

Recoherence by squeezed states in electron interferometry

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Coherent electrons coupled to the quantized electromagnetic field undergo decoherence which can be viewed as due either to fluctuations of the Aharonov-Bohm phase or to photon emission. When the electromagnetic field is in a squeezed vacuum state, it is possible for this decoherence to be reduced, leading to the phenomenon of recoherence. This recoherence effect requires electrons which are emitted at selected times during the cycle of the excited mode of the electromagnetic field. We show that there are bounds on the degree of recoherence which are analogous to quantum inequality restriction on negative energy densities in quantum field theory. We make some estimates of the degree of recoherence, and show that although small, it is in principle observable.

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I. INTRODUCTION

The interference of electrons is one of the most basic phenomena which illustrate the quantum nature of electrons. In recent years, technological advances have allowed electron interferometry to be used for a variety of investigations [1–5]. However, the quality of the interference pattern obtained with electrons is never as good as can be achieved with light or with neutral atoms. This can be attributed to the fact that charged particles interact more strongly with their environment than do photons or neutral atoms, and are hence more subject to loss of quantum coherence, or decoherence. This can arise from a variety of effects, such as interaction with random fields in the interferometer or with thermal radiation. Recently, Sonnentag and Hasselbach [5] observed decoherence as a result of dissipative interaction with image charge fields near an imperfectly conducting plate. In principle, these effects could be removed if there are no photons or classical fields in the interferometer. However, there is still a decoherence effect even when the quantized electromagnetic field is initially in its vacuum state [6–12]. This effect can be interpreted as arising from photon emission by the electrons. The emission of a photon with sufficiently short wavelength can reveal which path a particular electron takes and hence acts to destroy the interference pattern. (See Fig. 1.) An equivalent description is in terms of a fluctuating Aharonov-Bohm phase. For neutral particles with a magnetic moment, an analogous decoherence effect arises from Aharonov-Casher phase fluctuations [13].

In this paper, we will be concerned with the effects of squeezed photon states on the electron coherence. Squeezed states describe reduced quantum fluctuations in

one variable at the expense of increased fluctuations in the conjugate variable. One remarkable property is that they can exhibit locally negative energy densities. This phenomenon can be understood as a suppression of vacuum fluctuations. The normal ordered stress tensor operator is a difference between an expectation value in a given state

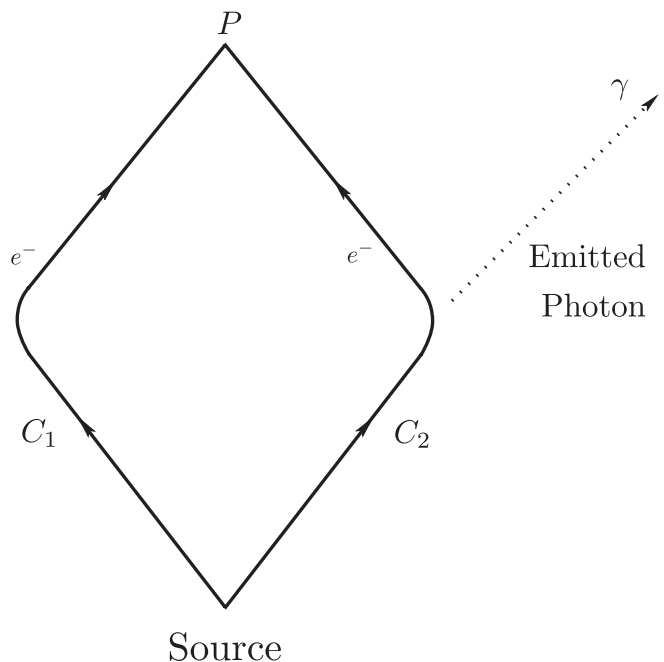


FIG. 1. An electron interference experiment in which the electrons may take either one of two paths, C_1 or C_2 , from the source to the point P where the interference pattern is formed. The emission of photons by the electrons tends to cause decoherence. The detection of an emitted photon with wavelength smaller than the path separation can reveal which path a particular electron takes, and hence causes decoherence.

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and that in the vacuum, a difference which can become negative. As will be detailed in the next section, electron decoherence due to a fluctuating electromagnetic field can be ascribed to fluctuations of the Aharonov-Bohm phase. As we will demonstrate, it is possible to use squeezed states of the quantized electromagnetic field to reduce these fluctuations, leading to a decrease in decoherence, which we will call “recoherence.” Squeezed states and coherent electrons were discussed in a somewhat different context by Vourdas and Sanders [14], who developed a procedure by which coherent electrons may be used to measure quantum states of the electromagnetic field.

The outline of this paper is as follows: In Sec. II, we outline the formalism of decoherence by a fluctuating Aharonov-Bohm phase. We then apply this formalism to the case of a single-mode squeezed state in Sec. III and calculate the degree of recoherence which is possible. These calculations are extended to multimode squeezed states in Sec. IV, and some numerical estimates are given in Sec. V. In Sec. VI, we summarize and discuss our results. Some of the properties of squeezed states are reviewed in the Appendix. Unless otherwise noted, we use Lorentz-Heaviside units with $\hbar = c = 1$.

II. GENERAL FORMALISM

Here we briefly review the effects of electromagnetic field fluctuations on electron coherence [6–19]. Consider a double slit interference experiment in which coherent electrons can take either one of two paths, as illustrated in Fig. 1. First consider the case of no field fluctuations. If the amplitudes for the electrons to take path C_1 and C_2 are ψ_1 and ψ_2 , respectively, to point P , then the mean number of electrons at P will be proportional to

$$n(P) = |\psi_1 + \psi_2|^2 = |\psi_1|^2 + |\psi_2|^2 + 2 \operatorname{Re}(\psi_1 \psi_2^*). \quad (1)$$

In the presence of a classical, nonfluctuating electromagnetic field described by vector potential A^μ , there will be an Aharonov-Bohm phase shift of the form [20]

$$\varphi_{AB} = e \oint_C dx^\mu A_\mu, \quad (2)$$

where the integral is taken around the closed path $C = C_1 - C_2$. This shifts the locations of the interference minima and maxima, but does not alter their relative amplitudes, the contrast.

