

Refinement of the Gribov-Zwanziger approach in the Landau gauge: Infrared propagators in harmony with the lattice results

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Recent lattice data have reported an infrared suppressed, positivity violating gluon propagator which is nonvanishing at zero momentum and a ghost propagator which is no longer enhanced. This paper discusses how to obtain analytical results which are in qualitative agreement with these lattice data within the Gribov-Zwanziger framework. This framework allows one to take into account effects related to the existence of gauge copies, by restricting the domain of integration in the path integral to the Gribov region. We elaborate to great extent on a previous short paper by presenting additional results, also confirmed by the numerical simulations. A detailed discussion on the soft breaking of the Becchi-Rouet-Stora-Tyutin symmetry arising in the Gribov-Zwanziger approach is provided.

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I. INTRODUCTION

As is well known, quantum chromodynamics (QCD) is confining at low energy. Confinement means that it is impossible to detect free quarks and gluons in the low momentum region as quarks form colorless bound states like baryons and mesons. Even if one omits the quarks, pure $SU(N)$ Yang-Mills gauge theory remains confining as gluons form bound states known as glueballs. Hitherto, confinement is still poorly understood. There is a widespread belief that the infrared behavior of the gluon and ghost propagator is deeply related to the issue of confinement and, therefore, these propagators have been widely investigated. In this paper, we shall use the following conventions for the gluon and the ghost propagator,

$$\langle A_\mu^a(-p)A_\nu^b(p) \rangle = \delta^{ab} \mathcal{D}(p^2) \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right), \quad (1)$$

$$\langle c^a(-p)\bar{c}^b(p) \rangle = \delta^{ab} \mathcal{G}(p^2).$$

Until recently, lattice results have shown an infrared suppressed, positivity violating gluon propagator which seemed to tend towards zero for zero momentum, i.e. $\mathcal{D}(0) = 0$, and a ghost propagator which was believed to be enhanced in the infrared [1,2], $\mathcal{G}(k^2 \approx 0) \sim 1/k^{2+\kappa}$ with $\kappa > 0$. Different analytical approaches were in agreement with these results (e.g. [3–10] to quote only a few). For instance, several works based on the Schwinger-Dyson or exact renormalization group equations reported an infrared enhanced ghost propagator and an infrared sup-

pressed, vanishing gluon propagator, obeying a power law behavior characterized by a unique infrared exponent, as stated by a sum rule discussed in [3–6]. The infrared propagators have also been studied from a thermodynamical viewpoint in [11]. Also the Gribov-Zwanziger action predicts an infrared enhanced ghost propagator and a zero-momentum vanishing gluon propagator [8,9]. This action was constructed in order to analytically implement the restriction to the Gribov region Ω , defined as the set of field configurations fulfilling the Landau gauge condition and for which the Faddeev-Popov operator,

$$\mathcal{M}^{ab} = -\partial_\mu (\partial_\mu \delta^{ab} + g f^{acb} A_\mu^c), \quad (2)$$

is strictly positive, namely

$$\Omega \equiv \{A_\mu^a, \partial_\mu A_\mu^a = 0, \mathcal{M}^{ab} > 0\}. \quad (3)$$

The boundary $\partial\Omega$ of the region Ω is called the (first) Gribov horizon. This restriction is necessary to avoid the appearance of Gribov copies in the Landau gauge related to gauge transformations [7]. However, this region Ω still contains a number of Gribov copies and is therefore still “larger” than the fundamental modular region (FMR), which is completely free of Gribov copies. Unfortunately, it is unknown how to treat the FMR analytically [12–15].

However, more recent lattice data [16–19] at larger volumes display an infrared suppressed, positivity violating gluon propagator, which is *nonvanishing* at zero momentum, i.e. $\mathcal{D}(0) \neq 0$, and a ghost propagator which is *no longer enhanced*, $\mathcal{G}(k^2 \approx 0) \sim 1/k^2$. This implies that the previous mentioned analytical approaches are not conclusive. It is worth pointing out that, recently, the authors of [20,21] have obtained a solution of the Schwinger-Dyson equations which is in agreement with the latest lattice data.

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Furthermore, as we have shown in a previous work [22], this agreement can also be found within the Gribov-Zwanziger approach. In this framework, we have added a novel mass term to the original Gribov-Zwanziger action. This new term corresponds to the introduction of a dimension 2 operator. We recall that by including condensates, which are the vacuum expectation value of certain local operators, one can take into account nonperturbative effects which play an important role in the infrared region. During the course of the current work, it shall become clear that we also have to add an additional vacuum term to the action, which will allow us to stay within the Gribov region Ω . The previous paper [22] only gave a brief account of the consequences of adding the mass operator to the original Gribov-Zwanziger action. For this reason, here we shall present an extensive study of the Gribov-Zwanziger action with the inclusion of the new parts.

The purpose of this paper is fourfold, and it is organized as follows. The first aim, discussed in Sec. II, is to give a detailed proof of the renormalizability of the extended action. Therefore, we first present an overview of the Gribov-Zwanziger action S_{GZ} in the Landau gauge which implements the restriction of the Gribov region Ω . Next, we add the local composite operator $S_m = \frac{m^2}{2} \int d^4x A_\mu^2$ to this action and we prove the renormalizability of this extended action, $S_{\text{GZ}} + S_m$. Subsequently, we show that by adding another term, $S_M = M^2 \int d^4x [(\bar{\varphi}\varphi - \bar{\omega}\omega) + \frac{2(N^2-1)}{g^2N} \varsigma\lambda^2]$, the renormalizability is not destroyed. In summary, Sec. II establishes the renormalizability of the action $S_{\text{GZ}} + S_m + S_M$. The second aim, investigated in Sec. III, is to demonstrate that this extra term enables us to obtain propagators which exhibit the desired behavior. In particular, the tree level gluon propagator is calculated explicitly and the ghost propagator is determined up to one loop. Both the ghost and the gluon propagator are in qualitative agreement with the latest lattice results. Up to this point, we have added this mass term by hand. Hence, a third aim is to obtain a dynamical value for M^2 . Section IV presents this dynamical value. An estimate for the one loop gluon propagator at zero momentum as well as for the ghost propagator at low momenta is given. Also, the positivity violation of the gluon propagator is scrutinized and compared with the available lattice data. The last aim is to

highlight the Becchi-Rouet-Stora-Tyutin (BRST) breaking of the Gribov-Zwanziger action, which is presented in detail in Sec. V. We already stress here that it is the restriction to the Gribov region Ω , implemented by the Gribov-Zwanziger action, which induces the explicit breaking of the BRST symmetry. Further, we provide a few remarks on the Maggiore-Schaden approach to the issue of the BRST breaking [23], and we revisit a few aspects of the Kugo-Ojima confinement criterion [24]. We end this paper with a discussion in Sec. VI.

II. THE EXTENDED ACTION AND THE RENORMALIZABILITY

A. The Gribov-Zwanziger action

We begin with an overview of the action constructed by Zwanziger [10] which implements the restriction to the Gribov region Ω [7] in Euclidean Yang-Mills theories in the Landau gauge. We start from the following action,

$$S_h = S_{\text{YM}} + \int d^4x (b^a \partial_\mu A_\mu^a + \bar{c}^a \partial_\mu D_\mu^{ab} c^b) + \gamma^4 \int d^4x h(x), \quad (4)$$

with S_{YM} the classical Yang-Mills action,

$$S_{\text{YM}} = \frac{1}{4} \int d^4x F_{\mu\nu}^a F_{\mu\nu}^a, \quad (5)$$

and $h(x)$ the so-called horizon function,

$$h(x) = g^2 f^{abc} A_\mu^b (\mathcal{M}^{-1})^{ad} f^{dec} A_\mu^e. \quad (6)$$

The parameter γ , known as the Gribov parameter, is not free and is determined by the horizon condition

$$\langle h(x) \rangle = d(N^2 - 1), \quad (7)$$

where d is the number of space-time dimensions. The nonlocal horizon function can be localized through a suitable set of additional fields. The complete localized action reads

$$S = S_0 + S_\gamma, \quad (8)$$

with

$$S_0 = S_{\text{YM}} + \int d^4x (b^a \partial_\mu A_\mu^a + \bar{c}^a \partial_\mu D_\mu^{ab} c^b) + \int d^4x (\bar{\varphi}_\mu^{ac} \partial_\nu (\partial_\nu \varphi_\mu^{ac} + g f^{abm} A_\nu^b \varphi_\mu^{mc}) - \bar{\omega}_\mu^{ac} \partial_\nu (\partial_\nu \omega_\mu^{ac} + g f^{abm} A_\nu^b \omega_\mu^{mc}) - g (\partial_\nu \bar{\omega}_\mu^{ac}) f^{abm} (D_\nu c)^b \varphi_\mu^{mc}),$$

$$S_\gamma = -\gamma^2 g \int d^4x \left(f^{abc} A_\mu^a \varphi_\mu^{bc} + f^{abc} A_\mu^a \bar{\varphi}_\mu^{bc} + \frac{4}{g} (N^2 - 1) \gamma^2 \right). \quad (9)$$

The fields $(\bar{\varphi}_\mu^{ac}, \varphi_\mu^{ac})$ are a pair of complex conjugate bosonic fields, while $(\bar{\omega}_\mu^{ac}, \omega_\mu^{ac})$ are anticommuting fields. Each of these fields has $4(N^2 - 1)^2$ components. We can easily see that the action S_0 displays a global $U(f)$

symmetry, $f = 4(N^2 - 1)$, with respect to the composite index $i = (\mu, c) = 1, \dots, f$, of the additional fields $(\bar{\varphi}_\mu^{ac}, \varphi_\mu^{ac}, \bar{\omega}_\mu^{ac}, \omega_\mu^{ac})$. Therefore, we simplify the notation of these fields by setting

$$(\bar{\varphi}_\mu^{ac}, \varphi_\mu^{ac}, \bar{\omega}_\mu^{ac}, \omega_\mu^{ac}) = (\bar{\varphi}_i^a, \varphi_i^a, \bar{\omega}_i^a, \omega_i^a), \quad (10)$$

so we get

$$\begin{aligned} S_0 &= S_{\text{YM}} + \int d^4x (b^a \partial_\mu A_\mu^a + \bar{c}^a \partial_\mu (D_\mu c)^a) \\ &+ \int d^4x (\bar{\varphi}_i^a \partial_\nu (D_\nu \varphi_i)^a - \bar{\omega}_i^a \partial_\nu (D_\nu \omega_i)^a \\ &- g(\partial_\nu \bar{\omega}_i^a) f^{abm} (D_\nu c)^b \varphi_i^m). \end{aligned} \quad (11)$$

Now we shall try to translate the horizon condition (7) into a more practical version [9]. The local action S and the nonlocal action S_h are related as follows:

$$\int dAdbdcd\bar{c}e^{-S_h} = \int dAdbdcd\bar{c}d\varphi d\bar{\varphi}d\omega d\bar{\omega}e^{-S}. \quad (12)$$

If we take the partial derivative of both sides with respect to γ^2 we obtain

$$-2\gamma^2 \langle h \rangle = \langle g f^{abc} A_\mu^a \varphi_\mu^{bc} \rangle + \langle g f^{abc} A_\mu^a \bar{\varphi}_\mu^{bc} \rangle. \quad (13)$$

Using this last expression and assuming that $\gamma \neq 0$, we can rewrite the horizon condition (7)

$$\langle g f^{abc} A_\mu^a \varphi_\mu^{bc} \rangle + \langle g f^{abc} A_\mu^a \bar{\varphi}_\mu^{bc} \rangle + 2\gamma^2 d(N^2 - 1) = 0. \quad (14)$$

We know that the quantum action Γ is obtained through the definition

$$e^{-\Gamma} = \int d\Phi e^{-S}, \quad (15)$$

where $\int d\Phi$ stands for the integration over all the fields. It is now easy to see that

$$\frac{\partial \Gamma}{\partial \gamma^2} = 0 \quad (16)$$

is exactly equivalent with Eq. (14). Therefore, Eq. (16) represents the horizon condition. We remark that the condition (16) also includes the solution $\gamma = 0$. However, $\gamma = 0$ would correspond to the case in which the restriction to the Gribov region would not have been implemented. As such, the value $\gamma = 0$ has to be disregarded as an artefact due to the reformulation of the horizon condition.

As it has been proven in [9], the Gribov-Zwanziger action S is renormalizable to all orders. In the next section, we shall give an overview of this renormalizability, but with the insertion of the local composite operator $A_\mu^a A_\mu^a$, to extend the action further. Obviously, the renormalizability of this extended action S' also includes the renormalizability of the ordinary Gribov-Zwanziger action S .

B. Adding the local composite operator $A_\mu^a A_\mu^a$

If we add the local composite operator $A_\mu^a A_\mu^a$ to (8) one can prove [25] that the following action is renormalizable to all orders

$$S' = S_0 + S_\gamma + S_{A^2}, \quad (17)$$

with

$$S_{A^2} = \int d^4x \left(\frac{\tau}{2} A_\mu^a A_\mu^a - \frac{\zeta}{2} \tau^2 \right), \quad (18)$$

with τ a new source and ζ a new parameter. We now go a little bit more into the details of the renormalization of this action, as it will be useful later. We remark that if we prove the renormalizability of the action S' , we have also proven the renormalizability of $S = S_0 + S_\gamma$ just by putting τ equal to zero. We will use the method of algebraic renormalization [26]. Roughly speaking, this means that we will embed the action S' into a larger action by adding new sources, so it will display a greater number of symmetries. These symmetries are important as they will imply constraints on the possible allowed counterterm. The larger the number of symmetries, the more limitations we will find on the counterterm. This will lead to a bigger possibility to absorb the counterterm into the original action, thereby proving the renormalizability. In the end, we give the sources the correct physical values, so we obtain the action S' again.

We shall now implement, step by step, this method of algebraic renormalization. Firstly, we introduce two local external sources M_μ^{ai}, V_μ^{ai} so we can treat $f^{abc} A_\mu^a \varphi_\mu^{bc}$ and $f^{abc} A_\mu^a \bar{\varphi}_\mu^{bc}$ as composite operators just like A_μ^2 . Hence, we replace the term S_γ by

$$\begin{aligned} S'_\gamma &= - \int d^4x (M_\mu^{ai} (D_\mu \varphi_i)^a + V_\mu^{ai} (D_\mu \bar{\varphi}_i)^a \\ &+ 4\gamma^4 (N^2 - 1)). \end{aligned} \quad (19)$$

If we set the sources to their physical values in the end

$$M_{\mu\nu}^{ab}|_{\text{phys}} = V_{\mu\nu}^{ab}|_{\text{phys}} = \gamma^2 \delta^{ab} \delta_{\mu\nu}, \quad (20)$$

we obtain, as requested, the term S_γ defined in (8).

Secondly, the algebraic renormalization procedure requires this action to be BRST invariant. Therefore, we further introduce three extra sources N_μ^{ai}, U_μ^{ai} , and η and replace $S'_\gamma + S_{A^2}$ by

$$\begin{aligned} S_s &= s \int d^4x \left(-U_\mu^{ai} (D_\mu \varphi_i)^a - V_\mu^{ai} (D_\mu \bar{\omega}_i)^a - U_\mu^{ai} V_\mu^{ai} \right. \\ &+ \left. \frac{1}{2} \eta A_\mu^a A_\mu^a - \frac{1}{2} \zeta \tau \eta \right) \\ &= \int d^4x \left(-M_\mu^{ai} (D_\mu \varphi_i)^a - g U_\mu^{ai} f^{abc} (D_\mu c)^b \varphi_i^c \right. \\ &+ U_\mu^{ai} (D_\mu \omega_i)^b - N_\mu^{ai} (D_\mu \bar{\omega}_i)^a - V_\mu^{ai} (D_\mu \bar{\varphi}_i)^a \\ &+ g V_\mu^{ai} f^{abc} (D_\mu c)^b \bar{\omega}_i^c - M_\mu^{ai} V_\mu^{ai} + U_\mu^{ai} N_\mu^{ai} \\ &+ \left. \frac{1}{2} \tau A_\mu^a A_\mu^a + \eta A_\mu^a \partial_\mu c^a - \frac{1}{2} \zeta \tau^2 \right), \end{aligned} \quad (21)$$

where the BRST transformations of all the fields and sources are:

$$\begin{aligned}
sA_\mu^a &= -(D_\mu c)^a, & sc^a &= \frac{1}{2}gf^{abc}c^b c^c, \\
s\bar{c}^a &= b^a, & sb^a &= 0, & s\varphi_i^a &= \omega_i^a, \\
s\omega_i^a &= 0, & s\bar{\omega}_i^a &= \bar{\varphi}_i^a, & s\bar{\varphi}_i^a &= 0,
\end{aligned} \tag{22}$$

and

$$\begin{aligned}
sU_\mu^{ai} &= M_\mu^{ai}, & sM_\mu^{ai} &= 0, & sV_\mu^{ai} &= N_\mu^{ai}, \\
sN_\mu^{ai} &= 0, & s\eta &= \tau, & s\tau &= 0.
\end{aligned} \tag{23}$$

We recall that the BRST operator s is nilpotent, meaning that $s^2 = 0$. We mention again that by replacing the sources with their physical values in the end

$$U_\mu^{ai}|_{\text{phys}} = N_\mu^{ai}|_{\text{phys}} = 0, \tag{24}$$

$$\eta|_{\text{phys}} = 0, \tag{25}$$

one recovers the original terms $S_\gamma + S_{A^2}$.

Finally, a term S_{ext} ,

$$S_{\text{ext}} = \int d^4x \left(-K_\mu^a (D_\mu c)^a + \frac{1}{2} g L^a f^{abc} c^b c^c \right), \tag{26}$$

was added, which is needed to define the nonlinear BRST transformations of the gauge and ghost fields. K_μ^a and L^a are two new sources, invariant under the BRST symmetry s and with

$$K_\mu^a|_{\text{phys}} = L^a|_{\text{phys}} = 0. \tag{27}$$

The enlarged action is thus given by

$$\Sigma = S_0 + S_s + S_{\text{ext}}, \tag{28}$$

and one easily sees that the action Σ is indeed BRST invariant. This action now enjoys a larger number of Ward identities summarized as follows:

(i) For the $U(f)$ invariance mentioned before we have

$$\begin{aligned}
U_{ij}\Sigma &= 0, \\
U_{ij} &= \int d^4x \left(\varphi_i^a \frac{\delta}{\delta \varphi_j^a} - \bar{\varphi}_j^a \frac{\delta}{\delta \bar{\varphi}_i^a} + \omega_i^a \frac{\delta}{\delta \omega_j^a} \right. \\
&\quad \left. - \bar{\omega}_j^a \frac{\delta}{\delta \bar{\omega}_i^a} + M_\mu^{ai} \frac{\delta}{\delta M_\mu^{aj}} - U_\mu^{aj} \frac{\delta}{\delta U_\mu^{ai}} \right. \\
&\quad \left. + N_\mu^{ai} \frac{\delta}{\delta N_\mu^{aj}} - V_\mu^{aj} \frac{\delta}{\delta V_\mu^{ai}} \right). \tag{29}
\end{aligned}$$

By means of the diagonal operator $Q_f = U_{ii}$, the i -valued fields and sources turn out to possess an additional quantum number. One can find all quantum numbers in Tables I and II.

(ii) The Slavnov-Taylor identity reads

$$S(\Sigma) = 0, \tag{30}$$

with

$$\begin{aligned}
S(\Sigma) &= \int d^4x \left(\frac{\delta \Sigma}{\delta K_\mu^a} \frac{\delta \Sigma}{\delta A_\mu^a} + \frac{\delta \Sigma}{\delta L^a} \frac{\delta \Sigma}{\delta c^a} + b^a \frac{\delta \Sigma}{\delta \bar{c}^a} \right. \\
&\quad \left. + \bar{\varphi}_i^a \frac{\delta \Sigma}{\delta \bar{\omega}_i^a} + \omega_i^a \frac{\delta \Sigma}{\delta \varphi_i^a} + M_\mu^{ai} \frac{\delta \Sigma}{\delta U_\mu^{ai}} \right. \\
&\quad \left. + N_\mu^{ai} \frac{\delta \Sigma}{\delta V_\mu^{ai}} \right). \tag{31}
\end{aligned}$$

(iii) The Landau gauge condition and the antighost equation are given by

$$\frac{\delta \Sigma}{\delta b^a} = \partial_\mu A_\mu^a, \tag{32}$$

$$\frac{\delta \Sigma}{\delta \bar{c}^a} + \partial_\mu \frac{\delta \Sigma}{\delta K_\mu^a} = 0. \tag{33}$$

TABLE I. Quantum numbers of the fields.

| | A_μ^a | c^a | \bar{c}^a | b^a | φ_i^a | $\bar{\varphi}_i^a$ | ω_i^a | $\bar{\omega}_i^a$ |
|---------------|-----------|-------|-------------|-------|---------------|---------------------|--------------|--------------------|
| Dimension | 1 | 0 | 2 | 2 | 1 | 1 | 1 | 1 |
| Ghost number | 0 | 1 | -1 | 0 | 0 | 0 | 1 | -1 |
| Q_f -charge | 0 | 0 | 0 | 0 | 1 | -1 | 1 | -1 |

TABLE II. Quantum numbers of the sources.

| | U_μ^{ai} | M_μ^{ai} | N_μ^{ai} | V_μ^{ai} | K_μ^a | L^a | τ | η |
|---------------|--------------|--------------|--------------|--------------|-----------|-------|--------|--------|
| Dimension | 2 | 2 | 2 | 2 | 3 | 4 | 2 | 2 |
| Ghost number | -1 | 0 | 1 | 0 | -1 | -2 | 0 | 0 |
| Q_f -charge | -1 | -1 | 1 | 1 | 0 | 0 | 0 | 0 |

(iv) The ghost Ward identity is

$$\mathcal{G}^a \Sigma = \Delta_{\text{cl}}^a, \quad (34)$$

with

$$\begin{aligned} \mathcal{G}^a = \int d^4x \left(\frac{\delta}{\delta c^a} + g f^{abc} \left(\bar{c}^b \frac{\delta}{\delta b^c} + \varphi_i^b \frac{\delta}{\delta \omega_i^c} \right. \right. \\ \left. \left. + \bar{\omega}_i^b \frac{\delta}{\delta \bar{\varphi}_i^c} + V_\mu^{bi} \frac{\delta}{\delta N_\mu^{ci}} + U_\mu^{bi} \frac{\delta}{\delta M_\mu^{ci}} \right) \right), \end{aligned} \quad (35)$$

and

$$\Delta_{\text{cl}}^a = g \int d^4x f^{abc} (K_\mu^b A_\mu^c - L^b c^c). \quad (36)$$

Notice that the term Δ_{cl}^a , being linear in the quantum fields A_μ^a, c^a , is a classical breaking.

