Magnetized baryonic matter in holographic QCD

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We investigate the properties of the Sakai-Sugimoto model at finite magnetic field and baryon chemical potentials. We show that in a finite magnetic field, there exists a spatially homogeneous configuration carrying a finite-baryon number density. At a low magnetic field and baryon chemical potential, the equation of state of the matter coincides with that obtained from the chiral perturbation theory Lagrangian with an anomalous term. We discuss the behavior of the system at larger magnetic fields.

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I. INTRODUCTION

Recently, gauge/gravity duality [1–3] has been used extensively to investigate properties of strongly coupled gauge theories at finite temperature and/or density. The popularity of this method is due to its ability to calculate in the strong coupling regime. The main disadvantage is that the models that can be solved, for example, the $\mathcal{N} = 4$ super Yang-Mills (SYM) theory, do not coincide with QCD, typically containing additional degrees of freedoms like adjoint fermions or scalars.

While finite-temperature $\mathcal{N} = 4$ SYM plasma has many features reminiscent of QCD plasma at temperatures not too large compared to the deconfinement temperature, it is more difficult to construct a holographic model of cold nuclear or quark matter. One problem is that at large N_c, nuclear matter is a crystal instead of a liquid. This fact finds reflection in the Sakai-Sugimoto model [4], where baryons are 5D instanton particles and nuclear matter is a crystal of such instantons [5], which is necessarily inhomogeneous [6].

In this paper, we investigate the possible gravity dual of magnetized nuclear matter. In a recent study [7], it was found that in the chiral limit of massless quarks, at any magnetic field the ground state of finite-density matter is not a crystal, but a spatially homogeneous phase. At low density such a phase is characterized by a finite gradient of the π^0 field, $\nabla\pi^0 \neq 0$ [in the parametrization where the chiral condensate is proportional to $\exp(i\pi^0\tau^3/f_\pi)$]. At finite quark masses this state becomes a stack of π^0 domain walls. The π^0 domain wall is locally stable in magnetic fields stronger than B_0, and is energetically more favorable than nuclear matter above a magnetic field B_1, where both B_0 and B_1 vanish in the chiral limit.

The treatment of Ref. [7] relies on the use of chiral perturbation theory, including the appropriate Wess-Zumino-Witten (WZW) term in the presence of an electromagnetic field and a baryon number chemical potential. It is valid only for sufficiently small magnetic field and baryon chemical potential.

In this paper, we search for a similar solution in the Sakai-Sugimoto model of holographic QCD [4]. The Sakai-Sugimoto model is an application of the AdS/CFT conjecture involving a system of N_f D8–$\overline{\mathrm{D8}}$ probe brane pairs in a D4 brane background of type IIA string theory. The model exhibits chiral symmetry breaking and thereby reproduces much of the low-energy physics of massless QCD, such as the octet of pseudoscalar Nambu-Goldstone bosons. Including the Chern-Simons (CS) term of the probe brane action is equivalent to including the effects of the axial anomaly on the field theory side. While the model has been used to investigate properties of the vacuum and the thermal state (with zero chemical potentials) in an external magnetic field [8,9], the case when both the magnetic field and the baryon chemical potentials are nonzero has not been considered.

Because the Sakai-Sugimoto model incorporates the axial anomaly into a theory of massless pions, one would expect to find, at least at small magnetic fields and baryon chemical potentials, a solution similar to the one found in [7]. The purpose of this paper is to demonstrate that solution.

We found, as expected, that at low B the results of Ref. [7] are reproduced. At larger values of the magnetic field, the quadratic approximation to the Dirac-Born-Infeld (DBI) action of the probe branes can no longer be trusted. However, we can still consider the quadratic action at large B as a bottom-up AdS/QCD theory [10–12]. In this sense, we unexpectedly discovered that in the opposite limit of large B the zero-temperature thermodynamics of matter with finite-baryon density is identical to the thermodynamics of free quarks, which fill energy levels in the lowest Landau level. It is rather surprising given that the supergravity limit corresponds to the strong coupling regime in field theory.

