Mass-deformed Bagger-Lambert theory and its BPS objects

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We find a 16-supersymmetric mass-deformed Bagger-Lambert theory with $SO(4) \times SO(4)$ global R symmetry. The R charge plays the role of the noncentral term in the superalgebra. This theory has one symmetric vacuum and two inequivalent broken sectors of vacua. Each sector of the broken symmetry has $SO(4)$ geometry. We find the $1/2$ BPS domain walls connecting the symmetric phase and any broken phase, and $1/4$ BPS supertubelike objects, which may appear as anyonic q-balls in the symmetric phase or vortices in the broken phase. We also discuss mass deformations, which reduce the number of supersymmetries.

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I. INTRODUCTION AND CONCLUSION

Recently there has been a spur of activity on the possible superconformal field theories for the multiple M2-branes. The so-called Bagger-Lambert theory [1–3] has 16 supersymmetries and $SO(8)$ global symmetry. However, the gauge matter coupling and the Chern-Simons term are not standard but given by the three-product structures of the $SO(4)$ group acting on vectors. (See also the work by Gustavsson [4].) Various aspects of this theory have been explored [5–7]. In particular, some detailed analyses of the vacuum structure were done to argue that this theory is the theory of two M2-branes on M-theory orbifolds [8,9].

In this work we find a mass deformation of the Bagger-Lambert theory without breaking any supersymmetry. We find that this mass-deformed theory is one example of the 3 dimensional supersymmetric field theory with the so-called ‘‘noncentral’’ term whose superalgebra has been studied before [10,11]. The global R charge is now $SO(4) \times SO(4)$ and is also the noncentral term in the superalgebra. We investigate the vacuum structure and find $1/2$ BPS and $1/4$ BPS domain walls and $1/4$ BPS localized solitons. This $1/4$ BPS localized solitons are basically supertubes which appear as q-balls in the symmetric phase and vortices in the broken phases.

It has been known for some time that the n-product object is closely related to a cross product of n $(n+1)$ dimensional vectors to one $(n+1)$ dimensional vector. The three-product object used by Bagger and Lambert is realized as the cross product of three 4 dimensional vectors or $SO(4)$. Since their work, there have been several attempts to extend this structure [12–16]. However, any concrete realization remains to be seen.

While the ordinary Yang-Mills–Chern-Simons theory can have only six supersymmetries [17], Lin and Maldacena have found a family of eight-supersymmetric Yang-Mills–Chern-Simons theories with noncentral terms [11]. This eight-supersymmetric theory is possible with an arbitrary gauge group. As it turns out, this theory has rich structures and allows one to include matter in other representations, as studied in [18]. However, we are not aware of any explicit construction of the 16 supersymmetric theory with a noncentral term, except the mass-deformed Bagger-Lambert theory studied here.

The theory with mass deformation has one symmetric vacuum and two broken phases where one of the global $SO(4)$ and the gauge $SO(4)$ are broken to the diagonal $SO(4)$. There is no massless particle in either phase. One can imagine the BPS domain walls connecting different sectors of vacua. Indeed, there are $1/2$ BPS domain walls connecting the symmetric phase and the broken phase, but there is no BPS object connecting two broken phases. However, we find a non-BPS domain configuration explicitly, as it satisfies a ‘‘fake’’ BPS equation.

The potential term suggests that there are attractive interactions between some particles. One naturally expects some sort of q-balls carrying R charges. Similarly, one may expect that there could be topological or nontopological vortices in the broken phase, as the vacuum manifold has $\pi_1(SO(4)) = Z_2$. Indeed, we find there are $1/4$ BPS q-balls in the symmetric phase and $1/4$ BPS vortices in the broken phase. For a certain ansatz, the corresponding $1/4$ BPS equations become that of the Abelian Chern-Simons theory [19–21], and so the previous analysis of the solitons is carried over to our case. From the study of the $1/4$ BPS domain walls which carry the R charges, one can see that the large-charge limit of the q-balls and vortices should be like many other field theoretic supertubes [22]. In this case both the interior and the exterior of the soliton would be vacua, and the boundary would be a domain wall carrying both R charges and linear momentum. Since the supertube has been proposed in Ref. [23], there has been enormous work done on this subject but without any direct relevance to this work.