(v) The linearly broken local constraints yield

$$\frac{\delta \Sigma}{\delta \bar{\varphi}^{ai}} + \partial_\mu \frac{\delta \Sigma}{\delta M_\mu^{ai}} = g f^{abc} A_\mu^b V_\mu^{ci}, \quad (37)$$

$$\begin{aligned} \frac{\delta \Sigma}{\delta \omega^{ai}} + \partial_\mu \frac{\delta \Sigma}{\delta N_\mu^{ai}} - g f^{abc} \bar{\omega}^{bi} \frac{\delta \Sigma}{\delta b^c} \\ = g f^{abc} A_\mu^b U_\mu^{ci}, \end{aligned} \quad (38)$$

$$\begin{aligned} \frac{\delta \Sigma}{\delta \bar{\omega}^{ai}} + \partial_\mu \frac{\delta \Sigma}{\delta U_\mu^{ai}} - g f^{abc} V_\mu^{bi} \frac{\delta \Sigma}{\delta K_\mu^c} \\ = -g f^{abc} A_\mu^b N_\mu^{ci}, \end{aligned} \quad (39)$$

$$\begin{aligned} \frac{\delta \Sigma}{\delta \varphi^{ai}} + \partial_\mu \frac{\delta \Sigma}{\delta V_\mu^{ai}} - g f^{abc} \bar{\varphi}^{bi} \frac{\delta \Sigma}{\delta b^c} \\ - g f^{abc} \bar{\omega}^{bi} \frac{\delta \Sigma}{\delta \bar{c}^c} - g f^{abc} U_\mu^{bi} \frac{\delta \Sigma}{\delta K_\mu^c} \\ = g f^{abc} A_\mu^b M_\mu^{ci}. \end{aligned} \quad (40)$$

(vi) The exact \mathcal{R}_{ij} symmetry reads

$$\mathcal{R}_{ij} \Sigma = 0, \quad (41)$$

with

$$\begin{aligned} \mathcal{R}_{ij} = \int d^4x \left(\varphi_i^a \frac{\delta}{\delta \omega_j^a} - \bar{\omega}_j^a \frac{\delta}{\delta \bar{\varphi}_i^a} + V_\mu^{ai} \frac{\delta}{\delta N_\mu^{ai}} \right. \\ \left. - U_\mu^{ai} \frac{\delta}{\delta M_\mu^{ai}} \right). \end{aligned} \quad (42)$$

(vii) The integrated Ward identity

$$\int d^4x \left(c^a \frac{\delta \Sigma}{\delta \omega^{ai}} + \bar{\omega}^{ai} \frac{\delta \Sigma}{\delta \bar{c}^a} + U_\mu^{ai} \frac{\delta \Sigma}{\delta K_\mu^a} \right) = 0. \quad (43)$$

When we turn to the quantum level, we can use these symmetries to characterize the most general allowed invariant counterterm Σ^c . Following the algebraic renormalization procedure [26], Σ^c is an integrated local polynomial in the fields and sources with dimension bounded by four, and with vanishing ghost number and Q_f -charge. The previous Ward identities imply the following constraints for Σ^c :

(i) The $U(f)$ invariance:

$$U_{ij} \Sigma^c = 0. \quad (44)$$

(ii) The linearized Slavnov-Taylor identity:

$$\mathcal{B}_\Sigma \Sigma^c = 0, \quad (45)$$

with \mathcal{B}_Σ the nilpotent linearized Slavnov-Taylor operator,

$$\begin{aligned} \mathcal{B}_\Sigma = \int d^4x \left(\frac{\delta \Sigma}{\delta K_\mu^a} \frac{\delta}{\delta A_\mu^a} + \frac{\delta \Sigma}{\delta A_\mu^a} \frac{\delta}{\delta K_\mu^a} + \frac{\delta \Sigma}{\delta L^a} \right. \\ \times \frac{\delta}{\delta c^a} + \frac{\delta \Sigma}{\delta c^a} \frac{\delta}{\delta L^a} + b^a \frac{\delta}{\delta \bar{c}^a} + \bar{\varphi}_i^a \frac{\delta}{\delta \bar{\omega}_i^a} \\ \left. + \omega_i^a \frac{\delta}{\delta \varphi_i^a} + M_\mu^{ai} \frac{\delta}{\delta U_\mu^{ai}} + N_\mu^{ai} \frac{\delta}{\delta V_\mu^{ai}} \right), \end{aligned} \quad (46)$$

and

$$\mathcal{B}_\Sigma \mathcal{B}_\Sigma = 0. \quad (47)$$

(iii) The Landau gauge condition and the antighost equation:

$$\frac{\delta \Sigma^c}{\delta b^a} = 0, \quad \frac{\delta \Sigma}{\delta \bar{c}^a} + \partial_\mu \frac{\delta \Sigma}{\delta K_\mu^a} = 0. \quad (48)$$

(iv) The ghost Ward identity:

$$\mathcal{G}^a \Sigma^c = 0. \quad (49)$$

(v) The linearly broken local constraints:

$$\begin{aligned} \frac{\delta \Sigma^c}{\delta \varphi^{ai}} + \partial_\mu \frac{\delta \Sigma^c}{\delta V_\mu^{ai}} - g f^{abc} \bar{\omega}^{bi} \frac{\delta \Sigma^c}{\delta \bar{c}^c} \\ - g f^{abc} U_\mu^{bi} \frac{\delta \Sigma^c}{\delta K_\mu^c} = 0, \end{aligned}$$

$$\frac{\delta \Sigma^c}{\delta \bar{\omega}^{ai}} + \partial_\mu \frac{\delta \Sigma^c}{\delta U_\mu^{ai}} - g f^{abc} V_\mu^{bi} \frac{\delta \Sigma^c}{\delta K_\mu^c} = 0,$$

$$\frac{\delta \Sigma^c}{\delta \omega^{ai}} + \partial_\mu \frac{\delta \Sigma^c}{\delta N_\mu^{ai}} = 0, \quad \frac{\delta \Sigma}{\delta \bar{\varphi}^{ai}} + \partial_\mu \frac{\delta \Sigma}{\delta M_\mu^{ai}} = 0.$$

(vi) The exact \mathcal{R}_{ij} symmetry:

$$\mathcal{R}_{ij} \Sigma^c = 0. \quad (50)$$

(vii) The integrated Ward identity

$$\int d^4x (c^a \frac{\delta \Sigma^c}{\delta \omega^{ai}} + \bar{\omega}^{ai} \frac{\delta \Sigma^c}{\delta \bar{c}^a} + U_\mu^{ai} \frac{\delta \Sigma^c}{\delta K_\mu^a}) = 0. \quad (51)$$

These constraints imply that Σ^c does not depend on the Lagrange multiplier b^a , and that the antighost \bar{c}^a and the i -valued fields φ_i^a , ω_i^a , $\bar{\varphi}_i^a$, $\bar{\omega}_i^a$ can enter only through the combinations [9,25]

$$\begin{aligned} \tilde{K}_\mu^a &= K_\mu^a + \partial_\mu \bar{c}^a - g f^{abc} \bar{U}_\mu^{bi} \varphi^{ci} - g f^{abc} V_\mu^{bi} \bar{\omega}^{ci}, \\ \tilde{U}_\mu^{ai} &= U_\mu^{ai} + \partial_\mu \bar{\omega}^{ai}, & \tilde{V}_\mu^{ai} &= V_\mu^{ai} + \partial_\mu \varphi^{ai}, \\ \tilde{N}_\mu^{ai} &= N_\mu^{ai} + \partial_\mu \omega^{ai}, & \tilde{M}_\mu^{ai} &= M_\mu^{ai} + \partial_\mu \bar{\varphi}^{ai}. \end{aligned} \quad (52)$$

The most general counterterm fulfilling the conditions (44)–(51) contains four arbitrary parameters, a_0 , a_1 , a_2 , a_3 and reads

$$\begin{aligned} \Sigma^c &= a_0 S_{YM} + a_1 \int d^4x (A_\mu^a \frac{\delta S_{YM}}{\delta A_\mu^a} + \tilde{K}_\mu^a \partial_\mu c^a \\ &+ \tilde{V}_\mu^{ai} \tilde{M}_\mu^{ai} - \tilde{U}_\mu^{ai} \tilde{N}_\mu^{ai}) + \int d^4x (\frac{a_2}{2} \tau A_\mu^a A_\mu^a \\ &+ \frac{a_3}{2} \zeta \tau^2 + (a_2 - a_1) \eta A_\mu^a \partial_\mu c^a). \end{aligned} \quad (53)$$

Once the most general counterterm has been determined, one can straightforwardly verify that it can be reabsorbed through a multiplicative renormalization of the fields, sources, and coupling constants. We also mention the renormalization factors, useful for later calculations. If we set $\phi = (A_\mu^a, c^a, \bar{c}^a, b^a, \varphi_i^a, \omega_i^a, \bar{\varphi}_i^a, \bar{\omega}_i^a)$ for all the fields and $\Phi = (K^{a\mu}, L^a, M_\mu^{ai}, N_\mu^{ai}, V_\mu^{ai}, U_\mu^{ai}, \tau, \eta)$ for the sources, and if we define

$$\begin{aligned} g_0 &= Z_g g, & \zeta_0 &= Z_\zeta \zeta, \\ \phi_0 &= Z_\phi^{1/2} \phi, & \Phi_0 &= Z_\Phi \Phi, \end{aligned} \quad (54)$$

one can determine

$$\begin{aligned} Z_g &= 1 + \eta \frac{a_0}{2}, & Z_A^{1/2} &= 1 + \eta \left(a_1 - \frac{a_0}{2} \right), \\ Z_\zeta &= 1 + \eta (-a_3 - 2a_2 + 4a_1 - 2a_0). \end{aligned} \quad (55)$$

These are the only independent renormalization constants. For example, the Faddeev-Popov ghosts (c^a , \bar{c}^a) and the i -valued fields (φ_i^a , ω_i^a , $\bar{\varphi}_i^a$, $\bar{\omega}_i^a$) have a common renormalization constant, determined by the renormalization constants Z_g and $Z_A^{1/2}$,

$$\begin{aligned} Z_c &= Z_{\bar{c}} = Z_\varphi = Z_{\bar{\varphi}} = Z_\omega = Z_{\bar{\omega}} = (1 - \eta a_0) \\ &= Z_g^{-1} Z_A^{-1/2}. \end{aligned} \quad (56)$$

The renormalization of the sources (M_μ^{ai} , N_μ^{ai} , V_μ^{ai} , U_μ^{ai}) is also determined by the renormalization constants Z_g and $Z_A^{1/2}$, being given by

$$Z_{\gamma^2} \equiv Z_M = Z_N = Z_V = Z_U = Z_g^{-1/2} Z_A^{-1/4}. \quad (57)$$

Also Z_τ is related to Z_g and $Z_A^{1/2}$ [25]:

$$Z_\tau = Z_g Z_A^{-1/2}. \quad (58)$$

Finally, Z_b , Z_K , and Z_L are also not independent as they are given by

$$Z_b = Z_A^{-1}, \quad Z_K = Z_c^{1/2}, \quad Z_L = Z_A^{1/2}. \quad (59)$$

C. Adding a new mass term

1. Extended action

We first explain the need for the inclusion of a new dynamical effect. According to the latest lattice results, the gluon propagator does not seem to vanish for zero momentum. This is incompatible with the actions (8) and (17), which both lead to a vanishing gluon propagator near the origin. The tree level gluon propagator in the Gribov-Zwanziger model reads [25]

$$\begin{aligned} \langle A_\mu^a(-p) A_\nu^b(p) \rangle &\equiv \delta^{ab} \mathcal{D}(p^2) \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \\ &= \delta^{ab} \frac{p^2}{p^4 + \lambda^4} \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right), \end{aligned} \quad (60)$$

where we have set

$$\lambda^4 = 2g^2 N \gamma^4. \quad (61)$$

One recognizes indeed that expression (60) vanishes at the origin due to the presence of Gribov parameter λ . In the A_μ^2 model the gluon propagator is modified in the following form:

$$\begin{aligned} \langle A_\mu^a(-p) A_\nu^b(p) \rangle &\equiv \delta^{ab} \mathcal{D}(p^2) \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \\ &= \delta^{ab} \frac{p^2}{p^4 + m^2 p^2 + \lambda^4} \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right), \end{aligned} \quad (62)$$

which reveals a further suppression near the origin and thus it still vanishes. We recall here that the fields ($\bar{\varphi}_\mu^{ac}$, φ_μ^{ac} , $\bar{\omega}_\mu^{ac}$, ω_μ^{ac}) were introduced to localize the horizon function [9], which implements the restriction to the Gribov region Ω . If we take a closer look at the action (11), we observe an $A\varphi$ -coupling at the quadratic level. One can suspect that a nontrivial effect in the φ -sector will immediately get translated into the gluon sector. For this reason, if we try to give a mass to the $\bar{\varphi}$, φ -fields without spoiling the renormalizability of the action, we might be able to modify the gluon propagator in the desired way. Implementing this idea means that we add a new term to the action (17) of the form $J \bar{\varphi}_i^a \varphi_i^a$, with J a new source. If we want to preserve the renormalizability we have to add the mass term in a BRST invariant way. Therefore, we consider the following extended action:

$$S'' = S' + S_{\bar{\varphi}\varphi}, \quad (63)$$

$$\begin{aligned}
 S_{\bar{\varphi}\varphi} &= \int d^4x (s(-J\bar{\omega}_i^a \varphi_i^a) + \rho J\tau) \\
 &= \int d^4x (-J(\bar{\varphi}_i^a \varphi_i^a - \bar{\omega}_i^a \omega_i^a) + \rho J\tau), \quad (64)
 \end{aligned}$$

with ρ a parameter and J a dimension 2 source, invariant under the BRST transformation

$$sJ = 0. \quad (65)$$

2. Renormalizability

The proof of the renormalizability of this action S'' can be easily done with the help of the Ward identities derived in the previous section. Again, we embed the action S'' into a larger action,

$$\Sigma' = \Sigma + S_{\bar{\varphi}\varphi}, \quad (66)$$

containing more symmetries. It is subsequently trivial to check that all Ward identities (29)–(42) remain unchanged up to potential harmless linear breaking terms and therefore the constraints (44)–(50) as well as the combinations (52) are preserved. Only the integrated Ward identity (51) is lost by the introduction of the new mass term. However, as we are using mass independent renormalization schemes, the value of the new mass dimension 2 source J cannot influence the explicit expression of the counterterm Σ^c already determined in Eq. (53). This is quite logical, as a mass term can only add its own renormalization factor, it does not affect the renormalization factors of other quantities, which can be equally well computed with $J = 0$. This implies that the counterterm $\Sigma^{c'}$ corresponding to the action Σ' is now given by

$$\Sigma^{c'} = \Sigma^c + \Sigma_{\bar{\varphi}\varphi}^c, \quad \Sigma_{\bar{\varphi}\varphi}^c = a_4 J\tau, \quad (67)$$

with a_4 an arbitrary parameter. This counterterm can be absorbed into the original action Σ' , hence we have proven the renormalizability of our extended action. If we define

$$J_0 = Z_J J, \quad \rho_0 = Z_\rho \rho, \quad (68)$$

we find

$$Z_J = Z_\varphi^{-1} = Z_g Z_A^{1/2}, \quad Z_\rho = 1 + \eta \left(a_4 - \frac{a_0}{2} - a_2 \right). \quad (69)$$

As the reader might have noticed, symmetries also do not prevent a term κJ^2 from occurring, with κ a new parameter, but we can argue that κ is in fact a redundant parameter, as no divergences in J^2 will occur. A term of this form is independent of the fields, hence it would only be necessary to get rid of the infinities in the functional energy, which we calculate by integrating the action over all the fields

$$\int d\Phi e^{-S''} = e^{-W(J)}. \quad (70)$$

Seen from another perspective, we need a counterterm $\propto J^2$ to remove possible divergences in the vacuum correla-

tors $\langle (\bar{\varphi}\varphi - \bar{\omega}\omega)_x (\bar{\varphi}\varphi - \bar{\omega}\omega)_y \rangle$ for $x \rightarrow y$. Such new divergences are typical when a local composite operator (LCO) of dimension 2 is added to the theory in 4D. An *a priori* arbitrary new coupling κ is then needed to reabsorb these divergences. In general, it can be made a unique function of g^2 such that $W(J)$ obeys a standard homogeneous linear renormalization group equation [25]. This is a good sign, as we do not want new independent couplings entering our action or results. A nice feature of the LCO under study, i.e. $(\bar{\varphi}\varphi - \bar{\omega}\omega)$, is that divergences $\propto J^2$ are in fact absent in the correlators, so there is even no need for the coupling κ here. The argument goes as follows. The Ward identities prohibit terms in $J\gamma^2$ from occurring. Notice that this is not a trivial point, as naively we expect it to occur from the dimensional point of view. It is only by making use of the extended action and its larger symmetry content that we can exclude a term $\propto J\gamma^2$ from the game. Hence, we can set $\gamma^2 = 0$ to find the vacuum divergence structure $\propto J^2$, as we will employ as usual mass independent renormalization schemes like the $\overline{\text{MS}}$ scheme. Now, there are two ways to understand that no divergences in J will occur. Firstly, at the level of the action, it is easily recognized that the term $g(\partial_\nu \bar{\omega}_i^a) f^{abm} (D_\nu c)^b \varphi_i^m$ in the action is irrelevant for the computation of the generating functional as the associated vertices cannot couple to anything without external ω - and c -legs. Thus forgetting about this term, the $(\bar{\varphi}, \varphi)$ - and $(\bar{\omega}, \omega)$ -integrations can be done exactly, and they neatly cancel due to the opposite statistics of both sets of fields. Hence, all J -dependence is in fact lost, and *a fortiori* no divergences arise. Secondly, for $\gamma^2 = 0$, the action $S''_{\gamma^2=0}$ is BRST invariant, $sS''_{\gamma^2=0} = 0$. Consequently, the vacuum correlators $\langle (\bar{\varphi}\varphi - \bar{\omega}\omega)_x \times (\bar{\varphi}\varphi - \bar{\omega}\omega)_y \rangle = \langle s[(\varphi\bar{\omega})_x (\bar{\varphi}\varphi - \bar{\omega}\omega)_y] \rangle = 0$. Therefore, we have again proven that no divergences in J appear. For $\gamma^2 \neq 0$, the BRST transformation s no longer generates a symmetry (see Sec. V), hence a nonvanishing result for the correlator $\langle (\bar{\varphi}\varphi - \bar{\omega}\omega)_x (\bar{\varphi}\varphi - \bar{\omega}\omega)_y \rangle$ or the condensate $\langle \bar{\varphi}\varphi - \bar{\omega}\omega \rangle$ is allowed. A nonvanishing vacuum expectation value (VEV) for our new mass operator is thus exactly allowed since the BRST is already broken by the restriction to the horizon. From the first viewpoint, the $(\bar{\varphi}, \varphi)$ - and $(\bar{\omega}, \omega)$ -integrations will no longer cancel against each other, giving room for J -dependent contributions in the generating functional, albeit without generating any new divergences.

