PHYSICAL REVIEW D **78,** 066003 (2008)

Radion effective potential in brane gas cosmology

Jin Young Kim*

Department of Physics, Kunsan National University, Kunsan 573-701, Korea

(Received 3 July 2008; published 2 September 2008)

We consider a cosmological solution which can explain anisotropic evolution of spatial dimensions and the stabilization of extra dimensions in brane gas formalism. We evaluate the effective potentials, induced by brane gas, bulk flux and supergravity particles, which govern the sizes of the observed three and the extra dimensions. It is possible that the wrapped internal volume can oscillate between two turning points or sit at the minimum of the potential while the unwrapped three-dimensional volume can expand monotonically. Including the supergravity particles makes the effective potential steeper as the internal volume shrinks.

DOI: 10.1103/PhysRevD.78.066003 PACS numbers: 11.25.−w, 98.80.Cq

I. INTRODUCTION

The idea that the space-time might be more than four dimensions is being considered widely from particle physics to cosmology. Obviously string theory provided a strong motivation for considering higher dimensions. Recently the development of string theory has led us to diverse cosmological scenarios, for example, *D*-brane inflation, moduli inflation, cyclic and ekpyrotic scenarios, mirage cosmology, and string or brane gas cosmology. One of the primary goals of string cosmology is to achieve string compactification which can produce inflation successfully.

Early in the study of cosmology based on string theory, it was realized that the presence of a gas of strings plays an important role in the evolution of the Universe in Refs. [1,2]. They suggested a mechanism to generate dynamically the spatial dimensionality of space-time and to explain the problem of initial singularity. With the symmetry of string theory, called T-duality, space-time has the topology of a nine-dimensional torus, and its dynamics is driven by a gas of fundamental strings. In string theory the winding modes can annihilate the antiwinding modes if they meet. Once the windings are removed from some dimensions, these dimensions can expand. Since strings have $(1 + 1)$-dimensional world volumes, they can intersect efficiently in $2(1 + 1)$ space-time dimensions or less. Thus, three spatial dimensions can become large with a gas of strings.

A gas of fundamental strings was generalized to a gas of various branes to accommodate the development of *D*-branes in string theory [3–5]. Many studies on this issue of string cosmology with string or brane gas were followed (see [6] and references therein for comprehensive reviews). The key point of replacing string gas with brane gas is that a hierarchy of scales can be achieved between wrapped and unwrapped dimensions. It has been checked whether the unwrapped configuration of branes can generate the inflation successfully [7–10]. Also it is known that string or brane gases of purely winding modes are not enough to stabilize the extra dimensions. For example, in 11-dimensional supergravity, Easter *et al.* [11] have succeeded in producing anisotropic expansion by selecting a certain wrapping matrix. However the radions (scales of the extra dimensions) were not stabilized.

Stabilization of the radion has been the focus of research in string or brane gas cosmology [12–26]. One way to obtain the stabilization of the extra dimensions is to introduce bulk fields [27–30]. In the previous work of the author [31], it is shown that the extra dimensions can be stabilized by including a bulk Ramond-Ramond (RR) flux in the brane gas formalism. For the specific configuration of brane gas and RR-flux where effectively six-dimensional brane gas is wrapping the extra dimensions and 4-form RR-flux is in the unwrapped dimensions, the flux can cause a logarithmic bounce to the effective potential as the volume of the extra dimensions shrinks.

Considering the quantum aspect of the string or brane gas, there will be a large amount of energy when winding modes and antiwinding modes of branes annihilate each other. For example, string winding modes and antiwinding modes can annihilate into unwound closed string loops which can be treated as supergravity particles or radiation. Thus it will be interesting to see how the simplified stabilization mechanism by brane gas and flux can be modified if we include supergravity particles. The purpose of this paper is to extend the previous study by including supergravity particles.

