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New non-Abelian black hole solutions in Born-Infeld gravity

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We introduce new black hole solutions to the Einstein-Yang-Mills-Born-Infeld (EYMBI), Einstein-Yang-Mills-Born-Infeld-Gauss-Bonnet (EYMBIGB), and Einstein-Yang-Mills-Born-Infeld-Gauss-Bonnet-Lovelock (EYMBIGBL) gravities in higher dimensions $N \geq 5$ to investigate the roles of Born-Infeld parameter β. It is shown that these solutions in the limits of $\beta \to 0$ and $\beta \to \infty$ represent pure gravity and gravity coupled with Yang-Mills fields, respectively. For $0 < \beta < \infty$ it yields a variety of black holes, supporting even regular ones at $r = 0$.

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I. INTRODUCTION

Historically, the Born-Infeld (BI) nonlinear electrodynamics model was formulated in 1934 [1]. Since it has been proposed as a viable model in low energy string theory, BI electrodynamics has attracted much attention from the fronts of both string theory and cosmology [2]. While classical electrodynamics due to Maxwell is a linear theory obeying the principle of superposition, in the latter these properties are not valid anymore. In this respect the BI electrodynamics is comparable with the other nonlinear theories of physics such as Yang-Mills (YM) and gravitation. Finding plane wave solutions in such a theory, for instance, in the presence of boundaries and/or background effects becomes a difficult task. Coupling of BI electrodynamics to gravity has given birth to a new theory known as the Einstein-Born-Infeld (EBI) gravity which found applications in string theory. The BI version of electromagnetism is already in the form of a string Lagrangian, i.e. square root of a determinant, living inside higher dimensional worlds of branes. The addition of a Higgs field and investigating its monopole solutions become equally attractive for the field theorists in the realm of confinement related problems [3]. We recall that the original BI electrodynamics was introduced in order to resolve the self-energy divergence in the Coulomb problem. With the advent of quantum electrodynamics this feature of the BI theory was almost forgotten. Coincidentally, besides other things, string theory was also introduced to eliminate divergences due to pointlike structures. The combination of these two theories (i.e. BI and string theory) is expected naturally to yield finite physical results. BI action in supergravity admits solitonic solutions known as D-branes which form the end points for open strings. In this paper, however, we shall not address ourselves to D-branes or dilatons, postponing these to a future study. A different theory, which will establish our strategy in this paper, is to consider the EBI action in which instead of the electromagnetic field we employ the non-Abelian YM field [4]. For this purpose we make use of a YM ansatz in the spherically symmetric space-time. Recently we have obtained such EYM black hole solutions and extended it to the higher dimensional Gauss-Bonnet (GB) and Lovelock theories [5]. Our method of solving the YM equations was to generalize the original Wu-Yang ansatz in $N = 4$ to higher dimensions ($N \geq 5$). In this ansatz the YM field is of magnetic type so that the invariant $F^{(a)}_{\lambda\sigma} {}^\star F^{(a)\lambda\sigma} = 0$ in the action, leaving behind the term $F^{(a)}_{\lambda\sigma} F^{(a)\lambda\sigma} \neq 0$. As expected, employing YM instead of the Maxwell field accumulates different types of nonlinearities to yield, altogether a highly nonlinear model of gravity *à la* BI formalism. In the proper limit $\beta \to \infty$, where β is called the BI parameter, we recover the Einstein-Hilbert action coupled with YM field in the standard way. The Einstein-Hilbert action constitutes the simplest geometrical theory which involves mass as its parameter. Its geometrical/topological extensions employ higher order invariants with more parameters that provide extra degrees of freedom in the theory. The Lovelock Lagrangian is the most general Lagrangian that admits second order equations without invoking ghost structures. By taking appropriate limits we recover all interesting cases obtained so far. It is remarkable that three highly nonlinear theories, such as BI, YM, and Lovelock gravity are brought together in a common Lagrangian which admits exact solutions.

In this paper we address the issue of black hole solutions in the EBI action by incorporating YM fields in higher dimensions. Naturally the BI parameter β modifies the black holes and their thermodynamics properties. Next, we consider the GB extension and search for new features brought in by the topological properties of the GB theory. The latter has the property that in the absence of a true cosmological constant Λ, asymptotically it produces an effective one, Λ_{eff}, to imitate the real one. In other words, the de Sitter (dS) and anti-de Sitter (AdS) spacetimes which are of utmost importance in the conformal field theory correspondence arise simply as boundary conditions of the space-time. Inclusion of the β parameter adds

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further degrees of freedom to the theory. We find, for example, that β can be employed to construct/regulate black hole horizons at wish. The extension to the third order Lovelock gravity, however, restricts our exact solution such that in the absence of a real cosmological constant it does not admit an effective one.

The paper is organized as follows: In Sec. II we introduce the Einstein-Yang-Mills-Born-Infeld (EYMBI) action, metric, YM ansätze, and the resulting field equations. In the same section we find exact solutions of the field equations in $N \geq 5$. Section III follows by introducing the action, field equations, and solutions for the $N \geq 5$ dimensional Einstein-Yang-Mills-Born-Infeld-Gauss-Bonnet (EYMBIGB) theory. In Sec. IV we follow the same patterns for the Einstein-Yang-Mills-Born-Infeld-Gauss-Bonnet-Lovelock (EYMBIGBL), in which the abbreviation L refers to the third order Lovelock gravity. The paper ends with concluding remarks in Sec. V.

II. FIELD EQUATIONS AND THE METRIC ANSATZ FOR EYMBI GRAVITY

The $N(= n + 1)$-dimensional action for Einstein-Yang-Mills-Born-Infeld gravity with a cosmological constant Λ is given by

$$S = \frac{1}{16\pi}\int_{\mathcal{M}} d^{n+1}x\sqrt{-g}\left(R - \frac{n(n-1)}{3}\Lambda + L(\mathbf{F})\right) + \frac{1}{8\pi}\int_{\partial\mathcal{M}} d^{n}x\sqrt{-\gamma}K(\gamma), \tag{1}$$

in which the YMBI Lagrangian $L(\mathbf{F})$ is given by

$$L(\mathbf{F}) = 4\beta^2\left(1 - \sqrt{1 + \frac{\mathrm{Tr}(F^{(a)}_{\lambda\sigma}F^{(a)\lambda\sigma})}{2\beta^2} + \frac{\mathrm{Tr}(F^{(a)}_{\lambda\sigma}\star F^{(a)\lambda\sigma})^2}{16\beta^4}}\right), \tag{2}$$

where

$$\mathrm{Tr}(.) = \sum_{a=1}^{n(n-1)/2}(.). \tag{3}$$

Herein we are interested in the magnetically charged YM ansatz in which $\mathrm{Tr}(F^{(a)}_{\lambda\sigma}\star F^{(a)\lambda\sigma}) = 0$ and therefore $L(\mathbf{F})$ reduces to the form

$$L(\mathbf{F}) = 4\beta^2\left(1 - \sqrt{1 + \frac{\mathrm{Tr}(F^{(a)}_{\lambda\sigma}F^{(a)\lambda\sigma})}{2\beta^2}}\right). \tag{4}$$

