Hawking radiation, entanglement, and teleportation in the background of an asymptotically flat static black hole

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(Received 15 December 2007; revised manuscript received 19 June 2008; published 10 September 2008)

The effect of the Hawking temperature on the entanglement and teleportation for the scalar field in a most general, static, and asymptotically flat black hole with spherical symmetry has been investigated. It has been shown that the same "initial entanglement" for the state parameter α and its "normalized partners" $\sqrt{1-\alpha^2}$ will be degraded by the Hawking effect with increasing Hawking temperature along two different trajectories except for the maximally entangled state. In the infinite Hawking temperature limit, corresponding to the case of the black hole evaporating completely, the state no longer has distillable entanglement for any α. It is interesting to note that the mutual information in this limit is equal to just half of the "initially mutual information." It has also been demonstrated that the fidelity of teleportation decreases as the Hawking temperature increases, which indicates the degradation of entanglement.

DOI: 10.1103/PhysRevD.78.065015 PACS numbers: 03.65.Ud, 03.67.Mn, 04.70.Dy, 97.60.Lf

I. INTRODUCTION

The quantum information theory in the relativistic framework has received considerable attention due to its theoretical importance and practical application [1–3]. Especially, more and more efforts have been expended on the study of quantum entanglement in a relativistic setting because people consider the entanglement to be a major resource for quantum information tasks such as quantum teleportation, quantum computation, and so on [4]. With the intention of studying the entanglement between accelerated observers, the fidelity of teleportation between two parties in relative uniform acceleration was discussed by Alsing *et al.* [5,6]. Ge *et al.* extended the gravitational field of the teleportation to the four and higher dimensional spacetimes, and even explicitly discussed what effects the shape of the cavity in which particles are confined has on the teleportation in a black hole spacetime [7,8]. In order to further investigate the observer-dependent character of the entanglement, Fuentes-Schuller *et al.* analyzed the entanglement between two modes of a noninteracting massless scalar field when one of the observers describing the state is uniformly accelerated [9]. And then Alsing *et al.* calculated the entanglement between two modes of a free Dirac field described by relatively accelerated parties in a flat spacetime [10]. Their results [9,10] also showed that the different type of field will have a qualitatively different effect on the degradation of entanglement produced by the Unruh effect [11,12]. More recently, Ahn *et al.* extended the investigation to the entanglement of a two-mode squeezed state in Riemannian spacetime [13], Ling *et al.* discussed the entanglement of an electromagnetic field in noninertial reference frames [14], and Adesso *et al.* investigated the distribution of entanglement between modes of a free scalar field from the perspective of observers in uniform acceleration [15].

As a further step along this line, we will provide an analysis of the entanglement for the scalar field in the spacetime of a most general, static, and asymptotically flat black hole with spherical symmetry. It seems to be an interesting study to consider the influences of the Hawking effect [16–18] on the quantum entangled states and show how the Hawking temperature will change the properties of the entanglement and teleportation. Choosing a generically entangled state as the initially entangled state for two observers in the flat region of this black hole, we will also try to see what effects the uncertain entangled state will have on the degradation of entanglement in our scheme due to the presence of an arbitrary state parameter. Our scheme proposes that the two observers, Alice and Bob, share an initially entangled state at the same initial point in flat Minkowski spacetime before the black hole is formed. After the coincidence of Alice and Bob, Alice stays stationary at the asymptotically flat region, while Bob falls in toward the mass and then hovers outside of it. Once Bob is safely hovering outside of the object at some constant acceleration, let it collapse to form a black hole. By Birkhoff's theorem [19] this will not change the metric outside of the black hole and therefore will not change Bob's acceleration. Thus, Bob's detector registers only thermally excited particles due to the Hawking effect [20,21]. In order to investigate the teleportation between two modes of a scalar field as detected by the two observers, we assume that Alice and Bob each hold an optical cavity which is small and perfect for the teleportation in the

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black hole spacetime. Just as suggested by Refs. [5,6], we further suppose that each cavity supports two orthogonal modes, with the same frequency, which are each excited to a single photon Fock state at the coincidence point for Alice and Bob. Different from the standard teleportation protocol, our scheme assumes that Bob hovers outside of the object before it collapses, and turns on his detector after the formation of the black hole. Then, Bob can check to see whether any thermal photons have been excited in his local cavity using the nonabsorbing detector.

The organization of this paper is as follows. In Sec. II we discuss the vacuum structure of the background spacetime and the Hawking effect for the scalar particles as experienced by the observer outside the black hole. In Sec. III we analyze the effects of the Hawking temperature on the entanglement between the modes for the different state parameter. In Sec. IV we describe the process of the teleportation between Alice and Bob, and calculate the fidelity of teleportation. We summarize and discuss our conclusions in the last section.

