are possible.

In addition, one should also consider solutions for which the sigma-model metric is nonflat. In these cases, the dilaton must have a vanishing longitudinal gradient.[8] To study the ability of a solitonic configuration to localize the

[8] A recent paper [33] has provided evidence for the existence of a codimension one soliton in closed string field theory. In that analysis, the dilaton did not vary longitudinally along the soliton, though the more general case was not considered directly.

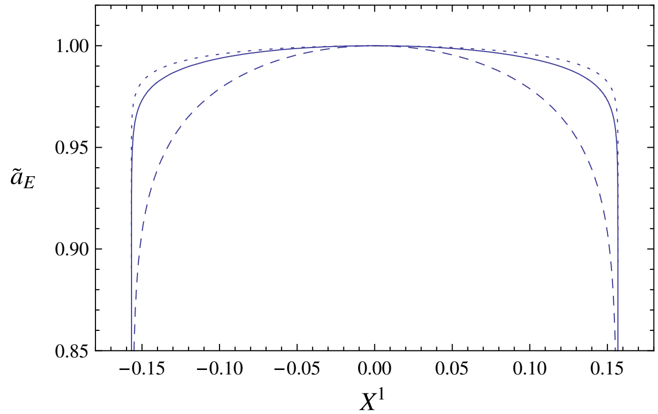

FIG. 13 (color online). An example of a solitonic tachyon configuration that localizes spacetime in a spatial direction, at $D = 30$ (dashed curve), $D = 100$ (solid curve), and $D = 150$ (dotted curve). Increasing D brings the scale factor closer to a constant over the finite region of the solution. The overall scale of the localized dimension is roughly constant for a given set of boundary conditions.

universe along a spatial direction, it is most natural to work in the Einstein frame, and a straightforward approach is to search for numerical solutions to the equations of motion that exhibit the qualitative properties of interest. If the dilaton and tachyon are arranged to increase in opposite coordinate directions (like all of the examples studied above), the warp factor in the Einstein frame $\tilde{a}_E$ will naturally acquire local support in a region where both the dilaton and tachyon are small.

One such set of numerical solutions is displayed for various D in Fig. 13. In these solutions, the dilaton increases in the negative X^1 direction, while the tachyon magnitude increases in the positive X^1 direction. This is depicted in Fig. 14. Along the negative X^1 axis, the growth of the dilaton drives the warp factor toward zero size, while along the positive X^1 direction the growth of the tachyon drives $\tilde{a}_E$ toward zero as well. The scale factor can exist at finite size in the region in between. Taken at face value, the overall scale of the confined dimension is essentially a function of initial conditions. For a given set of initial conditions, the dilaton and tachyon solutions do not vary considerably with shifting D (see Fig. 14). Of course, the string theory can become strongly coupled deep in the region of negative X^1, so the solution is subject to corrections there. Furthermore, α' corrections can become important in regions that exhibit singular (or near-singular) behavior. Again, these types of solutions should therefore be viewed only as toy models.

VI. SUMMARY AND CONCLUSIONS

We have seen that there is a unique two-derivative effective action that perturbatively supports the full class of solutions in Eq. (2.5). Finding this effective action amounts to finding a perturbative expansion for the function $f_1(T)$, defined as a prefactor of the Einstein-Hilbert term in the sigma-model frame. Demanding that the general action support the linearized solutions in Eq. (2.5) allowed us to solve for $f_1(T)$ to quadratic order.

Taken at face value, the solutions of interest generically encounter singularities as the tachyon evolves to become of order 1. This confirms the general expectations provided by [41]. At $T \approx 1$, however, we expect these solutions to become subject to higher-order corrections. We saw that the two-derivative effective action supports the null tachyon for any $f_1(T)$, and possible modifications from higher-order effects are restricted to affect the spacetime