If the electromagnetic field undergoes fluctuations, then the situation is different. In this case, the fluctuating Aharonov-Bohm phase causes a change in the contrast by a factor of

$$\Gamma = e^W, \quad (3)$$

where we define the *coherence functional* by

$$W = -\frac{1}{2} \langle \varphi_{AB}^2 \rangle \quad (4)$$

with the angular brackets denoting averaging over the

fluctuations. This functional can be expressed as

$$W = -2\pi\alpha \oint_C dx_\mu \oint_C dx'_\nu D^{\mu\nu}(x, x'), \quad (5)$$

where α is the fine-structure constant and

$$D^{\mu\nu}(x, x') = \frac{1}{2} \langle \{A^\mu(x), A^\nu(x')\} \rangle. \quad (6)$$

So far, we have not specified the source of the fluctuations, which could be thermal, quantum, or due to averaging over classical time variations [16, 19]. In this paper, we will be concerned with quantum fluctuations in a squeezed vacuum state.

III. SINGLE-MODE SQUEEZED VACUUM

A. Renormalized coherence functional, W_R

In this section, we consider the special case where the quantized electromagnetic field is in a state in which one mode is excited to a squeezed vacuum state, and all other modes remain in the ground state. We take the excited mode to be a plane wave in a box with periodic boundary conditions, with wave vector $\bar{\mathbf{k}}$ and polarization $\bar{\lambda}$, so the quantum state may be denoted by $|\zeta_{\bar{\lambda}\bar{\mathbf{k}}}\rangle$. The paths C_1 and C_2 are taken to be in the xz plane and we suppose that the electron wave packet is prepared to be highly localized about the classical trajectory and its dispersion can be ignored in the classical limit [8]. If the x component of the electron velocity is constant, and the trajectories C_1 and C_2 are chosen to be symmetric to one another with respect to the $z = 0$ plane, then in a comoving frame where the electron only has the sideways motion along the z axis, the quantity W can be greatly simplified to

$$W = -2\pi\alpha \oint_C dz \oint_C dz' D^{zz}(x, x'). \quad (7)$$

If we are only interested in the change of the fringe contrast due to the excitation of a particular squeezed vacuum mode, then after subtracting the vacuum contribution of all modes, we have the renormalized coherence functional given by

$$W_R = -\pi\alpha \oint_C dz \oint_C dz' \langle \zeta_{\bar{\lambda}\bar{\mathbf{k}}} | \{A^z(x), A^z(x')\} | \zeta_{\bar{\lambda}\bar{\mathbf{k}}}\rangle_R. \quad (8)$$

Here we use the subscript R to denote the renormalized quantity, which has the Minkowski vacuum term subtracted. In the Coulomb gauge, the z component of the vector potential in the plane-wave expansion takes the form

$$A^z(x) = \frac{1}{\sqrt{V}} \sum_{\mathbf{k}} \frac{1}{\sqrt{2\omega}} \sum_{\lambda=1}^2 \mathbf{e}_z \cdot \boldsymbol{\epsilon}_\lambda(\mathbf{k}) (a_{\lambda\mathbf{k}} e^{-ik \cdot x} + a_{\lambda\mathbf{k}}^\dagger e^{ik \cdot x}), \quad (9)$$

where \mathbf{e}_z is the unit vector along the z axis and $\boldsymbol{\epsilon}_\lambda$ are unit polarization vectors. The quantity V is the box normaliza-

tion volume and $\omega = |\mathbf{k}|$. If we further assume that the mode $(\bar{\lambda}, \bar{\mathbf{k}})$ is polarized in the z direction, and its wave vector is directed in the y direction, then the renormalized Hadamard function in a squeezed vacuum is given by

$$\langle \zeta_{\bar{\lambda}\bar{\mathbf{k}}} | \{A^z(x), A^z(x')\} | \zeta_{\bar{\lambda}\bar{\mathbf{k}}}\rangle_R = \frac{1}{V\bar{\omega}} [-\mu\nu e^{-i\bar{\omega}(t+t')} + |\nu|^2 e^{-i\bar{\omega}(t-t')} + \text{c.c.}], \quad (10)$$

where $\mu = \cosh r$, $\nu = e^{i\theta} \sinh r$, and $\zeta_{\bar{\lambda}\bar{\mathbf{k}}} = r e^{i\theta}$ is the complex squeeze parameter defined in the Appendix. Then Eq. (8) becomes

$$W_R = -\frac{\pi\alpha}{V\bar{\omega}} \oint_C dz \oint_C dz' [-\mu\nu e^{-i\bar{\omega}(t+t')} + |\nu|^2 e^{-i\bar{\omega}(t-t')} + \text{c.c.}], \quad (11)$$

This is the factor that accounts for the contrast change of the electron interference fringe in the present arrangement, where we shine a polarized beam in a single-mode squeezed vacuum state in the direction perpendicular to the plane of the electron paths.

To evaluate W_R , we pick a path which is twice differentiable,

$$z(t) = \frac{R}{T^4} (t^2 - T^2)^2, \quad (12)$$

where $2T$ and $2R$ can be thought of as the effective flight time and path separation, respectively. Electrons which start from the source at different times will experience different fluctuations. We can study this effect by letting t_0 be the electron emission time, in which case the quantity W_R becomes

$$\begin{aligned} W_R &= -\frac{4\pi\alpha}{V\bar{\omega}} \int_{-T}^T dt \int_{-T}^T dt' v_z v'_z [-\mu\eta e^{-i\bar{\omega}(t+t')-i2\bar{\omega}t_0+i\theta} \\ &\quad + \eta^2 e^{-i\bar{\omega}(t-t')} + \text{c.c.}] \\ &= -\frac{8\pi\alpha\eta}{V\bar{\omega}} [\mu \cos(2\bar{\omega}t_0 - \theta) + \eta] M, \end{aligned} \quad (13)$$

with $v_z = dz(t)/dt$, $\eta = \sinh r$, and

$$M = \left(\frac{16R}{\bar{\omega}^4 T^4}\right)^2 [(-3 + \bar{\omega}^2 T^2) \sin \bar{\omega} T + 3\bar{\omega} T \cos \bar{\omega} T]^2. \quad (14)$$

The quantity M does not depend on the electron emission time t_0 and is always positive definite, so the sign of W_R is solely determined by the quantity $\mu \cos(2\bar{\omega}t_0 - \theta) + \eta$.