II. VACUUM STRUCTURE AND HAWKING RADIATION OF SCALAR FIELD

It is well known that the spherically symmetric line element of a static and asymptotically flat black hole such as the Schwarzschild black hole, the Reissner-Nordström black hole [22], the Garfinkle-Horowitz-Strominger dilaton black hole [23], the Casadio-Fabbri-Mazzacurati brane black hole [24], and so on can be written in the form

$$ds^2 = f(r)dt^2 - \frac{1}{h(r)}dr^2 - R^2(r)(d\theta^2 + \sin\theta^2 d\varphi^2), \quad (1)$$

where the functions $f(r)$ and $h(r)$ vanish at the event horizon $r = r_+$ of the black hole. Throughout this paper we use $G = c = \hbar = \kappa_B = 1$. It is obvious that the surface gravity of the event horizon is determined by $\kappa = \sqrt{f'(r_+)h'(r_+)}/2$. Defining the tortoise coordinates r_* as $dr_* = dr/\sqrt{f(r)h(r)}$, we can rewrite the metric (1) as

$$ds^2 = f(r)(dt^2 - dr_*^2) - R^2(r)(d\theta^2 + \sin\theta^2 d\varphi^2). \quad (2)$$

The massless scalar field ψ satisfies the Klein-Gordon equation

$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^\mu}\left(\sqrt{-g}g^{\mu\nu}\frac{\partial\psi}{\partial x^\nu}\right) = 0. \quad (3)$$

After expressing the normal mode solution as [12,25]

$$\psi_{\omega lm} = \frac{1}{R(r)}\chi_{\omega l}(r)Y_{lm}(\theta,\varphi)e^{-i\omega t}, \quad (4)$$

we can easily get the radial equation

$$\frac{d^2\chi_{\omega l}}{dr_*^2} + [\omega^2 - V(r)]\chi_{\omega l} = 0, \quad (5)$$

with

$$V(r) = \frac{\sqrt{f(r)h(r)}}{R(r)}\frac{d}{dr}\left[\sqrt{f(r)h(r)}\frac{dR(r)}{dr}\right] + \frac{l(l+1)f(r)}{R^2(r)}, \quad (6)$$

where $Y_{lm}(\theta,\varphi)$ is a scalar spherical harmonic on the unit twosphere. Solving Eq. (5) near the event horizon, we obtain the incoming wave function which is analytic everywhere in the spacetime manifold [25]

$$\psi_{\mathrm{in},\omega lm} = e^{-i\omega v}Y_{lm}(\theta,\varphi), \quad (7)$$

and the outgoing wave functions for the inside and outside region of the event horizon

$$\psi_{\mathrm{out},\omega lm}(r<r_+) = e^{i\omega u}Y_{lm}(\theta,\varphi), \quad (8)$$

$$\psi_{\mathrm{out},\omega lm}(r>r_+) = e^{-i\omega u}Y_{lm}(\theta,\varphi), \quad (9)$$

where $v = t + r_*$ and $u = t - r_*$. Equations (8) and (9) are analytic inside and outside the event horizon, respectively, so they form a complete orthogonal family. In second quantizing the field Φ_{out} in the exterior of the black hole we can expand it as follows [12]:

$$\begin{aligned}\Phi_{\mathrm{out}} = \sum_{lm}\int d\omega[&b_{\mathrm{in},\omega lm}\psi_{\mathrm{out},\omega lm}(r<r_+) \\ &+ b^\dagger_{\mathrm{in},\omega lm}\psi^*_{\mathrm{out},\omega lm}(r<r_+) + b_{\mathrm{out},\omega lm}\psi_{\mathrm{out},\omega lm}(r>r_+) \\ &+ b^\dagger_{\mathrm{out},\omega lm}\psi^*_{\mathrm{out},\omega lm}(r>r_+)],\end{aligned} \quad (10)$$

where $b_{\mathrm{in},\omega lm}$ and $b^\dagger_{\mathrm{in},\omega lm}$ are the annihilation and creation operators acting on the vacuum of the interior region of the black hole, and $b_{\mathrm{out},\omega lm}$ and $b^\dagger_{\mathrm{out},\omega lm}$ are the annihilation and creation operators acting on the vacuum of the exterior region, respectively. Thus, the Fock vacuum state can be defined as

$$b_{\mathrm{in},\omega lm}|0\rangle_{\mathrm{in}} = b_{\mathrm{out},\omega lm}|0\rangle_{\mathrm{out}} = 0. \quad (11)$$