In Eqs. (1) and (2) R is the Ricci scalar, Λ is the cosmological constant, K is the trace of the extrinsic curvature $K^{\mu\nu}$ of boundary $\partial\mathcal{M}$ of the manifold $\mathcal{M}$, with induced metric γ_{ij}, and β is the BI parameter with the dimension of mass. Here the YM field is defined as

$$\mathbf{F}^{(a)} = \mathbf{d}\mathbf{A}^{(a)} + \frac{1}{2\sigma}C^{(a)}_{(b)(c)}\mathbf{A}^{(b)}\wedge\mathbf{A}^{(c)} \tag{5}$$

in which $C^{(a)}_{(b)(c)}$ stands for the structure constants of $\frac{n(n-1)}{2}$-parameter Lie group G and σ is a coupling constant. $\mathbf{A}^{(a)}$ are the $SO(n)$ gauge group YM potentials. We note that the internal indices $\{a, b, c, \ldots\}$ do not differ whether in covariant or contravariant form. Variation of the action with respect to the space-time metric $g_{\mu\nu}$ yields the field equations

$$G^{\mu}{}_{\nu} + \frac{n(n-1)}{6}\Lambda g^{\mu}{}_{\nu} = T^{\mu}{}_{\nu}, \tag{6}$$

$$T^{\mu}{}_{\nu} = \frac{1}{2}g^{\mu}{}_{\nu}L(\mathbf{F}) + g^{\mu\alpha}\frac{2\,\mathrm{Tr}(F^{(a)}_{\nu\lambda}F^{(a)\lambda}_{\alpha})}{\sqrt{1 + \frac{\mathrm{Tr}(F^{(a)}_{\lambda\sigma}F^{(a)\lambda\sigma})}{2\beta^2}}}, \tag{7}$$

where $G_{\mu\nu}$ is the Einstein tensor. Variation with respect to the gauge potentials $\mathbf{A}^{(a)}$ yields the YM equations

$$\mathbf{d}\left(\frac{\star\mathbf{F}^{(a)}}{\sqrt{1 + \frac{\mathrm{Tr}(F^{(a)}_{\lambda\sigma}F^{(a)\lambda\sigma})}{2\beta^2}}}\right) + \frac{1}{\sigma}C^{(a)}_{(b)(c)}\frac{1}{\sqrt{1 + \frac{\mathrm{Tr}(F^{(a)}_{\lambda\sigma}F^{(a)\lambda\sigma})}{2\beta^2}}}\mathbf{A}^{(b)}\wedge\star\mathbf{F}^{(c)} = 0, \tag{8}$$

where $\star$ means duality. Our metric ansatz for $N = n + 1$ is chosen as

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2_{n-1}, \tag{9}$$

in which $f(r)$ is our metric function and

$$d\Omega^2_{n-1} = d\theta_1^2 + \sum_{i=2}^{n-1}\prod_{j=1}^{i-1}\sin^2\theta_j d\theta_i^2, \tag{10}$$

where

$$0 \leq \theta_{n-1} \leq 2\pi, \qquad 0 \leq \theta_i \leq \pi, \qquad 1 \leq i \leq n-2.$$

A. Energy momentum tensor

In this subsection we calculate the energy momentum tensor defined by Eq. (7) in $N(= n + 1)$ dimensions. As we have recently introduced and used the higher dimensional version of the Wu-Yang ansatz in EYM theory of gravity [5] we write the gauge potential one-forms as

$$\mathbf{A}^{(a)} = \frac{Q}{r^2}(x_i dx_j - x_j dx_i), \qquad Q = \text{charge}, \qquad r^2 = \sum_{i=1}^{n}x_i^2,$$
$$2 \leq j+1 \leq i \leq n, \quad \text{and} \quad 1 \leq a \leq n(n-1)/2, \tag{11}$$

in which, by using (5), one gets the YM field two-forms satisfying the YM equations [5]. Nevertheless the energy momentum tensor defined by (7) is found after using

$$\mathrm{Tr}\,(F^{(a)}_{\lambda\sigma}F^{(a)\lambda\sigma}) = \frac{(n-1)(n-2)Q^2}{r^4}, \tag{12}$$

$$L(\mathbf{F}) = 4\beta^2\left(1 - \sqrt{1 + \frac{(n-1)(n-2)Q^2}{2\beta^2 r^4}}\right) \tag{13}$$

as

$$T^t_t = T^r_r = 2\beta^2\left(1 - \sqrt{1 + \frac{(n-1)(n-2)Q^2}{2\beta^2 r^4}}\right), \tag{14}$$

$$T^{\theta_i}_{\theta_i} = 2\beta^2\left(1 - \sqrt{1 + \frac{(n-1)(n-2)Q^2}{2\beta^2 r^4}}\right) + \frac{2(n-2)Q^2}{r^4\sqrt{1 + \frac{(n-1)(n-2)Q^2}{2\beta^2 r^4}}}, \tag{15}$$

where $1 \le i \le n-1$. One may easily show that, in the limit of $\beta \to 0$, the energy momentum tensor reduces to the pure gravity

$$T^t_t = T^r_r = T^{\theta_i}_{\theta_i} = 0 \tag{16}$$

and once $\beta \to \infty$, it becomes the EYM case [5]

$$T^a{}_b = -\frac{(n-1)(n-2)Q^2}{2r^4}\mathrm{diag}[1, 1, \kappa, \kappa, \ldots, \kappa] \quad \text{and} \quad \kappa = \frac{n-5}{n-1}. \tag{17}$$

In the sequel we shall use this energy momentum tensor to find black hole solutions to the EYMBI, EYMBIGB, and EYMBIGBL field equations with/without cosmological constant Λ.

B. EYMBI black hole solution in 5 dimensions

In five dimensions, the EYMBI field equations (6) after some calculation, can be written as

$$3rf' + 6(f-1) + 4(\Lambda - \beta^2)r^2 + 4\beta\sqrt{\beta^2 r^4 + 3Q^2} = 0, \tag{18}$$

$$[r^2 f'' + 4rf' + 2(f-1) + 4(\Lambda - \beta^2)r^2]\sqrt{\beta^2 r^4 + 3Q^2} + 4\beta(\beta^2 r^4 + Q^2) = 0, \tag{19}$$

which admits the following solution:

$$f(r) = 1 - \frac{2M + \beta(\beta - \sqrt{Q^2 + \beta^2})}{r^2} - \frac{(\Lambda - \beta^2)}{3}r^2 - \frac{\beta}{3}\sqrt{\beta^2 r^4 + 3Q^2} - \frac{Q^2}{r^2}\ln\left[\frac{(\beta r^2 + \sqrt{\beta^2 r^4 + 3Q^2})}{\sqrt{4\beta^2 + 3Q^2}}\right]. \tag{20}$$