B. Interpretation of W_R

An intriguing feature is that the values of W_R are not always negative, and they can be positive, depending on the parameters t_0 , μ , and η . It implies that the amplitude factor e^{W_R} may be larger than unity for some moments,

which in turn means that the contrast on the screen can be higher than it would otherwise be for the vacuum state. This is generally interpreted as enhancement of coherence, or recoherence. This contrast change cannot be observed right away when only one electron is released at each moment. We have to wait for a sufficiently long time so that enough electrons are accumulated to have visible patterns. However, since t_0 is related to the electron emission time and is assumed to be a random variable, if the time scale of the measurement is much longer than the flight time $2T$, then it is a long-time-averaged result that should be observed:

$$\bar{W}_R \equiv \lim_{\Xi \rightarrow \infty} \frac{1}{2\Xi} \int_{-\Xi}^{\Xi} dt_0 W_R = -\frac{8\pi\alpha}{V\bar{\omega}} \eta^2 M < 0. \quad (15)$$

Hence, this time-averaged value of W_R is always negative. This means that measurements which average over a long time will always find decoherence from the presence of the squeezed vacuum state, but measurements on shorter time scales have a chance to find $W_R > 0$, which means transient recoherence.

This feature bears a strong resemblance to the issue of negative energy density. It is known that quantum field theory has the remarkable property that local energy density can be negative even though the energy density is a positive definite quantity in classical physics. It is a general feature of both free and interacting theories that there exist states in which the energy density at a particular point can be arbitrarily negative [21]. Nonetheless, the total energy, integrated over all space, is required to be non-negative. It is also shown that there exist quantum inequalities [22–25] which constrain the magnitude and duration of the negative energy density and flux. Physically, the inequalities imply that the energy density seen by an observer cannot be arbitrarily negative for an arbitrarily long period of time. Marecki [26,27] has recently derived variants of the quantum inequalities for limiting the amount of squeezing which might be observed in photodetection experiments in quantum optics. Therefore it is interesting to know whether there exists a similar inequality on the quantity W_R , at least for the squeezed vacuum, to limit how positive it can be and for how long.

Define a new function $g(r, t)$ which includes all r dependence of the quantity W_R ,

$$g(r, t) = \eta [\mu \cos(\alpha + \beta t_0) + \eta], \quad (16)$$

with

$$\alpha = \bar{\omega} T - \theta, \quad \beta = 2\bar{\omega}. \quad (17)$$

Then W_R can be expressed in terms of $g(r, t)$ by

$$W_R = -\frac{8\pi\alpha}{V\bar{\omega}} M g(r), \quad (18)$$

and the behavior of $g(r, t)$ will tell us how the quantity W_R depends on the parameter r . From Fig. 2, we see that W_R is

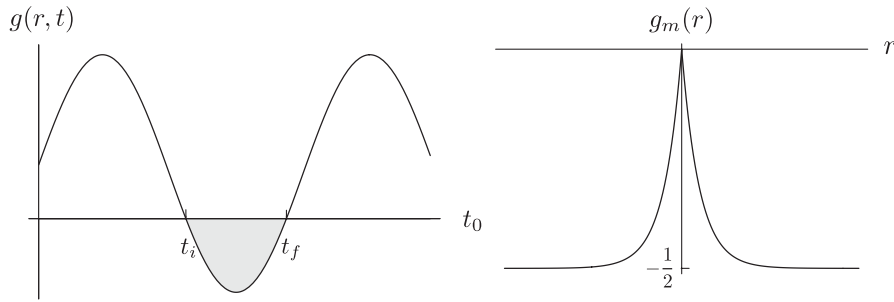


FIG. 2. The left figure shows the behavior of $g(r, t)$ defined in Eq. (16), as a function of the emissions time t_0 . The right figure shows how the minimum value of g as a function of t_0 , $g_m(r)$, depends on r .

positive only if the time variable $t_0 \bmod \pi/\omega$ lies in the time interval between t_i and t_f . If we can somehow collect only those electrons that are emitted during those moments, then we can guarantee a positive average value of the quantity W_R . Hence, the recoherence of the electron interference may be maintained and may remain strong enough to be observed. Next, we will discuss how to compute the averaged value, \tilde{W}_R , formed by averaging W_R over the interval in which it is positive.

C. Behavior of $g(r, t)$

Since only the function $g(r, t)$ will affect the overall sign of W_R , it is sufficient to calculate the time-averaged value of $g(r, t)$ between t_i and t_f . Then \tilde{W}_R is just proportional to this averaged value $\tilde{g}(r)$. Here we note that $g(r, t)$ is only defined for $r \geq 0$. From Fig. 2, the condition $g(r, t) = 0$ is satisfied when t_0 is equal to either t_i or t_f , and $g(r, t)$ is symmetric about those values of t_0 that satisfy $\alpha + \beta t_0 = (2n + 1)\pi$, with n an integer. Thus let

$$\pi - \xi = \alpha + \beta t_i, \quad \pi + \xi = \alpha + \beta t_f, \quad (19)$$

where t_i is assumed to be smaller than t_f . Then it is easy to see that ξ satisfies

$$\mu \cos(\pi \pm \xi) + \eta = 0 \Rightarrow \cos \xi = \frac{\eta}{\mu} < 1, \quad (20)$$

and from Eq. (19), we have

$$\Delta t = t_f - t_i = \frac{2\xi}{\beta} = \frac{2}{\beta} \cos^{-1}\left(\frac{\eta}{\mu}\right). \quad (21)$$

On the other hand, the integration of $\cos(\alpha + \beta t_0)$ over t_0 between t_i and t_f yields

$$\int_{t_i}^{t_f} dt_0 \cos(\alpha + \beta t_0) = -\frac{2}{\beta \mu}. \quad (22)$$