D. Modifying the effective action in order to stay within the horizon

1. Extended action

A very important fact is to check if it is still possible to stay within the Gribov region Ω , after adding this new mass term. This can be investigated with the help of the ghost propagator $\mathcal{G}(k^2)$, which can be easily read off from the Feynman diagrams depicted in Fig. 1,

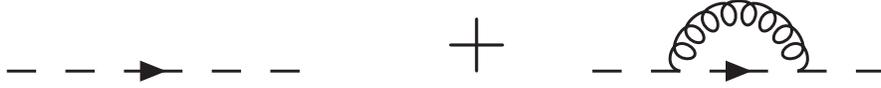


FIG. 1. The one loop corrected ghost propagator.

$$\begin{aligned}
\mathcal{G}^{ab}(k^2) &= \delta^{ab} \mathcal{G}(k^2) \\
&= \delta^{ab} \left(\frac{1}{k^2} + \frac{1}{k^2} \left[g^2 \frac{N}{N^2 - 1} \int \frac{d^4 q}{(2\pi)^4} \frac{(k-q)_\mu k_\nu}{(k-q)^2} \right. \right. \\
&\quad \left. \left. \times \langle A_\mu^a A_\nu^a \rangle \right] \frac{1}{k^2} \right) + O(g^4) \\
&= \delta^{ab} \frac{1}{k^2} (1 + \sigma(k^2)) + O(g^4), \tag{71}
\end{aligned}$$

with

$$\sigma(k^2) = \frac{N}{N^2 - 1} \frac{g^2}{k^2} \int \frac{d^4 q}{(2\pi)^4} \frac{(k-q)_\mu k_\nu}{(k-q)^2} \langle A_\mu^a A_\nu^a \rangle. \tag{72}$$

Going back to the original formulation of Gribov [7], being inside the region Ω , is equivalent to stating that

$$\sigma(k^2) \leq 1, \tag{73}$$

which is called the *no-pole condition*. In this case, the ghost propagator can be rewritten in the following form:

$$\mathcal{G}(k^2) = \frac{1}{k^2} \frac{1}{1 - \sigma(k^2)} + O(g^4), \tag{74}$$

which represents the fact that we are working at the level of the inverse propagator or equivalently, at the level of the 1-particle irreducible n -point functions, which are generated by the effective action Γ . This form is more natural, as we can now impose the gap equation (16), which is also formulated at the level of the effective action. However, in the next section, it shall become clear that the current action S'' does not guarantee us that we are located within the region Ω as $\sigma(0) \geq 1$. Therefore, we add a second term to the action, S_{en} , given by

$$S_{\text{en}} = 2 \frac{d(N^2 - 1)}{\sqrt{2g^2 N}} \int d^d x \mathfrak{s} \gamma^2 J \tag{75}$$

with \mathfrak{s} a new parameter. We have introduced the particular prefactor of $2 \frac{d(N^2 - 1)}{\sqrt{2g^2 N}}$ for later convenience. As it is a constant term, is it comparable with the term $(-\int d^d x 4(N^2 - 1)\gamma^4)$ in the original Gribov-Zwanziger formulation (9). Therefore, it can be responsible for allowing us to stay inside the Gribov horizon by enabling σ to be smaller than 1. The explicit calculation of σ will be done in the next section, but we can already intuitively sketch the reasoning why σ will be altered. As this new term is independent of the fields, it will only enter the expression for the vacuum energy. However, due to the gap equation (16), it will also enter in the expression of the ghost propagator

(and analogously any other quantity which contains γ^2). Recapitulating, the complete action now reads

$$S''' = S'' + S_{\text{en}} \tag{76}$$

with S'' given in Eq. (63).

2. Renormalizability

The renormalizability of S''' can be easily verified. Therefore, we replace S_{en} with

$$\Sigma_{\text{en}} = \int d^d x \mathfrak{s} \Theta J, \tag{77}$$

with Θ a color singlet and BRST invariant source, $s\Theta = 0$. In the end, we give Θ the physical value of

$$\Theta|_{\text{phys}} = 2 \frac{d(N^2 - 1)}{\sqrt{2g^2 N}} \gamma^2, \tag{78}$$

to return to the original action S''' . Again, we embed the action S''' into a larger action Σ'' ,

$$\Sigma'' = \Sigma' + \Sigma_{\text{en}}, \tag{79}$$

with Σ' given by (66). Firstly, as it is easily checked, the term Σ_{en} can only give rise to an additional harmless classical breaking in the Ward identities. Therefore, all the previous Ward identities will remain valid. Secondly, we have the following additional Ward identity:

$$\frac{\delta \Sigma''}{\delta \Theta} = \mathfrak{s} J, \tag{80}$$

which implies that the counterterm is independent from Θ . Taking these two arguments together, we can conclude that the counterterm will be exactly the same as before, given by (67). Therefore,

$$\frac{\mathfrak{s}_0 \gamma_0^2 J_0}{g_0} = \frac{\mathfrak{s} \gamma^2 J}{g} \tag{81}$$

and consequently, no new renormalization factor is necessary,

$$Z_{\mathfrak{s}} = Z_g Z_{\gamma^2}^{-1} Z_J^{-1}. \tag{82}$$

3. Boundary condition

Introducing a new parameter \mathfrak{s} requires a second gap equation in order to determine this new parameter. We recall that, in the case in which $M^2 = 0$ or equivalently in the original Gribov-Zwanziger formulation, we have

$$\sigma(k^2 \approx 0) = 1 - Ck^2, \tag{83}$$

with C a certain positive constant, which causes the enhancement of the ghost propagator $\mathcal{G}(k^2)$ at zero momentum,

$$\mathcal{G}(k^2 \approx 0) \sim \frac{1}{Ck^4}. \quad (84)$$

Therefore, we know that at zero momentum, slowly switching off M^2 will cause $\sigma(k^2 = 0)$ going to 1. It is therefore very natural to demand that this transition has to occur smoothly by imposing the following boundary condition:

$$\left. \frac{\partial \sigma(0)}{\partial M^2} \right|_{M^2=0} = 0. \quad (85)$$

In summary, we have now two gap equations. Firstly, the gap equation $\frac{\partial \Gamma}{\partial \gamma^2} = 0$ fixes γ^2 as a function of M^2 and secondly, demanding that $\left. \frac{\partial \sigma(0)}{\partial M^2} \right|_{M^2=0} = 0$ will uniquely fix s . This leaves us with one free parameter, M^2 , the fixation of which shall be discussed in Sec. IV.

III. THE MODIFIED GLUON AND GHOST PROPAGATOR

Now that we have constructed the action S''' , by adding two additional terms $S_{\bar{\varphi}\varphi}$ and S_{en} to the original Gribov-Zwanziger action, we investigate the gluon and the ghost propagator in detail. For the calculations, we have replaced the sources J and τ with the more conventional mass notations M^2 and m^2 respectively.

A. The gluon propagator

We shall first examine the tree level gluon propagator. In order to calculate this free gluon propagator we only need that part of the free action S'' containing the A -fields and the φ , $\bar{\varphi}$ -fields. This free action reads

$$\begin{aligned} S''_0 = \int d^4x & \left[\frac{1}{4} (\partial_\mu A_\nu^a - \partial_\nu A_\mu^a)^2 + \frac{1}{2\alpha} (\partial_\mu A_\mu^a)^2 \right. \\ & + \bar{\varphi}_\mu^{ab} \partial^2 \varphi_\mu^{ab} - \gamma^2 g (f^{abc} A_\mu^a \varphi_\mu^{bc} + f^{abc} A_\mu^a \bar{\varphi}_\mu^{bc}) \\ & \left. - M^2 \bar{\varphi}_\mu^{ab} \varphi_\mu^{ab} + \frac{m^2}{2} A_\mu^2 + \dots \right], \quad (86) \end{aligned}$$

where the limit $\alpha \rightarrow 0$ is understood in order to recover the

Landau gauge. The “ \dots ” stand for the constant terms $-d(N^2 - 1)\gamma^4$ and $2 \frac{4(N^2-1)}{\sqrt{2g^2N}} s \gamma^2 M^2$ and other terms in the ghost- and ω , $\bar{\omega}$ -fields irrelevant for the calculation of the gluon propagator. Next, we integrate out the φ - and $\bar{\varphi}$ -fields. As we are only interested in the gluon propagator, we simply use the equations of motion, $\frac{\partial S''_0}{\partial \varphi_\mu^{bc}} = 0$ and $\frac{\partial S''_0}{\partial \bar{\varphi}_\mu^{bc}} = 0$, which give

$$\varphi_\mu^{bc} = \bar{\varphi}_\mu^{bc} = \frac{1}{\partial^2 - M^2} \gamma^2 g f^{abc} A_\mu^a. \quad (87)$$

We use this result to rewrite S''_0 ,

$$\begin{aligned} S''_0 &= \int d^4x \left[\frac{1}{4} (\partial_\mu A_\nu^a - \partial_\nu A_\mu^a)^2 + \frac{1}{2\alpha} (\partial_\mu A_\mu^a)^2 + \frac{m^2}{2} A_\mu^2 \right. \\ & \quad + \gamma^4 g^2 f^{abc} A_\mu^a \frac{1}{\partial^2 - M^2} f^{dbc} A_\mu^d \\ & \quad \left. - 2\gamma^4 g \left(f^{abc} A_\mu^a \frac{1}{\partial^2 - M^2} g f^{dbc} A_\mu^d \right) + \dots \right] \\ &= \int d^4x \left[\frac{1}{4} (\partial_\mu A_\nu^a - \partial_\nu A_\mu^a)^2 + \frac{1}{2\alpha} (\partial_\mu A_\mu^a)^2 \right. \\ & \quad \left. + \frac{m^2}{2} A_\mu^2 - N \gamma^4 g^2 A_\mu^a \frac{1}{\partial^2 - M^2} A_\mu^a + \dots \right]. \quad (88) \end{aligned}$$

The last step is explained with the following relation:

$$f^{abc} f^{dbc} = N \delta^{ad}, \quad (89)$$

and we restrict ourselves to the color group $SU(N)$ throughout. We continue rewriting S''_0 so we can easily read the gluon propagator

$$\begin{aligned} S''_0 &= \int d^4x \left[\frac{1}{2} A_\mu^a \Delta_{\mu\nu}^{ab} A_\nu^b + \dots \right], \\ \Delta_{\mu\nu}^{ab} &= \left[\left(-\partial^2 + m^2 - \frac{2g^2 N \gamma^4}{\partial^2 - M^2} \right) \delta_{\mu\nu} \right. \\ & \quad \left. - \partial_\mu \partial_\nu \left(\frac{1}{\alpha} - 1 \right) \right] \delta^{ab}. \quad (90) \end{aligned}$$

The gluon propagator can be determined by taking the inverse of $\Delta_{\mu\nu}^{ab}$ and converting it to momentum space. Doing so, we find the following expression

$$\begin{aligned} \langle A_\mu^a(p) A_\nu^b(-p) \rangle &= \frac{1}{p^2 + m^2 + \frac{2g^2 N \gamma^4}{p^2 + M^2}} \left[\delta_{\mu\nu} - \frac{P_\mu P_\nu}{p^2} \right] \delta^{ab} \\ &= \underbrace{\frac{p^2 + M^2}{p^4 + (M^2 + m^2)p^2 + 2g^2 N \gamma^4 + M^2 m^2}}_{\mathcal{D}(p^2)} \left[\delta_{\mu\nu} - \frac{P_\mu P_\nu}{p^2} \right] \delta^{ab}. \quad (91) \end{aligned}$$

From this expression we can already make two observations:

(i) $\mathcal{D}(p^2)$ enjoys infrared suppression.

(ii) $\mathcal{D}(0) \propto M^2$, so the gluon propagator does not vanish at the origin. Even if we set $m^2 = 0$ we still find a nonvanishing gluon propagator, so we want to

stress that this different result is clearly due to the novel mass term proportional to $\bar{\varphi}\varphi - \bar{\omega}\omega$.

In Sec. IV B we shall uncover a third property, namely, that $\mathcal{D}(p^2)$ displays a positivity violation. Also this observation is in accordance with the latest lattice results [27].

B. The ghost propagator

The observation that $m^2 = 0$ does not qualitatively alter the gluon propagator will be repeated for the ghost propagator. Henceforth, we set $m^2 = 0$, which also improves the readability of the paper. However, all calculations could in principle be repeated with the inclusion of the mass m^2 .

We start with the expression for the ghost propagator. Substituting the expression of the gluon propagator, we find

$$\begin{aligned}\sigma(k^2) &= \frac{N}{N^2 - 1} \frac{g^2}{k^2} \int \frac{d^d q}{(2\pi)^d} \frac{(k - q)_\mu k_\nu}{(k - q)^2} \langle A_\mu^a A_\nu^a \rangle \\ &= N g^2 \frac{k_\mu k_\nu}{k^2} \int \frac{d^d q}{(2\pi)^d} \frac{1}{(k - q)^2} \frac{q^2 + M^2}{q^4 + M^2 q^2 + \lambda^4} \\ &\quad \times \left[\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right],\end{aligned}\quad (92)$$

where we have also used Eq. (61). As we are interested in the infrared behavior of this propagator, we expand the previous expression for small k^2 ,

$$\begin{aligned}\sigma(k^2 \approx 0) &= N g^2 \frac{k_\mu k_\nu}{k^2} \frac{d - 1}{d} \delta_{\mu\nu} \int \frac{d^d q}{(2\pi)^d} \frac{1}{q^2} \\ &\quad \times \frac{q^2 + M^2}{q^4 + M^2 q^2 + \lambda^4} + O(k^2) \\ &= N g^2 \frac{d - 1}{d} \int \frac{d^d q}{(2\pi)^d} \frac{1}{q^2} \frac{q^2 + M^2}{q^4 + M^2 q^2 + \lambda^4} \\ &\quad + O(k^2).\end{aligned}\quad (93)$$

For later use, let us rewrite $\sigma(0)$ as

$$\begin{aligned}\sigma(0) &= N g^2 \frac{d - 1}{d} \int \frac{d^d q}{(2\pi)^d} \frac{1}{q^4 + M^2 q^2 + \lambda^4} \\ &\quad + N g^2 M^2 \frac{d - 1}{d} \int \frac{d^d q}{(2\pi)^d} \frac{1}{q^2} \frac{1}{q^4 + M^2 q^2 + \lambda^4}.\end{aligned}\quad (94)$$

Notice that the first integral in the right-hand side of Eq. (94) diverges while the second integral is UV finite in 4D.

We continue with the derivation of the gap equations as we would like to write λ^2 as a function of M^2 , i.e. $\lambda^2(M^2)$, in expression (94). Firstly, we calculate the horizon condition (16) explicitly starting from the effective action. The one loop effective action $\Gamma_\gamma^{(1)}$ is obtained from the quadratic part of our action S''

$$e^{-\Gamma_\gamma^{(1)}} = \int d\Phi e^{-S''}. \quad (95)$$

This time, the terms $-d(N^2 - 1)\gamma^4$ and $2 \frac{d(N^2 - 1)}{\sqrt{2g^2 N}} \mathfrak{s} \gamma^2 M^2$ have to be maintained, as they will enter the horizon condition. After a straightforward calculation the one loop effective action in d dimensions yields,

$$\begin{aligned}\Gamma_\gamma^{(1)} &= -d(N^2 - 1)\gamma^4 + 2 \frac{d(N^2 - 1)}{\sqrt{2g^2 N}} \mathfrak{s} \gamma^2 M^2 + \frac{(N^2 - 1)}{2} \\ &\quad \times (d - 1) \int \frac{d^d q}{(2\pi)^d} \ln \frac{q^4 + M^2 q^2 + 2g^2 N \gamma^2}{q^2 + M^2}.\end{aligned}\quad (96)$$

Setting $\lambda^4 = 2g^2 N \gamma^4$ [see Eq. (61)], we rewrite the previous expression,

$$\begin{aligned}\mathcal{E}^{(1)} &= \frac{\Gamma_\gamma^{(1)}}{N^2 - 1} \frac{2g^2 N}{d} \\ &= -\lambda^4 + 2\mathfrak{s} \lambda^2 M^2 + g^2 N \frac{d - 1}{d} \int \frac{d^d q}{(2\pi)^d} \\ &\quad \times \ln \frac{q^4 + M^2 q^2 + \lambda^4}{q^2 + M^2},\end{aligned}\quad (97)$$

and apply the gap equation (16),

$$\begin{aligned}\frac{\partial \mathcal{E}^{(1)}}{\partial \lambda^2} &= 2\lambda^2 \left(-1 + \mathfrak{s} \frac{M^2}{\lambda^2} + g^2 N \frac{d - 1}{d} \int \frac{d^d q}{(2\pi)^d} \right. \\ &\quad \left. \times \frac{1}{q^4 + M^2 q^2 + \lambda^4} \right) = 0.\end{aligned}\quad (98)$$

Secondly, we impose the boundary condition (85) in order to obtain an explicit value for \mathfrak{s} . Instead of explicitly starting from expression (92) to fix \mathfrak{s} , there is a much simpler way to find the corresponding \mathfrak{s} . Therefore, we act with $\frac{\partial}{\partial M^2}$ on the gap equation (98). Subsequently setting $M^2 = 0$, gives

$$\mathfrak{s} \frac{1}{\lambda^2(0)} - \frac{d - 1}{d} g^2 N \int \frac{d^d q}{(2\pi)^d} \frac{1}{q^2} \frac{1}{q^4 + \lambda^4(0)} = 0, \quad (99)$$

where we imposed (85). Proceeding, we find

$$-\frac{d - 1}{d} g^2 N \int \frac{d^d q}{(2\pi)^d} \frac{1}{q^2} \frac{1}{q^4 + \lambda^4(0)} + \mathfrak{s} \frac{1}{\lambda^2(0)} = 0 \Rightarrow \mathfrak{s} = \lambda^2(0) \frac{3}{4} g^2 N \int \frac{d^d q}{(2\pi)^d} \frac{1}{q^2} \frac{1}{q^4 + \lambda^4(0)} \Rightarrow \mathfrak{s} = \frac{3g^2 N}{128\pi}, \quad (100)$$

which determines ς at the current order.

With the help of the latter two gap equations (98) and (100), we can rephrase the correction to the self energy of the ghost. Combining equation (94) and (98) we can write

$$\sigma(0) = 1 + M^2 g^2 N \frac{d-1}{d} \int \frac{d^d q}{(2\pi)^d} \frac{1}{q^2} \times \frac{1}{q^4 + M^2 q^2 + \lambda^4(M^2)} - \varsigma \frac{M^2}{\lambda^2(M^2)}. \quad (101)$$

From this expression, we can make several observations. Firstly, when $M^2 = 0$, from the previous expression it immediately follows that

$$\sigma(0) = 1, \quad (102)$$

which gives back the ordinary Gribov-Zwanziger result [7,8,10,25]. Indeed, from the previous expression, one

$$\begin{aligned} \sigma(0) &= 1 + M^2 g^2 N \frac{3}{4} \int \frac{d^4 q}{(2\pi)^4} \frac{1}{q^2} \frac{1}{q^4 + M^2 q^2 + \lambda^4(M^2)} - \frac{M^2}{\lambda^2(M^2)} \lambda^2(0) \frac{3}{4} g^2 N \int \frac{d^4 q}{(2\pi)^4} \frac{1}{q^2} \frac{1}{q^4 + \lambda^4(0)} \\ &= 1 + \frac{3}{4} \frac{M^2}{\lambda^2(M^2)} g^2 N \int \frac{d^4 p}{(2\pi)^4} \frac{1}{p^2} \frac{1}{p^4 + \frac{M^2}{\lambda^2(M^2)} p^2 + 1} - \frac{3}{4} \frac{M^2}{\lambda^2(M^2)} g^2 N \frac{3}{4} g^2 N \int \frac{d^4 p}{(2\pi)^4} \frac{1}{p^2} \frac{1}{p^4 + 1} \\ &= 1 - \frac{3x^2}{4} g^2 N \int \frac{d^4 p}{(2\pi)^4} \frac{1}{p^2} \frac{1}{(p^4 + x p^2 + 1)(p^4 + 1)}, \end{aligned} \quad (103)$$

with $x = \frac{M^2}{\lambda^2(M^2)} \geq 0$, hence $\sigma(0) \leq 1$. At this point, we can really appreciate the role of the novel vacuum term (75). It serves as a stabilizing term for the horizon condition. Indeed, without the term (75), we would end up outside of the Gribov region for some $k^2 > 0$, even for an infinitesimal¹ $M^2 > 0$. In this sense, the action S''' constitutes a refinement of the original Gribov-Zwanziger action, which is a smooth limiting case of S''' .

For later use, we can evaluate the integral in expression (101) as it is finite. The explicit one loop value for $\sigma(0)$ yields

$$\begin{aligned} \sigma(0) &= 1 + M^2 \frac{3g^2 N}{64\pi^2} \frac{1}{\sqrt{M^4 - 4\lambda^4}} [\ln(M^2 + \sqrt{M^4 - 4\lambda^4}) \\ &\quad - \ln(M^2 - \sqrt{M^4 - 4\lambda^4})] - \left(\frac{3g^2 N}{128\pi}\right) \frac{M^2}{\lambda^2(M^2)}, \end{aligned} \quad (104)$$

where we have substituted the value (100) for ς .

In summary, we have found a ghost propagator which is no longer enhanced. So far, we have fixed λ^2 in function of M^2 and we have found a constant value for ς . However, we have not yet fixed M^2 . This will be the task of the next section.

