Phantom energy accretion onto black holes in a cyclic universe

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Black holes pose a serious problem in cyclic or oscillating cosmology. It is speculated that, in the cyclic universe with phantom turnarounds, black holes will be torn apart by phantom energy prior to turnaround before they can create any problems. In this paper, using the mechanism of phantom accretion onto black holes, we find that black holes do not disappear before phantom turnaround. But the remanent black holes will not cause any problems due to Hawking evaporation.

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I. INTRODUCTION

The scenario of cyclic or oscillating universes is an attractive idea in theoretical cosmology since it is expected to avoid the initial singularity by providing an infinitely oscillating universe. This idea has a long history [1,2]. In recent years, there have been many discussions on such a topic [3–5]. Generally, however, cyclic models of the universe confront a serious problem: black holes. If black holes formed during expansion of the universe survive into the next cycle, they will grow even larger from cycle to cycle and act as defects in an otherwise nearly uniform universe. Eventually, the black holes will occupy the entire horizon volume, and then the cyclic models break away.

In [4], by assuming the existence of phantom dark energy and using the modified Friedmann equation, the authors suggested an oscillating cosmology. It is argued that any black holes produced in an expanding phase in the universe are torn apart before they can create problems during contraction. A rough calculation has been given. In general relativity, the source for a gravitational potential is the volume integral of $\rho + 3p$, where ρ is the energy density in the universe and p is the pressure. So an object of radius R and mass M is pulled apart when $-(4\pi/3) \times (\rho + 3p)R^3 \sim M$. Then a black hole of mass M and horizon radius $R = 2GM$ is pulled apart when the energy density of the universe has climbed up to a value $\rho_{\rm BH} \sim (3/32\pi)(M^2G^3|1 + 3w|)^{-1}$, where $p = w\rho$ is the equation of state (for phantom dark energy $w < -1$). Then the black holes will be torn apart before turnaround, if $\rho_{\rm BH} < \rho_c$, where ρ_c is the critical energy density in the cyclic model, namely, the energy density corresponding to the turnaround (and bounce).

However, it is obvious that the destruction of black holes is not an instantaneous event that just happened at $\rho \sim \rho_{\rm BH}$, but a process. At the same time, the qualitative analysis in [4] is too rough and cannot be taken as a mechanism of tearing up black holes. We need such a mechanism in order to know whether the analysis in [4] works or not. In [6], the authors have suggested a mechanism in which, by accreting the phantom energy, the mass of a black hole decreases at the rate $\dot{M} = 4\pi AG^2M^2(\rho + p)$, where A is a positive dimensionless constant. Replacing ρ and p by the effective energy density $\rho_{\rm eff} = \rho(1 - \rho/\rho_c)$ and the effective pressure $p_{\rm eff} = p(1 - 2\rho/\rho_c) - \rho^2/\rho_c$, respectively, the author of [7] used this mechanism, $\dot{M} = 4\pi AG^2M^2(\rho_{\rm eff} + p_{\rm eff})$, to study the destruction of black holes in cyclic models. The conclusion in [7] is that, in the expanding stage of the universe, through the phantom accretion, the masses of black holes first decrease and then increase. And at the turnaround, the initial values of the black hole masses are restored. So it is claimed that black hole cannot be torn up in the cyclic model of [4].

But in [8], the author argued that $\rho_{\rm eff}$ and $p_{\rm eff}$ are not proper physical quantities. Taking into account this view, in this paper we will study the destruction of black holes in the phantom cyclic universe by using $\dot{M} = 4\pi AG^2M^2(\rho + p)$. A similar application in brane cosmology has been discussed in [9].

The paper is organized as follows. In Sec. II, we analyze the phantom energy accretion onto black holes in the cyclic model of [4]. Section III contains conclusions and a discussion.

II. ACCRETION OF PHANTOM FLUID IN CYCLIC MODELS

Here, we consider that the dark energy fluid covers the whole space in the homogeneous and isotropic forms with dark energy density ρ and pressure p. For an asymptotic observer, the black hole mass M changes at the rate [6]

$$\dot{M} = 4\pi AM^2(\rho + p)/M_p^4. \tag{1}$$

Here and below, the dot denotes the derivative with respect to the cosmic time and $c = 1$, $M_p = G^{-1/2}$. Assuming the universe is dominated by dark energy, the expansion of the universe is governed by the Friedmann equation

$$H^2 = \frac{8\pi}{3M_p^2}\rho \tag{2}$$

and the local energy conservation law of dark energy

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$$\dot{\rho} + 3H(\rho + p) = 0, \tag{3}$$