In Sec. II, the necessary facets of the Sakai-Sugimoto model are reviewed. In Sec. III, the domain wall solution is presented. We give interpretations of the result in Sec. IV. Section V provides a concluding discussion.

II. REVIEW OF THE SAKAI-SUGIMOTO MODEL

In [13], a way to holographically model nonsupersymmetric pure Yang-Mills theory was presented. Although the initial motivation of the model involved the M-theory duality on $\mathrm{AdS}_7 \times S_4$, Witten argued that it could equivalently be described as the background of a stack of N_c D4 branes in type IIA supergravity, where one of the directions parallel to the D4 branes is compactified into a circle. Antiperiodic boundary conditions around the circle are imposed on the fermionic fields, giving the fermions a mass and breaking the supersymmetry. The scalar fields also acquire a mass at the one-loop level, leaving the $\mathrm{SU}(N_c)$ vector field as the only massless field in the theory, and thus reproducing nonsupersymmetric pure Yang-Mills theory at energies small compared to the Kaluza-Klein scale M_{KK}.

Sakai and Sugimoto added massless flavor to the theory by considering the addition of N_f D8–$\overline{\mathrm{D8}}$ probe branes to the background [4], where the probe branes are transverse to the circle of compactification. The essential idea of adding flavor to holographic systems via probe branes [14] is that if $N_f \ll N_c$, the backreaction of the probe branes on the geometry can be neglected, and the probe brane action consists simply of the Dirac-Born-Infeld action in the original background, plus the relevant Chern-Simons (CS) terms.

There is a $\mathrm{U}(N_f) \times \mathrm{U}(N_f)$ gauge symmetry living on the probe brane pairs, which provides a global chiral symmetry on the field theory side. The geometry of the D4 branes is cigar-shaped, and one finds from analysis of the DBI action that the D8–$\overline{\mathrm{D8}}$ branes merge at some value of the radial coordinate. Thus, the $\mathrm{U}(N_f) \times \mathrm{U}(N_f)$ symmetry is broken to a single $\mathrm{U}(N_f)$; this is a holographic manifestation of chiral symmetry breaking. It was shown in Refs. [4,15] that the DBI action of the probe branes reproduces much of the low-energy physics of QCD, including the chiral Lagrangian and qualitative features of the meson spectrum. Inclusion of the CS term for the probe branes incorporates the effects of the axial anomaly into the low-energy theory.

It is possible to construct baryons in this model [16–18]. Witten's baryon vertex appears as a D4 brane wrapping the S^4. Strings stretching between the D4 brane and the D8 branes will source the gauge field living on the D8s. The baryon number is then given in terms of the $\mathrm{SU}(N_f)$ valued field strengths F living on the probe branes as

$$N_B = \frac{1}{32\pi^2} \int d^3x dz \epsilon_{MNPQ} \operatorname{tr}[F_{MN} F_{PQ}], \qquad (1)$$

where $M, N, P, Q = 1, 2, 3, z$. The baryon number integral also shows up in the probe brane action in the CS term coupled to the time-component of the U(1) part of the gauge field, $\hat{A}$:

$$S_{\mathrm{CS}} \supset \frac{N_c}{64\pi^2} \epsilon_{MNPQ} \int d^4x dz \hat{A}_0 \operatorname{tr}[F_{MN} F_{PQ}]. \qquad (2)$$

Thus, $\hat{A}_0$ acts as a source for baryon number, and, in order to turn on a finite chemical potential, we will consider solutions with nontrivial $\hat{A}_0$.

We also wish to turn on an external magnetic field that couples to our flavor degrees of freedom. There is no proper U(1) gauge field in our theory. However, we can simulate the effects of an external field by weakly gauging U(1) subgroups of the global chiral symmetry. In real QCD with $N_f = 2$ flavors, the electric charge is related to both the third component of isospin and to the baryon number, $Q = I_3 + \frac{1}{2}B$. Therefore, in order to properly introduce an electromagnetic field into Sakai-Sugimoto, one would have to gauge both one of the components of I_3 and the baryon number potential. However, for simplicity, in this paper we will only consider gauging of the isospin component by looking for solutions with a nonzero $F^{(3)}$.