¹Notice that we must take $M^2 \geq 0$ to avoid unwanted tachyonic instabilities.

derived that the ghost propagator,

$$\mathcal{G}(k^2) = \frac{1}{k^2} \frac{1}{1 - \sigma(k^2)},$$

is enhanced and behaves like $1/k^4$, for $k^2 \approx 0$. Secondly, when $M^2 \neq 0$, we notice that the ghost propagator is no longer enhanced and behaves like $1/k^2$ as already found in [22], which is in qualitative agreement with the latest lattice results. This behavior is clearly due to the novel mass term $M^2 \int d^4 x (\bar{\varphi}_i^a \varphi_i^a - \bar{\omega}_i^a \omega_i^a)$. Thirdly, we see that the term in ς is crucial in order to obtain a $\sigma(0)$ which is smaller than 1. Omitting this term would result in $\sigma(0) > 1$ in the case that $M^2 \neq 0$. However, including this term, we can easily prove that $\sigma \leq 1$. Indeed, taking expression (101) and replacing ς with the integral in (100), we find

IV. A DYNAMICAL VALUE FOR M^2

Up to this point, we have only introduced the mass M^2 by hand, however it is recommendable to obtain a dynamical value for this parameter. We shall present two methods to find such a value. Firstly, we explain how to obtain a dynamical value for M^2 with the help of the effective action. However, as the calculations become too involved, we investigate a second method, the variational principle, and apply this to the ghost and gluon propagator, with more success.

A. The effective action and the gap equations

We first explain the idea behind the method before going into detailed calculations. In the previous section we have derived the gluon propagator. We recall that the mass term $m^2 A_\mu^2$ does not qualitatively change the form of the gluon and ghost propagators, therefore we have put $m = 0$ for our purpose. With $m = 0$, the tree level propagator (91) yields

$$\mathcal{D}(p^2) = \frac{p^2 + M^2}{p^4 + M^2 p^2 + 2g^2 N \gamma^4}. \quad (105)$$

Expanding the mass M^2 as a series in g^2 gives

$$M^2 = M_0^2 + g^2 M_1^2 + g^4 M_2^2 + \dots \quad (106)$$

We only need to consider M_0 , which is of order unity, as we are considering the tree level propagator. We know that at

the end of our calculations we have to set our sources equal to zero, or $J = M^2 = 0$. If we work at lowest order, this means we have to set $M_0 = 0$ (and the gluon propagator will not display the desired behavior). However, going one order higher gives

$$M_0^2 + g^2 M_1^2 = 0. \quad (107)$$

The last equation might imply that M_0^2 is no longer equal to zero, and consequently, the tree level gluon propagator will attain the desired form. Let us elaborate further on this aspect.

1. One loop effective potential

To implement the above-mentioned ideas, we shall first calculate the one loop energy functional. We start with the action (63), whereby setting $m = 0$ is equivalent with putting $\tau = 0$. We replace the mass M^2 again with the source J . In order to determine the one loop effective action, we first need the one loop energy functional $W_0(J)$ which we obtain from the quadratic part of the action,

$$e^{-W_0(J)} = \int d\Phi e^{-S''}. \quad (108)$$

From the previous expression we find for $W_0(J)$,

$$W_0(J) = -\frac{d(N^2 - 1)}{2g^2 N} \lambda^4 + \frac{d(N^2 - 1)}{g^2 N} \varsigma \lambda^2 J + \frac{(N^2 - 1)}{2} \times (d - 1) \int \frac{d^d p}{(2\pi)^d} \ln \left[p^2 \left(p^2 + \frac{\lambda^4}{p^2 + J} \right) \right]. \quad (109)$$

We shall work in the $\overline{\text{MS}}$ scheme, and use a notational shorthand:

$$m_1^2 = \frac{J - \sqrt{J^2 - 4\lambda^4}}{2}, \quad m_2^2 = \frac{J + \sqrt{J^2 - 4\lambda^4}}{2}, \quad (110)$$

whereby λ^2 is defined in Eq. (61). Evaluating the integrals in $W_0(J)$ gives

$$W_0(J) = -\frac{4(N^2 - 1)}{2g^2 N} \lambda^4 + \frac{d(N^2 - 1)}{g^2 N} \varsigma \lambda^2 M^2 + \frac{3(N^2 - 1)}{64\pi^2} \left(\frac{8}{3} \lambda^4 + m_1^4 \ln \frac{m_1^2}{\mu^2} + m_2^4 \ln \frac{m_2^2}{\mu^2} - J^2 \ln \frac{J}{\mu^2} \right). \quad (111)$$

This calculation is explained in detail in the Appendix.

As we have determined the energy functional $W_0(J)$, we can now calculate the one loop effective action via the Legendre transform of $W(J)$. If we define

$$\sigma(x) = \frac{\delta W(J)}{\delta J(x)} \quad \sigma_{\text{cl}} = \frac{d(N^2 - 1)}{g^2 N} \varsigma \lambda^2 \quad (112)$$

then

$$\hat{\sigma}(x) = \sigma(x) - \sigma_{\text{cl}} = -\frac{\int d\Phi (\bar{\varphi}\varphi - \bar{\omega}\omega) e^{-S''}}{\int d\Phi e^{-S''}}, \quad (113)$$

represents the expectation value of the local composite operator $-(\bar{\varphi}\varphi - \bar{\omega}\omega)$. The effective action is given by

$$\Gamma(\sigma) = W(J) - \int d^4 x J(x) \sigma(x), \quad (114)$$

or equivalently, as we prefer to work in the variable $\hat{\sigma}$,

$$\Gamma(\hat{\sigma}) = W(J) - \int d^4 x J(x) (\hat{\sigma}(x) + \sigma_{\text{cl}}). \quad (115)$$

Calculating $\Gamma(\hat{\sigma})$ by explicitly doing the inversion is a rather cumbersome task. In most cases one can perform a Hubbard-Stratonovich transformation to eliminate the term $J(\bar{\varphi}\varphi - \bar{\omega}\omega)$ from the action and introduce a new field σ' which couples linearly to J . This greatly simplifies the calculation. However, in this case, it seems impossible to do such a transformation as a required term in J^2 is missing. Hence, there is no other option than to actually perform the inversion. In order to calculate this inversion, we shall limit ourself to constant J and $\hat{\sigma}$ as we are mainly interested in the (space-time) independent vacuum expectation value of the operator $-(\bar{\varphi}\varphi - \bar{\omega}\omega)$ coupled to the source J . This vacuum expectation value is given by

$$\hat{\sigma}|_{J=0} = -\frac{\int d\Phi (\bar{\varphi}\varphi - \bar{\omega}\omega) e^{-S}}{\int d\Phi e^{-S}}, \quad (116)$$

where S represents the ordinary Gribov-Zwanziger action (8). As we already have calculated $W(J)$ up to one loop it is straightforward to verify that

$$\begin{aligned} \hat{\sigma} &= \frac{\partial}{\partial J} W_0(J) - \sigma_{\text{cl}} \\ &= \frac{1}{2} \frac{3(N^2 - 1)}{64\pi^2} J \left(2 \ln \frac{t}{4} + \left(\sqrt{1-t} + \frac{1}{\sqrt{1-t}} \right) \right. \\ &\quad \left. \times \ln \frac{1 + \sqrt{1-t}}{1 - \sqrt{1-t}} \right), \end{aligned} \quad (117)$$

whereby we shortened the notation by putting $t = 4\lambda^4/J^2$. From the previous expression we find for the condensate

$$\hat{\sigma}|_{J=0} = -\frac{3(N^2 - 1)}{64\pi} \lambda. \quad (118)$$

This is an important result, as it indicates that a nonzero value for the Gribov parameter γ will result in a non-vanishing condensate $\langle -(\bar{\varphi}\varphi - \bar{\omega}\omega) \rangle$ even at the perturbative level.

We are now ready to compute the effective action up to one loop along the lines of [28]. The energy functional can be written as a series in the coupling constant g^2 ,

$$W(J) = W_0(J) + g^2 W_1(J) + \dots = \sum_{i=0}^{\infty} (g^2)^i W_i(J). \quad (119)$$

As a consequence, looking at the definition (112), we can write

$$\hat{\sigma} = \hat{\sigma}_0(J) + g^2 \hat{\sigma}_1(J) + \dots = \sum_{i=0}^{\infty} (g^2)^i \hat{\sigma}_i(J), \quad (120)$$

where $\hat{\sigma}_i(J)$ corresponds to the i th order in g^2 (regarding J as of order unity). This is called the original series. The inverted series is defined as

$$J = J_0(\hat{\sigma}) + g^2 J_1(\hat{\sigma}) + \dots = \sum_{j=0}^{\infty} (g^2)^j J_j(\hat{\sigma}), \quad (121)$$

with $J_j(\hat{\sigma})$ the j th order coefficient. Substituting (121) into (120) gives,

$$\begin{aligned} \hat{\sigma} &= \sum_{i=0}^{\infty} (g^2)^i \hat{\sigma}_i \left[\sum_{j=0}^{\infty} (g^2)^j J_j(\hat{\sigma}) \right] \\ &= \hat{\sigma}_0(J_0(\hat{\sigma})) + g^2 (\hat{\sigma}'_0(J_0(\hat{\sigma})) \cdot J_1(\hat{\sigma}) \\ &\quad + \hat{\sigma}_1(J_0(\hat{\sigma}))) + \dots. \end{aligned} \quad (122)$$

By regarding $\hat{\sigma}$ as of the order unity and by comparing both sides of the last equation, one finds

$$\hat{\sigma} = \hat{\sigma}_0(J_0(\hat{\sigma})), \quad (123)$$

$$\begin{aligned} J_1(\hat{\sigma}) &= -\frac{\hat{\sigma}_1(J_0(\hat{\sigma}))}{\hat{\sigma}'_0(J_0(\hat{\sigma}))}. \\ &\vdots \end{aligned} \quad (124)$$

For the moment, as we are working at lowest order, we only need Eq. (123). We can invert this equation, so we find for $J_0(\hat{\sigma})$,

$$J_0(\hat{\sigma}) = \hat{\sigma}_0^{-1}(\hat{\sigma}), \quad (125)$$

meaning that we have to solve

$$\begin{aligned} \hat{\sigma} &\equiv \hat{\sigma}_0(J_0, \lambda) \\ &= \frac{1}{2} \frac{3(N^2 - 1)}{64\pi^2} J_0 \left(2 \ln \frac{t(\lambda, J_0)}{4} + \left(\sqrt{1 - t(\lambda, J_0)} \right. \right. \\ &\quad \left. \left. + \frac{1}{\sqrt{1 - t(\lambda, J_0)}} \right) \ln \frac{1 + \sqrt{1 - t(\lambda, J_0)}}{1 - \sqrt{1 - t(\lambda, J_0)}} \right), \end{aligned} \quad (126)$$

for J_0 , so we can write

$$J_0 = f(\hat{\sigma}, \lambda). \quad (127)$$

We immediately suspect that this inversion will not give rise to an analytical expression. Once we have found $f(\hat{\sigma}, \lambda)$, we substitute this expression into the effective action,

$$\Gamma(\hat{\sigma}, \lambda) = W(f(\hat{\sigma}, \lambda), \lambda) - f(\hat{\sigma}, \lambda) \hat{\sigma}. \quad (128)$$

At this point, as we have found an expression for the one loop effective action, we can implement two equations to fix $\hat{\sigma}$ and λ . Firstly, the minimization condition reads

$$\frac{\partial}{\partial \hat{\sigma}} \Gamma(\hat{\sigma}, \lambda) = 0, \quad (129)$$

and secondly, the horizon condition (16) can be translated as

$$\frac{\partial}{\partial \lambda} \Gamma(\hat{\sigma}, \lambda) = 0. \quad (130)$$

We start with the first gap equation. Replacing Γ by Eq. (115) leads to

$$\begin{aligned} \frac{\partial}{\partial \hat{\sigma}} \Gamma(\hat{\sigma}, \lambda) &= 0 \Rightarrow \frac{\partial W}{\partial J} \frac{\partial J}{\partial \hat{\sigma}} - \frac{\partial J}{\partial \hat{\sigma}} \hat{\sigma} - \frac{\partial J}{\partial \hat{\sigma}} \sigma_{\text{cl}} - J \\ &= 0 \Rightarrow J = 0 \Rightarrow f(\hat{\sigma}, \lambda) = 0. \end{aligned} \quad (131)$$

Since there are only 2 explicit scales, λ and $\hat{\sigma}$, present, the first gap equation can be used to express e.g. $\hat{\sigma}$ in terms of λ . For the sake of a numerical computation, we can therefore momentarily set $\lambda = 1$. From Fig. 2 one can obtain an estimate $\hat{\sigma}'$ of $f(\hat{\sigma}', 1) = 0$, with $\hat{\sigma}' = \frac{2}{3} 64\pi^2 \frac{\hat{\sigma}}{N^2 - 1}$. Doing so, we find $\hat{\sigma}' \approx -6.28$, so that

$$\hat{\sigma} \approx -6.28 \times \left(\frac{3(N^2 - 1)}{128\pi^2} \right) \lambda, \quad (132)$$

which of course corresponds to the already obtained perturbative solution (118). The second gap equation (130) must then consequently also give us back the perturbative solution. To check this, we first calculate the perturbative result for λ by taking the limit $J \rightarrow 0$ in expression (111)

$$\Gamma_0 = -\frac{2(N^2 - 1)}{g^2 N} \lambda^4 + \frac{3(N^2 - 1)}{64\pi^2} \left(\frac{8}{3} \lambda^4 - 2\lambda^4 \ln \frac{\lambda^2}{\bar{\mu}^2} \right). \quad (133)$$

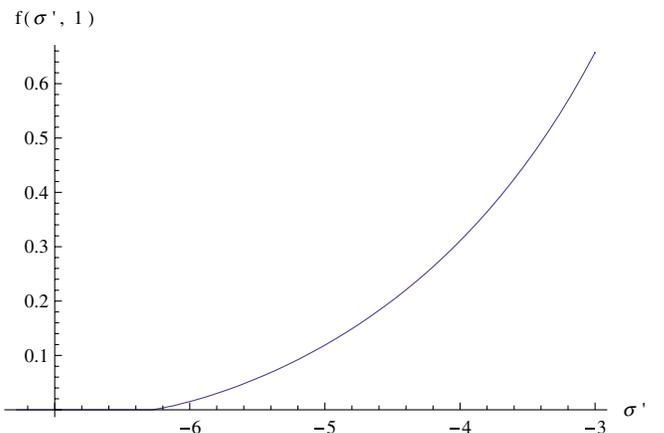


FIG. 2 (color online). A plot of $f(\hat{\sigma}', 1)$ in terms of $\hat{\sigma}' = \frac{2}{3} 64\pi^2 \frac{\hat{\sigma}}{N^2 - 1}$.

Next, we take the partial derivative with respect to λ which gives

$$\frac{\partial \Gamma_0}{\partial \lambda} = 4\lambda^3 \left(-\frac{2(N^2 - 1)}{g^2 N} + \frac{3(N^2 - 1)}{64\pi^2} \left(\frac{5}{3} - 2 \ln \frac{\lambda^2}{\bar{\mu}^2} \right) \right). \quad (134)$$

The natural choice for the renormalization constant is to set $\bar{\mu} = \lambda$ to kill the logarithms. Imposing the gap equation $\frac{\partial \Gamma_0}{\partial \lambda} = 0$ gives us

$$\frac{g^2 N}{16\pi^2} = \frac{8}{5}. \quad (135)$$

We remark that we have neglected the solution $\gamma = 0$, as explained in Sec. II A. From

$$g^2(\bar{\mu}^2) = \frac{1}{\beta_0 \ln \frac{\bar{\mu}^2}{\Lambda_{\overline{\text{MS}}}^2}}, \quad \text{with } \beta_0 = \frac{11}{3} \frac{N}{16\pi^2}, \quad (136)$$

and expression (135) we find an estimate for λ ,

$$\lambda^4 = e^{44/15}, \quad (137)$$

where we have worked in units $\Lambda_{\overline{\text{MS}}} = 1$. This perturbative solution is also in compliance with [25]. Now, we return to the effective action (128). We first take the partial derivative with respect to λ , afterwards we set $N = 3$, we explicitly replace g^2 by expression (136), and we use the minimizing condition (132). Numerically, we find the following value for λ^4 :

$$\lambda^4 = 1.41, \quad (138)$$

as one can read from Fig. 3. This is exactly the perturbative result (137). If we calculate the vacuum energy with this value for λ , we find from (133),

$$E_{\text{vac}} = \frac{3}{64} \frac{N^2 - 1}{\pi^2} e^{44/15}. \quad (139)$$

We notice that the vacuum energy is positive.

2. Intermediate conclusion

We can conclude at this point, that in the framework we have used, we recover only the perturbative solution. Unfortunately, at lowest order, one finds $J_0 = 0$ as explained in the beginning of this section, so we were unable to find a dynamical value for M^2 at first order. However, if we would be able to go one order higher, with $J_0 + g^2 J_1 = 0$, we might find $J_0 \neq 0$ and consequently the gluon propagator at tree level would attain the desired form (105). In addition, we might even discover a nonperturbative solution. Unfortunately, this is not as straightforward as at leading order. The main difficulty resides in the evaluation of two loop vacuum bubbles for the effective potential with three different mass scales. While the master integrals are known [29–31], the main complication is that the propagator of (91) with $m^2 = 0$ needs to be split into standard form but this introduces the masses of (110) which are either complex or negative. In either scenario the master two loop vacuum bubble is known for distinct positive masses and involves several dilogarithm functions. Therefore in our case for even the simplest of mass choices the resulting dilogarithms will be complex as well as being a complicated function of m_1^2 , m_2^2 , and λ . Moreover, this is prior to computing the full effective potential itself by adding all the relevant combinations of master integrals together. Therefore, it seems to us that while such a computation could be completed in principle, currently the resulting huge expression could not possibly lend itself to a tractable analysis similar to the relatively simple one we have carried out at one loop.

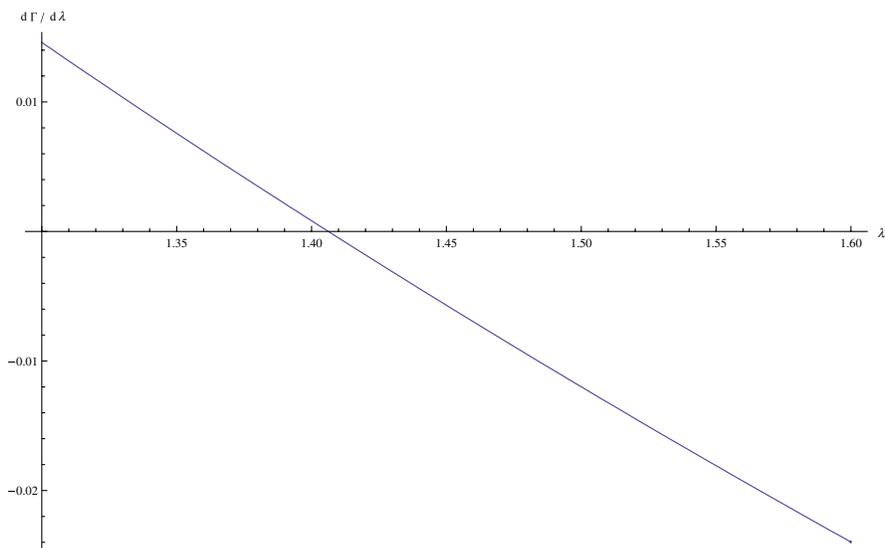


FIG. 3 (color online). The horizon function $\frac{\partial \Gamma}{\partial \lambda}$ for $N = 3$.

B. Applying the variational principle on the ghost propagator and the gluon propagator

In this section, we shall rely on variational perturbation theory in order to find a value for the hitherto arbitrary mass parameter M^2 .

Along the lines of [32], we introduce a formal loop counting parameter $\ell \equiv 1$ by replacing the action S with $\frac{1}{\ell}S$. At the same time, we replace all the fields Φ by $\sqrt{\ell}\Phi$. Symbolically,

$$S(\Phi, g) \rightarrow \frac{1}{\ell}S(\sqrt{\ell}\Phi, g). \quad (140)$$

It is readily derived that multiplying each field with a factor of $\sqrt{\ell}$ and performing an overall $1/\ell$ rescaling is the same as replacing the coupling g with $\sqrt{\ell}g$, so we can replace (140) with

$$S(\Phi, g) \rightarrow S(\Phi, \sqrt{\ell}g). \quad (141)$$

In this fashion, the free (quadratic) part of the action is ℓ -invariant, while every interaction term contains powers² of $\sqrt{\ell}$. The first order in the ℓ -expansion, obtained by setting $\ell = 0$, then corresponds to the free theory. More generally, the ℓ -expansion is equivalent with the loop expansion, where it is understood that we put the formal bookkeeping parameter $\ell = 1$ at the end.

The next step is to introduce the variational parameter M^2 into the theory. This is done in a specific way: we add the quadratic mass term $S_M \equiv M^2 \int d^4x [(\bar{\varphi}\varphi - \bar{\omega}\omega) + \frac{2(N^2-1)}{g^2N} \varsigma\lambda^2]$ to the action, but subtract it again at higher order in ℓ , i.e. we consider the action

$$S(\Phi, g) \rightarrow S(\Phi, \sqrt{\ell}g) + S_M - \ell^k S_M, \quad (142)$$

with $k > 0$. Since $\ell \equiv 1$, we did not change the actual starting action at all.