where $H \equiv \dot{a}/a$ is the Hubble parameter. For phantom dark energy with equation of state $w = p/\rho < -1$, $\rho \propto a^{-3(1+w)}$ increases as the universe expands. In order to avoid the big rip [10], we use the modified Friedmann equation

$$H^2 = \frac{8\pi}{3M_p^2}\rho\left(1 - \frac{\rho}{\rho_c}\right), \tag{4}$$

where ρ_c is the critical energy density of the order of the Planck density. The modified Friedmann equation (4) has been suggested with different setups [11–13]. Then, in the expanding phase of the universe, the phantom energy density ρ increases. When $\rho = \rho_c$, due to Eq. (4), a turnaround occurs. After turnaround, the universe begins to contract. In the contracting phase, the energy densities of other nonphantom components in the universe increase, and eventually dominate the evolution of the universe. When the dominant energy density reaches the critical energy density ρ_c again, a bounce occurs. Roughly, this is the scenario of oscillating cosmology in [4]. Now let us study the evolution of black hole masses due to the phantom energy accretion in the scenario.

A. Before turnaround

Using Eqs. (1) and (3), we get

$$\frac{dM}{M^2} = -\frac{4\pi A}{3HM_p^4}d\rho. \tag{5}$$

Before turnaround, we have

$$H = \sqrt{\frac{8\pi G}{3}}\sqrt{\rho\left(1 - \frac{\rho}{\rho_c}\right)}. \tag{6}$$

Substituting this equation into Eq. (5), we get

$$\frac{dM}{M^2} = -\frac{D}{\sqrt{\rho(1-\rho/\rho_c)}}d\rho, \tag{7}$$

with $D \equiv \sqrt{\frac{2\pi}{3}}\frac{A}{M_p^3}$. The integration of (7) gives

$$M = \frac{M_i}{1 + 2DM_i\sqrt{\rho_c}(\arcsin\sqrt{\rho/\rho_c} - \arcsin\sqrt{\rho_i/\rho_c})}, \tag{8}$$

with $\rho_i \le \rho \le \rho_c$. Here M_i and ρ_i denote, respectively, the black hole mass and the phantom energy density at the moment when the phantom energy begins to dominate the evolution of the universe. Generally, $\rho_i \ll \rho_c$. Then we obtain

$$M \simeq \frac{M_i}{1 + 2M_iD\sqrt{\rho_c}\arcsin\sqrt{\rho/\rho_c}}. \tag{9}$$

So the black hole mass at turnaround is

$$M_c \simeq \frac{M_i}{1 + \pi M_iD\sqrt{\rho_c}}. \tag{10}$$

This result means that black holes, by accreting phantom energy, do not disappear before turnaround. But our result is different from the result of [7]. In [7], it is claimed that, through phantom accretion, the black hole mass will decrease first, and then increase until restoring its initial mass at turnaround. Here, our result, Eq. (8), indicates that, through phantom accretion, the black hole mass always decreases as the universe expands and, at turnaround, reaches the minimum M_c in the expanding phase. For $M_i \gg M_p = G^{-1/2}$, M_c becomes independent of M_i,

$$M_c \simeq \frac{1}{\pi D\sqrt{\rho_c}}. \tag{11}$$

B. After turnaround

After turnaround, the universe begins to contract and the phantom energy density ρ decreases. Equation (8) cannot be used directly because H is negative in the contracting phase of the universe. Substituting the equation

$$H = -\sqrt{\frac{8\pi G}{3}}\sqrt{\rho\left(1 - \frac{\rho}{\rho_c}\right)} \tag{12}$$

into Eq. (5), we obtain

$$\frac{dM}{M^2} = -\frac{D}{\sqrt{\rho(1-\rho/\rho_c)}}d\rho. \tag{13}$$

The integration of the equation gives

$$M = \frac{M_c}{1 + DM_c\sqrt{\rho_c}(\pi - 2\arcsin\sqrt{\rho/\rho_c})}, \tag{14}$$

with $\rho \le \rho_c$. Then Eq. (14) shows that, after turnaround, as the universe contracts the phantom energy density decreases and the black hole masses continue to decrease. When $\rho \ll \rho_c$, the black hole mass is

$$M_f \simeq \frac{M_c}{1 + \pi DM_c\sqrt{\rho_c}}. \tag{15}$$

For $M_i \gg M_p$, using Eq. (11), we find that the final mass of the black holes is

$$M_f \simeq \frac{M_c}{2} \simeq \frac{1}{2\pi D\sqrt{\rho_c}}. \tag{16}$$

C. Destruction of black holes

The analysis above shows that, by accreting phantom energy, a black hole in the cyclic universe with phantom turnaround does not disappear, but has a remanent mass M_c at turnaround. This means that, through phantom energy accretion, black holes in the cyclic model of [4] cannot be pulled apart before turnaround. Then it seems that the argument of destruction of black holes in [4] is wrong and the problem of black holes remains.