Introducing the generalized lightlike Kruskal coordinates [25–28]

$$\begin{aligned}&U = -\frac{1}{\kappa}e^{-\kappa u}, \qquad V = \frac{1}{\kappa}e^{\kappa v}, \quad \text{if } r>r_+; \\ &U = \frac{1}{\kappa}e^{-\kappa u}, \qquad V = \frac{1}{\kappa}e^{\kappa v}, \quad \text{if } r<r_+,\end{aligned} \quad (12)$$

and noticing that near the event horizon

$$r_* \simeq \frac{1}{\sqrt{f'(r_+)h'(r_+)}}\ln(r-r_+), \quad (13)$$

we can obtain a complete basis of the outgoing modes according to the suggestion of Damour and Ruffini [25]

$$\begin{aligned}\psi_{\mathrm{I},\omega lm} = {}& e^{(\pi\omega/2\kappa)}\psi_{\mathrm{out},\omega lm}(r>r_+) \\ &+ e^{-(\pi\omega/2\kappa)}\psi^*_{\mathrm{out},\omega lm}(r<r_+),\end{aligned} \quad (14)$$

$$\psi_{\mathrm{II},\omega lm} = e^{-(\pi\omega/2\kappa)}\psi^*_{\mathrm{out},\omega lm}(r > r_+) + e^{(\pi\omega/2\kappa)}\psi_{\mathrm{out},\omega lm}(r < r_+). \quad (15)$$

Thus, we can also quantize the quantum field Φ_{out} in terms of $\psi_{\mathrm{I},\omega lm}$ and $\psi_{\mathrm{II},\omega lm}$ in the Kruskal spacetime as

$$\Phi_{\mathrm{out}} = \sum_{lm}\int d\omega[2\sinh(\pi\omega/\kappa)]^{-1/2}[a_{\mathrm{out},\omega lm}\psi_{\mathrm{I},\omega lm} + a^\dagger_{\mathrm{out},\omega lm}\psi^*_{\mathrm{I},\omega lm} + a_{\mathrm{in},\omega lm}\psi_{\mathrm{II},\omega lm} + a^\dagger_{\mathrm{in},\omega lm}\psi^*_{\mathrm{II},\omega lm}], \quad (16)$$

where the annihilation operator $a_{\mathrm{out},\omega lm}$ can be used to define the Kruskal vacuum outside the event horizon

$$a_{\mathrm{out},\omega lm}|0\rangle_K = 0. \quad (17)$$

According to Eqs. (10) and (16), we obtain the Bogoliubov transformations [28,29] for the particle creation and annihilation operators in the black hole and Kruskal spacetimes

$$a_{\mathrm{out},\omega lm} = \frac{b_{\mathrm{out},\omega lm}}{\sqrt{1-e^{-2\pi\omega/\kappa}}} - \frac{b^\dagger_{\mathrm{in},\omega lm}}{\sqrt{e^{2\pi\omega/\kappa}-1}},$$
$$a^\dagger_{\mathrm{out},\omega lm} = \frac{b^\dagger_{\mathrm{out},\omega lm}}{\sqrt{1-e^{-2\pi\omega/\kappa}}} - \frac{b_{\mathrm{in},\omega lm}}{\sqrt{e^{2\pi\omega/\kappa}-1}}. \quad (18)$$

We assume that the Kruskal vacuum $|0\rangle_K$ is related to the vacuum of the black hole $|0\rangle_{\mathrm{in}} \otimes |0\rangle_{\mathrm{out}}$ by

$$|0\rangle_K = \Upsilon(b_{\mathrm{in},\omega lm}, b^\dagger_{\mathrm{in},\omega lm}, b_{\mathrm{out},\omega lm}, b^\dagger_{\mathrm{out},\omega lm})|0\rangle_{\mathrm{in}} \otimes |0\rangle_{\mathrm{out}}. \quad (19)$$

From $[b_{\mathrm{in},\omega lm}, b^\dagger_{\mathrm{in},\omega lm}] = [b_{\mathrm{out},\omega lm}, b^\dagger_{\mathrm{out},\omega lm}] = 1$ and Eq. (17), we get [12,30]

$$\Upsilon \propto \exp(b^\dagger_{\mathrm{out},\omega lm} b^\dagger_{\mathrm{in},\omega lm} e^{-\pi\omega/\kappa}). \quad (20)$$

After properly normalizing the state vector, we obtain the Kruskal vacuum which is a maximally entangled two-mode squeezed state [29,30]

$$|0\rangle_K = \sqrt{1-e^{-2\pi\omega/\kappa}}\sum_{n=0}^{\infty} e^{-n\pi\omega/\kappa}|n\rangle_{\mathrm{in}} \otimes |n\rangle_{\mathrm{out}}, \quad (21)$$

and the first excited state

$$|1\rangle_K = a^\dagger_{\mathrm{out},\omega lm}|0\rangle_K = (1-e^{-2\pi\omega/\kappa})\sum_{n=0}^{\infty}\sqrt{n+1}\, e^{-n\pi\omega/\kappa}|n\rangle_{\mathrm{in}} \otimes |n+1\rangle_{\mathrm{out}}, \quad (22)$$

where $\{|n\rangle_{\mathrm{in}}\}$ and $\{|n\rangle_{\mathrm{out}}\}$ are the orthonormal bases for the inside and outside region of the event horizon, respectively. For the observer outside the black hole, he needs to trace over the modes in the interior region since he has no access to the information in this causally disconnected region. Therefore, when an outside observer travels through the Kruskal particle vacuum $|0\rangle_K$ of mode ω his detector registers a number of particles given by