II. BRANE GAS DYNAMICS WITH FLUX AND SUPERGRAVITY PARTICLES

We consider the ten-dimensional supergravity after the dilaton is stabilized. We start from the point that winding modes are annihilated in three spatial dimensions causing them to be free to expand while the brane gas remains in the extra six dimensions by the mechanism of Brandenberger and Vafa [1]. With a gas of branes only,

*jykim@kunsan.ac.kr

the extra dimensions will shrink to zero size and our assumptions of the brane gas cosmology are not valid anymore. To prevent this collapse we consider the RR-flux in the transverse dimensions. Thus the bulk effective action consists of graviton and gauge fields representing the four-form RR-flux. If we consider only the bosonic sector, the effective action can be written as [31]

$$S = \frac{1}{2\kappa^2}\int d^{D+1}x\sqrt{-g}\Big(R - \frac{1}{2\cdot 4!}F_4^2\Big), \tag{1}$$

where R is the scalar curvature, F_4 is the field strength of the bulk gauge field, $D = 9$, and κ is the ten-dimensional gravitational constant $\kappa^2 = \frac{1}{M_*^{D-1}}$ with M_* being the $(D+1)$-dimensional unification scale.

The gravitational equations of motion are given by

$$G^{MN} = -\kappa^2(T_g^{MN} + T_m^{MN}), \tag{2}$$

$$\nabla_M F^{MNIJ} = 0, \tag{3}$$

where T_g^{MN} is the energy-momentum tensor from the four-form RR-flux

$$T_g^{MN} = \frac{1}{12\kappa^2}\Big(F^M{}_{IJK}F^{NIJK} - \frac{1}{8}g^{MN}F_{IJKL}F^{IJKL}\Big), \tag{4}$$

and T_m^{MN} is the averaged energy-momentum tensor coming from all the other matter contributions. Also we have the Bianchi identity since F is an exact form

$$\nabla_{[I}F_{JKLM]} = 0. \tag{5}$$

We assume that background fields and matter sources are homogeneous within the dimensions where they exist. Then we can treat them as functions of time only. Considering the spatial section to be a D-dimensional torus T^D, we write the metric as

$$ds^2 = -dt^2 + \sum_{k=1}^{D} a_k^2(t)(dx^k)^2, \qquad (0 \le x_k \le 1). \tag{6}$$

The nonvanishing components of the Einstein tensor are

$$G^t{}_t = \frac{1}{2}\sum_{k\neq l}\frac{\dot a_k}{a_k}\frac{\dot a_l}{a_l}, \tag{7}$$

$$G^i{}_i = \sum_{k\neq i}\frac{\ddot a_k}{a_k} + \frac{1}{2}\sum_{k\neq l}\frac{\dot a_k}{a_k}\frac{\dot a_l}{a_l} - \sum_{k\neq i}\frac{\dot a_k}{a_k}\frac{\dot a_i}{a_i}. \tag{8}$$

As in [31], we assume that the RR-flux is confined to the $(3+1)$-dimensional submanifold

$$F^{\alpha\beta\gamma\delta} = \frac{\epsilon^{\alpha\beta\gamma\delta}}{\sqrt{-g_4}}F(t), \tag{9}$$

where the Greek indices belong to $\{0, 1, 2, 3\}$ and g_4 is the induced metric on the $(3+1)$-dimensional submanifold.

With this choice, the Bianchi identity is automatically satisfied, and the solution for $F(t)$ is given by

$$F(t) = \frac{2Qa_1a_2a_3}{V}, \tag{10}$$

where Q is an integration constant and V is the total spatial volume $V = \prod_{k=1}^{D} a_k$. Then the components of the energy-momentum tensor by the RR-flux are calculated as

$$(T_g)^t{}_t = -\frac{1}{\kappa^2}\Big(\frac{F(t)}{2}\Big)^2, \tag{11}$$

$$(T_g)^1{}_1 = (T_g)^2{}_2 = (T_g)^3{}_3 = -\frac{1}{\kappa^2}\Big(\frac{F(t)}{2}\Big)^2, \tag{12}$$

$$(T_g)^4{}_4 = \cdots = (T_g)^D{}_D = \frac{1}{\kappa^2}\Big(\frac{F(t)}{2}\Big)^2. \tag{13}$$