This is a black hole solution and M is an integration constant to be identified as the mass of the black hole. One can show that in the limit of $\beta \to \infty$, $L(\mathbf{F})$ and $f(r)$ reduce to the case of EYM as we mentioned above, i.e.,

$$\lim_{\beta\to\infty} L(\mathbf{F}) = \mathrm{Tr}(F^{(a)}_{\lambda\sigma}F^{(a)\lambda\sigma}) = \frac{6Q^2}{r^4}, \quad \lim_{\beta\to\infty} f(r) = 1 - \frac{2M}{r^2} - \frac{\Lambda}{3}r^2 - \frac{2Q^2\ln(r)}{r^2}, \tag{21}$$

while in the limit of $\beta \to 0$ they reduce to the pure gravity with the cosmological constant

$$\lim_{\beta\to 0} L(\mathbf{F}) = \mathrm{Tr}(F^{(a)}_{\lambda\sigma}F^{(a)\lambda\sigma}) = 0, \quad \lim_{\beta\to 0} f(r) = 1 - \frac{2M}{r^2} - \frac{\Lambda}{3}r^2. \tag{22}$$

The black hole solution (20) asymptotically behaves like a de Sitter space-time (anti-de Sitter) such that

$$\lim_{r\to\infty} f(r) = 1 - \frac{\Lambda}{3}r^2$$

and for $\Lambda = 0$, it is asymptotically flat. The Born-Infeld parameter β modifies the radius of the horizon, as we plot in Fig. 1. In fact, for $\beta = 0$ the solution matches with the pure gravity while for $\beta = \infty$ it gives the horizon of the EYM black hole. We notice that the BI parameter interpolates the horizon of the corresponding black hole, between the two extremal values of the radii of the horizons for $\beta = 0$ and $\beta = \infty$.

C. EYMBI black hole solution for $N \ge 5$ dimensions

In higher dimensions $N(= n+1)$, the EYMBI field equations become

$$(n-1)rg' + (n-1)(n-2)g + 4r^2\left(\frac{n(n-1)}{12}\Lambda - \beta^2\right) + 4\sqrt{r^4\beta^4 + \frac{(n-1)(n-2)\beta^2 Q^2}{2}} = 0,$$
$$\sqrt{\beta^2 r^4 + \frac{(n-1)(n-2)}{2}Q^2}\Big(r^2 g'' + 2(n-2)rg' + (n-3)(n-2)g + 4\left(\frac{(n-1)(n-2)}{12}\Lambda - \beta^2\right)r^2\Big) + 4\beta\left(\beta^2 r^4 + \frac{(n-2)(n-3)}{2}Q^2\right) = 0, \tag{23}$$

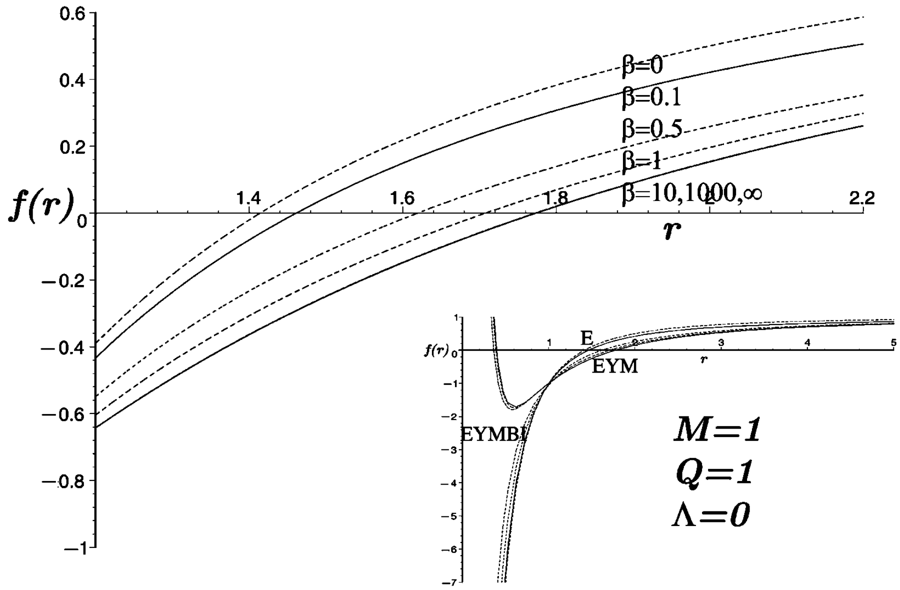

FIG. 1. Plots of $f(r)$ versus r, for $M = 1$, $Q = 1$, $\Lambda = 0$ and $\beta = 0, 0.1, 0.5, 1.0, 10, 1000$, and ∞. The role of β may be interpreted as an adjustment key to get any value for the radius of the horizon, between the extremal horizons of the corresponding pure gravity (E) ($\beta = 0$) and EYM ($\beta = \infty$) black holes.

where $g = f(r) - 1$. By defining a new radial coordinate $\rho = \beta r$ and introducing $\tilde{Q}^2 = \frac{(n-1)(n-2)}{2}\beta^2 Q^2$ and $\tilde{\Lambda} = 4(\frac{(n-1)(n-2)}{12\beta^2}\Lambda - 1)$, these equations can be rewritten in more convenient forms as

$$(n-1)\rho g' + (n-1)(n-2)g + \rho^2\tilde{\Lambda} + 4\sqrt{\rho^4 + \tilde{Q}^2} = 0, \tag{24}$$

$$\sqrt{\rho^4 + \tilde{Q}^2}(\rho^2 g'' + 2(n-2)\rho g' + (n-3)(n-2)g + \tilde{\Lambda}\rho^2) + 4\left(\rho^4 + \frac{n-3}{n-1}\tilde{Q}^2\right) = 0. \tag{25}$$

These admit the general solution

$$f(\rho) = 1 + g(\rho) = 1 - \frac{\tilde{M}}{\rho^{n-2}} - \frac{\tilde{\Lambda}}{(n-1)n}\rho^2 - \frac{4A(\rho)}{(n-1)\rho^{n-2}}, \tag{26}$$

$$A(\rho) = \int \sqrt{\rho^4 + \tilde{Q}^2}\rho^{n-3} d\rho = \frac{|\tilde{Q}|}{n-2}\rho^{n-2}{}_2F_1\left(\frac{n-2}{4}, \frac{-1}{2}, \frac{n+2}{4}, -\frac{\rho^4}{\tilde{Q}^2}\right), \tag{27}$$

where $\tilde{M}$ is an integration constant related to the mass of the black hole and ${}_2F_1$ stands for the hypergeometric function.