Putting the above results together, we have that the time average of the quantity $g(r, t)$ over the interval between t_i and t_f is given by

$$\tilde{g}(r) = \frac{1}{t_f - t_i} \int_{t_i}^{t_f} dt_0 \eta [\mu \cos(\alpha + \beta t_0) + \eta] \quad (23)$$

$$= -\frac{\eta}{\cos^{-1}(\eta/\mu)} + \eta^2, \quad (24)$$

and thus the average of W_R over the same interval is

$$\tilde{W}_R = -\frac{8\pi\alpha}{V\tilde{\omega}} M \eta \left[-\frac{1}{\cos^{-1}(\eta/\mu)} + \eta \right]. \quad (25)$$

In addition, the knowledge of the local extrema of the function $g(r, t)$ with respect to t_0 will prove useful. Its local minimum along the t_0 axis is given by

$$g_m(r) = \eta(-\mu + \eta) = -\frac{1}{2}(1 - e^{-2r}), \quad (26)$$

while the local maximum value of $g(r, t)$ is

$$g_M(r) = \eta(\mu + \eta) = \frac{1}{2}(e^{2r} - 1). \quad (27)$$

Because the function $\tilde{g}(r, t)$ and the extrema of $g(r)$ are monotonic functions of r , we may consider only two limiting values of r . In the limit that the parameter r approaches positive infinity, we have (see Fig. 3)

$$\begin{aligned} r \rightarrow +\infty \quad g_m(r) &\approx -\frac{1}{2} + \mathcal{O}(e^{-2r}), \\ g_M(r) &\approx \frac{1}{2}e^{2r} + \mathcal{O}(1), \end{aligned} \quad (28)$$

$$\Delta t \approx \frac{4}{\beta} e^{-r} + \mathcal{O}(e^{-3r}), \quad \tilde{g}(r) \approx -\frac{1}{3} + \mathcal{O}(e^{-2r}). \quad (29)$$

On the other hand, when the parameter approaches to 0^+ , we have

$$\begin{aligned} r \rightarrow 0^+ \quad g_m(r) &\approx -r + \mathcal{O}(r^2), \\ g_M(r) &\approx r + \mathcal{O}(r^2), \end{aligned} \quad (30)$$

$$\Delta t \approx \frac{\pi}{\beta} + \mathcal{O}(r), \quad \tilde{g}(r) \approx -\frac{2}{\pi}r + \mathcal{O}(r^2). \quad (31)$$

Thus when r gradually goes to zero, both the maximum and the minimum of the function $g(r, t)$ goes to zero from above and below, respectively. We intermediately know that $g(r, t)$ will be identically equal to zero in this limit. The width of the interval, Δt , approaches to a finite value

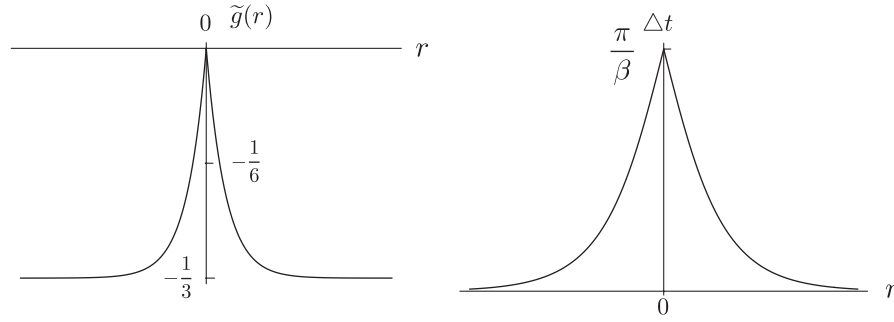


FIG. 3. The left figure shows how $\tilde{g}(r)$, the average of $g(r, t)$ over the interval when $g(r, t) < 0$, as a function of r . The right figure illustrates the width of this interval, Δt , also as a function of r .

π/β . Thus the average value $\tilde{g}(r)$ will be vanishing accordingly.

In contrast, the maximum of the function $g(r, t)$ grows exponentially as r increases, and the minimum decreases, approaching a lower bound of $-1/2$. The width of the interval, over which the average is performed, decreases to zero in the limit $r \rightarrow \infty$. Nonetheless, the average value $\tilde{g}(r)$ remains finite and is equal to $-1/3$ in this limit. This is the lower bound of the function $\tilde{g}(r)$.

D. Bound on recoherence and preservation of unitarity

In short, the function $g(r, t)$ is always bounded from below by a finite value of $-1/2$ while it is unbounded above. Furthermore, although the width of the integration interval vanishes as $r \rightarrow \infty$, the function $\tilde{g}(r)$ is still bounded between 0 and $-1/3$. Thus, we can see that there does exist a upper bound for the average value \tilde{W}_R given by

$$\begin{aligned} \max[\tilde{W}_R] &= \frac{8\pi\alpha}{3V\bar{\omega}} M \\ &= \frac{8\pi\alpha}{3V\bar{\omega}} \left(\frac{16R}{\bar{\omega}^4 T^4} \right)^2 [(-3 + \bar{\omega}^2 T^2) \sin \bar{\omega} T \\ &\quad + 3\bar{\omega} T \cos \bar{\omega} T]^2. \end{aligned} \quad (32)$$

We see that more often than not, $W_R < 0$, meaning that the photons in the squeezed state tend to increase decoherence above what is already present. However, by forming an interference pattern with carefully selected electron emitted in certain time intervals, we can reverse this tendency, and attain positive values of W_R , leading to recoherence. Note that the recoherence effect is maximal when r is large, corresponding to a large mean number of photons in the squeezed vacuum state, given by $\bar{n} = \eta^2 = \sinh^2 r$.