This corresponds to the energy-momentum tensor of a fluid with

$$\rho_g = \frac{1}{\kappa^2}\Big(\frac{F(t)}{2}\Big)^2, \qquad p_g^1 = p_g^2 = p_g^3 = -\rho_g, \qquad p_g^4 = \cdots = p_g^D = \rho_g. \tag{14}$$

For other sources of matter, first we consider the massless supergravity particles present in the early universe. The effect of this source can be expressed by a gas with energy density ρ_s and pressure p_s. We take the gas to be a homogeneous and isotropic perfect fluid with the equation of state $p_s = \rho_s/D$. The corresponding energy-momentum tensor is

$$(T_s)^t{}_t = -\rho_s, \tag{15}$$

$$(T_s)^1{}_1 = (T_s)^2{}_2 = \cdots = (T_s)^D{}_D = p_s. \tag{16}$$

If we assume that this energy-momentum tensor is covariantly conserved $\nabla_M T_s^{MN} = 0$, the energy density scales with time as

$$\rho_s(t) = \rho_s^0\Big(\frac{V_0}{V(t)}\Big)^{(D+1)/D}, \tag{17}$$

where ρ_s^0 and V_0 are the energy density and spatial volume at time t_0.

The second source of matter comes from a gas of branes, wrapped on the various cycles of the torus. The matter contribution of a single p-brane to the action, if the dilaton is stabilized, is represented by the Dirac-Born-Infeld (DBI) action

$$S_{\rm p} = -T_p\int d^{p+1}\xi\sqrt{-\det(\hat g_{\mu\nu} + \hat B_{\mu\nu} + 2\pi\alpha' F_{\mu\nu})}, \tag{18}$$

where T_p is the tension of the p-brane and $\hat g_{\mu\nu}$ is the induced metric to the brane

$$\hat g_{\mu\nu} = g_{MN}\frac{\partial X^M}{\partial\xi^\mu}\frac{\partial X^N}{\partial\xi^\nu}. \tag{19}$$

Here M, N are the indices of the $(D+1)$ dimensional bulk space-time and μ, ν are those of the brane. $\hat{B}_{\mu\nu}$ is the induced antisymmetric tensor field and $F_{\mu\nu}$ is the field strength tensor of gauge fields A_μ living on the brane. The fluctuations of the brane coordinates and other fields within the branes are negligible when the temperature is low enough and the radii is grown enough. So we neglect $\hat{B}_{\mu\nu}$ and $F_{\mu\nu}$ terms. Ignoring the running of the dilaton, we have absorbed the effect of the constant dilaton into the redefinition of brane tension so that the Einstein frame and the string frame are equivalent.

We start, after the thermal stage in the early universe by the mechanism of Brandenberger and Vafa, from the moment that three dimensions are completely unwrapped. We take the three dimensions in which the RR-flux exists as the unwrapped ones. The other $(D-3)$ dimensions are wrapped with gases of branes whose dimensions are less than or equal to $(D-3)$. Assuming each type of brane gas makes a comparable contribution, we consider a gas of effective $(D-3)$-branes whose tension we denote by T_{D-3}. Then the energy-momentum tensor for a gas of these branes can be written as

$$(T_B)^t{}_t = -\frac{T_{D-3}}{a_1a_2a_3}, \tag{20}$$

$$(T_B)^1{}_1 = (T_B)^2{}_2 = (T_B)^3{}_3 = 0, \tag{21}$$

$$(T_B)^4{}_4 = \cdots = (T_B)^D{}_D = -\frac{T_{D-3}}{a_1a_2a_3}. \tag{22}$$

Since the $SO(D)$ Poincare invariance is broken down to $SO(3)\times SO(D-3)$ by RR-flux and $(D-3)$-brane gas, we denote the scale factor of three-dimensional space by a and $(D-3)$-dimensional subspace by b. Then the density and pressure of the brane gas can be written as

$$\rho_B = \frac{T_{D-3}}{a^3}, \qquad p_B^1 = p_B^2 = p_B^3 = 0, \qquad p_B^4 = \cdots = p_B^D = -\frac{T_{D-3}}{a^3}. \tag{23}$$

Finally we include the cosmological constant term which can be interpreted as the space filling branes

$$(T_\Lambda)^M{}_N = \mathrm{diag}(-\rho_\Lambda, p_\Lambda), \tag{24}$$

where $\rho_\Lambda = \Lambda$ and $p_\Lambda = -\Lambda$.