While the theory is parity even, one suspects that the Chern-Simons term may still play a role in the dynamics,

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leading to anyons of fractional spin and statistics. Indeed, we find that the $1/4$ BPS localized solitons carry fractional spin in the symmetric phase. It would be interesting to find the role of the fractional spin and statistics in the mass-deformed theory as the possible theory of two M2-branes on orbifolds.

The geometry of type IIB string theory with 16 supersymmetries and with $S^1 \times SO(4) \times SO(4)$ symmetries has been described by the droplets of an incompressible fluid [24]. The theory of fermion droplets on a cylinder with a Fermi sea is argued to describe the 16-supersymmetric theory of M2-branes with a mass deformation whose vacuum structure describes the M2-branes polarized into M5-branes wrapping two possible S^3's [11,25–27]. The mass-deformed Bagger-Lambert theory may realize this picture. The theory of the fermion droplet on a torus is supposed to also be a massive 16-supersymmetric theory, but is not written explicitly yet. The dimensional reduction of the superalgebra to $1+1$ dimensions leads to the linearly realized supersymmetries on the light-cone world sheet of a string moving in the maximally supersymmetric type IIB plane wave [28]. The investigation of these pictures may lead to a more explicit realization of 16-supersymmetric theories in $2+1$ dimensions.

The plan of this work is as follows. The mass-deformed theory is introduced in Sec. II. The vacuum structure and the superalgebra are given. In Sec. III, the $1/2$ domain wall connecting the symmetric phase to a broken phase is discussed. In Sec. IV, we find the $1/4$ BPS q-balls, vortices, and supertubes. In the Appendix, we find mass deformations which reduce the number of supersymmetries.

(While this paper was being written, a massive deformation of the Bagger-Lambert theory was also proposed in Ref. [29].)

II. MASS-DEFORMED THEORY WITH $SO(4) \times SO(4)$ GLOBAL SYMMETRY

The Bagger-Lambert theory contains the $SO(4)$ bosonic variables X_I^a, where $a = 1, 2, 3, 4$ for the $SO(4)$ index and $I = 3, 4, \ldots 10$ for the $SO(8)$ index, as well as the eight-independent-component spinor field Ψ^a. The gauge field is A_μ^{ab}, which is an antisymmetric tensor of $SO(4)$ gauge symmetry. The spacetime signature is $(-++)$. We choose the 11 dimensional Gamma matrices which are 32-by-32 matrices such that Γ^0 is an antisymmetric real matrix and the rest are symmetric real matrices. We choose the convention for the spinor parameter so that

$$\Gamma^{012}\Psi = -\Psi, \qquad \Gamma_{34\ldots\overline{10}}\Psi = \Psi,$$
$$\Gamma^{012}\epsilon = \epsilon, \qquad \Gamma_{34\ldots 9\overline{10}}\epsilon = -\epsilon. \tag{1}$$

The gauge symmetry is

$$\delta X_I^a = f^{abcd}\Lambda^{cd}X_I^d \equiv \tilde{\Lambda}^{ab}X_I^b, \tag{2}$$

and the covariant derivative is

$$D_\mu X_I^a = \partial_\mu X^a + f^{abcd}A_\mu^{cd}X_I^b \equiv \partial_\mu X_I^a + \tilde{A}_\mu^{ab}X_I^b, \tag{3}$$

with the local gauge transformation given as

$$\delta\tilde{A}_\mu = -D_\mu\tilde{\Lambda}_\mu = -(\partial_\mu\tilde{\Lambda} + \tilde{A}_\mu^{ac}\tilde{\Lambda}^{cb} - \tilde{\Lambda}^{ac}\tilde{A}_\mu^{cb}). \tag{4}$$

The field strength given by $[D_\mu, D_\nu]X^a = \tilde{F}_{\mu\nu}^{ab}X^b$ leads to

$$\tilde{F}_{\mu\nu}^{ab} = \partial_\mu\tilde{A}_\nu^{ab} - \partial_\mu\tilde{A}_\nu^{ab} + \tilde{A}_\mu^{ac}\tilde{A}_\nu^{cb} - \tilde{A}_\nu^{ac}\tilde{A}_\mu^{cb}. \tag{5}$$

As the three-product structure has been realized only for the $SO(4)$ group, our $f^{abcd} = \epsilon^{abcd}$.