One might be concerned that $W_R > 0$ could lead to a violation of unitarity, but this is not the case, because the vacuum effect will always dominate and lead to $W = W_0 + W_R < 0$. It suffices to compute this combined contribution of one mode $(\bar{\lambda}, \bar{\mathbf{k}})$ to W . It is straightforward to find that, for this mode, $W_0(\bar{\lambda}, \bar{\mathbf{k}})$ is given by

$$W_0(\bar{\lambda}, \bar{\mathbf{k}}) = -\frac{4\pi\alpha}{V\bar{\omega}} M \quad (33)$$

for the same path configuration; while the maximal value of W_R is

$$W_R(\bar{\lambda}, \bar{\mathbf{k}}) = \frac{8\pi\alpha}{3V\bar{\omega}} M. \quad (34)$$

Therefore, we have the combined value of W for this mode $(\bar{\lambda}, \bar{\mathbf{k}})$ is negative:

$$W(\bar{\lambda}, \bar{\mathbf{k}}) = W_0(\bar{\lambda}, \bar{\mathbf{k}}) + W_R(\bar{\lambda}, \bar{\mathbf{k}}) = -\frac{4\pi\alpha}{3V\bar{\omega}} M < 0. \quad (35)$$

Since for the rest of the modes $(\lambda, \mathbf{k}) \neq (\bar{\lambda}, \bar{\mathbf{k}})$, we have $W(\lambda, \mathbf{k}) = W_0(\lambda, \mathbf{k}) < 0$, which in turn implies that

$$\sum_{\mathbf{k}, \lambda} W(\lambda, \mathbf{k}) < 0. \quad (36)$$

IV. MULTIMODE SQUEEZED VACUUM OF FINITE BANDWIDTH

So far, we have considered a single excited mode. Now we wish to extend our result to the case of many excited modes. Assume that the electromagnetic field is initially prepared in the state

$$|\nu\rangle = |0_1\rangle \cdots |0_r\rangle |\zeta_{r+1}\rangle \cdots |\zeta_{r+n}\rangle |0_{r+n+1}\rangle \cdots, \quad (37)$$

where $|\zeta_i\rangle$ is the squeezed vacuum state for mode i . Thus n modes, $r+1$ to $r+n$, are excited in squeezed vacuum states and the rest remain in the vacuum state. Here the subscripts in the bra and ket denote the mode labels. We assume that the excited modes are all linearly polarized in the same direction. The distribution of the wave vectors for the excited modes are also assumed to be sharply centered about some wave vector $\bar{\mathbf{k}}$, which is parallel to the y axis, so that the distribution forms a small cone with a solid angle $d\Omega$ about $\bar{\mathbf{k}}$, and coherence is maintained among these modes. Thus the quantity W_R is then given by

$$W_R = -\frac{2}{V} e^2 \sum_{\mathbf{k}} \omega [\mu_\omega \eta_\omega \cos(2\omega t_0 - \theta_\omega) + \eta_\omega^2] \left(\frac{16R}{\omega^3 T^2} \right)^2 \times \left[\sin\omega T + \frac{3\cos\omega T}{\omega T} - \frac{3\sin\omega T}{\omega^2 T^2} \right]^2, \quad (38)$$

where $\mathbf{k} \in \{\mathbf{k}_{r+1} \dots \mathbf{k}_{r+n}\}$ and $\omega = |\mathbf{k}|$. If the distribution of modes is dense enough, then it can be described by a smooth mode-distribution function $f(\mathbf{k})$, centered at $\bar{\mathbf{k}}$. If we further assume that $f(\mathbf{k})$ depends only on the frequency ω , we can rewrite the mode summation as an integration over the phase space volume,

$$\frac{1}{V} \sum_{\mathbf{k}} = \int \frac{d^3k}{(2\pi)^3} = \frac{d\Omega}{(2\pi)^3} \int_0^\infty d\omega \omega^2 f(\omega), \quad (39)$$

where $f(\omega)$ is the mode-distribution function, peaked at $\omega = \bar{\omega}$ with the width $\Delta\omega$. If $\Delta\omega \ll \bar{\omega}$, so the bandwidth is not overly wide, we may assume the squeeze parameters r_ω , θ_ω , μ_ω , and η_ω are constants, independent of frequency for all excited modes within the band, thus removing the subscript ω from now on. Therefore, W_R becomes

$$W_R = -2e^2 \left(\frac{16R}{T^2} \right)^2 \frac{d\Omega}{(2\pi)^3} \int_0^\infty d\omega f(\omega) \times [\mu\eta \cos(2\omega t_0 - \theta) + \eta^2] \times \frac{1}{\omega^3} \left[\sin\omega T + \frac{3\cos\omega T}{\omega T} - \frac{3\sin\omega T}{\omega^2 T^2} \right]^2. \quad (40)$$

If we only sample electrons which will contribute to recoherence, then according to the discussion in the previous section, the expression $\mu\eta \cos(2\omega t_0 - \theta) + \eta^2$ is replaced by $\tilde{g}(r)$, given by Eq. (24), which is independent of frequency. Note that time interval Δt over which we sample is inversely proportional to $\bar{\omega}$, but the bandwidth $\Delta\omega$ is independent of Δt , so long as $\Delta\omega \ll \bar{\omega}$. Moreover, if we assume that the mode-distribution function takes the form

$$f(\omega) = \begin{cases} 1, & \text{if } \bar{\omega} - \Delta\omega \leq \omega \leq \bar{\omega} + \Delta\omega, \\ 0, & \text{otherwise,} \end{cases} \quad (41)$$

then the quantity \tilde{W}_R reduces to

$$\tilde{W}_R = -2e^2 \left(\frac{16R}{T^2} \right)^2 \tilde{g}(r) \frac{d\Omega}{(2\pi)^3} \int_{\bar{\omega}-\Delta\omega}^{\bar{\omega}+\Delta\omega} d\omega \frac{1}{\omega^3} \times \left[\sin\omega T + \frac{3\cos\omega T}{\omega T} - \frac{3\sin\omega T}{\omega^2 T^2} \right]^2. \quad (42)$$