Now we insert the energy-momentum tensors, $(T_g)^{MN}$ and $(T_m)^{MN} = (T_s + T_B + T_\Lambda)^{MN}$, into the right-hand side of the Einstein Eq. (2). After some algebra, the time component of the Einstein equation can be expressed as, taking units in which $2\kappa^2 = 1$ for simplicity,

$$6\left(\frac{\dot a}{a}\right)^2 + (D-3)(D-4)\left(\frac{\dot b}{b}\right)^2 + 6(D-3)\frac{\dot a}{a}\frac{\dot b}{b} = \rho_s + 2\frac{Q^2}{b^{2(D-3)}} + \frac{T_{D-3}}{a^3} + \Lambda. \tag{25}$$

The spatial components for $SO(3)$ and $SO(D-3)$ subspaces are given by

$$\frac{\ddot a}{a} + 2\left(\frac{\dot a}{a}\right)^2 + (D-3)\frac{\dot a}{a}\frac{\dot b}{b} = -\frac{2(D-4)}{D-1}\frac{Q^2}{b^{2(D-3)}} + \frac{\rho_s}{2D} + \frac{D-2}{2(D-1)}\frac{T_{D-3}}{a^3} + \frac{\Lambda}{D-1}, \tag{26}$$

$$\frac{\ddot b}{b} + (D-4)\left(\frac{\dot b}{b}\right)^2 + 3\frac{\dot a}{a}\frac{\dot b}{b} = \frac{6}{D-1}\frac{Q^2}{b^{2(D-3)}} + \frac{\rho_s}{2D} - \frac{1}{2(D-1)}\frac{T_{D-3}}{a^3} + \frac{\Lambda}{D-1}. \tag{27}$$

The key parameters controlling the relative rates of the growth for a and b are their accelerations not their velocities. For the configuration that a exceeds b by many orders of magnitudes, the source terms in Eqs. (26) and (27) must produce slow-roll conditions for b making its acceleration small or negative, while keeping the acceleration of a positive.

III. EFFECTIVE POTENTIAL

To study the functional behavior analytically, we rewrite the equations of motion in terms of $\zeta = a^3$ and $\xi = b^{D-3}$ corresponding to the volumes of $SO(3)$ and $SO(D-3)$ subspaces. The constraint Eq. (25) can be written as

$$\rho_s = \frac{2}{3}\left(\frac{\dot\zeta}{\zeta}\right)^2 + \frac{D-4}{D-3}\left(\frac{\dot\xi}{\xi}\right)^2 + 2\frac{\dot\zeta}{\zeta}\frac{\dot\xi}{\xi} - 2\frac{Q^2}{\xi^2} - \frac{T_{D-3}}{\zeta} - \Lambda. \tag{28}$$

Using (28), the second derivative equations can be written as

$$\frac{1}{3}\frac{\ddot\zeta}{\zeta} + \frac{D-3}{3D}\frac{\dot\zeta}{\zeta}\frac{\dot\xi}{\xi} = \frac{1}{3D}\left(\frac{\dot\zeta}{\zeta}\right)^2 + \frac{D^2-3D+1}{2D(D-1)}\frac{T_{D-3}}{\zeta} + \frac{D-4}{2D(D-3)}\left(\frac{\dot\xi}{\xi}\right)^2 + \frac{-2D^2+7D+1}{D(D-1)}\frac{Q^2}{\xi^2} + \frac{D+1}{2D(D-1)}\Lambda, \tag{29}$$

$$\frac{1}{D-3}\frac{\ddot\xi}{\xi} + \frac{3}{D(D-3)}\frac{\dot\zeta}{\zeta}\frac{\dot\xi}{\xi} = \frac{1}{3D}\left(\frac{\dot\zeta}{\zeta}\right)^2 + \frac{-2D+1}{2D(D-1)}\frac{T_{D-3}}{\zeta} + \frac{D-4}{2D(D-3)}\left(\frac{\dot\xi}{\xi}\right)^2 + \frac{5D+1}{D(D-1)}\frac{Q^2}{\xi^2} + \frac{D+1}{2D(D-1)}\Lambda. \tag{30}$$