The supersymmetric Lagrangian for the Bagger-Lambert theory is

$$\begin{aligned}\mathcal{L} = &-\frac{1}{2}D_\mu X_I^a D^\mu X_I^a + \frac{i}{2}\bar{\Psi}^a\Gamma^\mu D_\mu\Psi^a \\ &+ \frac{\kappa}{2}\epsilon^{\mu\nu\rho}\Big(f^{abcd}A_\mu^{ab}\partial_\nu A_\rho^{cd} \\ &+ \frac{2}{3}f^{acde}f^{bcfg}A_\mu^{ab}A_\nu^{de}A_\rho^{fg}\Big) \\ &- \frac{i}{4\kappa}f^{abcd}\bar{\Psi}^a\Gamma_{IJ}\Psi^b X_I^c X_J^d \\ &- \frac{1}{12\kappa^2}\sum_{a,I,J,K}(f^{abcd}X_I^b X_J^c X_K^d)^2,\end{aligned} \tag{6}$$

where $\bar{\Psi}^a = \Psi^{a\dagger}\Gamma^0$. We propose the mass deformation of the theory to be

$$\begin{aligned}\mathcal{L}_m = &-\frac{m^2}{2}(X_I^a)^2 - \frac{i}{2}m\bar{\Psi}^a\Gamma_{3456}\Psi^a \\ &+ \frac{4m}{\kappa}f^{abcd}(X_3^aX_4^bX_5^cX_6^d + X_7^aX_8^bX_9^cX_{10}^d).\end{aligned} \tag{7}$$

The bosonic part of the mass deformation for 3, 4, 5, 6 was studied in [2,25]. In this work the broken phase of the theory is also identified as the M2-branes blowing up to M5-branes wrapping S^3. The supersymmetric transformation of the fields also gets deformed as

$$\delta X_I^a = i\bar{\epsilon}\Gamma_I\Psi^a, \tag{8}$$

$$\begin{aligned}\delta\Psi^a = &(\Gamma^\mu D_\mu + m\Gamma_{3456})X_I^a\Gamma_I\epsilon \\ &+ \frac{1}{6\kappa}\Gamma_{IJK}\epsilon f^{abcd}X_I^bX_J^cX_K^d,\end{aligned} \tag{9}$$

$$\delta A_\mu^{ab} = -\frac{i}{2\kappa}\bar{\epsilon}\Gamma_\mu\Gamma_I(\Psi^aX_I^b - \Psi^bX_I^a). \tag{10}$$

The Gauss law constraint arising from δA_0^{ab} is

$$\kappa\tilde{F}_{12}^{ab} - f^{abcd}X_I^cD_0X_I^d + \frac{i}{2}f^{abcd}\bar{\Psi}^c\Gamma^0\Psi^d = 0. \tag{11}$$

The quantization of the Chern-Simons coefficient is

$$2\pi\kappa = n, \tag{12}$$

with integer n. The original global $SO(8)$ symmetry is broken to $SO(4) \times SO(4)$ symmetries. Among the conserved charge for the original $SO(8)$ symmetry

$$R_{IJ} = \int d^2x \left(X_I^a D_0 X_J^a - X_J^a D_0 X_I^a + \frac{i}{2} \bar{\Psi}^a \Gamma^0 \Gamma_{IJ} \Psi^a \right), \tag{13}$$

only $SO(4) \times SO(4)$ symmetries, each of which rotates 3, 4, 5, 6 and 7, 8, 9, 10, respectively, will be preserved.

The bosonic potential of the theory can be written as

$$U(X) = \frac{1}{12\kappa^2} \sum_{a,J,K,L} (f^{abcd} X_J^b X_K^c X_L^d - \kappa m \delta^{3456}_{IJKL} X_I^a - \kappa m \delta^{789\overline{10}}_{IJKL} X_I^a)^2, \tag{14}$$

where $\delta^{3456}_{IJKL} = \epsilon_{IJKL}$ for $I, J, K, L \in \{3, 4, 5, 6\}$ and vanishes for other combinations, and δ^{78910}_{IJKL} is defined in a similar manner. There exist three independent sectors of vacua in the theory: the symmetric phase where $X_I^a = 0$ for all I and two broken phases where

$$(\mathbf{I})\ X_I^a = \sqrt{|\kappa m|} E_I^a, \qquad I = 3, 4, 5, 6, \tag{15}$$

$$(\mathbf{II})\ X_I^a = \sqrt{|\kappa m|} E_I^a, \qquad I = 7, 8, 9, \overline{10}. \tag{16}$$