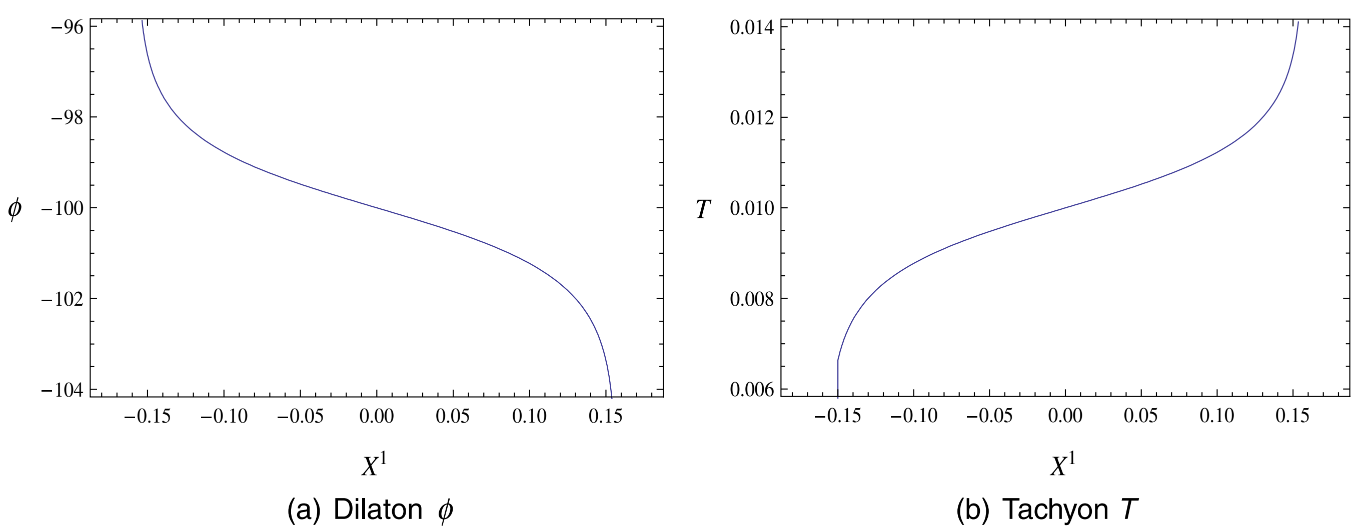

FIG. 14 (color online). Behavior of the dilaton and tachyon in a solitonic configuration ($D = 100$). With a given set of boundary conditions, the numerical solutions for the dilaton and tachyon do not vary visibly with varying dimension.

solution only through higher-order constraints to the function $f_1(T)$.

Considering the effective action for completely general $f_1(T)$, we showed that singularities in the string-frame action arise either from singularities of the metric itself, or from prefactors of the matter kinetic terms that arise from the Weyl transformation in Eq. (3.1). If we demand that the latter singularities are absent, and that the gravity sector remains unitary for all T, then the curvature singularities of the null tachyon system are resolved for all finite values of the tachyon itself. The resolution of these singularities suggests an interesting description of how cosmological time can be initiated or terminated in string theory. We expect there to be many applications of this sort of mechanism in more general cosmological models.

Of course, we should leave open the possibility that the framework of the spacetime effective action is simply insufficient to capture the physics of bulk tachyon condensation, and that a careful accounting of higher-order corrections to the action will reveal unavoidable singularities arising at finite values of the tachyon, even for exact solutions. It would certainly be interesting to try to constrain possible higher-order corrections to the effective action directly. The general expectation is that higher-order effects in conformal perturbation theory will amount to corrections to $f_1(T)$ beyond quadratic order. There is not a unique function that satisfies the conditions on $f_1(T)$ given in Sec. IV, but a test to determine whether corrections beyond $O(T^2)$ can resolve cosmological singularities is to see whether higher-order terms divert the function from crossing zero at $T = 1$. A promising hint of this would be that the next term in the series is positive definite for all T. In other words, one might expect general higher-order effects to yield a correction to $f_1(T)$ of the form

$$f_1(T) = 1 - T^2 + cT^n + O(T^{n+1}), \tag{6.1}$$

where c is positive definite and n is an even integer. It would clearly be valuable to test this prediction by direct methods.

ACKNOWLEDGMENTS

The author thanks Oren Bergman and Simeon Hellerman for helpful and interesting discussions, and for reading a draft of the manuscript; he thanks David Kutasov, Martin Schnabl, and Sav Sethi for useful discussions. I. S. is supported by the Marvin L. Goldberger Membership at the Institute for Advanced Study, and by U.S. National Science Foundation Grant No. PHY-0503584.