Removing the coupled first derivative terms ($\frac{\dot\zeta}{\zeta}\frac{\dot\xi}{\xi}$), we have

$$\frac{1}{D-3}\left\{\frac{\ddot{\zeta}}{\zeta}+\frac{D-6}{9}\left(\frac{\dot{\zeta}}{\zeta}\right)^2-\frac{9(D^2-3D+1)+(D-3)^2(2D-1)}{6D(D-1)}\frac{T_{D-3}}{\zeta}\right\}$$
$$=\frac{1}{3}\left\{\frac{\ddot{\xi}}{\xi}-\frac{(D-6)(D-4)}{2(D-3)^2}\left(\frac{\dot{\xi}}{\xi}\right)^2-\frac{(D-3)^2(5D+1)+9(2D^2-7D-1)}{D(D-1)(D-3)}\frac{Q^2}{\xi^2}-\frac{(D-6)(D+1)}{2(D-1)(D-3)}\Lambda\right\}. \quad (31)$$

Since the left-hand side is a function of ζ and the right-hand side is a function of ξ, we take the simplest case by equating them to a constant E to decouple the variable ζ and ξ

$$\frac{\ddot{\zeta}}{\zeta}+\frac{D-6}{D}\left(\frac{\dot{\zeta}}{\zeta}\right)^2-\frac{9(D^2-3D+1)+(D-3)^2(2D-1)}{6D(D-1)}\frac{T_{D-3}}{\zeta}-(D-3)E=0, \quad (32)$$

$$\frac{\ddot{\xi}}{\xi}-\frac{(D-6)(D-4)}{2(D-3)^2}\left(\frac{\dot{\xi}}{\xi}\right)^2-\frac{(D-3)^2(5D+1)+9(2D^2-7D-1)}{D(D-1)(D-3)}\frac{Q^2}{\xi^2}-\frac{(D-6)(D+1)}{2(D-1)(D-3)}\Lambda-3E=0. \quad (33)$$

Putting $D=9$, we have

$$\frac{\ddot{\zeta}}{\zeta}+\frac{1}{3}\left(\frac{\dot{\zeta}}{\zeta}\right)^2-\frac{41}{16}\frac{T_6}{\zeta}-6E=0, \quad (34)$$

$$\frac{\ddot{\xi}}{\xi}-\frac{5}{24}\left(\frac{\dot{\xi}}{\xi}\right)^2-\frac{47}{8}\frac{Q^2}{\xi^2}-\frac{5}{16}\Lambda-3E=0. \quad (35)$$

We remove the first-order derivative terms with $\zeta=f^{3/4}$, $\xi=g^{24/19}$, then the equations reduce to the motions of a particle with unit mass in one dimension

$$\ddot{f}-\frac{41}{12}T_6f^{1/4}-8Ef=0, \quad (36)$$

$$\ddot{g}-\frac{893}{192}Q^2g^{-(29/19)}-\frac{19}{24}\left(3E+\frac{5}{16}\Lambda\right)g=0. \quad (37)$$

Now we can analyze the behavior of the two subvolumes by considering the effective potential as in [31], $\ddot{f}=-\frac{dV_{\rm eff}(f)}{df}$, $\ddot{g}=-\frac{dV_{\rm eff}(g)}{dg}$. The effective potentials are calculated as

$$V_{\rm eff}(f)=-\frac{41}{15}T_6f^{5/4}-4Ef^2, \quad (38)$$

$$V_{\rm eff}(g)=\frac{16967}{1920}Q^2g^{-(10/19)}-\frac{19}{48}\left(3E+\frac{5}{16}\Lambda\right)g^2. \quad (39)$$

To make the $SO(3)$ subvolume become large indefinitely, E must be positive and the shape of $V_{\rm eff}(f)$ is given in Fig. 1. The behavior of the effective potential for the $SO(6)$ subvolume shows a bouncing behavior (Q^2 term) for small values of g. So the overall shape of the potential depends on the sign of $3E+\frac{5}{16}\Lambda$. For $3E+\frac{5}{16}\Lambda>0$, the shape is given in Fig. 2. In this case, both the unwrapped three dimensions and the wrapped six dimensions expand monotonically. For $3E+\frac{5}{16}\Lambda<0$, the shape is given by Fig. 3. In this case, the wrapped internal subvolume can oscillate between two turning points or sit at the minimum of the potential $g_{\rm min}$ while the unwrapped subvolume expands indefinitely.