$${}_K\langle 0|b^\dagger_{\mathrm{out},\omega lm} b_{\mathrm{out},\omega lm}|0\rangle_K = \frac{1}{e^{2\pi\omega/\kappa}-1} = \frac{1}{e^{\omega/T}-1}, \quad (23)$$

where we have defined the Hawking temperature as [31,32]

$$T = \frac{\kappa}{2\pi} = \frac{\sqrt{f'(r_+)h'(r_+)}}{4\pi}. \quad (24)$$

Equation (23) is well known as the Hawking effect [16–18], which shows that the observer in the exterior of the black hole detects a thermal Bose-Einstein distribution of particles as he traverses the Kruskal vacuum.

III. QUANTUM ENTANGLEMENT IN BACKGROUND OF A BLACK HOLE

Now we assume that Alice has a detector which only detects mode $|n\rangle_A$ and Bob has a detector sensitive only to mode $|n\rangle_B$, and they share a generically entangled state at the same initial point in flat Minkowski spacetime before the black hole is formed:

$$|\Psi\rangle = \alpha|0\rangle_A|0\rangle_B + \sqrt{1-\alpha^2}|1\rangle_A|1\rangle_B, \quad (25)$$

where α is some real number which satisfies $|\alpha| \in (0, 1)$, α and $\sqrt{1-\alpha^2}$ are the so-called normalized partners. After the coincidence of Alice and Bob, Alice remains at the asymptotically flat region but Bob freely falls in toward the mass with his detector and then hovers outside of it before it collapses to form a black hole. Obviously, there will be some thermal effects due to changes in Bob's acceleration, but there will be no Hawking radiation. Note that such effects can be negligible, or at least will disperse after some time. Then, once Bob is safely hovering outside of the object at some constant acceleration, let it collapse to form a black hole. By Birkhoff's theorem [19] this will not change the metric outside of the black hole and therefore will not change Bob's acceleration. Thus, Bob's detector registers only thermally excited particles due to the Hawking effect [20,21]. The states corresponding to mode $|n\rangle_B$ must be specified in the coordinates of the black hole in order to describe what Bob sees in this curved spacetime. Thus, using Eqs. (21) and (22), we can rewrite Eq. (25) in terms of Minkowski modes for Alice and black hole modes for Bob. Since Bob is causally disconnected from the interior region of the black hole, we will take the trace over the states in this region and obtain the mixed density matrix between Alice and Bob in the exterior region

$$\rho_{AB} = (1 - e^{-\omega/T}) \sum_{n=0}^{\infty} \rho_n e^{-n\omega/T},$$
$$\begin{aligned}\rho_n = {}& \alpha^2 |0n\rangle\langle 0n| + (n+1)(1-\alpha^2)(1-e^{-\omega/T}) \\ &\times |1(n+1)\rangle\langle 1(n+1)| \\ &+ \alpha\sqrt{(n+1)(1-\alpha^2)(1-e^{-\omega/T})}|0n\rangle\langle 1(n+1)| \\ &+ \alpha\sqrt{(n+1)(1-\alpha^2)(1-e^{-\omega/T})}|1(n+1)\rangle\langle 0n|,\end{aligned} \tag{26}$$

where $|nm\rangle = |n\rangle_A |m\rangle_{B,\text{out}}$.

It should be noted that we do not trace over the states located inside the event horizon for Alice, even though she now is also causally disconnected from the interior region of the black hole. As a matter of fact, the causal structure of spacetime keeps every observer exterior from the black hole disconnected from its interior. Why do we not trace over the degree of freedom of Alice in the region inaccessible to her? We now present the reasons as follows. On the one hand, we can justify what we are doing by theory. For a Schwarzschild black hole, the mass of which is assumed to be of the order of a solar mass ($M_{\text{bh}} \sim M_{\odot}$), the magnitude of acceleration near this black hole that Bob needs is about 10^{13} m/s^2, which is much larger than that of Alice's needs (almost equal to zero) in the asymptotical region. Thus, we argue that Alice's acceleration effects can be neglected, whereas Bob's cannot. On the other hand, we can think about it from experiment. Though we do not observe the black hole directly, impressive progress in optical, radio, and X-ray astronomy greatly bolsters the evidence for supermassive black holes in the centers of galaxies [33]. Thus, the Earth can be argued to be an asymptotical region far from black holes, as far as we know. The standard quantum field theory works fine for the earthbound experiments, so we have at least some circumstantial empirical evidence that tracing over the black hole interior can be neglected in asymptotical regions.