However, we maintain the strategy of performing an expansion in powers of ℓ . Since the mass term is split up into 2 parts $\sim(1 - \ell^k)M^2$, both parts will enter the ℓ -expansion in a different way. At the end, we must set $\ell = 1$ again. If we could compute an arbitrary quantity \mathcal{Q} exactly, the M^2 -independence would of course be apparent since the theory is not altered. However, at any finite order in ℓ , a residual M^2 -dependence will enter the result for \mathcal{Q} due to the reexpanded powers series in ℓ . Said otherwise, we have partially resummed the perturbative series for \mathcal{Q} by making use of the parameter ℓ . The hope is that some nontrivial information, encoded by the operator coupled to $1 - \ell^k$, will emerge in the final expression for \mathcal{Q} . One query remains: how to handle the M^2 which appears in the approximate \mathcal{Q} ? Therefore we can rely on the lore of minimal sensitivity [33]: we know that the exact \mathcal{Q} cannot depend on M^2 , hence it is very natural to demand that also

²We recall that the perturbative expansion is one in powers of g^2 , and thus in integer powers of ℓ .

at a finite order $\frac{\partial \mathcal{Q}}{\partial M^2} = 0$, leading to a dynamical optimal value for the yet free parameter M^2 .

The described method of variationally introducing extra parameters into a quantum field theory provides us with a powerful tool to study nontrivial dynamical effects in an approximate fashion, yet the calculational efforts do not exceed those of conventional perturbation theory.

We still have to choose a value for k . We recall that the constant term S_{en} was introduced in order to stay within the horizon. Therefore, we want to retain this term when we are applying the variational principle. However, we are working up to first order, meaning that we shall expand the quantity \mathcal{Q} up to first order in ℓ and subsequently set $\ell = 1$. Hence, taking $k = 1$ is not a good option as the constant term would vanish and have no influence. Therefore, a better option is to take e.g. $k = 2$, to assure the consistency of the variational setup with the restriction to the Gribov region. In this way, we are simply coupling the variational parameter M^2 directly to the theory.

1. The ghost propagator

We start from the expression (74) of the ghost propagator

$$\mathcal{G}(k^2) = \frac{1}{k^2} \frac{1}{1 - \sigma(k^2)}, \quad (143)$$

and apply the variational principle on the ghost propagator near zero momentum. We have,

$$\sigma(k^2 \approx 0) = Ng^2 \frac{d-1}{d} \int \frac{d^d q}{(2\pi)^d} \frac{1}{q^2} \frac{q^2 + M^2}{q^4 + M^2 q^2 + \lambda^4} + O(k^2). \quad (144)$$

As explained above, we replace $g^2 \rightarrow \ell g^2$ and $M^2 \rightarrow (1 - \ell^2)M^2$. Subsequently, we expand $\mathcal{G}(k^2)_{k^2 \approx 0}$ in powers of ℓ corresponding to a reordered loop expansion. As we have calculated the ghost propagator up to one loop, we only need to expand the above expression to the first power of ℓ ,

$$\sigma(0) = Ng^2 \ell \frac{d-1}{d} \int \frac{d^d q}{(2\pi)^d} \frac{1}{q^2} \frac{q^2 + M^2}{q^4 + M^2 q^2 + \lambda^4}. \quad (145)$$

As indicated earlier, setting $\ell = 1$ gives

$$\sigma(0) = Ng^2 \frac{d-1}{d} \int \frac{d^d q}{(2\pi)^d} \frac{1}{q^2} \frac{q^2 + M^2}{q^4 + M^2 q^2 + \lambda^4}, \quad (146)$$

which is exactly the same as (144). This expression not only depends on M^2 , but also on λ^2 . However, we already know that λ^2 and M^2 are not independent variables, as they are related through the gap equation (98),

$$-1 + \varsigma \frac{M^2}{\lambda^2} + g^2 N \frac{d-1}{d} \int \frac{d^d q}{(2\pi)^d} \frac{1}{q^4 + M^2 q^2 + \lambda^4} = 0. \quad (147)$$

Following the variational principle, we replace M^2 with $(1 - \ell^2)M^2$ and g^2 with ℓg^2 , expand the equation up to order ℓ^1 , and set $\ell = 1$ in the end. Doing so, we recover again expression (147). At this point, it can be clearly seen that $k = 1$ in Eq. (142) would cancel the effect of the constant term $\frac{sM^2}{\lambda^2}$, while $k = 2$ is a better choice.³ Evaluating the integral in expression (147), we find

$$0 = -1 + \frac{Ng^2}{64\pi^2} \left(\frac{5}{2} + 3 \frac{m_1^2}{\sqrt{M^4 - 4\lambda^4}} \ln \frac{m_1^2}{\bar{\mu}^2} - 3 \frac{m_2^2}{\sqrt{M^4 - 4\lambda^4}} \ln \frac{m_2^2}{\bar{\mu}^2} \right) + s \frac{M^2}{\lambda^2}. \quad (148)$$

This integral could be similarly calculated as done in the Appendix, or one could start from the effective action (128) and derive this equation with respect to λ^2 . We recall that from the boundary condition (100), we have already determined $s = \frac{3g^2N}{128\pi}$.

We still require an appropriate value for $\bar{\mu}$. Therefore, we fix $\bar{\mu}^2 = \frac{3}{2}|M^2 + \sqrt{M^4 - 4\lambda^4}|$ which was chosen as in [25]. We have opted for this specific renormalization scale $\bar{\mu}^2$ which shall result in an acceptably small effective expansion parameter $\frac{g^2N}{16\pi^2}$. Consequently, from Eq. (136), we find

$$\frac{g^2N}{16\pi^2} = \frac{3}{11 \ln\left(\frac{3}{2}|M^2 + \sqrt{M^4 - 4\lambda^4}|\right)} \quad (149)$$

in units of $\Lambda_{\overline{\text{MS}}} = 1$.

In summary, as $\sigma(0)$ remained the same after applying the variational principle, we can take the expression (104) for $\sigma(0)$,

$$\begin{aligned} \sigma(0) = 1 + M^2 \frac{3g^2N}{64\pi^2} \frac{1}{\sqrt{M^4 - 4(\lambda^2(M^2))^2}} & [\ln(M^2 \\ & + \sqrt{M^4 - 4(\lambda^2(M^2))^2}) - \ln(M^2 \\ & - \sqrt{M^4 - 4(\lambda^2(M^2))^2})] - \left(\frac{3g^2N}{128\pi} \right) \frac{M^2}{\lambda^2(M^2)}, \end{aligned} \quad (150)$$

where $\lambda^2(M^2)$ is determined by the gap equation,

$$0 = -1 + \frac{Ng^2}{64\pi^2} \left(\frac{5}{2} + 3 \frac{m_1^2}{\sqrt{M^4 - 4\lambda^4}} \ln \frac{m_1^2}{\bar{\mu}^2} - 3 \frac{m_2^2}{\sqrt{M^4 - 4\lambda^4}} \ln \frac{m_2^2}{\bar{\mu}^2} \right) + \frac{3g^2N}{128\pi} \frac{M^2}{\lambda^2}. \quad (151)$$

Before continuing the analysis, let us first have a look at the gap equation. The gap equation solved for λ^2 as a function of M^2 is depicted in Fig. 4. We find two emerging branches, displayed by a continuous and a dashed line. The former solution exists in the interval $[0, 1.53]$, while the

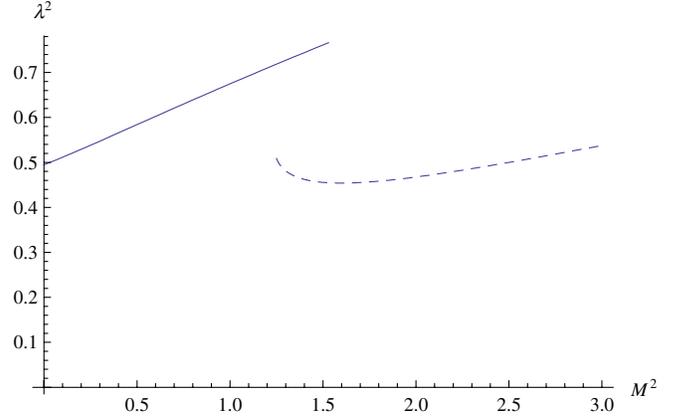


FIG. 4 (color online). λ^2 in function of M^2 in units $\Lambda_{\overline{\text{MS}}} = 1$.

latter one only exists in $[1.25, \infty[$. As the latter branch does not exist around $M^2 = 0$, we shall not consider this solution because the boundary condition (85) demands a smooth transition for the $M^2 \rightarrow 0$ limit.

We can now have a closer look at the ghost propagator or equivalently $\sigma(0)$. We have graphically depicted $\sigma(0)$ in Fig. 5. Firstly, from the figure, we see that $\sigma(0)$ is nicely smaller than 1 for all M^2 in the interval $[0, 1.53]$. This is a remarkable fact as it implies that we have managed to stay within the horizon. Secondly, we notice that the boundary condition $\frac{\partial \sigma(0)}{\partial M^2} \Big|_{M^2=0} = 0$ is indeed fulfilled, which is a nice check on our result. We can now apply the minimal sensitivity approach on the quantity $\sigma(0)$. From Fig. 5 we immediately see that there is no extremum. However, looking at the derivative of $\sigma(0)$ with respect to M^2 in Fig. 6, we do find a point of inflection at $M^2 = 0.37 \Lambda_{\overline{\text{MS}}}^2$. Demanding $\frac{\partial^2 \sigma(0)}{\partial M^2} = 0$ is an alternative option when no extremum is found [33]. Taking this value for M^2 , we find

$$\sigma(0) = 0.93. \quad (152)$$

The effective coupling is given by

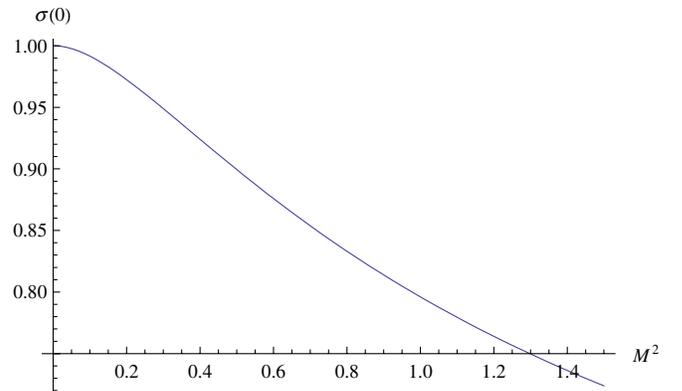


FIG. 5 (color online). $\sigma(0)$ drawn in function of M^2 in units $\Lambda_{\overline{\text{MS}}} = 1$.

³Actually, every value for k , with $k \geq 2$ is allowed.

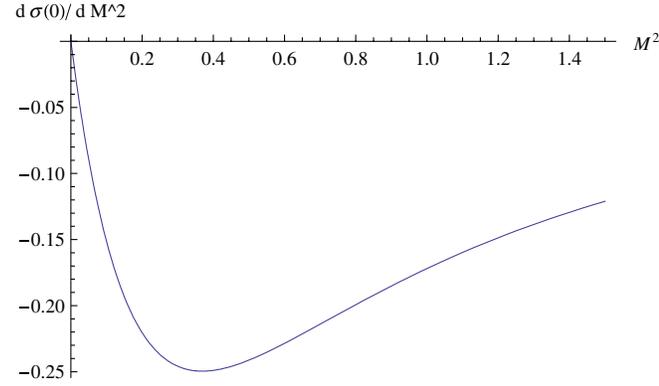


FIG. 6 (color online). $\frac{d\sigma(0)}{dM^2}$ drawn in function of M^2 in units $\Lambda_{\overline{\text{MS}}} = 1$.

$$\frac{g^2 N}{16\pi^2} = 0.53, \quad (153)$$

which is smaller than 1.

2. The gluon propagator

In order to apply the variational principle to the gluon propagator, we require its one loop correction. Given the rather complicated form of the propagator, obtaining the full exact expression for its one loop correction is not possible. Indeed to appreciate how cumbersome such an expression could be one has only to examine the

$M^2 = m^2 = 0$ case [34], where all the one loop corrections to the propagators are given explicitly. However, despite this we can still achieve our main aim of studying the low momentum behavior of the gluon propagator corrections directly in the zero momentum limit without knowledge of the full correction. In [34] this limit for the gluon propagator was deduced from the exact one loop computation. However, the resulting expression tallied with that obtained via the vacuum bubble expansion of the underlying 2-point functions. The latter is a much easier technique to apply and given the equivalence of the expressions it justifies its application to our case when $M^2 \neq 0$. Briefly one expands the 2-point functions relevant to the gluon propagator construction in powers of the external momentum p^2 . Though the expansion is truncated at some order such as $O((p^2)^2)$. The accompanying Feynman integrals are massive vacuum bubbles which are essentially trivial to compute at one loop. However, our situation is complicated significantly by the fact that there is mixing in the quadratic part of the $\{A_\mu^a, \varphi_\mu^{ab}\}$ sector of the tree action. Therefore in addition to the gluon propagator (72), we require the propagators of the remaining fields. For this derivation here we use the conventions and notation of the article [34] for an arbitrary color group, where the $M^2 = 0$ problem was discussed at length. There it is evident that one has to consider the full $\{A_\mu^a, \varphi_\mu^{ab}\}$ part of the momentum space action in order to invert the quadratic sector to derive all the propagators. In the Landau gauge we find the set of propagators for our situation is

$$\begin{aligned} \langle A_\mu^a(p) A_\nu^b(-p) \rangle &= \frac{\delta^{ab}(p^2 + M^2)}{[(p^2)^2 + M^2 p^2 + C_A \gamma^4]} P_{\mu\nu}(p), & \langle A_\mu^a(p) \bar{\varphi}_\nu^{bc}(-p) \rangle &= -\frac{f^{abc} \gamma^2}{\sqrt{2}[(p^2)^2 + M^2 p^2 + C_A \gamma^4]} P_{\mu\nu}(p), \\ \langle \varphi_\mu^{ab}(p) \bar{\varphi}_\nu^{cd}(-p) \rangle &= -\frac{\delta^{ac} \delta^{bd}}{(p^2 + M^2)} \eta_{\mu\nu} + \frac{f^{abe} f^{cde} \gamma^4}{(p^2 + M^2)[(p^2)^2 + M^2 p^2 + C_A \gamma^4]} P_{\mu\nu}(p), \end{aligned} \quad (154)$$

where the presence of $1/\sqrt{2}$ was a key ingredient in ensuring that ghost enhancement correctly emerged in the $M^2 = 0$ case [34]. Therefore we are confident that our extension here will include the previous valid analysis and therefore will provide a useful check.

For the one loop propagator corrections one has to first compute the corrections to all the 2-point functions which were relevant for the derivation of (154). From [34] this is of the form

$$\begin{pmatrix} p^2 \delta^{ac} & -\gamma^2 f^{acd} \\ -\gamma^2 f^{cab} & -(p^2 + M^2) \delta^{ac} \delta^{bd} \end{pmatrix} + \begin{pmatrix} X \delta^{ac} & U f^{acd} \\ N f^{cab} & Q \delta^{ac} \delta^{bd} + W f^{ace} f^{bde} + R f^{abe} f^{cde} + S d_A^{abcd} \end{pmatrix} a + O(a^2), \quad (155)$$

which is written with respect to the basis $\{\frac{1}{\sqrt{2}} A_\mu^a, \varphi_\mu^{ab}\}$ and as we work in the Landau gauge the common Lorentz structure $P_{\mu\nu}(p)$ has been factored off. The first matrix corresponds to the tree part of the action and the quantities X, U, N, Q, W, R , and S represent the one loop corrections

and we have used the shorthand coupling constant $a = g^2/(16\pi^2)$. The totally symmetric object d_A^{abcd} is defined by [35]

$$d_A^{abcd} = \frac{1}{6} \text{Tr}(T_A^a T_A^b T_A^c T_A^d), \quad (156)$$

where $(T_A^a)_{bc} = -if^{abc}$ is the adjoint representation of the color group generators. At this stage we note that (155) represents a formal definition and no vacuum bubble ex-

pansion has been performed. To one loop one can formally invert (155) to obtain the one loop corrections to all the propagators (154) which is

$$\left(\begin{array}{l} \frac{(p^2+M^2)}{[(p^2)^2+M^2p^2+C_A\gamma^4]}\delta^{cp} - \frac{\gamma^2}{[(p^2)^2+M^2p^2+C_A\gamma^4]}f^{cpq} \\ - \frac{\gamma^2}{[(p^2)^2+M^2p^2+C_A\gamma^4]}f^{pcd} - \frac{1}{(p^2+M^2)}\delta^{cp}\delta^{dq} + \frac{\gamma^4}{(p^2+M^2)[(p^2)^2+M^2p^2+C_A\gamma^4]}f^{cdr}f^{pqr} \end{array} \right) + \left(\begin{array}{l} A\delta^{cp} \\ E f^{pcd} \end{array} \begin{array}{l} C f^{cpq} \\ G \delta^{cp}\delta^{dq} + J f^{cpe}f^{dqe} + K f^{cde}f^{pqe} + L d_A^{cdpq} \end{array} \right) a + O(a^2). \quad (157)$$

The objects A , C , E , G , K , J , and L are related to the quantities of the one loop matrix of (155). However, as we are focusing in this article on the gluon propagator at zero momentum then we only need the relation for A and note that the formal correction at one loop for this is

$$A = -\frac{1}{[(p^2)^2 + M^2p^2 + C_A\gamma^4]^2} \times \left[(p^2 + M^2)^2 X - C_A\gamma^2(N + U)(p^2 + M^2) + C_A\gamma^4 \left(Q + C_A R + \frac{1}{2} C_A W \right) \right]. \quad (158)$$

As noted above we could in principle compute the exact form of each of the 2-point functions contributing to (158) but ultimately as we will take the $p^2 \rightarrow 0$ limit this would be unnecessarily overcomplicated. Instead we compute those pieces of (158) which remain at leading order in the vacuum bubble expansion.

For this we need to determine the 14 contributing Feynman diagrams. These were generated using the QGRAF package [36] and converted into FORM input language where FORM is a symbolic manipulation language [37]. The vacuum bubble expansion written in FORM was applied to each integral and expressions obtained for all the 2-point functions. As these depend on M^2 and γ^2 we were able to check that our expressions agreed with those already determined in the $M^2 = 0$, $\gamma^2 \neq 0$ case of [34]. Moreover, we also checked the explicit Slavnov-Taylor identities for the renormalization of the new mass operator in the $\overline{\text{MS}}$ scheme by applying the MINCER algorithm [38] written in FORM [39] to the Green's function where the operator $(\bar{\varphi}_\mu^{ab} \varphi_\mu^{ab} - \bar{\omega}_\mu^{ab} \omega_\mu^{ab})$ is inserted in an ω 2-point function. The resulting renormalization constants were crucial to not only ensuring that our conventions were consistent but also that our 2-point function vacuum bubble expansion is correctly finite after being fully renormalized. The upshot of our computations is the observation that for the gluon propagator in the zero-momentum limit only X is required for the leading (momentum independent) term of (158). Thus we finally obtain

$$\mathcal{D}^{(1)}(0) = \frac{M^2}{\lambda^4} - \frac{g^2 N}{16\pi^2} \frac{M^4}{\lambda^8} \left[\frac{M^4}{\lambda^4} \frac{9}{16} \sqrt{M^4 - 4\lambda^4} \ln \frac{m_2^2}{m_1^2} + \frac{M^6}{\lambda^4} \left(\frac{9}{16} \ln \frac{\lambda^4}{M^4} \right) - \frac{15}{16} M^2 \lambda^4 \frac{1}{M^4 - 4\lambda^4} + \frac{3}{2} \lambda^4 \frac{1}{\sqrt{M^4 - 4\lambda^4}} \ln \frac{m_2^2}{m_1^2} \right] + \frac{15}{8} \lambda^8 \frac{1}{(\sqrt{M^4 - 4\lambda^4})^3} \ln \frac{m_2^2}{m_1^2} + M^2 \left(\frac{9}{8} - \frac{21}{16} \ln \frac{\lambda^4}{M^4} \right) - \frac{3}{16} \sqrt{M^4 - 4\lambda^4} \ln \frac{m_2^2}{m_1^2} \quad (159)$$

for the one loop correction at zero momentum where all mass variables correspond to renormalized ones. We note that unlike the $M^2 = 0$ case the nonzero freezing akin to tree order is driven by the gluon 2-point function correction. By contrast in the $M^2 = 0$ situation the gluon suppression at one loop derives from the φ 2-point which is related to the horizon condition and the gap equation. Also in this case A will be $O(p^2)$ and not $O(1)$ to retain suppression at one loop [34].