The $\{E_3^a, E_4^b, E_5^c, E_7^d\}$ and $\{E_7^a, E_8^b, E_9^c, E_{\overline{10}}^d\}$ are two sets of four dimensional orthonormal frames of $\mathbf{R}^4$ such that

$$E_I^a E_J^a = \delta_{IJ}, \qquad E_I^a E_I^b = \delta^{ab}, \qquad f^{abcd} E_I^a E_J^b E_K^c E_L^d = \mathrm{sgn}(\kappa m)(\delta^{3456}_{IJKL} + \delta^{789\overline{10}}_{IJKL}). \tag{17}$$

These vacua have zero energy and are fully supersymmetric. In one of the broken phases, the $SO(4)$ gauge group and one $SO(4)$ of the global group get locked together into a single $SO(4)$ symmetry. The vacuum structure of each broken sector is the manifold of the $SO(4)$ gauge group, which is six dimensional. As the first homotopy group of the $SO(4)$ manifold is Z_2, one may expect Z_2 vortices in the broken phase. Of course, we can mod out the global gauge symmetry to get a point for each vacuum.

The superalgebra can be checked easily by the commutation relation

$$[\delta_\eta, \delta_\epsilon] X_I^a = 2a^\mu \partial_\mu X_I^a + 2 f^{abcd} \Lambda^{cd} X_I^b + 2m \sum_J S_{IJ} X_J^a, \tag{18}$$

where the parameters for the translation, gauge transformation, and the global $SO(4)$ rotations are given, respectively, as

$$a^\mu = i\bar{\eta}\Gamma^\mu \epsilon, \qquad \Lambda^{cd} = i\bar{\epsilon}\Gamma^\mu \epsilon A_\mu^{cd} - \frac{1}{2\kappa} i\bar{\eta}\Gamma_{JK}\epsilon X_J^c X_K^d, \qquad S_{IJ} = (\delta^{3456}_{IJKL} + \delta^{789\bar{1}0}_{IJKL}) i\bar{\eta}\Gamma_{KL}\epsilon. \tag{19}$$

As the supercharge is not invariant under the global $SO(4) \times SO(4)$, their charge is called a noncentral term. Indeed these noncentral terms are essential in reducing the supersymmetric representation of the massive particles in each phase.

The spectrum of the particles in the symmetric phase is trivial in the weak coupling constant limit $n \gg 1$. There are 32 scalar particles of mass m for the field X_I^a and 32 fermionic particles of mass m for the field Ψ. In the broken phase (I), the Higgs mechanism leads to massive vector bosons. The global rotation $SO(4)_1$ rotating 3, 4, 5, 6 variables and the gauge symmetry get soldered together. In the broken phase, all particles have mass $2m$. There are six massive vector bosons, 32 massive fermions, and 26 scalar bosons. As the theory is parity even, there are (3, 16, 26, 16, 3) of elementary particles for the spin $(1, 1/2, 0, -1/2, -1)$. Because of the Chern-Simons term, elementary particles in the symmetric phase may carry fractional statistics.

III. DOMAIN WALLS

Let us first consider the domain wall connecting the symmetric vacuum to the asymmetric vacuum of the first broken phase (I). As we assume that $X_{7,8,9,\overline{10}} = 0$, we introduce a "superpotential"

$$W = \frac{m}{2} \sum_{I=3,4,5,6} (X_I^a)^2 - \frac{1}{\kappa} f^{abcd} X_3^a X_4^b X_5^c X_6^d, \tag{20}$$

and then the bosonic potential becomes $U = |W_I^a|^2/2$ with

$$W_I^a = \frac{\partial W}{\partial X_I^a} = m X_I^a - \frac{1}{6\kappa} f^{abcd} \delta^{3456}_{IJKL} X_J^b X_K^c X_L^d. \tag{21}$$