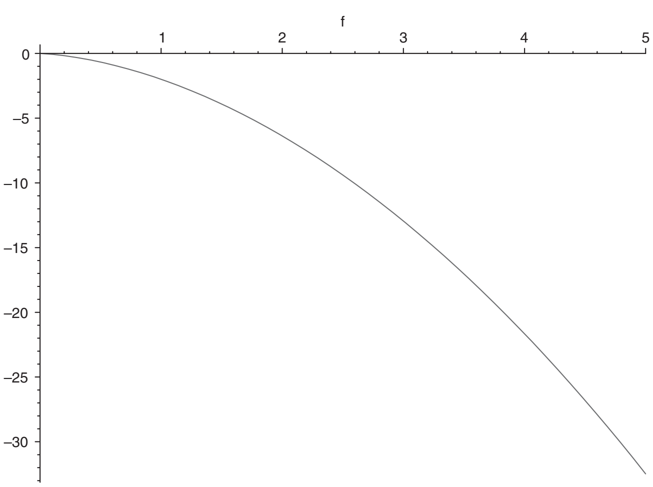

FIG. 1. Typical shape of effective potential $V_{\rm eff}(f)$ for unwrapped subvolume for $E>1$. The plot is for $T_6=15/41$ and $E=1/4$.

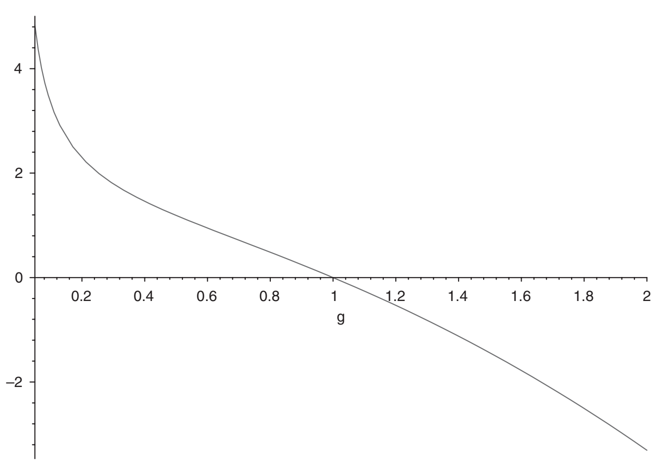

FIG. 2. Typical shape of effective potential $V_{\rm eff}(g)$ for $3E+\frac{5}{16}\Lambda>0$. The plot is for $Q^2=1920/16967$ and $3E+\frac{5}{16}\Lambda=48/19$.

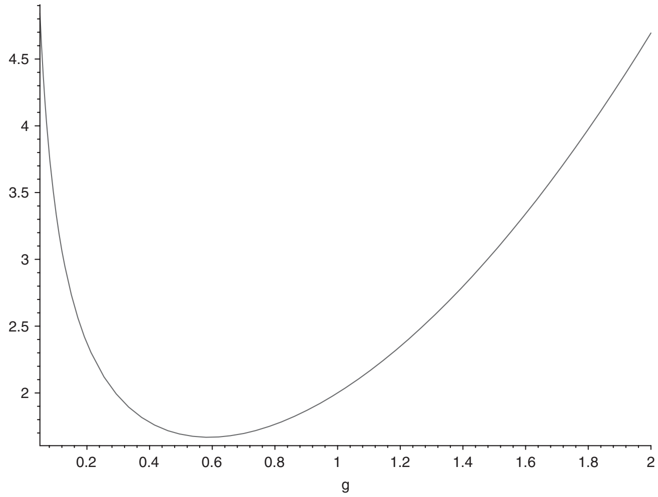

FIG. 3. Typical shape of effective potential $V_{\mathrm{eff}}(g)$ for $3E + \frac{5}{16}\Lambda < 0$. The plot is for $Q^2 = 1920/16967$ and $3E + \frac{5}{16}\Lambda = -48/19$.