III. SOLUTION WITH FINITE MAGNETIC FIELD AND CHEMICAL POTENTIAL

We will first establish our conventions and notation.

In this paper, we will consider the case of two flavor D8 branes. The branes and the antibranes will be maximally separated around the circle of compactification, as in the original treatment by Sakai and Sugimoto [4].

The effective action of the probe branes is written in terms of a $\mathrm{U}(2) = \mathrm{U}(1)_B \times \mathrm{SU}(2)$ five-dimensional gauge field. It will often be convenient to distinguish the fifth spacetime index z from the four-dimensional boundary indices. Thus, uppercase roman letters $M, N\ldots$ run over all five spacetime directions, whereas lowercase greek letters $\mu, \nu\ldots$ run only over 0, 1, 2, 3. We will use a combination of form and component notation to describe the gauge fields. The U(2)-valued form fields are given by

$$\mathcal{A} = \mathcal{A}_M dx^M, \qquad (3)$$

$$\mathcal{F} = d\mathcal{A} + i\mathcal{A} \wedge \mathcal{A}. \qquad (4)$$

The gauge fields can be decomposed into the U(1) part $\hat{A}$ and the SU(2) part A as

$$\mathcal{A} = A + \frac{\hat{A}}{2} = A^{(i)} T^i + \frac{\hat{A}}{2}\mathbf{1}, \qquad (5)$$

where the T^i are the SU(2) generators normalized as $\operatorname{tr}[T^i T^j] = \frac{1}{2}\delta^{ij}$ and the numerical factor on the U(1) piece is to ensure that the U(1) generator is normalized in the same manner. Likewise, $\mathcal{F}$ is decomposed in terms of F and $\hat{F}$:

$$\mathcal{F} = F^{(i)}T^i + \frac{\hat{F}}{2}. \tag{6}$$

In component form,

$$F^{(i)}_{MN} = \partial_M A^{(i)}_N - \partial_N A^{(i)}_M - \epsilon^{ijk} A^{(j)}_M A^{(k)}_N, \tag{7}$$

$$\hat{F}_{MN} = \partial_M \hat{A}_N - \partial_N \hat{A}_M. \tag{8}$$

The effective theory of the branes is described by the action [4]

$$S = S_{\rm YM} + S_{\rm CS}, \tag{9a}$$

$$S_{\rm YM} = -\kappa \int d^4x dz\, \mathrm{tr}\Big[\frac{1}{2} h(z) \mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} + M^2_{\rm KK} k(z) \mathcal{F}_{\mu z}\mathcal{F}^{\mu z}\Big] + \mathcal{O}(\mathcal{F}^3), \tag{9b}$$

$$S_{\rm CS} = \frac{N_c}{24\pi^2} \int \mathrm{tr}\Big[\mathcal{A}\mathcal{F}^2 - \frac{i}{2}\mathcal{A}^3\mathcal{F} - \frac{1}{10}\mathcal{A}^5\Big], \tag{9c}$$

where

$$\kappa = \frac{\lambda N_c}{216\pi^3}, \tag{10}$$

and $h(z)$ and $k(z)$ are defined as

$$h(z) = (1 + z^2)^{-1/3}, \qquad k(z) = 1 + z^2. \tag{11}$$

In "bottom-up" AdS/QCD models where the chiral symmetry is spontaneously broken by the boundary conditions at the IR brane, the action has the same form as (9), but with different functions $h(z)$ and $k(z)$. For example, in the model considered in [10], $h(z) = \mathrm{const}$ and $k(z) \sim \cosh(bz)$, where b is a constant.

The Yang-Mills action (9b) arises from expanding the DBI action for the probe branes to second order in field strengths. For large values of the field strength, this expansion will no longer be valid. We will see later that turning on a magnetic involves setting $F^{(3)}_{12} = B$. It can be shown that the cubic terms involving B can be dropped as long as B satisfies the inequality

$$\frac{27\pi}{2\lambda} \frac{B}{M^2_{\rm KK}} \ll 1. \tag{12}$$

Alternatively, the action (9b) can be interpreted as a bottom-up effective action. From this perspective, B is allowed to be arbitrarily large.