We apply the variational principle to the gluon propagator in a completely similar manner as in the case of the ghost propagator. Therefore, we replace M^2 with $(1 - \ell^2)M^2$ and g^2 with ℓg^2 in the expression (159), expand up to order ℓ^1 , and set $\ell = 1$. Doing so, we find the original

expression (159) for the gluon propagator back. Firstly, we try to apply the principle of minimal sensitivity. Therefore, we have depicted the gluon propagator in Fig. 7. First, we notice that $\mathcal{D}^{(1)}(0)$ is positive for all $M^2 \in [0, 1.53]$. Unfortunately, we do find neither a minimum nor a point of inflection in this interval. Therefore, we shall take the value of M^2 , which was obtained in the study of the ghost propagator (see previous section). Hence, setting $M^2 = 0.37 \Lambda_{\overline{\text{MS}}}^2$ gives

$$\mathcal{D}^{(1)}(0) = \frac{0.63}{\Lambda_{\overline{\text{MS}}}^2} = \frac{11.65}{\text{GeV}^2}. \quad (160)$$

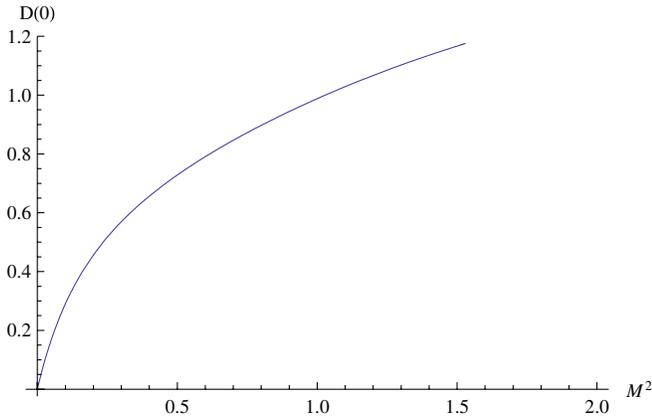


FIG. 7 (color online). The gluon propagator $\mathcal{D}^{(1)}(0)$ drawn in function of M^2 in units $\Lambda_{\overline{\text{MS}}} = 1$.

Evidently, the effective coupling is still smaller than 1 [cf. (153)].

In summary, the infrared value of the ghost propagator and the zero momentum gluon propagator seem to be reasonable. We find a nonenhanced ghost propagator and a gluon propagator which is nonzero at zero momentum. Our results for the gluon and ghost propagator are of a qualitative nature as we are only working in a first order approximation. In order to improve these numerical results, higher order calculations are recommendable. This is however far beyond the scope of the present article.

3. The temporal correlator: Violation of positivity

With the help of the variational technique, we can also show that the gluon propagator displays a violation of positivity. If we rewrite the gluon propagator in the Källén-Lehmann spectral representation,

$$\mathcal{D}(p^2) = \int_0^{+\infty} dM_p^2 \frac{\rho(M_p^2)}{p^2 + M_p^2}, \quad (161)$$

$\rho(M_p^2)$ should be a positive function in order to interpret the fields in terms of stable particles. If $\rho(M_p^2) < 0$ for certain M_p^2 , $\mathcal{D}(p^2)$ is positivity violating. As a practical way to uncover this property, one defines the temporal correlator [2]

$$\begin{aligned} \mathcal{C}(t) &= \int_0^{+\infty} dM_p \rho(M_p^2) e^{-M_p t} \\ &= \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{-ip't} \mathcal{D}(p^2) dp. \end{aligned} \quad (162)$$

Consequently, if we can show that $\mathcal{C}(t)$ becomes negative for certain t , $\rho(M_p^2)$ cannot be positive for all M_p^2 , resulting in a positivity violating gluon propagator. If the gluon propagator vanishes at zero momentum, $\mathcal{D}(0) = 0$, one can immediately verify from (161) that $\rho(M_p^2)$ cannot be a positive quantity. However, having $\mathcal{D}(0) \neq 0$ does not exclude a positivity violation as we shall soon find out.

We can now apply the variational technique on the temporal correlator. At tree level, this $\mathcal{C}(t)$ is given by

$$\mathcal{C}(t, M^2) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{-ipt} \frac{p^2 + M^2}{p^4 + M^2 p^2 + (\lambda^2(M^2))^2} dp, \quad (163)$$

where $\lambda^2(M^2)$ is still determined by the gap equation (151). Replacing $M^2 \rightarrow (1 - \ell^2)M^2$ and $g^2 \rightarrow \ell g^2$ is redundant in this case, as we only have the tree level gluon propagator $\mathcal{D}(p^2)$ at our disposal. We shall now implement the minimal sensitivity principle as follows: for each different value of t , we minimize the temporal correlator with respect to M^2 . $\mathcal{C}(t)$ displays a minimum at $M_{\min}^2 \neq 0$, for $t \geq 6/\Lambda_{\overline{\text{MS}}}$. In Table III, some values for $M_{\min}^2(t)$ for different t are presented. For $t \leq 6$, we have taken $M^2 = 0$; it is clearly visible from the table below that $M_{\min}^2 \rightarrow 0$ for decreasing t . The corresponding $\mathcal{C}(t, M_{\min}^2)$ is depicted in Fig. 8. Both the x -axis and y -axis are shown in units fm ($1/\Lambda_{\overline{\text{MS}}} = 0.847$ fm), in order to compare our results with [27,40]. Not only do we find a positivity violating gluon propagator as $\mathcal{C}(t)$ becomes negative, but even the shape of this function is consistent with the lattice results⁴ [27,40]. Moreover, in [27,40], the positivity violation starts from $t \sim 1.5$ fm, in good agreement with our results. Finally, Fig. 9 displays the corresponding values of $g^2 N/16\pi^2$. We can conclude that the previous results are reliable for $t \leq 8$ as $g^2 N/16\pi^2$ is smaller than 1.

C. A remark about the strong coupling constant

A renormalization group invariant definition of an effective strong coupling constant g_{eff}^2 can be written down from the knowledge of the gluon and ghost propagators as

$$g_{\text{eff}}^2(p^2) = g^2(\bar{\mu}^2) \tilde{\mathcal{D}}(p^2, \bar{\mu}^2) \tilde{\mathcal{G}}^2(p^2, \bar{\mu}^2) \quad (164)$$

(see e.g. [3]). $\tilde{\mathcal{D}}$ and $\tilde{\mathcal{G}}$ stand for the gluon and ghost form factor, defined by

$$\tilde{\mathcal{D}}(p^2) = p^2 \mathcal{D}(p^2), \quad \tilde{\mathcal{G}}(p^2) = p^2 \mathcal{G}(p^2). \quad (165)$$

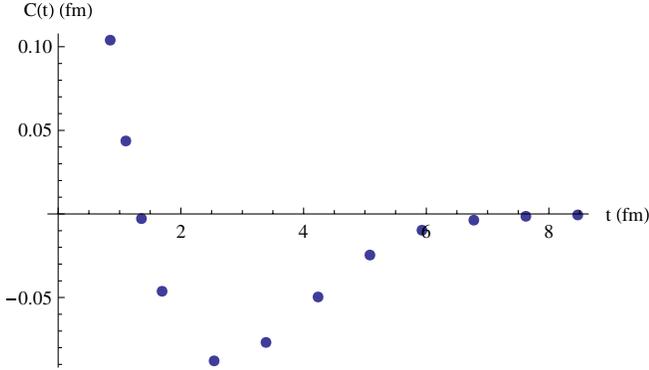
The definition (164) represents a kind of nonperturbative extension of the nonrenormalization of the ghost-gluon vertex. At the perturbative level, this is assured by the Ward identity (56), $Z_g = Z_c^{-1} Z_A^{-1/2}$. Usually, this is assumed to remain valid at the nonperturbative level. Although this cannot be proven, this hypothesis has been corroborated by lattice studies like [41,42].

In recent years, there was accumulating evidence that $g_{\text{eff}}^2(p^2)$ would reach an infrared fixed point different from zero: see e.g. [3–5,10] for a Schwinger-Dyson analysis, [34,43] in the ordinary Gribov-Zwanziger approach, and [44,45] for lattice results. These studies are mostly done in

⁴Reference [27] included quarks, while [40] considered gluodynamics as we are studying in this work.

TABLE III. Some M_{\min}^2 for different t in units $\Lambda_{\overline{\text{MS}}} = 1$.

| t | 6 | 7 | 8 | 9 | 10 |
|--------------|---|------|------|------|------|
| M_{\min}^2 | 0 | 0.16 | 0.35 | 0.51 | 0.65 |

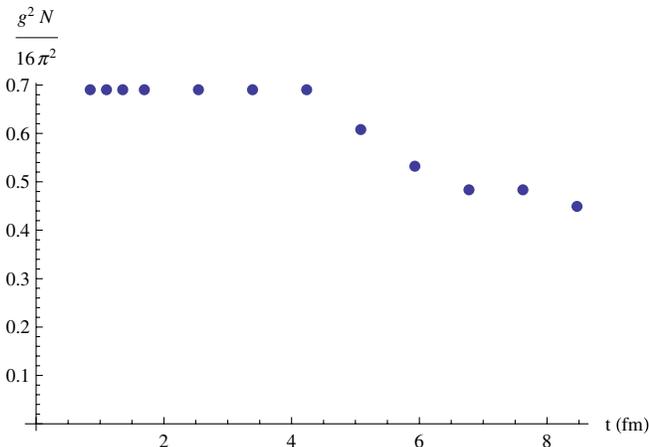
FIG. 8 (color online). $C(t)$ (fm) in function of t (fm).

a momentum subtraction renormalization scheme, with the exception of [34] where the $\overline{\text{MS}}$ scheme was employed. The manifestation of this infrared fixed point was motivated in Schwinger-Dyson studies and the ordinary Gribov-Zwanziger case by means of the power law behavior of the form factors,

$$\tilde{\mathcal{D}}(p^2)_{p^2 \approx 0} \propto (p^2)^{2\alpha}, \quad \tilde{\mathcal{G}}(p^2)_{p^2 \approx 0} \propto (p^2)^{-\alpha}, \quad (166)$$

being expressible in terms of a single exponent α . The Schwinger-Dyson community heralded in a variety of studies the value $\alpha \approx 0.595$, whereas the Gribov-Zwanziger scenario gives $\alpha = 1$. Anyhow, substituting a behavior like (166) into the definition (164) leads to $g_{\text{eff}}^2(p^2)_{p^2 \approx 0} \propto (p^2)^0$, opening the door for a finite value.

However, once again quoting the more recent large volume lattice data of [16,18,19,41], the power law behavior (166) seems to be excluded in favor of

FIG. 9 (color online). $g^2 N / (16\pi^2)$ in function of t (fm).

$$\tilde{\mathcal{D}}(p^2)_{p^2 \approx 0} \propto p^2, \quad \tilde{\mathcal{G}}(p^2)_{p^2 \approx 0} \propto (p^2)^0, \quad (167)$$

leading to a vanishing infrared effective strong coupling constant at zero momentum since $g_{\text{eff}}^2(p^2)_{p^2 \approx 0} \propto p^2$. The refined analysis in this paper of the extended Gribov-Zwanziger action, including an additional dynamical effect, allows us to draw a similar conclusion up to the one loop level, i.e. an infrared vanishing g_{eff}^2 . Certain lattice studies also pointed towards this particular scenario [46].

V. THE BRST BREAKING IN THE GRIBOV-ZWANZIGER THEORY

We recall here that the Gribov-Zwanziger action (8) is not invariant under the BRST transformation (22). Indeed, if we take the BRST variation of the action (8), one finds a breaking term Δ_γ given by

$$\Delta_\gamma \equiv sS = g\gamma^2 \int d^4x f^{abc} (A_\mu^a \omega_\mu^{bc} - (D_\mu^{am} c^m)(\bar{\varphi}_\mu^{bc} + \varphi_\mu^{bc})). \quad (168)$$

We see that the presence of the Gribov parameter γ prevents the action from being invariant under the BRST symmetry. Nevertheless, this fact does not prevent the use of the Slavnov-Taylor identity to prove the renormalizability of the theory, which is very remarkable. Since the breaking Δ_γ is soft, i.e. it is of dimension 2 in the fields, it can be neglected in the deep ultraviolet, where we recover the usual notion of exact BRST invariance as well as of BRST cohomology for defining the physical subspace [47]. However, in the nonperturbative infrared region, the breaking term cannot be neglected and the BRST invariance is lost. In the following, we shall present a detailed analysis of this breaking and of its consequences. In particular, we shall be able to prove that the origin of this breaking can be traced back to the properties of the Gribov region Ω . Moreover, it turns out that the existence of this breaking enables us to give an elementary algebraic proof of the fact that the Gribov parameter γ is a physical parameter of the theory, entering thus the expression of the correlation functions of gauge invariant operators.

A. The transversality of the gluon propagator

The reader might wonder whether the gluon propagator still remains transverse in the presence of the Gribov horizon. As the gluon propagator is the connected 2-point function, we ought to consider the generator Z^c of connected Green functions, which can be constructed from the quantum effective action⁵ Γ by means of a Legendre transformation. The renormalizability of the theory entails that Γ obeys the renormalized version of the Ward identity (32), or

⁵This is the generator of the 1PI Green functions.

$$\frac{\delta\Gamma}{\delta b^a} = \partial_\mu A_\mu^a. \quad (169)$$

Introducing sources $I^a(J_\mu^a)$ for the fields $b^a(A_\mu^a)$ and performing the Legendre transformation, the identity (169) translates into

$$I^a = \partial_\mu \frac{\delta Z^c}{\delta J_\mu^a}. \quad (170)$$

Acting with $\frac{\delta}{\delta J_\mu^b}$ on this expression, and by setting all sources equal to zero, we retrieve

$$0 = \partial_\mu^x \frac{\delta^2 Z^c}{\delta J_\mu^a(x) \delta J_\mu^b(y)} \Big|_{I,J=0} = \partial_\mu^x \langle A_\mu^a(x) A_\nu^b(y) \rangle, \quad (171)$$

which expresses nothing else but the transversality of the gluon propagator.

B. The BRST breaking and its consequences on the Slavnov-Taylor identity

Let us present here a few considerations on the consequences stemming from the BRST breaking Δ_γ appearing

in the left-hand side of Eq. (168) of the Slavnov-Taylor identity. Our argument will follow [48]. We start from the generalized Slavnov-Taylor identity (31) which is fulfilled by the enlarged action Σ (28). The quantum effective action $\Gamma = \Sigma + \hbar\Gamma^{(1)} + \dots$ obeys the quantum version of this Slavnov-Taylor identity [26],

$$\begin{aligned} \mathcal{S}(\Gamma) = \int d^4x \left(\frac{\delta\Gamma}{\delta K_\mu^a} \frac{\delta\Gamma}{\delta A_\mu^a} + \frac{\delta\Gamma}{\delta L^a} \frac{\delta\Gamma}{\delta c^a} + b^a \frac{\delta\Gamma}{\delta \bar{c}^a} \right. \\ \left. + \bar{\varphi}_i^a \frac{\delta\Gamma}{\delta \bar{\omega}_i^a} + \omega_i^a \frac{\delta\Gamma}{\delta \varphi_i^a} + M_\mu^{ai} \frac{\delta\Gamma}{\delta U_\mu^{ai}} + N_\mu^{ai} \frac{\delta\Gamma}{\delta V_\mu^{ai}} \right) = 0. \end{aligned} \quad (172)$$

We now pass to the Gribov-Zwanziger action, defined by giving the sources (M, N, U, V) their physical values (20) and (24). As a consequence, the physical quantum effective action Γ_{phys} will now obey a broken Slavnov-Taylor identity,

$$\begin{aligned} \mathcal{S}(\Gamma_{\text{phys}}) &= \int d^4x \left(\frac{\delta\Gamma_{\text{phys}}}{\delta K_\mu^a} \frac{\delta\Gamma_{\text{phys}}}{\delta A_\mu^a} + \frac{\delta\Gamma_{\text{phys}}}{\delta L^a} \frac{\delta\Gamma_{\text{phys}}}{\delta c^a} + b^a \frac{\delta\Gamma_{\text{phys}}}{\delta \bar{c}^a} + \bar{\varphi}_i^a \frac{\delta\Gamma_{\text{phys}}}{\delta \bar{\omega}_i^a} + \omega_i^a \frac{\delta\Gamma_{\text{phys}}}{\delta \varphi_i^a} \right) \\ &= - \int d^4x \left(M_\mu^{ai} \frac{\delta\Gamma}{\delta U_\mu^{ai}} + N_\mu^{ai} \frac{\delta\Gamma}{\delta V_\mu^{ai}} \right) \Big|_{\text{phys}} \\ &= -g\gamma^2 \left[\int d^4x f^{abc} A_\mu^a \omega_\mu^{bc} \cdot \Gamma_{\text{phys}} \right] + g\gamma^2 \left[\int d^4x f^{abc} (D_\mu^{am} c^m) (\bar{\varphi}_\mu^{bc} + \varphi_\mu^{bc}) \cdot \Gamma_{\text{phys}} \right] = -[\Delta_\gamma \cdot \Gamma_{\text{phys}}], \end{aligned} \quad (173)$$

whereby $[\Delta_\gamma \cdot \Gamma_{\text{phys}}]$ represents the generator of the 1PI Green functions with the insertion of the composite operator Δ_γ . Expression (173) generalizes at the quantum level the broken identity of Eq. (168). Once having a Slavnov-Taylor identity like (173) at our disposal, we can obtain relations between different Green functions by acting on it with test operators $\frac{\delta^n}{\delta\Psi(x_1)\dots\delta\Psi(x_n)}$, with Ψ any field, and by setting all fields and sources equal to zero at the end. The breaking term in the right-hand side of expression (173) will be translated into an extra contribution. In particular, we shall obtain

$$\begin{aligned} \frac{\delta^n[\mathcal{S}(\Gamma_{\text{phys}})]}{\delta\Psi(x_1)\dots\delta\Psi(x_n)} \Big|_{\text{fields, sources}=0} \\ = - \frac{\delta^n[\Delta_\gamma \cdot \Gamma_{\text{phys}}]}{\delta\Psi(x_1)\dots\delta\Psi(x_n)} \Big|_{\text{fields, sources}=0}. \end{aligned} \quad (174)$$

One sees thus that the right-hand side of the foregoing expression, corresponding to a 1PI Green function with the insertion of the composite operator Δ_γ and with n amputated external legs of the type $\Psi(x_1), \dots, \Psi(x_n)$, gives precisely the modification of the relationships among the Green functions due to the Gribov horizon. To our under-

standing, the contributions stemming from the right-hand side of Eq. (174) should be correctly taken into account when checking the validity of the Slavnov-Taylor identities or when invoking Slavnov-Taylor related identities in computations when the restriction to the Gribov horizon is understood.

It is worth noticing that the breaking term of (174) will certainly vanish if the chain $\Psi(x_1)\dots\Psi(x_n)$ has a ghost number different from $+1$. Indeed, the action preserves ghost number and the breaking term Δ_γ itself carries a nonvanishing ghost charge of $+1$, so that the operator $\frac{\delta^n}{\delta\Psi(x_1)\dots\delta\Psi(x_n)}$ must have ghost number -1 in order to allow for a nonvanishing contribution (174).

In summary, we emphasize that the broken Slavnov-Taylor identity (173) does in fact maintain a powerful predictive character. It allows us to establish relationships among various Green functions of the theory in a way which takes into account the presence of the Gribov horizon. At the same time, there exist Green functions for which the breaking of the Slavnov-Taylor identity is harmless. In particular, this is the case when considering gauge invariant operators built up with only the gauge fields A_μ^a . For these Green functions, the physical quantum action

Γ_{phys} behaves as it fulfills the unbroken Slavnov-Taylor identity, namely

$$\mathcal{S}(\Gamma_{\text{phys}}) = 0. \quad (175)$$

The gauge invariance of the correlator $\langle F^2(x)F^2(y) \rangle$ implies in fact that no useful information can be extracted for it from the Slavnov-Taylor identities. In a loose way of speaking, $\langle F^2(x)F^2(y) \rangle$ lives on its own and is not related to other Green functions. To formally prove this, one should add the operator $F^2(x)$ to the action with a (BRST invariant) scalar source $K(x)$, the (broken) Slavnov-Taylor identity (173) will remain unchanged. Hence, similarly as in the previous subsection, by performing a Legendre transformation to pass to Z^c and by acting with $\frac{\delta}{\delta K(x_i)}$ on that identity and again setting all sources to zero, it will follow that there is a trivially vanishing breaking term due to ghost charge conservation.

C. A few words on unitarity

Certainly, the BRST breaking and its consequences on the Green functions of the theory deserve further investigation. In this respect one could attempt to evaluate some gauge invariant correlation function like, for instance, $\langle F^2(x)F^2(y) \rangle$ in order to see if, despite the presence of the BRST breaking and of a positivity violating gluon propagator, this gauge invariant correlation function might display a real pole in momentum space. A first hint that something like this might happen has been given by a tree level computation in [8].

As one can easily figure out, the presence of the BRST breaking Δ_γ is related to the lack of unitarity in the gluon sector. To our understanding, this is a manifestation of gluon confinement: unitarity is jeopardized in the gluon sector because gluons are confined. This is also apparent from the positivity violation exhibited by the gluon propagator, which does not allow for a physical interpretation of the elementary gluon excitations. One might have the tendency to believe that the existence of the soft breaking Δ_γ of the BRST symmetry is a welcome feature, in particular, signaling that, in a confining theory, physics in the infrared region is not necessarily definable in the same way as in the deep ultraviolet, where the BRST breaking could be neglected and one recovers usual perturbation theory. As we already stated in the beginning of this section, in the ultraviolet, we also recover the usual notion of the BRST cohomology [47], allowing to prove that the ghost degrees of freedom cancel against 2 unphysical gluon polarizations, leaving over only 2 physical transverse polarizations, endowed with a positive norm. In the confining regime, it is unknown what the analogue of this scenario might be. The absence of the BRST symmetry in the infrared does not necessarily entail that the theory is not unitary. Certainly, the \mathcal{S} -matrix of the excitations of the physical spectrum has to be unitary. But as gluons are *not* the excitations belonging to the physical spectrum, unitarity is not to be expected

in the sector described by the elementary gluon fields. From this perspective, the question of what the number of physical gluon polarizations might be in the nonperturbative confining infrared sector loses its context.