The energy density along the wall becomes

$$E = \frac{1}{2} \int dy (D_y X_I^a - \beta W_I^a)^2 + \beta \mathcal{T}, \tag{22}$$

where $y = x^2$, $\beta = \pm 1$, and

$$\mathcal{T} = \int dy \partial_y W |\kappa m^2|. \tag{23}$$

Thus the tension of the domain wall becomes

$$|\mathcal{T}| = |\kappa m^2|. \tag{24}$$

The BPS equation is

$$D_y X_I^a - \beta W_I^a = 0. \tag{25}$$

We assume $\kappa, m > 0$ for convenience, and use the ansatz

$$A_2^{ab} = 0, \qquad X_I^a = \sqrt{\kappa m} f(y) \mathrm{diag}(1, 1, 1, 1), \tag{26}$$

where the row indices are I and the column indices are a. The BPS equation becomes

$$\partial_y f - \beta m (f - f^3) = 0, \tag{27}$$

whose solution is

$$f = \sqrt{\frac{1+\beta\tanh(my)}{2}}. \quad (28)$$

One can also consider the simple generalization of the above domain wall which connects the symmetric phase to the second broken vacuum (**II**). Clearly, both domain wall solutions are 1/2 BPS configurations. The above configuration is 1/2 BPS, as the invariant condition for $\delta\Psi$ is

$$\Gamma_{23456}\epsilon = \epsilon. \quad (29)$$

One can wonder whether there is any domain wall configuration which interpolates two broken phases. Indeed, we can find an interpolating configuration which satisfies the first order equations, but not the BPS as the supersymmetric condition is not satisfied. We combine the above ansatz and

$$X_I^a = g(y)\sqrt{\kappa m}\,\mathrm{diag}(1,1,1,1), \qquad I = 7, 8, 9, \overline{10}. \quad (30)$$

We impose the wrong BPS condition on the epsilon $\Gamma_{23456}\epsilon = \Gamma_{2789\overline{10}}\epsilon = \epsilon$ and $(\Gamma_{37}+\Gamma_{48}+\Gamma_{59}+\Gamma_{6\overline{10}})\epsilon = 0$, to get the wrong BPS equations

$$\begin{aligned} f' + m(f^3 - f + fg^2) &= 0, \\ -g' + m(g^3 - g + gf^2) &= 0, \end{aligned} \quad (31)$$

whose 1-parameter family of the solutions are

$$\begin{aligned} f^2 &= \frac{(1+a)(1+\tanh my)^2}{4(1+a\tanh^2 my)}, \\ g^2 &= \frac{(1+a)(1-\tanh my)^2}{4(1+a\tanh^2 my)}. \end{aligned} \quad (32)$$

We expect that the two walls are repulsive, and so the above configuration will not remain static in time.

IV. *Q*-BALLS, VORTICES, AND SUPERTUBES

As the 3, 4, 5, 6 particles can attract each other and condense into one of the broken phases, we expect that there can be a lump of these 3, 4, 5, 6 particles or, equally, lumps of the 7, 8, 9, $\overline{10}$ particles. Because of the noncentral terms, one may have BPS *q*-balls or nontopological solitons carrying *R* charges. Indeed, we will see here that there exist *q*-balls in the symmetric phase and vortices in the broken phase, whose equations are identical but with different boundary conditions.

We expect that spatial coordinates and $X_{3,4,5,6}$ get involved and $X_{7,8,9,\overline{10}} = 0$. We thus introduce a 1/4 BPS condition on the spinor parameter such as

$$\Gamma_{1234}\epsilon = -\alpha\epsilon, \qquad \Gamma_{1256}\epsilon = -\beta\epsilon, \quad (33)$$

where $\alpha, \beta = \pm 1$. This implies that $\Gamma_{3456}\epsilon = -\alpha\beta\epsilon$. The variation of the spinor field vanishes if the time-derivative parts and the potential parts are matched so that

$$\begin{aligned} D_0X_3^a + \beta W_4^a &= 0, \qquad D_0X_4^a - \beta W_3^a = 0, \\ D_0X_5^a + \alpha W_6^a &= 0, \qquad D_0X_6^a - \alpha W_5^a = 0, \end{aligned} \quad (34)$$

and the spatial derivatives are matched so that

$$\begin{aligned} D_1X_3^a - \alpha D_2X_4^a &= 0, \qquad D_2X_3^a + \alpha D_1X_4^a = 0, \\ D_1X_5^a - \beta D_2X_6^a &= 0, \qquad D_2X_5^a + \beta D_1X_6^a = 0. \end{aligned} \quad (35)$$

We can go over the energy and find a BPS bound on the energy,

$$\mathcal{E} \geq m(\beta R_{34} + \alpha R_{56}) = m(|R_{34}| + |R_{56}|), \quad (36)$$

once we use the Gauss law and the signs α and β are chosen so that the equality in the right side holds. This bound is saturated by the configurations which satisfy Eqs. (35). For the α, $\beta = 1$, the total R charge for the BPS configurations becomes

$$R_{34} + R_{56} = m\sum_{I=3,4,5,6}(X_I^a)^2 - \frac{4}{\kappa}f^{abcd}X_3^aX_4^bX_5^cX_6^d, \quad (37)$$

which vanishes for the symmetric phase and the broken phase.