In terms of the U(1) and SU(2) pieces, the action reads [16]

$$S_{\rm YM} = -\frac{\kappa}{2} \int d^4x dz \Big[\frac{h(z)}{2}(F^{(i)}_{\mu\nu}F^{(i)\mu\nu} + \hat{F}_{\mu\nu}\hat{F}^{\mu\nu}) + M^2_{\rm KK} k(z)(F^{(i)}_{\mu z}F^{(i)\mu z} + \hat{F}_{\mu z}\hat{F}^{\mu z})\Big] \tag{13}$$

$$S_{\rm CS} = \frac{N_c}{24\pi^2}\int\Big(\frac{3}{2}\hat{A}\,\mathrm{tr}F^2 + \frac{1}{4}\hat{A}\hat{F}^2 + \frac{1}{2}d\Big(\hat{A}\,\mathrm{tr}\Big[2FA - \frac{i}{2}A^3\Big]\Big)\Big). \tag{14}$$

In [16], a localized soliton solution is found whose size is $\mathcal{O}(1/\sqrt{\lambda})$. In this case, the scaling of the gauge field solution allows the equations of motion to be expanded as a series in λ. In our case, there is no such scaling in our solution, so we must work with the full equations of motion. Variation of the above action leads to the equations

$$\frac{8}{\alpha}[h(z)\partial_M \hat{F}^{MN} + M^2_{\rm KK}\partial_z(k(z)\hat{F}^{zN})] = -\epsilon^{N\nu\rho\sigma\lambda}(F^{(i)}_{\nu\rho}F^{(i)}_{\sigma\lambda} + \hat{F}_{\nu\rho}\hat{F}_{\sigma\lambda}), \tag{15}$$

$$\frac{8}{\alpha}M^2_{\rm KK}k(z)\partial_M \hat{F}^{Mz} = -\epsilon^{z\nu\rho\sigma\lambda}(F^{(i)}_{\nu\rho}F^{(i)}_{\sigma\lambda} + \hat{F}_{\nu\rho}\hat{F}_{\sigma\lambda}), \tag{16}$$

$$\frac{4}{\alpha}[h(z)D_M F^{(i)MN} + M^2_{\rm KK}\partial_z(k(z)F^{(i)zN})] = -\epsilon^{N\nu\rho\sigma\lambda}\hat{F}_{\nu\rho}F^{(i)}_{\sigma\lambda}, \tag{17}$$

$$\frac{4}{\alpha}M^2_{\rm KK}k(z)D_M F^{(i)Mz} = -\epsilon^{z\nu\rho\sigma\lambda}\hat{F}_{\nu\rho}F^{(i)}_{\sigma\lambda}, \tag{18}$$

where the covariant derivative acting on field strengths is

$$D_\mu F^{(i)\mu\nu} = \partial_\mu F^{(i)\mu\nu} + \epsilon^{ijk}A^{(j)}_\mu F^{(k)\mu\nu}. \tag{19}$$

and we have defined, for future convenience,

$$\alpha = \frac{N_c}{16\pi^2\kappa} = \frac{27\pi}{2\lambda}. \tag{20}$$

We wish to turn on a magnetic field and a baryon number chemical potential and to look for a solution homogeneous in Minkowski space. As discussed above, in order to turn on the magnetic field we will assume a nonzero $F_{12}^{(3)}$, and to generate a baryon number chemical potential we will assume a nonzero $\hat{A}_0$. Recall that in terms of the five-dimensional gauge fields, the baryon number is given by (1),

$$N_B = \frac{1}{64\pi^2}\int d^3xdz\epsilon^{MNPQ}F_{MN}^{(i)}F_{PQ}^{(i)} \tag{21}$$

where M, N, P, Q are not zero. We see that in order to have nonzero baryon number, if $F_{12}^{(3)}$ is nonzero then $F_{3z}^{(3)}$ must also be nonzero. Because we want a solution that is homogeneous in the four-dimensional coordinates, all these quantities will only depend on z. Let us assume all other fields are zero. To reduce clutter, we will henceforth denote $F^{(3)}$ simply by F.