III. FIELD EQUATIONS AND THE METRIC ANSATZ FOR EYMBIGB GRAVITY

The EYMBIGB action in $N(= n + 1)$ dimensions may be written as

$$S = \frac{1}{16\pi}\int_{\mathcal{M}} d^{n+1}x\sqrt{-g}\left(R - \frac{n(n-1)}{3}\Lambda + \alpha\mathcal{L}_{\rm GB} + L(\mathbf{F})\right) + \frac{1}{8\pi}\int_{\partial\mathcal{M}} d^n x\sqrt{-\gamma}K(\gamma), \tag{28}$$

where the terms are as before; α is the GB parameter (or the second order Lovelock gravity term) and $\mathcal{L}_{\rm GB}$ is given by

$$\mathcal{L}_{\rm GB} = R_{\mu\nu\gamma\delta}R^{\mu\nu\gamma\delta} - 4R_{\mu\nu}R^{\mu\nu} + R^2. \tag{29}$$

Variation of the new action with respect to the space-time metric $g_{\mu\nu}$ yields the field equations

$$G^E_{\mu\nu} + \alpha G^{\rm GB}_{\mu\nu} + \frac{n(n-1)}{6}\Lambda g_{\mu\nu} = T_{\mu\nu}, \tag{30}$$

where

$$G^{\rm GB}_{\mu\nu} = 2(-R_{\mu\sigma\kappa\tau}R^{\kappa\tau\sigma}{}_{\nu} - 2R_{\mu\rho\nu\sigma}R^{\rho\sigma} - 2R_{\mu\sigma}R^{\sigma}{}_{\nu} + RR_{\mu\nu}) - \tfrac{1}{2}\mathcal{L}_{\rm GB}g_{\mu\nu}, \tag{31}$$

in which $T_{\mu\nu}$ is given in Eq. (7) and the YM field equations were presented in Eq. (8).

A. EYMBIGB black hole solution in $N = 5$ dimensions

In five dimensions, Eq. (30) leads to a set of two equations as follows:

$$(12\alpha g - 3r^2)g' - 6rg - 4r^3(\Lambda - \beta^2) - 4\beta r\sqrt{\beta^2 r^4 + 3Q^2} = 0, \quad (32)$$

$$[(4\alpha g - r^2)g'' + 4(\alpha g' - r)g' - 2g - 4r^2(\Lambda - \beta^2)] \times \sqrt{\beta^2 r^4 + 3Q^2} - 4\beta(\beta^2 r^4 + Q^2) = 0, \quad (33)$$

and these equations admit an exact solution in the form of

$$f_\pm(r) = 1 + g = 1 + \frac{r^2}{4\alpha}\Big\{1 \pm \Big[1 + \frac{8\alpha}{3}\Big(\Lambda + \beta^2\Big(\sqrt{1 + \frac{3Q^2}{\beta^2 r^4}} - 1\Big)\Big) + \frac{16\alpha}{r^4}\Big(\alpha + M + \frac{1}{2}\beta(\beta - \sqrt{Q^2 + \beta^2}) + \frac{Q^2}{2}\ln\Big[\frac{(\beta r^2 + \sqrt{\beta^2 r^4 + 3Q^2})}{\sqrt{4\beta^2 + 3Q^2}}\Big]\Big)\Big]^{1/2}\Big\} \quad (34)$$

in which M is an integration constant and will be identified as the mass of the black hole.

We notice that this solution has the following limits:

$$\lim_{\beta\to\infty} f_\pm(r) = 1 + \frac{r^2}{4\alpha}\Big\{1 \pm \Big[1 + \frac{8\alpha\Lambda}{3} + \frac{16\alpha(\alpha + M)}{r^4} + \frac{16\alpha Q^2 \ln r}{r^4}\Big]^{1/2}\Big\} \quad (35)$$

which is the solution of EYMGB gravity [5] and $\lim_{\beta\to 0} f(r)$ exists if and only if $Q = 0$, and one can show that

$$\lim_{\beta\to 0} f_\pm(r) = 1 + \frac{r^2}{4\alpha}\Big\{1 \pm \sqrt{1 + \frac{8\alpha\Lambda}{3} + \frac{16\alpha(\alpha + M)}{r^4}}\Big\}, \quad (36)$$

which is the case of EGB gravity. We comment that one may check that $\lim_{\alpha\to 0} f_-(r)$ will produce the solution of EYMBI gravity which was given by Eq. (20). In Fig. 2 we plot Eq. (34) for different values of β and fixed values for the mass, charge, and cosmological constant. We comment on this figure that again β provides such a flexibility to the black hole to have any value for the radius of the horizon between the two extremal values [i.e. the minimum value is the radius of the horizon of the pure gravity black hole ($\beta = 0$) and the maximum value corresponds with the horizon of the EYMGB black hole ($\beta = \infty$)].

The positive branch of the solution is defined once $\alpha \neq 0$, and for the positive value for α, the metric function $f_+(r)$ is positive. One may find the asymptotic behavior of the metric function at large r to show that

$$\lim_{r\to\infty} f_+(r) = 1 - \frac{\Lambda_{\rm eff}}{3} r^2, \quad (37)$$

where

$$\Lambda_{\rm eff} = -\frac{1 + \sqrt{1 + \frac{8\alpha}{3}\Lambda}}{4\alpha}, \qquad \alpha \neq 0, \qquad \Lambda \geq -\frac{3}{8\alpha}. \quad (38)$$

This implies that $f_+(r)$ is an asymptotically anti-de Sitter (A-AdS)-non-black hole solution with an effective cosmological constant $\Lambda_{\rm eff}$. Finally we comment that the positive

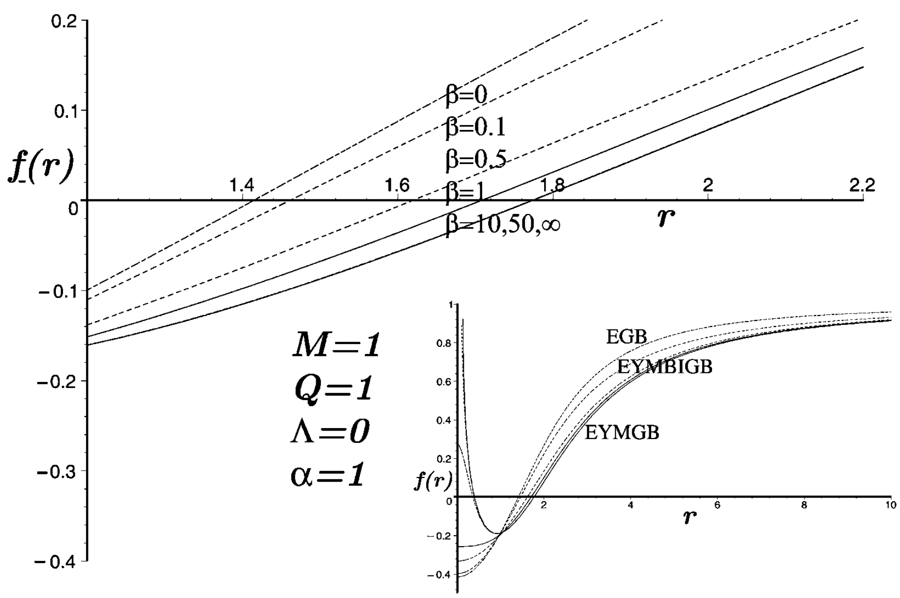

FIG. 2. Plots of $f_-(r)$ versus r, for $M = 1$, $Q = 1$, $\Lambda = 0$ and $\beta = 0, 0.1, 0.5, 1.0, 10, 50$, and ∞. The role of β may be interpreted as a regulator to get any value for the radius of the horizon, between the horizons corresponding to pure gravity with a GB term (EGB) ($\beta = 0$) and EYMGB ($\beta = \infty$) black holes. The smaller figure shows that by the choice of β it is possible to obtain black holes which are regular at $r = 0$.