[1] A. Sen, Int. J. Mod. Phys. A **20**, 5513 (2005).
[2] W. Taylor and B. Zwiebach, arXiv:hep-th/0311017.
[3] P.-J. De Smet, arXiv:hep-th/0109182.
[4] K. Ohmori, arXiv:hep-th/0102085.
[5] S. Hellerman and M. Schnabl, arXiv:0803.1184.
[6] A. Adams, J. Polchinski, and E. Silverstein, J. High Energy Phys. 10 (2001) 029.
[7] C. Vafa, arXiv:hep-th/0111051.
[8] J. A. Harvey, D. Kutasov, E. J. Martinec, and G. W. Moore, arXiv:hep-th/0111154.
[9] A. Dabholkar, Phys. Rev. Lett. **88**, 091301 (2002).
[10] R. Gregory and J. A. Harvey, Classical Quantum Gravity **20**, L231 (2003).
[11] M. Headrick, J. High Energy Phys. 03 (2004) 025.
[12] A. Adams, X. Liu, J. McGreevy, A. Saltman, and E. Silverstein, J. High Energy Phys. 10 (2005) 033.
[13] T. Suyama, J. High Energy Phys. 05 (2005) 065.
[14] Y. Okawa and B. Zwiebach, J. High Energy Phys. 03 (2004) 056.
[15] S. Hellerman and X. Liu, arXiv:hep-th/0409071.
[16] S. Hellerman and I. Swanson, Phys. Rev. D **77**, 126011 (2008).
[17] S. Hellerman and I. Swanson, J. High Energy Phys. 09 (2007) 096.
[18] S. Hellerman and I. Swanson, Phys. Rev. Lett. **99**, 171601 (2007).
[19] B. Zwiebach, Nucl. Phys. **B390**, 33 (1993).
[20] M. Saadi and B. Zwiebach, Ann. Phys. (N.Y.) **192**, 213 (1989).
[21] T. Kugo, H. Kunitomo, and K. Suehiro, Phys. Lett. B **226**, 48 (1989).
[22] T. Kugo and K. Suehiro, Nucl. Phys. **B337**, 434 (1990).
[23] M. Kaku, Phys. Rev. D **38**, 3052 (1988).
[24] M. Kaku and J. D. Lykken, Phys. Rev. D **38**, 3067 (1988).
[25] V. A. Kostelecky and S. Samuel, Phys. Rev. D **42**, 1289 (1990).
[26] A. Belopolsky and B. Zwiebach, Nucl. Phys. **B442**, 494 (1995).
[27] A. Belopolsky, Nucl. Phys. **B448**, 245 (1995).
[28] N. Moeller, J. High Energy Phys. 11 (2004) 018.
[29] N. Moeller, J. High Energy Phys. 03 (2007) 043.
[30] N. Moeller, J. High Energy Phys. 09 (2007) 118.
[31] O. Bergman and S. S. Razamat, J. High Energy Phys. 01 (2005) 014.
[32] H. Yang and B. Zwiebach, J. High Energy Phys. 09 (2005) 054.
[33] N. Moeller, arXiv:0804.0697.
[34] O. Bergman and S. Hirano, Nucl. Phys. **B744**, 136 (2006).
[35] O. Aharony and E. Silverstein, Phys. Rev. D **75**, 046003 (2007).
[36] S. Hellerman and I. Swanson, J. High Energy Phys. 07 (2008) 022.
[37] S. Hellerman and I. Swanson, arXiv:0709.2166.

[38] S. Hellerman and I. Swanson, arXiv:0710.1628.
[39] E. Witten, Nucl. Phys. **B195**, 481 (1982).
[40] O. Bergman and S. S. Razamat, J. High Energy Phys. 11 (2006) 063.
[41] H. Yang and B. Zwiebach, J. High Energy Phys. 08 (2005) 046.
[42] D. Kutasov and V. Niarchos, Nucl. Phys. **B666**, 56 (2003).
[43] R. H. Brandenberger, A. R. Frey, and S. Kanno, Phys. Rev. D **76**, 083524 (2007).
[44] D. Green, A. Lawrence, J. McGreevy, D. R. Morrison, and E. Silverstein, Phys. Rev. D **76**, 066004 (2007).