The second and fourth equations of motion are then trivially satisfied, whereas the first and third become

$$\frac{M_{\mathrm{KK}}^2}{\alpha}\partial_z(k(z)\hat{F}^{z0}) = -F^{12}F^{3z} \tag{22}$$

$$\frac{M_{\mathrm{KK}}^2}{\alpha}(\partial_z(k(z)F^{3z}) = -\hat{F}^{z0}F^{12}. \tag{23}$$

The covariant derivative has reduced to a partial derivative because $A^{(j)}$ is zero for $j = 1, 2$.

We notice that due to the Bianchi identity and the requirement that fields depend only on z, F_{12} has to be a constant, $F_{12} = B$. The equations can be solved exactly for any function $k(z)$. For $k(z) = 1 + z^2$ as in the Sakai-Sugimoto model, the general solution is

$$F^{3z} = c_1\frac{\exp(\tilde{B}\arctan z)}{1+z^2} + c_2\frac{\exp(-\tilde{B}\arctan z)}{1+z^2}, \tag{24}$$

$$\hat{F}^{z0} = -c_1\frac{\exp(\tilde{B}\arctan z)}{1+z^2} + c_2\frac{\exp(-\tilde{B}\arctan z)}{1+z^2}. \tag{25}$$

where $\tilde{B} = \alpha B/M_{\mathrm{KK}}^2$.

In the presence of a finite chemical potential, the thermodynamic ground state will minimize the free energy $H - \mu N_B$. Minimizing this quantity will give us the values of c_1 and c_2 in the ground state. Under our Ansatz, the baryon number (1) reduces to

$$N_B = \frac{1}{8\pi^2}\int d^3xdzBF^{3z} = \frac{V}{4\pi^2\alpha}M_{\mathrm{KK}}^2(c_1+c_2)\sinh\frac{\pi\tilde{B}}{2} \tag{26}$$

where V is the volume of the three dimensional space.

The energy of the configuration is given by

$$\frac{\kappa}{2}\int d^3xdz\bigg(\frac{1}{2}h(z)B^2 + M_{\mathrm{KK}}^2k(z)(F_{3z}F^{3z} - \hat{F}_{z0}\hat{F}^{z0})\bigg). \tag{27}$$

The piece proportional to B gives an infinite contribution. This is the expected divergent energy of a space-filling magnetic field. This piece is independent of the constants c_1 and c_2, so does not affect our minimization problem.

Performing the integrals involved in the energy we write

$$H - \mu N_B = VM_{\mathrm{KK}}^2\sinh\frac{\pi\tilde{B}}{2}\bigg(\frac{2\kappa}{\tilde{B}}\cosh\frac{\pi\tilde{B}}{2}(c_1^2+c_2^2) - \frac{\mu}{4\alpha\pi^2}(c_1+c_2)\bigg). \tag{28}$$

It is simple to minimize this with respect to c_1 and c_2. The solution is

$$c_1 = c_2 = \frac{\mu B}{16\pi^2\kappa M_{\mathrm{KK}}^2\cosh\frac{\pi\tilde{B}}{2}}. \tag{29}$$

Plugging into (24) and (25), we can write the ground state solutions entirely in terms of the parameters of the problem,

$$F^{3z} = \frac{27\pi}{\lambda N_c}\frac{\mu B}{M_{\mathrm{KK}}^2}\frac{\cosh(\frac{27\pi}{2\lambda}\frac{B}{M_{\mathrm{KK}}^2}\arctan z)}{\cosh(\frac{27\pi^2}{4\lambda}\frac{B}{M_{\mathrm{KK}}^2})(1+z^2)} \tag{30}$$

and

$$\hat{F}^{z0} = -\frac{27\pi}{\lambda N_c}\frac{\mu B}{M_{\mathrm{KK}}^2}\frac{\sinh(\frac{27\pi}{2\lambda}\frac{B}{M_{\mathrm{KK}}^2}\arctan z)}{\cosh(\frac{27\pi^2}{4\lambda}\frac{B}{M_{\mathrm{KK}}^2})(1+z^2)}. \tag{31}$$