D. The BRST breaking as a tool to prove that the Gribov parameter is a physical parameter

The breaking term (168) has also the interesting consequence that it allows us to give a simple algebraic proof of the fact that the Gribov parameter γ is a physical parameter of the theory, and that as such it can enter the explicit expression of gauge invariant correlation functions like for instance $\langle F^2(x)F^2(y) \rangle$ or the vacuum condensate $\langle F^2 \rangle$. In fact, by taking the derivative of both sides of Eq. (168) with respect to γ^2 one gets

$$\begin{aligned} s \frac{\partial S}{\partial \gamma^2} &= \frac{1}{\gamma^2} \Delta_\gamma \\ &= g \int d^4x f^{abc} (A_\mu^a \omega_\mu^{bc} - (D_\mu^{am} c^m)(\bar{\varphi}_\mu^{bc} + \varphi_\mu^{bc})), \end{aligned} \quad (176)$$

from which, keeping in mind that the BRST operator s as defined in Eq. (22) is still nilpotent, it immediately follows that $\frac{\partial S}{\partial \gamma^2}$ cannot be cast in the form of a BRST exact variation, namely

$$\frac{\partial S}{\partial \gamma^2} \neq s \hat{\Delta}_\gamma, \quad (177)$$

for some local integrated dimension 2 quantity $\hat{\Delta}_\gamma$. From Eq. (177) it becomes then apparent that the Gribov parameter γ^2 is a physical parameter, as much as the gauge coupling constant g , for which a similar equation holds. Furthermore, it is worth underlining that, due to the form of the BRST operator s , the presence of the soft breaking Δ_γ is, in practice, the unique way to ensure that the Gribov parameter indeed is a physical parameter and not an unphysical one, as it would be the case of a gauge parameter entering the gauge fixing term. Let us suppose that the part of the action S_γ containing the Gribov parameter would be left invariant by the BRST transformation (22), namely

$$sS_\gamma = 0, \quad (178)$$

instead of inducing the breaking term Δ_γ . Since S_γ depends on the auxiliary fields $(\bar{\varphi}_\mu^{ac}, \varphi_\mu^{ac}, \bar{\omega}_\mu^{ac}, \omega_\mu^{ac})$ which constitute a set of BRST doublets⁶ [26], it would follow from Eq. (178) that a local integrated polynomial \hat{S}_γ would exist such that

⁶We remind here that a BRST doublet is given by a pair (α, β) transforming as: $s\alpha = \beta$, $s\beta = 0$. It can be shown that a BRST doublet has always vanishing cohomology, meaning that any invariant quantity, $sF(\alpha, \beta) = 0$, has necessarily the form of an exact BRST cocycle, namely $F(\alpha, \beta) = s\hat{F}(\alpha, \beta)$.

$$S_\gamma = s\hat{S}_\gamma. \quad (179)$$

Subsequently, taking the derivative of both sides of expression (179) with respect to γ^2 , one would obtain

$$\frac{\partial S_\gamma}{\partial \gamma^2} = s \frac{\partial \hat{S}_\gamma}{\partial \gamma^2}, \quad (180)$$

a relation implying that γ^2 would have the same meaning as an unphysical gauge parameter.⁷ In turn, this would imply that correlation functions of gauge invariant operators would be completely independent from γ^2 . We see thus that the presence of the soft breaking term Δ_γ plays an important role, ensuring that γ^2 is a relevant parameter of the theory. The same conclusion also holds when the Gribov-Zwanziger action is supplemented by the BRST invariant mass term (63). The existence of the breaking Δ_γ thus seems to be an important ingredient to introduce a nonperturbative mass gap in a local and renormalizable way.

A question which arises almost naturally is whether it might be possible to modify the BRST operator, i.e. $s \rightarrow s_m$, in such a way that the new operator s_m would be still nilpotent, while defining an exact symmetry of the action $s_m S'' = 0$. Although we are not going to give a formal proof, we can present a simple argument discarding such a possibility. We have already observed that the BRST transformation (22) defines an exact symmetry of the action when $\gamma = 0$, which corresponds to the physical situation in which the restriction to the Gribov region has not been implemented. Hence, it appears that one should search for possible modifications of the BRST operator which depends on γ , namely

$$s_m = s + s_\gamma, \quad (181)$$

whereby

$$s_\gamma = \gamma\text{-dependent terms}, \quad (182)$$

so as to guarantee a smooth limit when γ is set to zero. However, taking into account the fact that γ has mass dimension 1, that all auxiliary fields ($\tilde{\varphi}_\mu^{ac}$, φ_μ^{ac} , $\tilde{\omega}_\mu^{ac}$, ω_μ^{ac}) have dimension 1 too, and that the BRST operator s does not alter the dimension of the fields,⁸ it does not seem possible to introduce extra γ -dependent terms in the BRST transformation of the fields ($\tilde{\varphi}_\mu^{ac}$, φ_μ^{ac} , $\tilde{\omega}_\mu^{ac}$, ω_μ^{ac}) while preserving locality, Lorentz covariance as well as color group structure.

⁷One easily shows that in this case, $\frac{\partial \langle \mathcal{G} \rangle}{\partial \gamma^2} = -\langle s(\frac{\partial \hat{S}}{\partial \gamma^2} \mathcal{G}) \rangle = 0$ for any gauge invariant operator \mathcal{G} .

⁸It is understood that the usual canonical dimensions are assigned to the fields A_μ^a , b^a , c^a , \bar{c}^a [26], as shown in Table I. It is apparent that the BRST operator s does not alter the dimension of the fields.

E. Tracing the origin of the BRST breaking

Having clearly seen the explicit loss of the BRST symmetry, it would be instructive to point out more precisely where this breaking originates. We recall that the BRST transformation of the gluon field A_μ is in fact constructed from the infinitesimal gauge transformations. Indeed, for an infinitesimal gauge parameter ω^a , the corresponding gauge transformation is determined by

$$\delta_\omega A_\mu^a = D_\mu^{ab} \omega^b, \quad (183)$$

which can be compared with the BRST transformation (22). Based on this identification, we shall present our argument using infinitesimal gauge transformations. In particular, we shall establish the following proposition: any infinitesimal gauge transformation of field configurations belonging to the Gribov region Ω necessarily gives rise to configurations which lie *outside* of Ω . We can distinguish two cases.

- (i) *The field A_μ is not located close to the boundary $\partial\Omega$.*

Let us consider a gauge configuration A_μ which belongs to the Gribov region Ω but not close to its boundary $\partial\Omega$ (the horizon), thus $\partial_\mu A_\mu = 0$ and $-\partial_\mu D_\mu(A) > 0$. Next, consider the field \tilde{A}_μ obtained from A_μ through an infinitesimal gauge transformation with parameter ω ,

$$\tilde{A}_\mu = A_\mu + D_\mu(A)\omega. \quad (184)$$

This configuration \tilde{A}_μ cannot belong to Ω . Suppose the contrary, then $\partial_\mu \tilde{A}_\mu = 0 = \partial_\mu A_\mu$ would lead to

$$\partial_\mu D_\mu(A)\omega = 0, \quad (185)$$

in contradiction with the hypothesis that A_μ is not located on the boundary $\partial\Omega$, thus there are no zero modes ω allowing for (185) to hold.

- (ii) *The field A_μ is located close to the boundary $\partial\Omega$.*

In this case, we can even make a more precise statement. If A_μ lies very close to the boundary $\partial\Omega$, we can decompose it as

$$A_\mu = a_\mu + C_\mu, \quad (186)$$

with $C_\mu \in \partial\Omega$, thus C_μ lies on the horizon. The shift a_μ is a small (infinitesimal) perturbation. Obviously, $\partial_\mu C_\mu = \partial_\mu a_\mu = 0$. Subsequently, we find

$$\tilde{A}_\mu = C_\mu + a_\mu + D_\mu(C)\omega + \dots \quad (187)$$

for the gauge transformed field at lowest order in the infinitesimal quantities ω and a_μ . Since $C_\mu \in \partial\Omega$ and by identifying ω with the zero mode corresponding to C_μ , we find

$$\partial_\mu \tilde{A}_\mu = \partial_\mu D_\mu(C)\omega = 0, \quad (188)$$

showing that \tilde{A}_μ is transverse. The field \tilde{A}_μ also lies very close to the boundary $\partial\Omega$. However, as it follows from Gribov's original statement⁹ [7], it is located on the side of the horizon opposite to that of the field A_μ , i.e. it lies outside of the Gribov region Ω .

We can conclude thus that any infinitesimal transformation of a gauge field configuration which belongs to the Gribov region Ω results in another configuration which lies outside Ω . Since the BRST transformation of the gluon field is naturally obtained from the infinitesimal gauge transformations, it is apparent that the breaking of the BRST symmetry looks almost as a natural reflection of the previous result.

We can also offer a pictorial depiction of what is happening. We recall that the Gribov region Ω is convex, bounded in all directions in field space, that every gauge field has an equivalent representant within Ω , that the origin $A_\mu = 0$ belongs¹⁰ to Ω [13,49,50], and that every gauge configuration near the horizon $\partial\Omega$ has a copy on the other side of $\partial\Omega$ [7]. The first four quoted properties are important to make Ω a suitable domain of integration in the path integral, i.e. we can restrict the whole space of A_μ -configurations to Ω as proposed by Gribov. However, implementing this restriction in A_μ space jeopardizes the BRST invariance. As we have seen, if we move throughout A_μ space with a BRST transformation (cf. infinitesimal gauge transformations), we must unavoidably cross the horizon $\partial\Omega$. Hence, restricting the fields within the horizon breaks the BRST invariance.

F. The Maggiore-Schaden construction revisited

The authors of Ref. [23] attempted to interpret the BRST breaking as a kind of *spontaneous* symmetry breaking. We shall now reexamine this proposal and conclude that, instead, the BRST breaking has to be considered as an *explicit* symmetry breaking, where we shall present a few arguments which have not been considered in [23]. Although this discussion might seem to be only of a rather academic interest, there is nevertheless a big difference between a *spontaneously* or *explicitly* broken continuous symmetry, since only in the former case a Goldstone mode would emerge. For the benefit of the reader, we shall first explain in detail the approach of [23]. One starts by adding the following BRST exact term to the Yang-Mills action:

⁹For the benefit of the reader we quote here Gribov's statement, proven in [7]: for each field A_μ belonging to the Gribov region Ω and located near the boundary $\partial\Omega$, i.e. $A_\mu = C_\mu + a_\mu$, there exists an equivalent field \tilde{A}_μ , $\tilde{A}_\mu = C_\mu + a_\mu + D_\mu(C)\omega$, near the boundary $\partial\Omega$, located, however, on the other side of the horizon, outside of the region Ω .

¹⁰This means that perturbation theory belongs to Ω .

$$S_1 = s \int d^4x (\bar{c}^a \partial_\mu A_\mu^a + \bar{\omega}_\mu^{ac} \partial_\nu D_\nu^{ab} \varphi_\mu^{bc}), \quad (189)$$

with s , the same nilpotent BRST operator as defined in (22). The first term represents the Landau gauge fixing, while the second term is a BRST exact piece in the fields $(\varphi, \omega, \bar{\varphi}, \bar{\omega})$. Of course, from expression (189), it follows that s defines a symmetry of the action $S_{\text{YM}} + S_1$. As a consequence, the nilpotent operator s allows us to define two doublets (φ, ω) and $(\bar{\varphi}, \bar{\omega})$. This doublet structure implies that we can exclude these fields from the physical subspace [26,47], which makes $S_{\text{YM}} + S_1$ equivalent to the ordinary Yang-Mills gauge theory. Next, Maggiore and Schaden introduced a set of shifted fields, which—translated to our conventions—are given by

$$\begin{aligned} \varphi_\mu^{ab} &= \varphi_\mu'^{ab} + \gamma^2 \delta^{ab} x_\mu, & \bar{\varphi}_\mu^{ab} &= \bar{\varphi}_\mu'^{ab} + \gamma^2 \delta^{ab} x_\mu, \\ \bar{c}^a &= \bar{c}'^a + g \gamma^2 f^{abc} \bar{\omega}_\mu^{bc} x_\mu, & b^a &= b'^a + g \gamma^2 f^{abc} \bar{\varphi}_\mu^{bc} x_\mu. \end{aligned} \quad (190)$$

All fields $(\varphi_\mu'^{ab}, \bar{\varphi}_\mu'^{ab}, \bar{c}'^a, b'^a)$ have vanishing vacuum expectation value (VEV), namely

$$\langle \varphi_\mu'^{ab} \rangle = \langle \bar{\varphi}_\mu'^{ab} \rangle = \langle \bar{c}'^a \rangle = \langle b'^a \rangle = 0. \quad (191)$$

Along with these new fields $(\varphi_\mu'^{ab}, \bar{\varphi}_\mu'^{ab}, \bar{c}'^a, b'^a)$, one introduces a modified nilpotent BRST operator \tilde{s} given by

$$\begin{aligned} \tilde{s} \bar{c}'^a &= b'^a, & \tilde{s} b'^a &= 0, & \tilde{s} \varphi_\mu'^{ab} &= \omega_\mu^{ab}, \\ \tilde{s} \bar{\varphi}_\mu'^{ab} &= 0, & \tilde{s} A_\mu^a &= -D_\mu^{ab} c^b, & \tilde{s} \omega_\mu^{ab} &= 0, \end{aligned} \quad (192)$$

which looks exactly like (22). However, we emphasize that by introducing these new fields, the BRST operator \tilde{s} will give rise to an explicit x -dependence when acting on the field $\bar{\omega}_\mu^{ab}$:

$$\tilde{s} \bar{\omega}_\mu^{ab} = \bar{\varphi}_\mu'^{ab} + \gamma^2 \delta^{ab} x_\mu. \quad (193)$$

Furthermore, by taking the vacuum expectation value of both sides of Eq. (193), one gets

$$\langle \tilde{s} \bar{\omega}_\mu^{ab} \rangle = \gamma^2 \delta^{ab} x_\mu, \quad (194)$$

from which the authors of [23] infer that the BRST operator \tilde{s} suffers from spontaneous symmetry breaking. Notice also that (194) gives a VEV to a quantity with a free Lorentz index.

With the introduction of the shifted fields, we can rewrite the action S_1 as

$$\begin{aligned} S_1 &= \tilde{s} \int d^4x (\bar{c}'^a \partial_\mu A_\mu^a + \bar{\omega}_\mu^{ac} \partial_\nu D_\nu^{ab} \varphi_\mu^{bc} \\ &\quad + g \gamma^2 f^{abc} \bar{\omega}_\nu^{bc} x_\nu \partial_\mu A_\mu^a + \gamma^2 \bar{\omega}_\mu^{ac} \partial_\nu D_\nu^{ab} \delta^{bc} x_\mu). \end{aligned} \quad (195)$$

The last two terms can be simplified, leading to

$$S_1 = \tilde{s} \int d^4x (\bar{c}^{la} \partial_\mu A_\mu^a + \bar{\omega}_\mu^{ac} \partial_\nu D_\nu^{ab} \varphi_\mu^{bc} - g\gamma^2 \bar{\omega}_\mu^{ab} f_{abc} A_\mu^c). \quad (196)$$

If we calculate this action explicitly, we recover the original Gribov-Zwanziger action, without the constant part $4\gamma^4(N^2 - 1)$. For this reason one adds $-\gamma^2 \tilde{s} \int d^4x \partial_\nu \bar{\omega}_\mu^{aa}$ to the action S_1 . Doing so, one finds

$$\begin{aligned} S_1 &= \tilde{s} \int d^4x (\bar{c}^{la} \partial_\mu A_\mu^a + \bar{\omega}_\mu^{ac} \partial_\nu D_\nu^{ab} \varphi_\mu^{bc} - g\gamma^2 \bar{\omega}_\mu^{ab} f_{abc} A_\mu^c - \gamma^2 \partial_\mu \bar{\omega}_\mu^{aa}) \\ &= \int d^4x [b^{la} \partial_\mu A_\mu^a + \bar{c}^{la} \partial_\mu (D_\mu^{ab} c^b)] + \int d^4x [\bar{\varphi}_\mu^{lac} \partial_\nu D_\nu^{ab} \varphi_\mu^{bc} + \gamma^2 x_\mu \partial_\nu D_\nu^{ab} \varphi_\mu^{ba} + \bar{\omega}_\mu^{ac} \partial_\nu (g f^{akb} D_\nu^{kd} c^d \varphi_\mu^{bc}) \\ &\quad - \bar{\omega}_\mu^{ac} \partial_\nu D_\nu^{ab} \omega_\mu^{bc}] + \int d^4x [-g\gamma^2 \bar{\varphi}_\mu^{lab} f_{abc} A_\mu^c - g\gamma^2 \bar{\omega}_\mu^{ab} f_{abc} D_\mu^{cd} c^d - 4\gamma^4(N^2 - 1)]. \end{aligned} \quad (197)$$

If we naively assume that we can perform a partial integration, we find after dropping the surface terms,

$$S_{\text{YM}} + S_1 = (8) - g\gamma^2 f^{abc} \int d^4x \bar{\omega}_\mu^{ab} D_\mu^{cd} c^d. \quad (198)$$

The last expression reveals that one has recovered the Gribov-Zwanziger action from an exact \tilde{s} -variation with the addition of an extra term $(-g\gamma^2 f^{abc} \int d^4x \bar{\omega}_\mu^{ab} D_\mu^{cd} c^d)$. However, this term is irrelevant as we shall explain now. Assume that we want to compose an arbitrary Feynman diagram without any external $\bar{\omega}$ -leg and thereby using the action (198). The second term from this action can never contribute to this Feynman diagram as it contains an external $\bar{\omega}$. Indeed, this leg requires an ω -leg, which in its turn is always accompanied by an $\bar{\omega}$ -leg. Hence, the action (198) is equivalent to the standard Gribov-Zwanziger action (8) when we exclude the diagrams containing external $\bar{\omega}$ -legs.¹¹

Although at first sight this construction might seem useful, it turns out that a few points have been overlooked. Let us investigate this in more detail. Firstly, we point out that rather delicate assumptions have been made concerning the partial integration. To reveal the obstacle, we perform once more the partial integration explicitly,

$$\begin{aligned} S_1 &= \int d^4x [b^{la} \partial_\mu A_\mu^a + \bar{c}^{la} \partial_\mu (D_\mu^{ab} c^b)] + \int d^4x [-\partial_\nu \bar{\varphi}_\mu^{lac} D_\nu^{ab} \varphi_\mu^{bc} - \gamma^2 \delta^{\mu\nu} D_\nu^{ab} \varphi_\mu^{ba} - (\partial_\nu \bar{\omega}_\mu^{ac}) g f^{akb} D_\nu^{kd} c^d \varphi_\mu^{bc} \\ &\quad + (\partial_\nu \bar{\omega}_\mu^{ac}) D_\nu^{ab} \omega_\mu^{bc}] + \int d^4x [-g\gamma^2 \bar{\varphi}_\mu^{lab} f_{abc} A_\mu^c - g\gamma^2 \bar{\omega}_\mu^{ab} f_{abc} D_\mu^{cd} c^d - 4\gamma^4(N^2 - 1)] \\ &= (8) - g\gamma^2 f^{abc} \int d^4x \bar{\omega}_\mu^{ab} D_\mu^{cd} c^d. \end{aligned} \quad (201)$$

In this case, we do not encounter the problem of non-vanishing surface terms. To recapitulate, if we first let the BRST variation act on the action (196), and then perform a partial integration, we find a different result than perform-

$$\int d^4x \gamma^2 x_\mu \partial_\nu D_\nu^{ab} \varphi_\mu^{ba} = \text{surface term} - \int d^4x \gamma^2 \delta_{\mu\nu} D_\nu^{ab} \varphi_\mu^{ba}. \quad (199)$$

Normally, one drops the surface terms, as the fields vanish at infinity. However in this case, as x_μ does not vanish at infinity, it is not sure if the surface terms $\propto x_\mu$ will be zero. One would have to impose extra conditions on the fields to justify the dropping of the surface terms. On the other hand, when we do not perform the partial integration to avoid the surface terms, we are facing an explicit, unwanted x -dependence in the action, resulting in an explicit breaking of translation invariance.