branch of the solution with a negative value for α is a black hole solution which asymptotically behaves like dS, i.e.

$$\lim_{r\to\infty} f_+(r) = 1 - \frac{\Lambda_{\rm eff}}{3} r^2, \tag{39}$$

where

$$\Lambda_{\rm eff} = \frac{1+\sqrt{1-\frac{8|\alpha|}{3}\Lambda}}{4|\alpha|}, \qquad \alpha \neq 0, \qquad \Lambda \leq \frac{3}{8|\alpha|}. \tag{40}$$

Such an analysis for the negative branch of the solution also gives the same results but $\Lambda_{\rm eff} = -\frac{1-\sqrt{1+\frac{8\alpha}{3}\Lambda}}{4\alpha}$. In this case for $\Lambda = 0$, one gets $\Lambda_{\rm eff} = 0$, which is visible from Fig. 2.

B. EYMBIGB black hole solution for $N \geq 5$ dimensions

In the previous chapter we presented a black hole solution for EYMBIGB in 5 dimensions. Our attempt in this chapter is to give a general black hole solution to Eq. (30). One can show that the general EYMBIGB equation in $N(= n+1)$ dimensions can be written as

$$\frac{1}{2r^4}[(r^3 - 2\alpha(n-3)(n-2)rg)g' + (n-2)r^2 g - (n-2)(n-3)(n-4)\alpha g^2](n-1) + \frac{n(n-1)}{6}\Lambda = 2\beta^2\left(1-\sqrt{1+\frac{(n-1)(n-2)Q^2}{2\beta^2 r^4}}\right), \tag{41}$$

where $g = g(r) = f(r) - 1$. Again we set $\rho = \beta r$, $\tilde{\alpha} = (n-3)(n-2)\beta^2\alpha$, $\tilde{Q}^2 = \frac{(n-1)(n-2)}{2}\beta^2 Q^2$, and $\tilde{\Lambda} = 4(\frac{n(n-1)}{12\beta^2}\Lambda - 1)$ to get the above equation in a more convenient form as

$$4\rho^2\sqrt{\rho^4+\tilde{Q}^2} + (\rho^2 - 2\tilde{\alpha}g)\rho(n-1)g' - \tilde{\alpha}(n-1)(n-4)g^2 + \rho^2(n-1)(n-2)g + \tilde{\Lambda}r^4 = 0. \tag{42}$$

This equation admits the following solution:

$$f_\pm(\rho) = 1 + g(\rho) = 1 + \frac{\rho^2}{2\tilde{\alpha}}\left(1 \pm \sqrt{1 + \frac{4\tilde{\alpha}\tilde{\Lambda}}{n(n-1)} + \frac{4\tilde{\alpha}(\tilde{M} + 4A(\rho))}{(n-1)\rho^n}}\right), \tag{43}$$

where $\tilde{M}$ is an integration constant to be identified as the mass of the black hole and $A(\rho)$ is defined in Eq. (27). We comment that $\lim_{\alpha\to 0} f_-(\rho)$ gives the EYMBI black hole solution given by (20) while in the case of $f_+(\rho)$, α cannot be zero. In the latter case one gets

$$\lim_{r\to\infty} f_+(\rho) = 1 - \frac{\tilde{\Lambda}_{\rm eff}}{3}\rho^2, \qquad \lim_{r\to\infty} f_+(r) = 1 - \frac{\Lambda_{\rm eff}}{3} r^2, \tag{44}$$

where

$$\Lambda_{\rm eff} = \beta^2\tilde{\Lambda}_{\rm eff} = \begin{cases} \frac{3\beta^2}{2\tilde{\alpha}}\left(1+\sqrt{1+\frac{4\tilde{\alpha}(\tilde{\Lambda}+4)}{n(n-1)}}\right), & \tilde{\alpha} > 0, (\tilde{\Lambda}+4) \geq -\frac{n(n-1)}{4\tilde{\alpha}} \\ -\frac{3\beta^2}{2|\tilde{\alpha}|}\left(1+\sqrt{1-\frac{4|\tilde{\alpha}|(\tilde{\Lambda}+4)}{n(n-1)}}\right), & \tilde{\alpha} < 0, (\tilde{\Lambda}+4) \leq \frac{n(n-1)}{4|\tilde{\alpha}|} \end{cases} \tag{45}$$

which implies for $\tilde{\alpha} > 0 (\tilde{\alpha} < 0)$, the solution is AdS (A-AdS) with a β-independent effective cosmological constant $\Lambda_{\rm eff}$. Similar to the five-dimensional case the negative branch of the solution admits a $\Lambda_{\rm eff} = \frac{3\beta^2}{2\tilde{\alpha}} \times (1-\sqrt{1+\frac{4\tilde{\alpha}(\tilde{\Lambda}+4)}{n(n-1)}})$, with proper values for $\tilde{\Lambda}$ and $\tilde{\alpha}$. In this case it is also easy to show that $\Lambda_{\rm eff}$ is β-independent and for $\Lambda = 0$ ($\tilde{\Lambda} = -4$, therefore) the effective cosmological constant vanishes.

IV. FIELD EQUATIONS AND THE METRIC ANSATZ FOR EYMBIGB-LOVELOCK GRAVITY

In this section we consider a more general action which involves, besides the GB term, the third order Lovelock term. The EYMBIGBL action in $N(= n+2)$ dimensions (we notice that in the case of EYMBIGBL $n = N-2$ and therefore it differs from before which was chosen as $n = N-1$), is given by

$$S = \frac{1}{16\pi}\int_{\mathcal{M}} d^{n+2}x\sqrt{-g}\left(R - \frac{n(n+1)}{3}\Lambda + \alpha_2\mathcal{L}_{\rm GB} + \alpha_3\mathcal{L}_{(3)} + L(\mathbf{F})\right) + \frac{1}{8\pi}\int_{\partial\mathcal{M}} d^{n+1}x\sqrt{-\gamma}K(\gamma), \tag{46}$$

where α_2 and α_3 are the second and third order Lovelock parameters, and [6]

$$\mathcal{L}_{(3)} = 2R^{\mu\nu\sigma\kappa}R_{\sigma\kappa\rho\tau}R^{\rho\tau}{}_{\mu\nu} + 8R^{\mu\nu}{}_{\sigma\rho}R^{\sigma\kappa}{}_{\nu\tau}R^{\rho\tau}{}_{\mu\kappa} + 24R^{\mu\nu\sigma\kappa}R_{\sigma\kappa\nu\rho}R^{\rho}{}_{\mu} + 3RR^{\mu\nu\sigma\kappa}R_{\sigma\kappa\mu\nu} + 24R^{\mu\nu\sigma\kappa}R_{\sigma\mu}R_{\kappa\nu} + 16R^{\mu\nu}R_{\nu\sigma}R^{\sigma}{}_{\mu} - 12RR^{\mu\nu}R_{\mu\nu} + R^3, \tag{47}$$