To understand the 1/4 BPS configuration, let us use the ansatz

$$X_I^a = \begin{pmatrix} \phi_1 & -\phi_2 & 0 & 0 \\ \phi_2 & \phi_1 & 0 & 0 \\ 0 & 0 & \phi_1 & -\phi_2 \\ 0 & 0 & \phi_2 & \phi_1 \end{pmatrix}, \quad \tilde{A}_\mu^{ab} \equiv \begin{pmatrix} 0 & -A_\mu & 0 & 0 \\ A_\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & -A_\mu \\ 0 & 0 & A_\mu & 0 \end{pmatrix}, \quad (38)$$

where the row indices for X_I^a are I and the column indices a. Introducing a complex scalar $\phi = \phi_1 + i\phi_2$, the above 1/4 BPS equations and the Gauss law reduce to

$$(D_1 + iD_2)\phi = 0, \qquad F_{12} = \frac{2}{\kappa^2}|\phi|^2(m\kappa - |\phi|^2), \quad (39)$$

where $D_i\phi = \partial_i\phi - iA_i\phi$ and $F_{12} = \partial_1A_2 - \partial_2A_1$. This is the BPS equation for *q*-balls and vortices in the self-dual Chern-Simons Higgs theory with fractional spin. Indeed, the angular momentum for the BPS configuration is

$$\begin{aligned} J_{12} &= -\int d^2x(x_1D_0X_I^aD_2X_I^a - x_2D_0X_I^aD_1X_I^a) \\ &= \int d^2x x_i\partial_iW. \end{aligned} \quad (40)$$

Q-balls can have vortices in the interior region. In particular, one can show that the angular momentum for *q*-balls is

$$J_{12} = \frac{(R_{34} + R_{56})^2}{4\pi\kappa}, \tag{41}$$

which shows that q-balls can carry fractional angular momentum [21]. (For the vortices, there would be a sign flip.) This makes the details of the representation of the superalgebra interesting.

As in Ref [22], the q-balls and vortices in the large-charge limit can be regarded as supertubes in the symmetric phase or the broken phase. For this interpretation, it is important that the domain wall interpolating the symmetric phase and one of the broken phases attracts the R charges to the wall and forms a composite object, and a novel BPS bound on the domain wall is found. We show that this is true again for the theory under consideration.

We can consider additional R charges on the domain walls. Let us again impose the supersymmetric conditions

$$e^{\theta\Gamma_{134}}\Gamma_{1234}\epsilon = -\alpha\epsilon, \qquad e^{\theta\Gamma_{134}}\Gamma_{1256}\epsilon = -\beta\epsilon, \tag{42}$$

where α, $\beta = \pm 1$. This leads again to $\Gamma_{3456}\epsilon = -\alpha\beta\epsilon$. Here the angle θ is arbitrary and will be fixed for BPS configurations. Note that $\theta = 0$ is the BPS condition for q-balls and vortices, and $\theta = \pi/2$ is that for the domain wall with R charges. The supersymmetric BPS equations for the $1/4$ BPS domain wall carrying R charges consist of eight equations as follows:

$$\begin{aligned}
&D_0X_3^a + \beta W_4^a\cos\theta - \alpha D_1X_3^a\sin\theta = 0,\\
&D_0X_4^a - \beta W_3^a\cos\theta - \alpha D_1X_4^a\sin\theta = 0,\\
&D_0X_5^a + \alpha W_6^a\cos\theta - \alpha D_1X_5^a\sin\theta = 0,\\
&D_0X_6^a - \alpha W_5^a\cos\theta - \alpha D_1X_6^a\sin\theta = 0,\\
&D_2X_3^a + \alpha D_1X_4^a\cos\theta - \beta W_3^a\sin\theta = 0,\\
&D_2X_4^a - \alpha D_1X_3^a\cos\theta - \beta W_4^a\sin\theta = 0,\\
&D_2X_5^a + \beta D_1X_6^a\cos\theta - \beta W_5^a\sin\theta = 0,\\
&D_2X_6^a - \beta D_1X_5^a\cos\theta - \beta W_6^a\sin\theta = 0.
\end{aligned} \tag{43}$$