In principle, the integration on the right-hand side can be carried out exactly; however, for simplicity, we only show the result of the integral to the order $\mathcal{O}(\Delta\omega/\bar{\omega})$,

$$\int_{\bar{\omega}-\Delta\omega}^{\bar{\omega}+\Delta\omega} d\omega \frac{1}{\omega^3} \left[\sin\omega T + \frac{3\cos\omega T}{\omega T} - \frac{3\sin\omega T}{\omega^2 T^2} \right]^2 = \frac{2}{\bar{\omega}^6 T^4} [\bar{\omega}^2 T^2 \sin\bar{\omega} T + 3\bar{\omega} T \cos\bar{\omega} T - 3\sin\bar{\omega} T]^2 \frac{\Delta\omega}{\bar{\omega}} + \mathcal{O}\left(\frac{\Delta\omega^2}{\bar{\omega}^2}\right), \quad (43)$$

where we have assumed $\Delta\omega T \ll 1$ and $\Delta\omega/\bar{\omega} \ll 1$. Therefore, the leading contribution of \tilde{W}_R is given by

$$\tilde{W}_R = -e^2 \frac{R^2}{T^2} \tilde{g}(r) \frac{d\Omega}{(2\pi)^3} \left(\frac{32}{\bar{\omega}^3 T^3} \right)^2 [\bar{\omega}^2 T^2 \sin\bar{\omega} T + 3\bar{\omega} T \cos\bar{\omega} T - 3\sin\bar{\omega} T]^2 \frac{\Delta\omega}{\bar{\omega}}. \quad (44)$$

V. SOME NUMERICAL ESTIMATES

A. Single mode in a cavity

Our treatment of a single excited mode in Sec. III assumed periodic boundary conditions for simplicity. However, the result should be useful for making an order-of-magnitude estimate of the effect in a cavity with more realistic boundary conditions. First, define the function

$$F(x) = \left(\frac{32}{x^3} \right)^2 [(x^2 - 3) \sin x + 3x \cos x]^2, \quad (45)$$

in terms of which we can write Eq. (44) as

$$\tilde{W}_R = -e^2 \frac{R^2}{T^2} \tilde{g}(r) \frac{d\Omega}{(2\pi)^3} F(\bar{\omega} T) \frac{\Delta\omega}{\bar{\omega}}. \quad (46)$$

The function $F(x)$ has a maximum value of $F(3.34) \approx 96.4$ at $x \approx 3.34$, and for large arguments is approximately

$$F(x) \approx \frac{1024}{x^2} \sin^2 x, \quad x \gg 1. \quad (47)$$

Let $\lambda = 2\pi/\bar{\omega}$ be the wavelength of the excited mode. If we assume that the averaged coherence functional, \tilde{W}_R , attains its maximum value given in Eq. (32), then we can express this value as

$$\tilde{W}_R \approx \frac{\alpha}{12\pi^2} \frac{\lambda^3}{V} \left(\frac{R}{T} \right)^2 F(2\pi T/\lambda). \quad (48)$$

If we assume $2\pi T \gg \lambda$, and use the large argument form for F , Eq. (47), we can write

$$\tilde{W}_R \approx 8 \times 10^{-4} \frac{\lambda^3}{V} \left(\frac{R}{T} \right)^2 \left(\frac{\lambda}{T} \right)^2. \quad (49)$$

For a rough estimate, let us take $V \approx \lambda^3$ and $R \approx \lambda$, corresponding to the lowest frequency mode in the cavity and a path separation of the order of the cavity size. This leads to

$$\tilde{W}_R \approx 10^{-3} \left(\frac{R}{T} \right)^4. \quad (50)$$

Nonrelativistic motion requires $T \gg R$. If, for example, we take $R/T \approx 1/10$, we would get the estimate $\tilde{W}_R \approx 10^{-7}$. However, it is plausible that a treatment which allows for relativistic motion of the electrons would yield a larger result, perhaps approaching the limiting value of $\tilde{W}_R \approx 10^{-3}$ which arises from Eq. (50) when $R \approx T$. This is a topic for future study.

B. Multiple modes in empty space

Now let us return to the main result of Sec. IV, Eq. (44), which describes the effect of a finite bandwidth of excited modes without a cavity. This expression may be written in terms of the function F as

$$\tilde{W}_R = -\frac{\alpha}{2\pi^2} \bar{g}(r) \left(\frac{R}{T} \right)^2 \frac{\Delta\omega}{\bar{\omega}} F(\bar{\omega}T) d\Omega. \quad (51)$$

Suppose that $\bar{g}(r) \approx -1/3$ and we integrate over a small but finite solid angle $\Delta\Omega$. Then we have the estimate

$$\tilde{W}_R \approx 10^{-4} \left(\frac{R}{T} \right)^2 \frac{\Delta\omega}{\bar{\omega}} F(\bar{\omega}T) \Delta\Omega. \quad (52)$$

If we further assume that $\bar{\omega}T \approx 3$, so that F attains its maximum value of about 10^2 , then we get the estimate

$$\tilde{W}_R \approx 10^{-2} \left(\frac{R}{T} \right)^2 \frac{\Delta\omega}{\bar{\omega}} \Delta\Omega. \quad (53)$$

All of the factors in the above expression, R/T , $\Delta\omega/\bar{\omega}$, and $\Delta\Omega$, should be small compared to unity for our analysis to be strictly valid. If we take all three of these factors to be of order 10^{-1} , then we would obtain $\tilde{W}_R \approx 10^{-6}$. Again, it may be possible to do better with an analysis which removes the restrictions on these factors.

VI. DISCUSSION AND CONCLUSIONS

Coherent electrons can undergo decoherence due to coupling to the quantized electromagnetic field, even if no real photons are initially present. The effect can be given two complementary descriptions in terms of either a fluctuating Aharonov-Bohm phase, or of photon emission. In general, the presence of real photons increases the degree of decoherence. However, as we have seen, it is possible to temporarily decrease the decoherence if the photons are in a squeezed vacuum state. This recoherence requires that the electrons be selected to pass through the interferometer in the correct phase relative to the excited mode or modes of the electromagnetic field. An interference pattern formed from such selected electrons can have a slightly increased contrast compared to the case where no photons are initially present. This can be interpreted as a transient suppression of Aharonov-Bohm phase fluctuations, analogous to the suppression of vacuum fluctuations which can lead to negative energy densities. Just as there

are quantum inequalities which limit negative energy density, we have found limits on the amount of recoherence possible in a squeezed vacuum state. Although the recoherence effect is small, it is in principle observable, but probably beyond the limits of present technology.