In [31], the existence of the $(3 + 1)$-form RR-flux in the unwrapped subspace induces a logarithmic bounce to the effective potential for small values of ξ, and this term prevents the internal subvolume from collapsing. Including the supergravity particles into the analysis makes the bounce steeper than the case with only RR-flux. The reason can be understood by looking at the signs of the pressures in Eqs. (14), (16), and (23). The brane gas wrapping the internal dimensions exerts negative pressure and makes the internal subvolume contract. However, RR-flux and supergravity particles exert positive pressure to prevent the internal subvolume from collapsing. The internal volume can be stabilized by the competition of positive and negative pressures, and our result realized this possibility.

IV. CONCLUSION

We have studied the anisotropic evolution of spatial dimensions and the stability of the extra dimensions with particular emphasis on the role of supergravity particles. We took a perfect fluid form for their energy-momentum tensor which drives expansion. Assuming dilaton is stabilized, we focused on the late-time behavior of brane gas cosmology where windings of branes are completely removed from three dimensions and RR-flux exists in these unwrapped dimensions. We investigated how the existence of supergravity particles affects the asymmetric evolution of the Universe by reducing the Einstein equations to the motions of a particle in the one-dimensional effective potential. The shape of the potential for thethree-dimensional subvolume is barrier-type so that it can expand indefinitely. However the shape of the potential for the extra dimensional subvolume can be well-type so that it can oscillate between two turning points or be fixed at the minimum of the potential.

In most approaches to the stabilization in string cosmology, it is crucial that the dilaton runs to a weak coupling. The scale factor in the Einstein frame is a linear combination of string frame dilaton and scale factors. If the dilaton is not fixed, this can cause serious problems at late-time evolution of the Universe since the Newton constant is not fixed. The extra dimensions can be unstable as far as the dilaton evolves. It will be an important challenge to include the running of the dilaton into the stabilization of the radion.

Recently it has been shown that dilaton stabilization and radion stabilization are compatible by Danos, Frey, and Brandenberger [32]. They identified a stable fixed point corresponding to the dilaton sitting at the minimum of the potential and the radion taking on the value at which the enhanced symmetry states are massless. The stability of this fixed point was analyzed by studying the linearized equations of motion around the fixed point. The solution shows a damped oscillatory behavior confirming the compatibility of two types of moduli stabilization. Despite the promising result, we have to be very careful when we stabilize both dilaton and radion simultaneously. Cremonini and Watson [22] have discussed the stabilization of moduli in 11-dimensional supergravity. They found that the production rate of the Bogomol'nyi-Prasad-Sommerfield bound states could be significant for a modified brane spectrum with enhanced symmetry. These states can lead to a stabilization by an attractor mechanism. However, the stabilization drives the evolution to a region where a perturbative description of the string dynamics fails. That is, the supergravity approximation is not valid in this region. It is important not to forget the string theory origin of the low-energy effective action.

Realizing the transitions between the different thermodynamic phases of string gas is important in string cosmology. In [33], it is pointed out that the dilaton field may obstruct the transition from the Hagedorn phase of hot and dense string gas to an expanding Friedmann-Robertson-Walker phase of dilute string gas. They categorized the possible branches of the solutions according to the sign of the dimensionally reduced effective dilaton and noticed that the branch changing is impossible as long as the energy density of the Universe is not negative. Finding the appropriate energy sources which enable the phase changing seems another challenge in string/brane gas cosmology.

ACKNOWLEDGMENTS

This work was supported by research funds of Kunsan National University.