Another way of looking at the problem consists of performing a partial integration on the second term of the action (196) *before* applying the BRST variation \tilde{s} . Doing so, we find

$$S_1 = \tilde{s} \int d^4x (\bar{c}^{la} \partial_\mu A_\mu^a - \partial_\nu \bar{\omega}_\mu^{ac} D_\nu^{ab} \varphi_\mu^{bc} - g\gamma^2 \bar{\omega}_\mu^{ab} f_{abc} A_\mu^c - \gamma^2 \partial_\mu \bar{\omega}_\mu^{aa}). \quad (200)$$

Subsequently, applying the BRST variation gives

ing these two operations the other way around. This difference is exactly given by the surface term from Eq. (199). This discrepancy arises of course from the explicit x -dependence introduced in the BRST transformation \tilde{s} , giving nontrivial contributions. For example, we introduced a term $-\gamma^2 \tilde{s} \int d^4x \partial_\nu \bar{\omega}_\mu^{aa}$ which might seem to be zero since we are looking at the integral of a complete derivative (thus usually taken to be a vanishing surface

¹¹For our purposes these diagrams are irrelevant, e.g. the vacuum energy, the gluon, and ghost propagator, etc.

term), but when the BRST variation is taken first, a non-trivial integrated piece remains.

Apparently, to find the correct Gribov-Zwanziger action with the Maggiore-Schaden argument, there is some kind of a “hidden working hypothesis” that (200) is the correct action to start with, and that partial integration is not always allowed.¹² The fact that there seems to be a kind of “preferred” action to start with is just a signal that there is a problem with the boundary conditions for some of the fields and hence surface terms when integrating.

Even if one forgets about the previous criticism, a second problem arises. In the Gribov-Zwanziger approach, we recall that the parameter γ is not free and is determined by the horizon condition (16). As it has been explained in Sec. II A, the solution $\gamma = 0$ is excluded. In Eq. (137), we found a solution for $\gamma \neq 0$. This gave rise to a positive vacuum energy $E_{\text{vac}} > 0$, as one can see from Eq. (139); see also [25]. However, according to Maggiore-Schaden argument, at one loop order the stable solution should be that corresponding to $\gamma = 0$ [23], as, if $\gamma = 0$, the vacuum energy would be vanishing, i.e. $E_{\text{vac}} = 0$, which is energetically favored over a positive vacuum energy. This delivers a contradiction with the Gribov-Zwanziger approach, as the restriction to the Gribov region requires that $\gamma \neq 0$, thus giving a positive energy $E_{\text{vac}} > 0$ at one loop.

To end this section, let us now consider the new operator $\int d^4x(\bar{\varphi}_\mu^{ab} \varphi_\mu^{ab} - \bar{\omega}_\mu^{ab} \omega_\mu^{ab})$ within the Maggiore-Schaden approach. We observe that we obtain an explicit x -dependence if we rewrite this operator in terms of the new fields,

$$\begin{aligned} \int d^4x(\bar{\varphi}_\mu^{ab} \varphi_\mu^{ab} - \bar{\omega}_\mu^{ab} \omega_\mu^{ab}) &= \int d^4x(\bar{\varphi}_\mu^{ab} \varphi_\mu^{ab} - \bar{\omega}_\mu^{ab} \omega_\mu^{ab} \\ &+ \gamma^2 x_\mu \varphi_\mu^{aa} - \gamma^2 x_\mu \bar{\varphi}_\mu^{aa} \\ &- \gamma^4 x_\mu x_\mu (N^2 - 1)). \end{aligned} \quad (202)$$

However, this x -dependence is necessary so that (202) would be invariant under the new BRST symmetry \tilde{s} ,

$$\begin{aligned} \tilde{s} \int d^4x(\bar{\varphi}_\mu^{ab} \varphi_\mu^{ab} - \bar{\omega}_\mu^{ab} \omega_\mu^{ab} + \gamma^2 x_\mu \varphi_\mu^{aa} \\ - \gamma^2 x_\mu \bar{\varphi}_\mu^{aa} - \gamma^4 x_\mu x_\mu (N^2 - 1)) = 0. \end{aligned} \quad (203)$$

A second option is to introduce the BRST \tilde{s} -exact mass operator

$$\begin{aligned} \tilde{s} \int d^4x(\bar{\omega}_\mu^{ab} \varphi_\mu^{ab}) &= \int d^4x(\bar{\varphi}_\mu^{ab} \varphi_\mu^{ab} - \bar{\varphi}_\mu^{ab} \varphi_\mu^{ab} \\ &+ \gamma^2 x_\mu \varphi_\mu^{aa}), \end{aligned} \quad (204)$$

which also displays an explicit x -dependence.

¹²If it would be allowed, one would be able to cross from the second action (200) to the first one (196), but as we have just shown, these two starting actions are inequivalent.

Finally, let us consider a third and last possible option. If we would have started with the following mass operator,

$$\int d^4x(\bar{\varphi}_\mu^{ab} \varphi_\mu^{ab} - \bar{\omega}_\mu^{ab} \omega_\mu^{ab}), \quad (205)$$

which does not contain an x -dependence, this operator is not left invariant by the symmetry \tilde{s} . In fact,

$$\tilde{s} \int d^4x(\bar{\varphi}_\mu^{ab} \varphi_\mu^{ab} - \bar{\omega}_\mu^{ab} \omega_\mu^{ab}) = -\gamma^2 \int d^4x x_\mu \omega_\mu^{aa}. \quad (206)$$

One sees that, with the introduction of the new mass operator, the Maggiore-Schaden construction will always give rise to an explicit breaking of translation invariance if the BRST invariance \tilde{s} has to be preserved. We thus conclude that the Maggiore-Schaden construction cannot be implemented in the presence of the new operator and even without the new mass operator we have collected a few arguments from which the frame of a possible spontaneous symmetry breaking cannot be applied to the Gribov-Zwanziger action.

G. A few remarks on the Kugo-Ojima confinement criterion

In this section we shall take a closer look at the Kugo-Ojima confinement criterion [24] in relation to the Gribov-Zwanziger action. In the literature, it is usually stated that the Kugo-Ojima confinement criterion is realized when the Gribov-Zwanziger scenario is realized. A key ingredient in the criterion is $u(0) = -1$, whereby $u(0)$ is the value at zero momentum of a specific Green function. u is related to the ghost propagator in the Landau gauge according to [51]

$$\mathcal{G}(p^2)_{p^2 \approx 0} = \frac{1}{p^2} \frac{1}{1 + u(p^2)}. \quad (207)$$

From this expression, it is obvious that an infrared enhanced ghost propagator results in $u(0) = -1$, thereby fulfilling the criterion. Let us recall here that the derivation of the Kugo-Ojima criterion is based on the assumption of an exact BRST invariance and is written down in a Minkowskian rather than a Euclidean space-time. This has a few repercussions:

- (i) At a nonperturbative level, some care should be taken when passing from Euclidean to Minkowskian space-time. According to our understanding, it is not clear whether a Wick rotation can always be implemented; e.g., the gluon propagator (105) can exhibit two complex conjugate poles, so one should be careful of not crossing these poles when the contour is Wick rotated. Clearly, there could be potential caveats when considering a more complicated gluon propagator.
- (ii) A more crucial shortcoming is the following. As we have emphasized in the foregoing section, the restriction to the Gribov region inevitably leads to a

breaking of the BRST symmetry which, however, was the very starting point of the Kugo-Ojima analysis. In addition, parts of the Kugo-Ojima study rely on analyzing the charge of the global color current and the expression of a piece of it in terms of the BRST symmetry generator. In our opinion, as the Gribov-Zwanziger action is essentially differing from the usual Faddeev-Popov fixed (Landau gauge) action due to new fields, extra interactions and especially another symmetry content, the Kugo-Ojima analysis cannot simply be applied to the Gribov-Zwanziger formalism, although both might superficially seem to be in accordance with each other. Therefore, it seems to us that one cannot verify the Kugo-Ojima criterion ($u(0) = -1$) when the restriction to the Gribov horizon is taken into account.¹³

- (iii) The latest lattice data point towards a ghost propagator which is no longer enhanced, so that the condition $u(0) = -1$ does not seem to be realized anyhow.

VI. DISCUSSION

Our starting point was the original localized Gribov-Zwanziger action S_{GZ} and the observation of the new lattice data, which shows an infrared suppressed, positivity violating gluon propagator, nonvanishing at the origin and a ghost propagator which is no longer enhanced. However, the propagators corresponding to the original Gribov-Zwanziger action are not in accordance with these new lattice data. Hence, we have searched for a solution by looking at nonperturbative effects like condensates. Therefore, we have added two extra terms to the Gribov-Zwanziger action, $S_M = M^2 \int d^4x [(\bar{\varphi}\varphi - \bar{\omega}\omega) + \frac{2(N^2-1)}{g^2N} \varsigma\lambda^2]$. A first intuitive argument why we added the first term, $M^2 \int d^4x (\bar{\varphi}\varphi - \bar{\omega}\omega)$, was the following. In the Gribov-Zwanziger action, S_{GZ} , an $A\varphi$ -coupling is already present at the quadratic level. Therefore, altering the φ -sector will be translated to the A -sector, thus modifying the gluon propagator. Secondly, this condensate is already present perturbatively as, at lowest order, we have found

$$\langle \bar{\varphi}\varphi - \bar{\omega}\omega \rangle = \frac{3(N^2-1)}{64\pi} \lambda^2, \quad (208)$$

with $\lambda^4 = 2g^2N\gamma^4$. This implies that the condensate is nonvanishing for $\gamma \neq 0$ already in the original Gribov-Zwanziger action. It was therefore very natural to add this operator to the theory. The second pure vacuum term, $M^2 \int d^4x \frac{2(N^2-1)}{g^2N} \varsigma\lambda^2$, was added in order to stay within the horizon or equivalently, to keep $\sigma(0)$ smaller than 1 when

¹³This would also include Schwinger-Dyson results which implemented the restriction to the Gribov region by suitable boundary conditions.

the horizon condition is implemented. We have fixed ς by imposing $\frac{\partial\sigma(0)}{\partial M^2} \Big|_{M^2=0} = 0$; this ensures a smooth limit to the original Gribov-Zwanziger action.

The extended Gribov-Zwanziger action $S_{\text{GZ}} + S_M$ has many interesting features. Not only is this action renormalizable, it is also remarkable that no new renormalization factors are necessary for the proof of its renormalizability, meaning that only two independent renormalization factors are required. As an extra feature, we have also shown that $S_{\text{GZ}} + S_M + S_{A^2}$, with $S_{A^2} = \frac{m^2}{2} \int d^4x A_\mu^2$, is renormalizable.

Another important observation is that the gluon propagator is already modified at tree level. We have found

$$\mathcal{D}(p^2) = \frac{p^2 + M^2}{p^4 + (M^2 + m^2)p^2 + \lambda^4 + M^2m^2}. \quad (209)$$

This type of propagator is in qualitative agreement with the most recent lattice data, which was the starting point of our analysis. In Sec. IV the gluon propagator at zero momentum was also presented at one loop [see Eq. (159)], where we switched off the effects related to A^2 by setting $m^2 = 0$. By virtue of the novel mass M^2 , $\mathcal{D}(p^2) \neq 0$ at zero momentum. Also the one loop ghost propagator is modified. At small momenta we have obtained

$$\mathcal{G}(p^2)_{p^2 \approx 0} = \frac{1}{p^2} \frac{1}{1 - \sigma}, \quad (210)$$

with

$$\begin{aligned} \sigma(0) = 1 + M^2 \frac{3g^2N}{64\pi^2} \frac{1}{\sqrt{M^4 - 4\lambda^4}} [\ln(M^2 + \sqrt{M^4 - 4\lambda^4}) \\ - \ln(M^2 - \sqrt{M^4 - 4\lambda^4})] - \left(\frac{3g^2N}{128\pi} \right) \frac{M^2}{\lambda^2}. \end{aligned} \quad (211)$$

We see that the ghost propagator is clearly no longer enhanced, again in accordance with the most recent lattice data.

Up to this point, the mass M^2 was put in by hand. However, we have treated $(\bar{\varphi}_i^a \varphi_i^a - \bar{\omega}_i^a \omega_i^a)$ as a composite operator coupled to the source $J = M^2$. In this way, we have been able to find nonperturbative effects induced by this composite operator without altering the original Gribov-Zwanziger action, and making the mass M^2 dynamical. We have developed two methods to find such nonperturbative effects. The first method uses the well-known principles of the effective action formalism. Unfortunately, the calculations become intractable. Therefore, we have implemented a second method, the variational principle. Intuitively, we have included effects of the mass term without altering the original Gribov-Zwanziger action by performing a suitable resummation. With the help of this technique, we have found in the $\overline{\text{MS}}$ scheme that $\sigma(0)$, the one loop correction to $(p^2 \mathcal{G}(p^2))_{p^2 \approx 0}^{-1}$ is given by¹⁴

¹⁴We set $\Lambda_{\overline{\text{MS}}} = 0.233 \text{ GeV}$, the value reported in [52].

$$\sigma(0) = 0.93, \quad (212)$$

resulting in a nonenhanced ghost propagator. Simultaneously, for the one loop gluon propagator at zero momentum, we have found

$$\mathcal{D}^{(1)}(0) = \frac{0.63}{\Lambda_{\overline{\text{MS}}}^2} \sim \frac{11.65}{\text{GeV}^2}, \quad (213)$$

which is nonzero. The corresponding value for the coupling constant is smaller than 1 [see Eq. (153)], which is acceptable for a perturbative expansion. We have also checked the positivity violation of the gluon propagator with the help of the variational technique and again, our results were in nice agreement with lattice results: not only is the shape of the temporal correlator $\mathcal{C}(t)$, displayed in Fig. 8 in qualitative agreement, also the value of the point $t \sim 1.5$ fm at which the violation of positivity starts is consistent with the results reported in lattice investigations. Using the plots displayed in [17] which were also obtained in the $SU(3)$ case, one can extract a rough lattice estimate for the quantities (212) and (213),

$$\mathcal{D}^{\text{lattice}}(0) \sim \frac{13}{\text{GeV}^2}, \quad (214)$$

$$p^2 \mathcal{G}(p^2)_{p^2=0}^{\text{lattice}} \sim 5 \Leftrightarrow \sigma(0)^{\text{lattice}} \sim 0.8. \quad (215)$$

We notice that our lowest order approximations (212) and (213) are qualitatively compatible with the current lattice values.

To conclude, we would like to emphasize that the original Gribov-Zwanziger action already breaks the BRST symmetry. Because of this breaking, it is unclear at present how to define the observables of the theory in the nonperturbative infrared region. According to our understanding, this breaking cannot be interpreted as a spontaneous breaking, according to the proposal of [23]. In fact, we have argued that the BRST breaking is a natural consequence of introducing the restriction to the Gribov region. In addition, we have underlined that the presence of the BRST breaking term in the Gribov-Zwanziger action provides a consistent way to ensure that the restriction to the Gribov region can have physical consequences, i.e. that the Gribov parameter γ enters the expectation value of physical, gauge invariant correlators. In the absence of such a breaking term, the Gribov mass parameter would play the role of an unphysical gauge parameter. The presence of the breaking is thus a necessary tool within the Gribov-Zwanziger approach, allowing for the introduction of a nonperturbative mass parameter in a local and renormalizable way. Finally, we have also commented on the Kugo-Ojima confinement criterion. Since it is fundamentally based on the concept of an exact BRST symmetry, it cannot be straightforwardly related to the Gribov-Zwanziger framework due to the breaking.

In summary, this paper presented the 4D analysis of the gluon and the ghost propagator within the Gribov-Zwanziger framework. By comparing these results with recent lattice data, we have found a good qualitative agreement. The ghost and gluon propagator have also been extensively studied on the lattice in 2 and 3 dimensions [16,18,19,53]. The 3D and 2D analysis of the extended Gribov-Zwanziger action, and a comparison with the lattice data, is currently under consideration.

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APPENDIX: THE ONE LOOP EFFECTIVE POTENTIAL

We explain in detail how we obtained Eq. (111). We start by evaluating the integral appearing in $W(J)$. We recall that this integral originates from

$$\begin{aligned} & \int dA_\mu \exp -\frac{1}{2} \int d^4x A_\mu^a (\Delta_{\mu\nu}^{ab}) A_\nu^b \\ & = \left[\det \left[-\left(\partial^2 + \frac{2g^2 N \gamma^4}{\partial^2 - M^2} \right) \delta_{\mu\nu} - \partial_\mu \partial_\nu \left(1 - \frac{1}{\alpha} \right) \right] \right]^{-1/2} \\ & = e^{-(1/2) \text{Tr} \ln Q_{\mu\nu}^{ab}}, \end{aligned} \quad (\text{A1})$$

with $Q_{\mu\nu}^{ab} = -\left(\partial^2 + \frac{2g^2 N \gamma^4}{\partial^2 - M^2} \right) \delta_{\mu\nu} - \partial_\mu \partial_\nu \left(1 - \frac{1}{\alpha} \right)$. From this expression it follows that we need to calculate $\frac{1}{2} \text{Tr} \ln Q_{\mu\nu}^{ab}$ to obtain the energy functional

$$\begin{aligned} \frac{1}{2} \text{Tr} \ln Q_{\mu\nu}^{ab} & = \frac{N^2 - 1}{2} (d - 1) \text{Tr} \ln \left(-\partial^2 - \frac{2g^2 N \gamma^4}{\partial^2 - M^2} \right) \\ & = \frac{N^2 - 1}{2} (d - 1) \{ \text{Tr} \ln(-\partial^2(\partial^2 - M^2)) \\ & \quad - 2g^2 N \gamma^4 \} - \text{Tr} \ln(-\partial^2 + M^2). \end{aligned} \quad (\text{A2})$$

The second part is a standard integral and evaluated as

$$\text{Tr} \ln(-\partial^2 + M^2) = \frac{-\Gamma(-d/2)}{(4\pi)^{d/2}} \frac{1}{(M^2)^{-d/2}}, \quad (\text{A3})$$

with Γ the Euler gamma function. Using dimensional regularization $d = 4 - \epsilon$ we obtain

$$\begin{aligned}
& -\frac{N^2-1}{2}(d-1)\text{Tr}\ln(-\partial^2+M^2) \\
& = -3\frac{N^2-1}{64\pi^2}M^4\left(-\frac{5}{6}-\frac{2}{\epsilon}+\ln\frac{M^2}{\bar{\mu}^2}\right). \quad (\text{A4})
\end{aligned}$$

We recall that we work in the $\overline{\text{MS}}$ scheme. Next, we try to convert the first part in to the standard form,

$$\begin{aligned}
\frac{N^2-1}{2}(d-1)\text{Tr}\ln(-\partial^2(\partial^2-M^2)-2g^2N\gamma^4) & = \frac{N^2-1}{2}(d-1)\text{Tr}\ln(-\partial^2+m_1^2)+\text{Tr}\ln(-\partial^2+m_2^2) \\
& = \frac{N^2-1}{2}(d-1)\left[\frac{-\Gamma(-d/2)}{(4\pi)^{d/2}}\frac{1}{(m_1^2)^{-d/2}}+\frac{-\Gamma(-d/2)}{(4\pi)^{d/2}}\frac{1}{(m_2^2)^{-d/2}}\right] \\
& = \frac{N^2-1}{2}(d-1)\left[\frac{-\Gamma(-d/2)}{(4\pi)^{d/2}}\frac{1}{(m_1^2)^{-d/2}}+\frac{-\Gamma(-d/2)}{(4\pi)^{d/2}}\frac{1}{(m_2^2)^{-d/2}}\right] \\
& = 3\frac{N^2-1}{64\pi^2}\left(m_1^4\left(-\frac{5}{6}-\frac{2}{\epsilon}+\ln\frac{m_1^2}{\bar{\mu}^2}\right)+m_2^4\left(-\frac{5}{6}-\frac{2}{\epsilon}+\ln\frac{m_2^2}{\bar{\mu}^2}\right)\right)+O(\epsilon), \quad (\text{A5})
\end{aligned}$$

where we have used the notational shorthand (110). We still have to calculate the first and the second term of (109). For the first term, we recall that

$$\gamma_0^4 = Z_{\gamma^2}^2 \gamma^4, \quad \text{with } Z_{\gamma^2}^2 = 1 + \frac{3}{2} \frac{g^2 N}{16\pi^2} \frac{1}{\epsilon}, \quad (\text{A6})$$

with Z_{γ^2} defined in (57), so we find

$$\begin{aligned}
-d(N^2-1)\gamma_0^4 & = -4(N^2-1)\gamma^4 - 4\frac{3}{2}(N^2-1)\frac{g^2 N}{16\pi^2}\frac{1}{\epsilon}\gamma^4 \\
& \quad + \frac{3}{2}\frac{g^2 N}{16\pi^2}\gamma^4(N^2-1). \quad (\text{A7})
\end{aligned}$$

The second term is invariant under renormalization and therefore given by

$$\frac{d(N^2-1)}{g^2 N} s \lambda^2 J. \quad (\text{A8})$$

From Eqs. (A4), (A5), and (A7) we see that the infinities cancel out nicely, so that the functional energy reads

$$\begin{aligned}
W^{(1)}(J) & = -\frac{4(N^2-1)}{2g^2 N}\lambda^4 + \frac{d(N^2-1)}{g^2 N} s \lambda^2 J \\
& \quad + \frac{3(N^2-1)}{64\pi^2}\left(\frac{8}{3}\lambda^4 + m_1^4 \ln\frac{m_1^2}{\bar{\mu}^2} + m_2^4 \ln\frac{m_2^2}{\bar{\mu}^2}\right. \\
& \quad \left. - J^2 \ln\frac{J}{\bar{\mu}^2}\right), \quad (\text{A9})
\end{aligned}$$

which is exactly expression (111).

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