It is clear that the partial transpose criterion provides a sufficient condition for the existence of entanglement in this case [34]: if at least one eigenvalue of the partial transpose of the density matrix is negative, the density matrix is entangled; but a state with positive partial transpose can still be entangled. It is a well-known bound or nondistillable entanglement [35,36]. Interchanging Alice's qubits ($|mn\rangle\langle pq| \to |pn\rangle\langle mq|$), we get the matrix representation of the partial transpose in the $(n, n+1)$ block

$$(\rho_{AB}^{T_A})_{n,n+1} = e^{-n\omega/T}(1-e^{-\omega/T}) \begin{pmatrix} n(1-\alpha^2)(e^{\omega/T}-1) & \alpha\sqrt{(n+1)(1-\alpha^2)(1-e^{-\omega/T})} \\ \alpha\sqrt{(n+1)(1-\alpha^2)(1-e^{-\omega/T})} & \alpha^2 e^{-\omega/T} \end{pmatrix}, \tag{27}$$

and its eigenvalues

$$\lambda_{\pm}^{n} = \frac{e^{-n\omega/T}(1-e^{-\omega/T})}{2} \times \Big[\zeta_n \pm \sqrt{\zeta_n^2 + 4\alpha^2(1-\alpha^2)(1-e^{-\omega/T})}\Big], \tag{28}$$

where $\zeta_n = \alpha^2 e^{-\omega/T} + n(1-\alpha^2)(e^{\omega/T}-1)$. Obviously the eigenvalue λ_{-}^{n} is always negative for finite value of the Hawking temperature. Hence, this mixed state is always entangled for any finite value of T. It should be noted that in the limit $T \to \infty$, the negative eigenvalue will go to zero. In order to discuss this further, we will use the logarithmic negativity which serves as an upper bound on the entanglement of distillation [35,36]. This entanglement monotone is defined as $N(\rho_{AB}) = \log_2 ||\rho_{AB}^{T_A}||$, where $||\rho_{AB}^{T_A}||$ is the trace norm of the partial transpose $\rho_{AB}^{T_A}$. Thus, we obtain the logarithmic negativity for this case

$$N(\rho_{AB}) = \log_2\Big[\alpha^2(1-e^{-\omega/T}) + \sum_{n=0}^{\infty} e^{-n\omega/T}(1-e^{-\omega/T}) \times \sqrt{\zeta_n^2 + 4\alpha^2(1-\alpha^2)(1-e^{-\omega/T})}\Big]. \tag{29}$$

The trajectories of the logarithmic negativity $N(\rho_{AB})$ versus T for different α in Fig. 1 just show how the Hawking temperature T would change the properties of the entanglement.

For the Hawking temperature of zero, corresponding to the case of a supermassive or an almost extreme black hole, $N(\rho_{AB}) = \log_2(1 + 2|\alpha|\sqrt{1-\alpha^2})$. In the range $0 < |\alpha| \le 1/\sqrt{2}$ the larger α, the stronger the initial entanglement; but in the range $1/\sqrt{2} \le |\alpha| < 1$ the larger α, the weaker

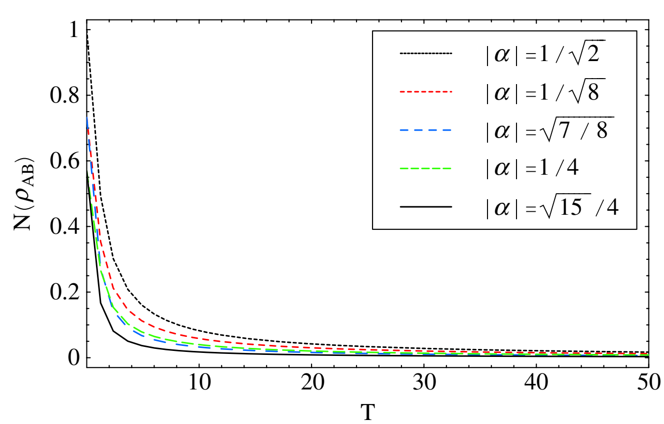

FIG. 1 (color online). The logarithmic negativity as a function of the Hawking temperature T with the fixed ω for different α.

the initial entanglement. For finite Hawking temperature, the monotonous decrease of $N(\rho_{AB})$ with increasing T for five different α means that the initial entanglement is lost to the thermal fields generated by the Hawking effect. This result agrees well with Hawking's original argument [16–18], which says that smaller black holes are at a higher temperature and so radiate more violently than massive black holes. Figure 1 also shows that when the initial entanglement is stronger, we lose it more rapidly. But it is surprisingly found that the same initial entanglement for α and its normalized partner $\sqrt{1-\alpha^2}$ will be degraded along two different curves except for the maximally entangled state, i.e., $|\alpha| = 1/\sqrt{2}$. This phenomenon, due to the coupling of α and the exponential functions related to T, just shows the inequivalence of the quantization for a scalar field in the black hole and Kruskal spacetimes. The logarithmic negativity is exactly zero for any α in the limit $T \to \infty$, which indicates that the state has no longer distillable entanglement for the arbitrary values of α when the black hole evaporates completely.