[1] R. Brandenberger and C. Vafa, Nucl. Phys. **B316**, 391 (1989).
[2] A. A. Tseytlin and C. Vafa, Nucl. Phys. **B372**, 443 (1992); A. A. Tseytlin, Classical Quantum Gravity **9**, 979 (1992).
[3] M. Maggiore and A. Riotto, Nucl. Phys. **B548**, 427 (1999).
[4] C. Park, S. J. Sin, and S. Lee, Phys. Rev. D **61**, 083514 (2000); S. Lee and S.-J. Sin, J. Korean Phys. Soc. **32**, 102 (1998); C. Park and S.-J. Sin, Phys. Rev. D **57**, 4620 (1998); J. Korean Phys. Soc. **34**, 463 (1999).
[5] S. Alexander, R. Brandenberger, and D. Easson, Phys. Rev. D **62**, 103509 (2000).
[6] T. Battefeld and S. Watson, Rev. Mod. Phys. **78**, 435 (2006).
[7] R. Brandenberger, D. A. Easson, and A. Mazumdar, Phys. Rev. D **69**, 083502 (2004); R. Danos, A. R. Frey, and A. Mazumdar, Phys. Rev. D **70**, 106010 (2004); T. Biswas, R. Brandenberger, D. A. Easson, and A. Mazumdar, Phys. Rev. D **71**, 083514 (2005).
[8] T. Biswas, J. High Energy Phys. 02 (2004) 039.
[9] A. Campos, Phys. Rev. D **68**, 104017 (2003); Phys. Lett. B **586**, 133 (2004).
[10] B. A. Bassett, M. Borunda, M. Serone, and S. Tsujikawa, Phys. Rev. D **67**, 123506 (2003); M. Borunda and L. Boubekeur, J. Cosmol. Astropart. Phys. 10 (2006) 002.
[11] R. Easther, B. R. Greene, M. G. Jackson, and D. Kabat, Phys. Rev. D **67**, 123501 (2003).
[12] S. Watson and R. Brandenberger, J. Cosmol. Astropart. Phys. 11 (2003) 008.
[13] S. P. Patil and R. Brandenberger, Phys. Rev. D **71**, 103522 (2005); J. Cosmol. Astropart. Phys. 01 (2006) 005.
[14] S. Watson, Phys. Rev. D **70**, 066005 (2004).
[15] T. Rador, Eur. Phys. J. C **49**, 1083 (2007).
[16] A. Chatrabhuti, Int. J. Mod. Phys. A **22**, 165 (2007).
[17] J. Y. Kim, Phys. Rev. D **70**, 104024 (2004).
[18] Y-K. E. Cheung, S. Watson, and R. Brandenberger, J. High Energy Phys. 05 (2006) 025.
[19] A. Kaya, Phys. Rev. D **72**, 066006 (2005).
[20] A. Berndsen, T. Biswas, and J. M. Cline, J. Cosmol. Astropart. Phys. 08 (2005) 012; D. A. Easson and M. Trodden, Phys. Rev. D **72**, 026002 (2005).
[21] S. P. Patil, arXiv:hep-th/0504145.
[22] S. Cremonini and S. Watson, Phys. Rev. D **73**, 086007 (2006).
[23] T. Battefeld and S. Watson, J. Cosmol. Astropart. Phys. 06 (2004) 001.
[24] S. Kanno and J. Soda, Phys. Rev. D **72**, 104023 (2005).
[25] T. Rador, Eur. Phys. J. C **52**, 683 (2007).
[26] S. Arapoglu, A. Karakci, and A. Kaya, Phys. Lett. B **645**, 255 (2007).
[27] S. Alexander, J. High Energy Phys. 10 (2003) 013.
[28] A. Flachi, J. Garriga, O. Pujolas, and T. Tanaka, J. High Energy Phys. 08 (2003) 053.
[29] J. J. Friess, S. S. Gubser, and I. Mitra, Nucl. Phys. **B689**, 243 (2004).
[30] A. Campos, Phys. Rev. D **71**, 083510 (2005).
[31] J. Y. Kim, Phys. Lett. B **652**, 43 (2007).
[32] R. J. Danos, A. R. Frey, and R. H. Brandenberger, Phys. Rev. D **77**, 126009 (2008).
[33] N. Kaloper and S. Watson, Phys. Rev. D **77**, 066002 (2008).