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APPENDIX: PROPERTIES OF THE SQUEEZED VACUUM STATE

The single-mode squeezed vacuum state $|\zeta\rangle$ is defined by

$$|\zeta\rangle = S(\zeta)|0\rangle,$$

where the squeeze operator $S(\zeta)$ is [28,29]

$$S(\zeta) = \exp\left[\frac{1}{2}(\zeta^* a^2 - \zeta a^{\dagger 2})\right].$$

The operators a^\dagger and a are creation and annihilation operators, respectively, satisfying the commutation relation $[a, a^\dagger] = 1$. The vacuum state $|0\rangle$ is annihilated by the action of a , that is, $a|0\rangle = 0$. The squeeze parameter $\zeta = re^{i\theta}$ is an arbitrary complex number with $r, \theta \in \Re$.

With the help of the operator expansion theorem,

$$e^{\lambda A} B e^{-\lambda A} = B + \lambda[A, B] + \frac{\lambda^2}{2!}[A, [A, B]] + \dots, \quad (A1)$$

we readily find for the unitary transformation of the operator a by $S(\zeta)$,

$$\begin{aligned} S^\dagger(\zeta) a S(\zeta) &= \mu a - \nu a^\dagger, \\ \text{and } S^\dagger(\zeta) a^\dagger S(\zeta) &= \mu a^\dagger - \nu^* a, \end{aligned} \quad (A2)$$

where

$$\mu = \cosh r \quad \nu = e^{i\theta} \sinh r, \quad (A3)$$

and $\mu^2 - |\nu|^2 = 1$.

The expectation value of a in the squeezed vacuum is given by

$$\langle \zeta | a | \zeta \rangle = \langle 0 | S^\dagger(\zeta) a S(\zeta) | 0 \rangle = 0 \quad (A4)$$

from Eq. (A2), and the expectation value of a^\dagger is

$$\langle \zeta | a^\dagger | \zeta \rangle = 0. \quad (A5)$$

Moreover, we have

$$\langle \zeta | a^2 | \zeta \rangle = -\mu\nu \quad (A6)$$

$$\langle \zeta | a^{\dagger 2} | \zeta \rangle = -\mu\nu^* \quad (A7)$$

$$\langle \zeta | a^\dagger a | \zeta \rangle = |\nu|^2. \quad (A8)$$

From Eq. (A8), it is apparent that the squeezed vacuum state is not a vacuum state at all, and it has $|\nu|^2$ photons on the average.

Next we evaluate the energy density of the electromagnetic fields in a single-mode squeezed vacuum state, as well as the total energy. It is assumed that only one of the modes of the electromagnetic fields is excited to the squeezed vacuum state while the rest of the modes remain in the vacuum state. This excited mode is denoted by the wave vector $\bar{\mathbf{k}}$ and polarization $\bar{\lambda}$. Thus the squeezed vacuum state is created by

$$|\zeta_{\bar{\lambda}\bar{\mathbf{k}}}\rangle = S(\zeta_{\bar{\lambda}\bar{\mathbf{k}}})|0_{\bar{\lambda}\bar{\mathbf{k}}}\rangle, \quad (\text{A9})$$

where $S(\zeta_{\bar{\lambda}\bar{\mathbf{k}}})$ is the squeeze operator for mode $(\bar{\lambda}, \bar{\mathbf{k}})$,

$$S(\zeta_{\bar{\lambda}\bar{\mathbf{k}}}) = \exp\left[\frac{1}{2}(\zeta_{\bar{\lambda}\bar{\mathbf{k}}}^* a^2 - \zeta_{\bar{\lambda}\bar{\mathbf{k}}} a^{\dagger 2})\right]. \quad (\text{A10})$$

Here $\zeta_{\bar{\lambda}\bar{\mathbf{k}}} = r e^{i\theta}$ is an arbitrary complex number. The energy density ϱ of the electromagnetic fields in a single-mode squeezed state is given by the expectation value of the corresponding energy density operator ρ in the squeezed vacuum state $|\zeta_{\bar{\lambda}\bar{\mathbf{k}}}\rangle$, that is, $\varrho(x) = \langle \zeta_{\bar{\lambda}\bar{\mathbf{k}}} | \rho(x) | \zeta_{\bar{\lambda}\bar{\mathbf{k}}}\rangle$, where the energy density operator is

$$\rho(x) = \frac{1}{2}[E(x)^2 + B(x)^2]. \quad (\text{A11})$$

Let the vector potential $A(x)$ take the form of the plane-wave expansion

$$A(x) = \frac{1}{\sqrt{V}} \sum_{\mathbf{k}} \frac{1}{\sqrt{2\omega}} \sum_{\lambda=1}^2 \boldsymbol{\varepsilon}_{\lambda}(\mathbf{k})(a_{\lambda\mathbf{k}} e^{-ik \cdot x} + a_{\lambda\mathbf{k}}^{\dagger} e^{ik \cdot x}), \quad (\text{A12})$$

where $\boldsymbol{\varepsilon}_{\lambda}(\mathbf{k})$ is the unit polarization vectors. V is the