In order to estimate the total amount of correlations between Alice and Bob, we will analyze the mutual information which is defined as [37]

$$I(\rho_{AB}) = S(\rho_A) + S(\rho_B) - S(\rho_{AB}), \tag{30}$$

where $S(\rho) = -\mathrm{Tr}(\rho \log_2 \rho)$ is the entropy of the density matrix ρ. From Eq. (26), we can give the entropy of this joint state

$$S(\rho_{AB}) = -\sum_{n=0}^{\infty} e^{-n\omega/T}(1-e^{-\omega/T})[\alpha^2 + (n+1)(1-\alpha^2) \times (1-e^{-\omega/T})]\log_2 e^{-n\omega/T}(1-e^{-\omega/T}) \times [\alpha^2 + (n+1)(1-\alpha^2)(1-e^{-\omega/T})]. \tag{31}$$

Tracing over Alice's states for the density matrix ρ_{AB}, we get Bob's density matrix in exterior region of the event horizon

$$\rho_B = (1-e^{-\omega/T})\sum_{n=0}^{\infty} e^{-n\omega/T}[\alpha^2 |n\rangle\langle n| + (n+1) \times (1-\alpha^2)(1-e^{-\omega/T})|n+1\rangle\langle n+1|], \tag{32}$$

and its entropy

$$S(\rho_B) = -\sum_{n=0}^{\infty} e^{-n\omega/T}(1-e^{-\omega/T})[\alpha^2 + n(1-\alpha^2) \times (e^{\omega/T}-1)]\log_2 e^{-n\omega/T}(1-e^{-\omega/T}) \times [\alpha^2 + n(1-\alpha^2)(e^{\omega/T}-1)]. \tag{33}$$

We can also obtain Alice's density matrix by tracing over Bob's states

$$\rho_A = \alpha^2 |0\rangle\langle 0| + (1-\alpha^2)|1\rangle\langle 1|, \tag{34}$$

whose entropy can be expressed as

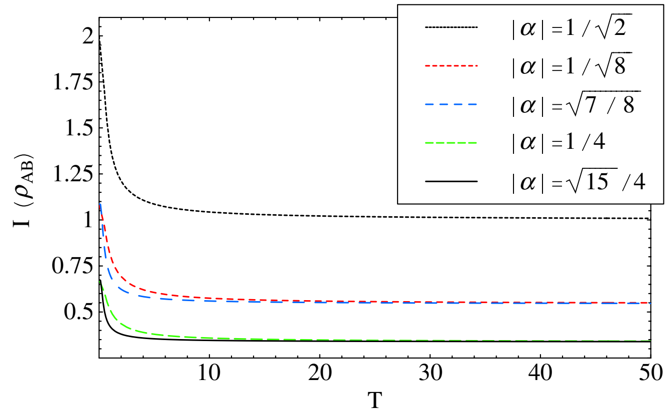

FIG. 2 (color online). The mutual information as a function of the Hawking temperature T with the fixed ω for different α.

$$S(\rho_A) = -[\alpha^2 \log_2 \alpha^2 + (1-\alpha^2)\log_2(1-\alpha^2)]. \tag{35}$$

Thus, we draw the behaviors of the mutual information $I(\rho_{AB})$ as a function of the Hawking temperature T for different values of the state parameter α in Fig. 2.

Figure 2 shows that for the Hawking temperature of zero, the initially mutual information is equal to

$$I_i(\rho_{AB}) = -2[\alpha^2 \log_2 \alpha^2 + (1-\alpha^2)\log_2(1-\alpha^2)]. \tag{36}$$

In the range $0 < |\alpha| \leq 1/\sqrt{2}$, the larger α, the stronger $I_i(\rho_{AB})$; but in the range $1/\sqrt{2} \leq |\alpha| < 1$, the larger α, the weaker $I_i(\rho_{AB})$. As the Hawking temperature increases, the mutual information becomes smaller. It is interesting to note that except for the maximally entangled state, the same initially mutual information for α and $\sqrt{1-\alpha^2}$ will be degraded along two different trajectories. However, in the infinite Hawking temperature limit $T \to \infty$, i.e., the black hole evaporates completely, the mutual information converges to the same value again

$$I_f(\rho_{AB}) = -[\alpha^2 \log_2 \alpha^2 + (1-\alpha^2)\log_2(1-\alpha^2)], \tag{37}$$

which is equal to just half of $I_i(\rho_{AB})$. Thus, we conclude that

$$I_f(\rho_{AB}) = \tfrac{1}{2} I_i(\rho_{AB}), \tag{38}$$

which is independent of the state parameter α. Obviously if $I_i(\rho_{AB})$ is higher, it is degraded to a higher degree in this limit. Since the distillable entanglement in the infinite Hawking temperature limit is zero, we are safe to say that the total correlations consist of classical correlations plus bound entanglement in this limit.