is the third order Lovelock Lagrangian. Variation of the new action with respect to the space-time metric $g_{\mu\nu}$ yields the field equations

$$G_{\mu\nu} + \alpha_2 G^{\rm GB}_{\mu\nu} + \alpha_3 G^{(3)}_{\mu\nu} + \frac{n(n+1)}{6}\Lambda g_{\mu\nu} = T_{\mu\nu}, \quad (48)$$

where

$$\begin{aligned} G^{(3)}_{\mu\nu} = &-3(4R^{\tau\rho\sigma\kappa}R_{\sigma\kappa\lambda\rho}R^{\lambda}{}_{\nu\tau\mu} - 8R^{\tau\rho}{}_{\lambda\sigma}R^{\sigma\kappa}{}_{\tau\mu}R^{\lambda}{}_{\nu\rho\kappa} \\ &+ 2R_{\nu}{}^{\tau\sigma\kappa}R_{\sigma\kappa\lambda\rho}R^{\lambda\rho}{}_{\tau\mu} - R^{\tau\rho\sigma\kappa}R_{\sigma\kappa\tau\rho}R_{\nu\mu} \\ &+ 8R^{\tau}{}_{\nu\sigma\rho}R^{\sigma\kappa}{}_{\tau\mu}R^{\rho}{}_{\kappa} + 8R^{\sigma}{}_{\nu\tau\kappa}R^{\tau\rho}{}_{\sigma\mu}R^{\kappa}{}_{\rho} \\ &+ 4R_{\nu}{}^{\tau\sigma\kappa}R_{\sigma\kappa\mu\rho}R^{\rho}{}_{\tau} - 4R_{\nu}{}^{\tau\sigma\kappa}R_{\sigma\kappa\tau\rho}R^{\rho}{}_{\mu} \\ &+ 4R^{\tau\rho\sigma\kappa}R_{\sigma\kappa\tau\mu}R_{\nu\rho} + 2RR_{\nu}{}^{\kappa\tau\rho}R_{\tau\rho\kappa\mu} \\ &+ 8R^{\tau}{}_{\nu\mu\rho}R^{\rho}{}_{\sigma}R^{\sigma}{}_{\tau} - 8R^{\sigma}{}_{\nu\tau\rho}R^{\tau}{}_{\sigma}R^{\rho}{}_{\mu} \\ &- 8R^{\tau\rho}{}_{\sigma\mu}R^{\sigma}{}_{\tau}R_{\nu\rho} - 4RR^{\tau}{}_{\nu\mu\rho}R^{\rho}{}_{\tau} + 4R^{\tau\rho}R_{\rho\tau}R_{\nu\mu} \\ &- 8R^{\tau}{}_{\nu}R_{\tau\rho}R^{\rho}{}_{\mu} + 4RR_{\nu\rho}R^{\rho}{}_{\mu} - R^2R_{\nu\mu}) - \tfrac{1}{2}\mathcal{L}_{(3)}g_{\mu\nu}. \end{aligned} \quad (49)$$

Equation (48), after making substitutions, reads

$$\begin{aligned} 12\rho^4\sqrt{\rho^4+\tilde{Q}^2} &+ 3n\rho(\rho^4 - 2\rho^2\tilde{\alpha}_2 g + 3\tilde{\alpha}_3 g^2)g' \\ &+ 3n\tilde{\alpha}_3(n-5)g^3 - 3n\rho^2\tilde{\alpha}_2(n-3)g^2 \\ &+ 3n\rho^4(n-1)g + \tilde{\Lambda}\rho^6 = 0, \end{aligned} \quad (50)$$

where $g = g(\rho) = f(\rho) - 1$, $\rho = \beta r$, $\tilde{\alpha}_2 = \beta^2(n-1)\times(n-2)\alpha_2$, $\tilde{\alpha}_3 = \beta^4(n-1)(n-2)(n-3)(n-4)\alpha_3$, $\tilde{Q}^2 = n(n-1)\beta^2Q^2/2$, and $\tilde{\Lambda} = \frac{n(n+1)}{\beta^2}\Lambda - 12$.

A. 7-dimensional EYMBIGBL black hole solution

The latter equation (50) in 7 dimensions which is the minimum dimensionality of space-time to see the effect of the third order Lovelock gravity, by setting $n = 5$, reads

$$\begin{aligned} 12\rho^3\sqrt{\rho^4+\tilde{Q}^2} &+ 15(\rho^4 - 2\rho^2\tilde{\alpha}_2 g + 3\tilde{\alpha}_3 g^2)g' \\ &- 30\rho\tilde{\alpha}_2 g^2 + 60\rho^3 g + \tilde{\Lambda}\rho^5 = 0. \end{aligned} \quad (51)$$

This admits a solution

$$\begin{aligned} f(\rho) &= 1 + g(\rho) \\ &= 1 + \frac{\tilde{\alpha}_2}{3\tilde{\alpha}_3}\rho^2 + \frac{\sqrt[3]{\xi}}{30\tilde{\alpha}_3} - \frac{10(3\tilde{\alpha}_3 - \tilde{\alpha}_2^2)\rho^4}{3\tilde{\alpha}_3\sqrt[3]{\xi}}, \end{aligned} \quad (52)$$

where we have used the following abbreviations:

$$\begin{aligned} \xi &= -4500\tilde{\alpha}_2\rho^6\left(\tilde{\alpha}_3 - \frac{2}{9}\tilde{\alpha}_2^2\right) - 150\left(\tilde{\Lambda}\tilde{\alpha}_3\rho^6 - 2\sqrt{\chi} + 72\left(A + \frac{m}{12}\right)\tilde{\alpha}_3\right)\tilde{\alpha}_3, \\ \chi &= 300\left(\tilde{\alpha}_3 - \frac{1}{4}\tilde{\alpha}_2^2\right)\rho^{12} + 15(\tilde{\Lambda}\rho^6 + 6m + 72A)\tilde{\alpha}_2\rho^6 \times\left(\tilde{\alpha}_3 - \frac{2}{9}\tilde{\alpha}_2^2\right) + \frac{1}{4}\tilde{\alpha}_3^2(\tilde{\Lambda}\rho^6 + 6m + 72A)^2, \\ A &= \int\rho^3\sqrt{\rho^4+\tilde{Q}^2}d\rho = \frac{1}{6}(\rho^4+\tilde{Q}^2)^{3/2}. \end{aligned} \quad (53)$$