The energy can also be found in terms of fundamental quantities by plugging in the values of c_1 and c_2 into our earlier expression for H. Doing so gives

$$\epsilon \equiv \frac{E}{V} = \frac{\mu^2 B}{4\pi^2 N_c}\tanh\bigg(\frac{27\pi^2}{4\lambda}\frac{B}{M_{\mathrm{KK}}^2}\bigg). \tag{32}$$

One can express the energy density in terms of the baryon number density n_B,

$$\epsilon = \pi^2 N_c\frac{n_B^2}{B}\coth\bigg(\frac{27\pi^2}{4\lambda}\frac{B}{M_{\mathrm{KK}}^2}\bigg). \tag{33}$$

The asymptotics of (33) at small B is

$$\epsilon = \frac{4\lambda N_c}{27}\frac{n_B^2 M_{\mathrm{KK}}^2}{B^2}. \tag{34}$$

and at large B is

$$\epsilon = \pi^2 N_c\frac{n_B^2}{B}. \tag{35}$$

Note, however, that in order to obtain the large B asymptotics we must assume that $B/(\lambda M_{\mathrm{KK}}^2) \gg 1$. This is precisely the limit in which we can no longer trust the quadratic approximation to the DBI action. Thus, in order to consider this limit, we must be thinking in the context of a bottom-up AdS/QCD model, where higher order terms in F are suppressed from the beginning.

IV. INTERPRETATION OF RESULTS

We now try to interpret our results for small and large magnetic field B. At small magnetic fields, one can use the chiral perturbation theory. The Hamiltonian describing the interaction of the π^0 field with the magnetic field and baryon chemical potential is [7]

$$H' \equiv H - \mu N_B = \int d\mathbf{x}\left(\frac{1}{2}(\nabla\pi^0)^2 - \frac{\mu}{4\pi^2 f_\pi}\boldsymbol{B}\cdot\nabla\pi^0\right). \tag{36}$$

The minimum of H' is achieved at

$$\nabla\pi^0 = \frac{\mu}{4\pi^2 f_\pi}\boldsymbol{B}, \tag{37}$$

at which the energy and baryon number densities are

$$\epsilon = \frac{\mu^2 B^2}{32\pi^4 f_\pi^2}, \qquad n_B = \frac{\mu B^2}{16\pi^4 f_\pi^2}. \tag{38}$$

The relationship between ϵ and n_B is

$$\epsilon = 8\pi^4 f_\pi^2 \frac{n_B^2}{B^2}. \tag{39}$$

Now using the value for the pion decay constant found in [4],

$$f_\pi^2 = \frac{\lambda N_c}{54\pi^4} M_{\mathrm{KK}}^2, \tag{40}$$

we reproduce the low-B asymptotics of Eq. (33) exactly.

We now show that at very large B the thermodynamic relation between ϵ and n_B, obtained in the approximation where one replaces the DBI action by the Maxwell action, approaches that of a free noninteracting gas. Consider a system of free, noninteracting quarks of two flavors u and d in an external magnetic field coupled to isospin. The charges of the quarks are $1/2$ and $-1/2$. In a magnetic field, the transverse motion is quantized. The lowest Landau level consist of two branches,

$$E = \pm k_z, \tag{41}$$

each having degeneracy in the transverse direction equal to $|e|B/(2\pi) = B/(4\pi)$.

A baryon chemical potential μ corresponds to a quark chemical potential $\mu_q = \mu/N_c$. The quarks fill energy levels below μ_q, leading to a nonzero baryon density. At small chemical potentials (or, equivalently, large magnetic fields) the filled energy levels are all in the lowest Landau level. The total baryon number is then

$$n_B = N_f \frac{B}{4\pi}\int_{-\mu_q}^{\mu_q}\frac{dk_z}{2\pi} = \frac{\mu B}{2\pi^2 N_c}. \tag{42}$$

Inverting the relation,

$$\mu = \frac{2\pi^2 N_c}{B} n_B, \tag{43}$$

and integrating over n_B, one finds the energy density as a function of the baryon number density

$$\epsilon = \pi^2 N_c \frac{n_B^2}{B}. \tag{44}$$

This coincides exactly with the thermodynamics of our bottom-up model at large B. Therefore, we conclude that at large B, the equation of state of matter at finite-baryon density is identical to that of free quarks.