IV. QUANTUM TELEPORTATION IN BACKGROUND OF A BLACK HOLE

In this section we will concentrate on a particular quantum information task: quantum teleportation. We assume that Alice and Bob each hold an optical cavity, at rest in their local frame. Each cavity supports two orthogonal modes (labeled by A_i and B_i with $i = 1, 2$), with the same frequency, which are each excited to a single photon Fock state at the coincidence point for Alice and Bob. We ignore the polarization of these modes and model the photons by the massless modes of a scalar field as suggested by Refs. [5,6]. Considering the textbook teleportation protocol [38], we let Alice and Bob share a maximally entangled state, i.e., an entangled Bell state in flat Minkowski spacetime

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|\mathbf{0}\rangle_A|\mathbf{0}\rangle_B + |\mathbf{1}\rangle_A|\mathbf{1}\rangle_B), \tag{39}$$

where the logical states $|\mathbf{0}\rangle_A$ and $|\mathbf{1}\rangle_A$ are defined in terms of the physical Fock states for Alice's cavity by the dual-rail basis states [5,6]

$$|\mathbf{0}\rangle_A = |1\rangle_{A_1}|0\rangle_{A_2}, \qquad |\mathbf{1}\rangle_A = |0\rangle_{A_1}|1\rangle_{A_2}, \tag{40}$$

with similar expressions for Bob's cavity. It should be noted that $|1\rangle_{A_1}$ and $|1\rangle_{A_2}$ are single photon excitations of the Minkowski vacuum states in Alice's cavity. Our construction implicitly assumes that we have chosen a modal decomposition of the Minkowski vacuum based on intracavity and extra-cavity modes, which is a legitimate alternative to the usual way of quantizing the vacuum in terms of plane wave modes [39,40]. Once the cavities are loaded with a photon, we also assume each cavity is perfect and cannot emit the photon.

Recalling the usual teleportation protocol with the unknown state [38]

$$|\varphi\rangle = a|\mathbf{0}\rangle + b|\mathbf{1}\rangle, \tag{41}$$

we assume that Alice has an additional cavity which contains this single qubit (41) with dual-rail encoding by a photon excitation of a two-mode Minkowski vacuum state. This will allow Alice to make a joint measurement on the two orthogonal modes of each cavity. For the usual teleportation protocol between two Minkowski observers Alice and Bob, after Alice's measurement, Bob's state will be projected according to the measurement outcome. We can give the final state received by Bob

$$|\varphi_{ij}\rangle = x_{ij}|\mathbf{0}\rangle + y_{ij}|\mathbf{1}\rangle, \tag{42}$$

with four possible conditional state amplitudes $(x_{00}, y_{00}) = (a, b)$, $(x_{01}, y_{01}) = (b, a)$, $(x_{10}, y_{10}) = (a, -b)$, and $(x_{11}, y_{11}) = (-b, a)$. Once receiving the classical information of the result of Alice's measurement, Bob can apply a unitary transformation to verify the protocol. Obviously the fidelity of the teleported state is unity in this idealized situation.

Alice now wishes to perform the same teleportation protocol with the noninertial observer Bob. We assume that prior to their coincidence, Alice and Bob ensure that all photons are removed from their cavities. When Alice and Bob instantaneously share a maximally entangled state at the asymptotically flat region, we suppose that the two cavities overlap and simultaneously a four photon source excites a two photon state in each cavity. Then Alice remains there but Bob falls in toward the mass and then hovers outside of it. Once Bob is safely hovering outside of the object at some constant acceleration, let it collapse to form a black hole. Then, Bob turns on his detector after the formation of the black hole. Bob can check to see whether any thermal photons have been excited in his local cavity using the nonabsorbing detector. It should be noted that the common frequency of both Alice's and Bob's cavity is just the frequency ω of Eqs. (21) and (22) [5,6]. For Bob, the observer locates near the event horizon of a black hole, and he needs to trace over the modes in the interior region since he is causally disconnected from this region. Thus, when Alice sends the result of her measurement to Bob, Bob's state can be projected into

$$\begin{aligned}\rho_{ij} &= \sum_{k=0}^{\infty}\sum_{l=0}^{\infty} {}_{\rm in}\langle k, l|\varphi_{ij}\rangle\langle\varphi_{ij}|k, l\rangle_{\rm in} \\ &= (1 - e^{-\omega/T})^3 \sum_{n=0}^{\infty}\sum_{m=0}^{n}\{e^{-(n-1)\omega/T}[(n-m)|x_{ij}|^2 \\ &\quad + m|y_{ij}|^2]|m, n-m\rangle_{\rm out}\langle m, n-m| \\ &\quad + [\sqrt{(m+1)(n-m+1)}x_{ij}y_{ij}^* \\ &\quad \times e^{-n\omega/T}|m, n-m+1\rangle_{\rm out}\langle m+1, n-m| + {\rm H.c.}]\},\end{aligned} \tag{43}$$

where $|m, n-m\rangle_{\rm out} = |m\rangle_{B_1} \otimes |n-m\rangle_{B_2}$ is a state of n total excitations in the exterior region product state, with $0 \le m \le n$ excitations in the leftmost mode. Equation (43) can be rewritten as

$$\begin{aligned}\rho_{ij} &= \sum_{n=0}^{\infty} p_n\rho_{ij,n}, \quad \text{with} \quad p_0 = 0, \\ p_n &= (1 - e^{-\omega/T})^3 e^{-(n-1)\omega/T} \quad \text{for } n \ge 1.\end{aligned} \tag{44}$$