The metric function (52) at large values for ρ (and r therefore) reads

$$f(\rho) = 1 - \frac{\tilde{\Lambda}_{\rm eff}}{3}\rho^2, \qquad f(r) = 1 - \frac{\Lambda_{\rm eff}}{3}r^2, \quad (54)$$

where

$$\Lambda_{\rm eff} = \beta^2\tilde{\Lambda}_{\rm eff} = \beta^2\left(\frac{10(3\tilde{\alpha}_3 - \tilde{\alpha}_2^2)}{\tilde{\alpha}_3\eta^{1/3}} - \frac{10\tilde{\alpha}_2 + \eta^{1/3}}{10\tilde{\alpha}_3}\right) \quad (55)$$

in which

$$\begin{aligned} \eta &= 200(5\tilde{\alpha}_2^3 - \tilde{\alpha}_3^2) - 150\tilde{\alpha}_3(30\tilde{\alpha}_2 + \tilde{\Lambda}\tilde{\alpha}_3 - \sqrt{\chi}), \\ \chi &= (\tilde{\Lambda}+12)^2\tilde{\alpha}_3^2 + 20\tilde{\alpha}_2(\tilde{\Lambda}+12)\left(3\tilde{\alpha}_3 - \frac{2}{3}\tilde{\alpha}_2^2\right) + 300(4\tilde{\alpha}_3 - \tilde{\alpha}_2^2). \end{aligned} \quad (56)$$

One can show that $\Lambda_{\rm eff}$ is β-independent and for the case of zero cosmological constant (i.e. $\Lambda = 0$ or $\tilde{\Lambda} = -12$), $\Lambda_{\rm eff}$ vanishes.

As a specific choice, for technical reasons, we set $3\tilde{\alpha}_3 - \tilde{\alpha}_2^2 = 0$ (i.e. $\tilde{\alpha}_3 = \tilde{\alpha}_2^2/3$), then this solution reduces to the simpler form

$$f(\rho) = 1 + \frac{\rho^2}{\tilde{\alpha}_2}\left(1 - \sqrt[3]{1 + \frac{\tilde{\Lambda}\tilde{\alpha}_2}{30} + \frac{\tilde{\alpha}_2(2(\rho^4+\tilde{Q}^2)^{3/2} + \tilde{M})}{5\rho^6}}\right), \quad (57)$$

which is an asymptotically flat black hole solution. This solution may be expressed as an explicit function of β

$$f(r) = 1 + \frac{r^2}{\hat{\alpha}_2}\left(1 - \sqrt[3]{1 + \left(\Lambda - \frac{2}{5}\beta^2\right)\hat{\alpha}_2 + \frac{3\hat{\alpha}_2 M}{r^6} + \frac{2\hat{\alpha}_2\beta^2}{5}\left(1 + \frac{10Q^2}{\beta^2 r^4}\right)^{3/2} - \frac{4\hat{\alpha}_2 Q^3\sqrt{10}}{\beta r^6}}\right) \quad (58)$$

where $\hat{\alpha}_2 = 12\alpha_2$. This expression, clearly in the two extremal limits, gives

$$\lim_{\beta\to 0} f(r) = 1 + \frac{r^2}{\hat{\alpha}_2}\left(1 - \sqrt[3]{1 + \Lambda\hat{\alpha}_2 + \frac{3\alpha_2 M}{r^6}}\right), \quad (59)$$

$$\lim_{\beta\to\infty} f(r) = 1 + \frac{r^2}{\hat{\alpha}_2}\left(1 - \sqrt[3]{1 + \Lambda\hat{\alpha}_2 + \frac{3\hat{\alpha}_2 M}{r^6} + \frac{6\hat{\alpha}_2 Q^2}{r^4}}\right). \tag{60}$$

From (58) it is observed that asymptotically ($r \to \infty$) we obtain an effective cosmological constant given by $\Lambda_{\rm eff} = \frac{3}{\hat{\alpha}_2}[(1 + \Lambda\hat{\alpha}_2)^{1/3} - 1]$ which vanishes for $\Lambda = 0$. Equation (58) and its extremal limits are plotted in Figs. 3 and 4 for different values for Λ. It is clear that for $\Lambda = 0$, $f(r)$ is an asymptotically flat black hole while for $\Lambda \neq 0$, $f(r)$ would be either AdS or A-AdS depending on the values of $\hat{\alpha}_2$ and Λ.

B. EYMBIGBL black hole solution for $N(= n + 2) \geq 7$ dimensions

In higher dimensions $N(= n + 2) \geq 7$, in general, the master equation given by (50) admits a solution as

$$f(\rho) = 1 + \frac{\tilde{\alpha}_2}{3\tilde{\alpha}_3}\rho^2 + \frac{\sqrt[3]{\xi}}{6n\tilde{\alpha}_3\rho^{n-5}} - \frac{2(3\tilde{\alpha}_3 - \tilde{\alpha}_2^2)n\rho^{n-5}}{3\tilde{\alpha}_3\sqrt[3]{\xi}}, \tag{61}$$

where

$$\xi = -\frac{36n^2\rho^{2(n-5)}}{n+1}\left\{\tilde{\alpha}_2 n(n+1)\rho^{n+1}\left(\tilde{\alpha}_3 - \frac{2}{9}\tilde{\alpha}_2^2\right) + \left(\tilde{\Lambda}\tilde{\alpha}_3\rho^{n+1} + 12(n+1) \times \left(-\frac{\sqrt{\chi}}{36} + \left(A + \frac{m}{12}\right)\tilde{\alpha}_3\right)\right)\tilde{\alpha}_3\right\}, \tag{62}$$

$$\chi = \frac{1}{(n+1)^2}\left\{(-3\tilde{\alpha}_2^2 + 12\tilde{\alpha}_3)n^2(n+1)^2\rho^{2(n+1)} + 216n(n+1)\tilde{\alpha}_2\left(\tilde{\alpha}_3 - \frac{2}{9}\tilde{\alpha}_2^2\right) \times \left[\frac{\tilde{\Lambda}}{12}\rho^{(n+1)} + \left(A + \frac{m}{12}\right)(n+1)\right]\rho^{(n+1)} + 1296\tilde{\alpha}_3^2\left[\frac{\tilde{\Lambda}}{12}\rho^{(n+1)} + \left(A + \frac{m}{12}\right)(n+1)\right]^2\right\}, \tag{63}$$

$$A = \int \rho^{n-2}\sqrt{\rho^4 + \tilde{Q}^2}d\rho = \frac{|\tilde{Q}|}{n-1}\rho^{n-1}{}_2F_1\left(\frac{n-1}{4}, \frac{-1}{2}, \frac{n+3}{4}, -\frac{\rho^4}{\tilde{Q}^2}\right). \tag{64}$$

The case of $\tilde{\alpha}_3 = \tilde{\alpha}_2^2/3$ may be considered in this solution and this leads us to

$$f(\rho) = 1 + \frac{\rho^2}{\tilde{\alpha}_2}\left(1 - \sqrt[3]{1 + \frac{\tilde{\Lambda}\tilde{\alpha}_2}{n(n+1)} + \frac{\tilde{\alpha}_2(12A + \tilde{M})}{n\rho^{n+1}}}\right), \tag{65}$$

where A is given in Eq. (27). One may use the asymptotic form of $A(\rho) = \rho^{n+1}/(n+1)$ to write

$$\lim_{r\to\infty} f(\rho) = 1 - \frac{\tilde{\Lambda}_{\rm eff}}{3}\rho^2, \qquad \lim_{r\to\infty} f(r) = 1 - \frac{\Lambda_{\rm eff}}{3}r^2, \tag{66}$$