However, it has been argued in [4] that for a black hole with mass $M = 10^5 M_p$, it Hawking evaporates in a time $\tau \sim (25\pi M^3/M_p^4) \sim 10^{-27}$ sec and causes no problems. Let us estimate the value of M_c. Defining a dimensionless constant $\tilde{D} \equiv \sqrt{\frac{2\pi}{3}}A$, we can rewrite Eq. (10) as

$$M_c \simeq \frac{M_i}{1 + \pi M_i \tilde{D} \sqrt{\rho_c}/M_p^3}. \tag{17}$$

$\tilde{D}$ may be taken as a constant of order unity and $\rho_c \sim M_p{}^4$. Then we have $M_c \sim \frac{M_i}{1+M_i/M_p}$. This implies that, for a black hole with $M_i \gg M_p$, the remanent mass M_c is about at the order of the Planck mass M_p and the remanent black hole Hawking evaporates in a time $\tau \sim 10^{-43}$ sec of the order of the Planck time. Then, fortunately, the remanent black holes do not cause problems. Now we know that, in the cyclic model with phantom energy turnarounds, black hole masses decrease due to the phantom energy accretion. Before turnaround, black holes cannot be torn apart, but the remanent black holes with masses $M_c \sim M_p$ remain at turnaround. However, the remanents do not cause problems in the cyclic model because of Hawking evaporation.

III. CONCLUSION AND DISCUSSION

In cyclic or oscillating cosmology, black holes pose a serious problem. In [4], an oscillating cosmology with phantom energy turnaround is suggested, and it is argued roughly that black holes are torn apart before turnaround. In [6], a successful mechanism in which the black hole masses decrease due to phantom energy accretion is obtained. In this paper, by using the result of [6], we have surveyed the destruction of black holes in cyclic cosmology with phantom energy turnaround.

Similar work has been done in [7]. The author's conclusion is that, through phantom accretion, the black hole mass will decrease first, and then increase until restoring its initial mass at turnaround. Then the author claimed that the problem of black holes remains in the cyclic model with phantom turnaround. However, this conclusion is obtained by using the effective energy density and pressure which are unphysical variables [8], rather than the energy density and pressure. Using the energy density and pressure, we find that, due to phantom energy accretion, the mass of a black hole always decreases before turnaround, and at turnaround it reaches the remanent mass M_c. After turnaround the remanent black hole mass continues to decrease. For black holes with mass much more massive than the Planck mass, the black hole masses approach $M_c/2$ asymptotically. Of course, our evaluation after turnaround is not very rigid, because, as the universe contracts, phantom energy will eventually become subdominant in the universe.

So the remanent mass M_c implies that black holes in the cyclic model cannot be pulled apart by phantom energy before turnaround. Here, we note that, if the Friedmann equation (2) is used, no turnaround occurs, but the big rip does occur. Then we get

$$M = \frac{M_i}{1 + 2DM_i(\sqrt{\rho} - \sqrt{\rho_i})}.$$

So, in this case, as $\rho \to \infty$, black holes disappear, $M \to 0$. But, in cyclic cosmology, the modified Friedmann equation (4) is used. The big rip is avoided, and then, through phantom accretion, black holes cannot be eliminated. This result can be obtained in another way. Using only Eq. (1), we can obtain

$$\frac{1}{M} - \frac{1}{M_i} = -\int_{t_i}^{t} 4\pi A(\rho + p)dt.$$

The black hole mass M will not be zero unless the integration on the right-hand side is divergent. However, a key property of cyclic cosmology is that the energy density and pressure are always well defined. So, it is impossible for black holes in the cyclic model to be eliminated by accreting the phantom energy.

However, fortunately, we find that the remanent masses of black holes at turnaround do not cause problems. The reason is that these remanent black holes Hawking evaporate in a time $\tau \sim 10^{-43}$.

So our analysis indicates that, although through phantom energy accretion black holes do not disappear before turnaround, they do not cause problems in the cyclic models with phantom turnarounds.

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