Since what we are concerned with is to which extent $|\varphi_{ij}\rangle$ might deviate from unitarity, so upon receiving the result (i, j) of Alice's measurement, Bob can apply the rotation operators (a unitary transformation in his local frame) restricted to the one-excitation sector of his state spanned by $\{|\mathbf{0}\rangle_{\rm out}, |\mathbf{1}\rangle_{\rm out}\} = \{|0, 1\rangle_{\rm out}, |1, 0\rangle_{\rm out}\}$ to turn this portion of his density matrix into the exterior region analogue of the state in Eq. (41) [5–8]

$$|\varphi\rangle_{\rm out} = a|\mathbf{0}\rangle_{\rm out} + b|\mathbf{1}\rangle_{\rm out}. \tag{45}$$

Thus, we can obtain the fidelity of Bob's final state with $|\varphi\rangle_{\rm out}$

$$F \equiv {}_{\rm out}\langle\varphi|\rho_{ij}|\varphi_{ij}\rangle_{\rm out} = (1 - e^{-\omega/T})^3. \tag{46}$$

From Fig. 3, we can see that the fidelity of teleportation depends on the Hawking temperature T. It has been found that the fidelity decreases as the Hawking temperature increases, which just indicates the entanglement degradation obtained in the previous section, because the state

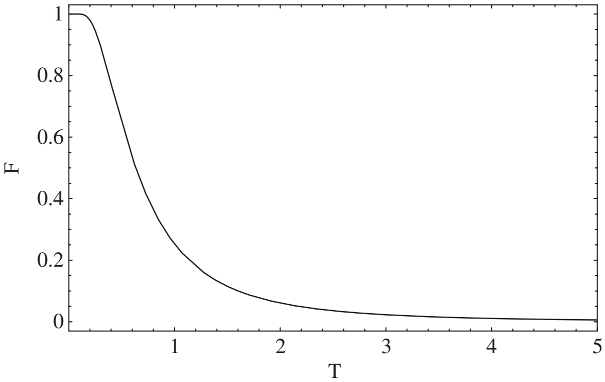

FIG. 3. The fidelity of teleportation as a function of the Hawking temperature T with the fixed ω for a maximally entangled state.

fidelity in conventional teleportation protocol is related to the entanglement.

V. SUMMARY

We have analytically discussed the effect of the Hawking temperature on the entanglement between two modes of a scalar field as detected by Alice who stays stationary at an asymptotically flat region and Bob who locates near the event horizon in the background of a most general, static and asymptotically flat black hole with spherical symmetry. It is shown that the entanglement is degraded by the Hawking effect with increasing Hawking temperature. It is found that the stronger the initial entanglement, which corresponds to the Hawking temperature of zero, i.e., the case of a supermassive or an almost extreme black hole, the faster it loses. It is found that the same initial entanglement for the state parameter α and its normalized partners $\sqrt{1-\alpha^2}$ will be degraded along two different trajectories as the Hawking temperature increases except for the maximally entangled state $\alpha = 1/\sqrt{2}$, which just shows the inequivalence of the quantization for a scalar field in the black hole and Kruskal spacetimes. In the infinite Hawking temperature limit $T \rightarrow \infty$, corresponding to the case of the black hole evaporating completely, the state has no longer distillable entanglement for the arbitrary values of α. Further analysis shows that the mutual information is degraded to a nonvanishing minimum value which is dependent of α with increasing Hawking temperature. However, it is interesting to note that the mutual information in the infinite Hawking temperature limit is equal to just half of the initially mutual information, which is independent of α. We have also investigated the scheme of teleportation in this black hole spacetime. It has been demonstrated that the fidelity of teleportation decreases as the Hawking temperature increases, which just indicates the entanglement degradation because the state fidelity in conventional teleportation protocol is related to the entanglement.

ACKNOWLEDGMENTS

The authors would like to thank the anonymous referee for his/her insightful and constructive criticism and suggestions, which allowed us to improve the manuscript significantly. This work was supported by the National Natural Science Foundation of China under Grant No. 10675045; the FANEDD under Grant No. 200317; and the Hunan Provincial Natural Science Foundation of China under Grant No. 08JJ3010.

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