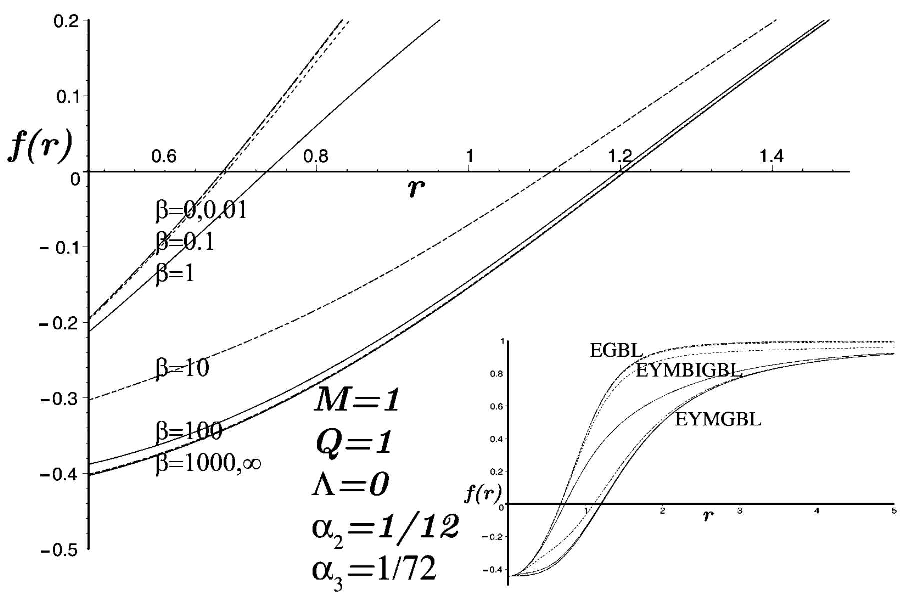

FIG. 3. Plots of $f(r)$ versus r, for fixed values of $M = 1$, $Q = 1$, $\Lambda = 0$, $\alpha_2 = 1/12$, $\alpha_3 = 1/72$, and $\beta = 0$, 0.01, 0.1, 1, 10, 100, 1000, and ∞. Different values of β from 0 to ∞, correspond to different black hole solutions between EGBL gravity and EYMBIGBL. By setting $\Lambda = 0$, the metric function represents an A-F black hole and therefore independent of β, all cases converge to a constant (= 1).

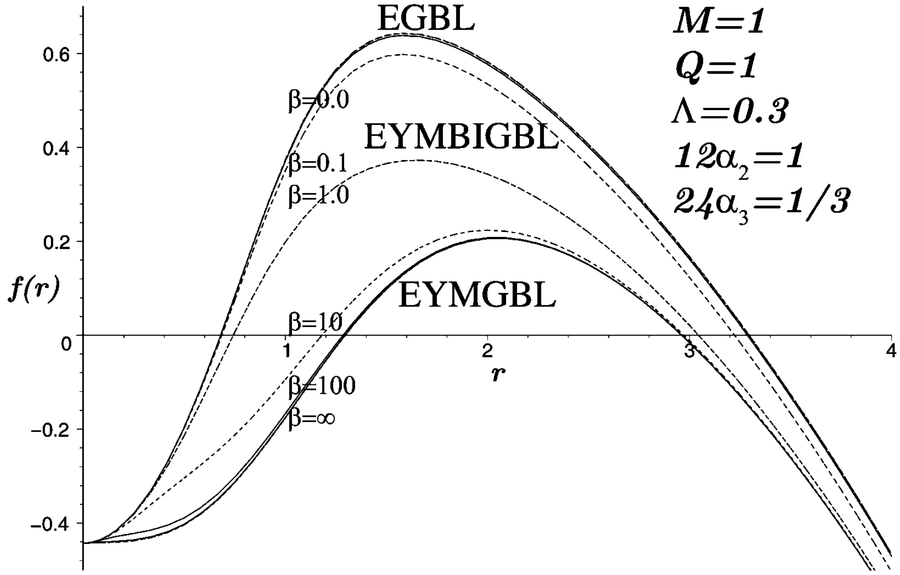

FIG. 4. Plots of $f(r)$ versus r, for fixed values of $M = 1$, $Q = 1$, $\Lambda = 0.3$, $\alpha_2 = 1/12$, $\alpha_3 = 1/72$, and $\beta = 0, 0.01, 0.1, 1, 10, 100, 1000$, and ∞. Different values of β from 0 to ∞, correspond to different black hole solutions between EGBL gravity and EYMBIGBL. By setting $\Lambda = 0.3$, the metric function represents an AdS black hole and therefore independent of β, all cases diverge to a $-\infty$.

$$\Lambda_{\text{eff}} = \beta^2 \tilde{\Lambda}_{\text{eff}} = -\frac{3\beta^2}{\tilde{\alpha}_2}\left(1 - \sqrt[3]{1 + \frac{(\tilde{\Lambda} + 12)\tilde{\alpha}_2}{n(n+1)}}\right), \quad (67)$$

herein Λ_{eff} is independent of β and vanishes for $\Lambda = 0$ (i.e. $\tilde{\Lambda} = -12$). Finally we comment that for arbitrary Lovelock parameters and $\Lambda \neq 0$, Λ_{eff} is also defined which is β-independent and vanishes for $\Lambda = 0$.

V. CONCLUSION

In this work we have found black hole solutions to the field equations of EYMBI, EYMBIGB, and EYMBIGBL theories of gravity. We have explicitly shown that these black hole solutions are the interpolated solutions between pure gravity and gravity coupled with the YM non-Abelian gauge potentials. It is the first time that a higher dimensional non-Abelian gauge field is considered exactly within such a context in higher dimensions. The BI parameter plays the role of an adjustment key from the pure gravity toward EYM solutions. We exploit this property of β as an interpolating parameter between the two different sets to show by numerical calculations that the construction of regular black holes becomes possible. Our results have been supported by some figures. Although our treatment of the third order Lovelock parameter α_3 is constrained by the GB parameter α_2, this seemed to be the only way to compactify our expressions. Asymptotically ($r \to \infty$) once $\Lambda = 0$, in the most general case $\alpha_2 \neq 0 \neq \alpha_3$, by analytical calculation, it can be proved that it gives a flat spacetime, while for $\alpha_3 = 0$ we have dS/AdS, depending on the sign of α_2. (We notice that in the case of the EYMBIGB black hole the positive branch of the general solution provided us to have AdS and A-AdS solutions depending on the relevant parameters.) In the most general version (i.e. EYMBIGBL) of the theory we have constructed 5 parametric black hole solutions consisting of (M, Q, α_2, α_3, and β). It is our belief that with the dilatonic extension these additional parameters will enrich string theory significantly.

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