$$\begin{aligned} \langle \zeta_{\bar{\lambda}\bar{\mathbf{k}}} | \rho(x) | \zeta_{\bar{\lambda}\bar{\mathbf{k}}}\rangle &= \frac{1}{2V} \left\{ \sum_{(\lambda, \mathbf{k}) \neq (\bar{\lambda}, \bar{\mathbf{k}})} \frac{\omega}{2} (\boldsymbol{\varepsilon}_{\lambda}(\mathbf{k}) \cdot \boldsymbol{\varepsilon}_{\lambda}(\mathbf{k}) + [\boldsymbol{\kappa} \times \boldsymbol{\varepsilon}_{\lambda}(\mathbf{k})] \cdot [\boldsymbol{\kappa} \times \boldsymbol{\varepsilon}_{\lambda}(\mathbf{k})]) + \frac{\bar{\omega}}{2} (\boldsymbol{\varepsilon}_{\bar{\lambda}}(\bar{\mathbf{k}}) \cdot \boldsymbol{\varepsilon}_{\bar{\lambda}}(\bar{\mathbf{k}}) + [\bar{\boldsymbol{\kappa}} \times \boldsymbol{\varepsilon}_{\bar{\lambda}}(\bar{\mathbf{k}})] \right. \\ &\quad \left. \cdot [\bar{\boldsymbol{\kappa}} \times \boldsymbol{\varepsilon}_{\bar{\lambda}}(\bar{\mathbf{k}})]) \times [(2|\nu|^2 + 1) + \mu\nu e^{-2i\bar{\mathbf{k}} \cdot x} + \mu\nu^* e^{2i\bar{\mathbf{k}} \cdot x}] \right\}. \end{aligned} \quad (\text{A18})$$

We notice that

$$\boldsymbol{\varepsilon}_{\lambda}(\mathbf{k}) \cdot \boldsymbol{\varepsilon}_{\lambda}(\mathbf{k}) + [\boldsymbol{\kappa} \times \boldsymbol{\varepsilon}_{\lambda}(\mathbf{k})] \cdot [\boldsymbol{\kappa} \times \boldsymbol{\varepsilon}_{\lambda}(\mathbf{k})] = 2. \quad (\text{A19})$$

After subtracting the vacuum contribution, we have the renormalized energy density ϱ_R in a squeezed vacuum given by

$$\begin{aligned} \varrho_R(x) &= \frac{1}{V} \left[|\nu|^2 + \frac{1}{2}(\mu\nu e^{-2i\bar{\mathbf{k}} \cdot x} + \mu\nu^* e^{2i\bar{\mathbf{k}} \cdot x}) \right] \bar{\omega} \\ &= \frac{1}{V} \eta [\mu \cos(2\bar{\mathbf{k}} \cdot x - \theta) + \eta] \bar{\omega}, \end{aligned} \quad (\text{A20})$$

where $\eta = \sinh r$. Note that this can be negative when the condition $\cos(2\bar{\mathbf{k}} \cdot x - \theta) < 0$ is met. Note that the factor which governs the sign of ϱ_R is of the same form as $g(r, t)$ defined in Eq. (16), apart from the fact that the cosine function depends upon different variables in the two cases.

normalization volume and $\omega = |\mathbf{k}|$. The commutation relations between the creation and the annihilation operators are

$$[a_{\lambda\mathbf{k}}, a_{\lambda'\mathbf{k}'}^{\dagger}] = \delta_{\lambda\lambda'} \delta_{\mathbf{k}\mathbf{k}'}, \quad (\text{A13})$$

$$[a_{\lambda\mathbf{k}}, a_{\lambda'\mathbf{k}'}] = [a_{\lambda\mathbf{k}}^{\dagger}, a_{\lambda'\mathbf{k}'}^{\dagger}] = 0. \quad (\text{A14})$$

Then the electric field $E(x)$ and the magnetic field $B(x)$ are given, respectively, by

$$E(x) = \frac{i}{\sqrt{V}} \sum_{\mathbf{k}} \sqrt{\frac{\omega}{2}} \sum_{\lambda=1}^2 \boldsymbol{\varepsilon}_{\lambda}(\mathbf{k})(a_{\lambda\mathbf{k}} e^{-ik \cdot x} - a_{\lambda\mathbf{k}}^{\dagger} e^{ik \cdot x}), \quad (\text{A15})$$

$$B(x) = \frac{i}{\sqrt{V}} \sum_{\mathbf{k}} \frac{1}{\sqrt{2\omega}} \sum_{\lambda=1}^2 \mathbf{k} \times \boldsymbol{\varepsilon}_{\lambda}(\mathbf{k})(a_{\lambda\mathbf{k}} e^{-ik \cdot x} - a_{\lambda\mathbf{k}}^{\dagger} e^{ik \cdot x}). \quad (\text{A16})$$

Hence, the energy density operator is given by

$$\begin{aligned} \rho(x) &= -\frac{1}{2V} \sum_{\mathbf{k}\mathbf{k}'} \sqrt{\frac{\omega}{2}} \sqrt{\frac{\omega'}{2}} \sum_{\lambda\lambda'=1}^2 (\boldsymbol{\varepsilon}_{\lambda}(\mathbf{k}) \cdot \boldsymbol{\varepsilon}_{\lambda'}(\mathbf{k}')) \\ &\quad + [\boldsymbol{\kappa} \times \boldsymbol{\varepsilon}_{\lambda}(\mathbf{k})] \cdot [\boldsymbol{\kappa}' \times \boldsymbol{\varepsilon}_{\lambda'}(\mathbf{k}')] \\ &\quad \times (a_{\lambda\mathbf{k}} e^{-ik \cdot x} - a_{\lambda\mathbf{k}}^{\dagger} e^{ik \cdot x})(a_{\lambda'\mathbf{k}'} e^{-ik' \cdot x} - a_{\lambda'\mathbf{k}'}^{\dagger} e^{ik' \cdot x}), \end{aligned} \quad (\text{A17})$$

where $\boldsymbol{\kappa}$ is a unit vector along the direction of the wave vector. It is straightforward to evaluate the expectation value of the energy density operator

Accordingly, the renormalized total energy E_R in the squeezed vacuum state is given by integrating the renormalized energy density over all quantization volume. If the quantization volume is sufficiently large, or the periodic boundary conditions are used for convenience, then the term proportional to $\cos(\cdot \cdot)$ will vanish and we have

$$E_R = \int_V d^3x \varrho_R(x) = \eta^2 \bar{\omega}. \quad (\text{A21})$$

The spatial average of the renormalized energy density is then given by

$$\bar{\varrho}_R = \frac{E_R}{V} = \frac{1}{V} \eta^2 \bar{\omega}, \quad (\text{A22})$$

which is always positive.

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