Once these equations are satisfied, the energy density integrated along the x^2 direction becomes

$$\mathcal{E} = m(-\beta R_{34} - \alpha R_{56})\cos\theta + (\alpha\mathcal{P} + \beta\mathcal{T})\sin\theta, \tag{44}$$

where the linear momentum density and the signed wall tension are, respectively,

$$\mathcal{P} = \int dy D_0X_I^aD_1X^a, \tag{45}$$

$$\mathcal{T} = \int dy\partial_y W. \tag{46}$$

As argued before, $|\mathcal{T}| = \kappa m^2$. The BPS bound would be

$$\mathcal{E} \geq \sqrt{m^2(\beta R_{34} + \alpha R_{56})^2 + (\beta\mathcal{T} + \alpha\mathcal{P})^2} \tag{47}$$

for all α, β. Indeed, when $|\mathcal{T}| = |\mathcal{P}|$, we can choose α and β so that $\beta\mathcal{T} + \alpha\mathcal{P} = 0$, so that the energy density along the wall is just given by the R charges, and there is a momentum flow along the wall which is given by the wall tension. This is exactly the supertube condition. In the supertube limit ($\sin\theta = 0$), the above BPS equations become those for the $1/4$ BPS q-balls and vortices.

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APPENDIX: MASS DEFORMATION WITH LESS SUPERSYMMETRIES

One can introduce mass deformations which partially break supersymmetry. Instead of the single mass term in (7), we introduce three mass parameters m, m' m'' so that the fermionic mass term becomes

$$\mathcal{L}_{fm} = -\frac{i}{2}\bar{\Psi}^a(m\Gamma_{3456} + m'\Gamma_{3478} + m''\Gamma_{349\overline{10}})\Psi^a. \tag{A1}$$

Again, only the fermionic supersymmetric transformation gets modified by the additional expression

$$\delta_m\Psi^a = (m\Gamma_{3456} + m'\Gamma_{3478} + m''\Gamma_{349\overline{10}})\Gamma_I\epsilon X_I^a. \tag{A2}$$

We impose three constraints on the ϵ parameter,

$$\begin{aligned}
\Gamma_{5678}\epsilon = -\alpha\epsilon, \qquad \Gamma_{569\overline{10}}\epsilon = -\beta\epsilon,\\
\Gamma_{789\overline{10}}\epsilon = -\alpha\beta\epsilon.
\end{aligned} \tag{A3}$$

Only two of them are independent, and so the number of supersymmetries is reduced to four. (If $m'' = 0$, the number would be eight.) We then introduce the following bosonic interactions $\mathcal{L}_{\text{deformed}} = \mathcal{L}_{\text{bm}} + \mathcal{L}_{\text{pot}}$ for supersymmetric completion,

$$\begin{aligned}
\mathcal{L}_{\text{bm}} &= -\frac{1}{2}m_{IJ}^2X_I^aX_J^a,\\
\mathcal{L}_{\text{pot}} &= \frac{4m}{\kappa}f_{abcd}(X_3^aX_4^bX_5^cX_6^d + X_7^aX_8^bX_9^cX_{\overline{10}}^d)\\
&\quad + \frac{4m'}{\kappa}f_{abcd}(X_3^aX_4^bX_7^cX_8^d + X_5^aX_6^bX_9^cX_{\overline{10}}^d)\\
&\quad + \frac{4m''}{\kappa}f_{abcd}(X_3^aX_4^bX_9^cX_{\overline{10}}^d + X_5^aX_6^bX_7^cX_8^d),
\end{aligned} \tag{A4}$$

where the mass matrices for the bosonic fields X_I^a are diagonal and given by

$$
\begin{aligned}
(m^2)_{33} &= (m^2)_{44} = (m + \alpha m' + \beta m'')^2, \\
(m^2)_{55} &= (m^2)_{66} = (m - \alpha m' - \beta m'')^2, \\
(m^2)_{77} &= (m^2)_{88} = (m - \alpha m' + \beta m'')^2, \\
(m^2)_{99} &= (m^2)_{\overline{10}\,\overline{10}} = (m + \alpha m' - \beta m'')^2.
\end{aligned} \tag{A5}
$$

Our approach for the mass deformation seems to have some analogy with the mass deformation of the supergravity solution for $AdS_4 \times S^7$ geometry in Ref. [25].

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