Moreover, by redoing the previous calculation, one can show that this feature is independent of the choice of the function $k(z)$, given that the integral $\int_0^\infty dz k^{-1}(z)$ is convergent. This condition is satisfied in the model of [10] where $k(z) \sim \cosh(bz)$. Therefore, in bottom-up holographic models of QCD where the action contains only the Maxwell and Chern-Simons parts, and where chiral symmetry breaking is due to a boundary condition at $z = 0$, the equation of state at very high magnetic field approaches that of a free gas.

V. CONCLUSIONS

To conclude, we have found a solution to the field equations in the Sakai-Sugimoto model that corresponds to matter at finite-baryon density in an external magnetic field. In contrast to the case without a magnetic field, it is possible to write down a solution that is completely homogeneous in space. At small B and small chemical potential, the result can be obtained from the chiral Lagrangian with the Chern-Simons term. The solution continues to exist for any value of B, although for larger values of B the solution can no longer be viewed as arising from the full Sakai-Sugimoto model. We can, however, interpret our solution at large B as arising from a bottom-up AdS/QCD model. At large B the system behaves, from the point of view of zero-temperature thermodynamics, as a system of free quarks.

How can one explain the latter fact? Right now we have only some vague ideas of how it can be understood. In magnetic fields, the fermions move in Larmor orbits whose radius shrinks as $B \to \infty$. This fact may explain why interaction between quarks do not seem to play any role at large B.

Clearly, the situation should be investigated further. One question one can ask is whether the state is stable with respect to small perturbations. At small μ, the energy per baryon is small and the system is clearly more stable than the ordinary Skyrmion crystal. We leave the investigation of the stability of the system at large B and μ to future work.

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[1] J. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998); Int. J. Theor. Phys. **38**, 1113 (1999).
[2] S. S. Gubser, I. R. Klebanov, and A. M. Polyakov, Phys. Lett. B **428**, 105 (1998).
[3] E. Witten, Adv. Theor. Math. Phys. **2**, 253 (1998).
[4] T. Sakai and S. Sugimoto, Prog. Theor. Phys. **113**, 843 (2005).
[5] K. Y. Kim, S. J. Sin, and I. Zahed, J. High Energy Phys. 01 (2008) 002; arXiv:0712.1582.
[6] M. Rozali, H. H. Shieh, M. Van Raamsdonk, and J. Wu, J. High Energy Phys. 01 (2008) 053.
[7] D. T. Son and M. A. Stephanov, Phys. Rev. D **77**, 014021 (2008).
[8] O. Bergman, G. Lifschytz, and M. Lippert, J. High Energy Phys. 05 (2008) 007.
[9] C. V. Johnson and A. Kundu, arXiv:0803.0038.
[10] D. T. Son and M. A. Stephanov, Phys. Rev. D **69**, 065020 (2004).
[11] J. Erlich, E. Katz, D. T. Son, and M. A. Stephanov, Phys. Rev. Lett. **95**, 261602 (2005).
[12] L. Da Rold and A. Pomarol, Nucl. Phys. **B721**, 79 (2005).
[13] E. Witten, Adv. Theor. Math. Phys. **2**, 505 (1998).
[14] A. Karch and E. Katz, J. High Energy Phys. 06 (2002) 043.
[15] T. Sakai and S. Sugimoto, Prog. Theor. Phys. **114**, 1083 (2005).
[16] H. Hata, T. Sakai, S. Sugimoto, and S. Yamato, arXiv:hep-th/0701280.
[17] D. K. Hong, M. Rho, H-U. Yee, and P. Yi, Phys. Rev. D **76**, 061901 (2007).
[18] D. K. Hong, M. Rho, H-U. Yee, and P. Yi, J. High Energy Phys. 09 (2007) 063.
[19] O. Bergman, G. Lifschytz, and M. Lippert, arXiv:0806.0366.