Magnetic branes in third order Lovelock-Born-Infeld gravity

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Considering both the nonlinear invariant terms constructed by the electromagnetic field and the Riemann tensor in gravity action, we obtain a new class of $(n + 1)$-dimensional magnetic brane solutions in third order Lovelock-Born-Infeld gravity. This class of solutions yields a spacetime with a longitudinal nonlinear magnetic field generated by a static source. These solutions have no curvature singularity and no horizons but have a conic geometry with a deficit angle δ. We find that, as the Born-Infeld parameter decreases, which is a measure of the increase of the nonlinearity of the electromagnetic field, the deficit angle increases. We generalize this class of solutions to the case of spinning magnetic solutions and find that, when one or more rotation parameters are nonzero, the brane has a net electric charge which is proportional to the magnitude of the rotation parameters. Finally, we use the counterterm method in third order Lovelock gravity and compute the conserved quantities of these spacetimes. We found that the conserved quantities do not depend on the Born-Infeld parameter, which is evident from the fact that the effects of the nonlinearity of the electromagnetic fields on the boundary at infinity are wiped away. We also find that the properties of our solution, such as deficit angle, are independent of Lovelock coefficients.

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I. INTRODUCTION

Actions of Lovelock gravity and Born-Infeld electrodynamics have been the subject of wide interest in recent years. This is due to the fact that both of them emerge in the low-energy limit of string theory [1]. On the gravity side string theories in their low-energy limit give rise to effective models of gravity in higher dimensions which involve higher curvature terms, while on the electrodynamics side the effective action for the open string ending on D-branes can be written in a Born-Infeld form. While Lovelock gravity [2] was proposed to have field equations with at most second order derivatives of the metric [3], the nonlinear electrodynamics was proposed, by Born and Infeld, with the aim of obtaining a finite value for the self-energy of a pointlike charge [4]. Lovelock gravity reduces to Einstein gravity in four dimensions and also in the weak field limit, while the Lagrangian of the Born-Infeld (BI) electrodynamics reduces to the Maxwell Lagrangian in the weak field limit. There have been considerable works on both of these theories. In Lovelock gravity, there have been some attempts for understanding the role of the higher curvature terms from various points of view. For example, exact static spherically symmetric black hole solutions of the second order Lovelock gravity have been found in Ref. [5] and of the Gauss-Bonnet-Maxwell model in Ref. [6]. An exact static solution in third order Lovelock gravity was introduced in Ref. [7] and of the charged rotating black brane solution in Ref. [8], and the magnetic

solutions with longitudinal and angular magnetic field were considered in Ref. [9]. All of these works were in the presence of a linear electromagnetic field. The first attempt to relate the nonlinear electrodynamics and gravity has been done by Hoffmann [10]. He obtained a solution of the Einstein equations for a pointlike Born-Infeld charge, which is devoid of the divergence of the metric at the origin that characterizes the Reissner-Nordström solution. However, a conical singularity remained there, as it was later objected by Einstein and Rosen. The spherically symmetric solutions in Einstein-Born-Infeld gravity with or without a cosmological constant have been considered by many authors [11,12].

In this paper, we are dealing with the issue of the spacetimes generated by static and spinning brane sources which are horizonless and have nontrivial external solutions. These kinds of solutions have been investigated by many authors in four dimensions. Static uncharged cylindrically symmetric solutions of Einstein gravity in four dimensions were considered in Ref. [13]. Similar static solutions in the context of cosmic string theory were found in Ref. [14]. All of these solutions [13,14] are horizonless and have a conical geometry; they are everywhere flat except at the location of the line source. The extension to include the electromagnetic field has also been done [15,16]. In three dimensions, the rotating magnetic solutions of Einstein gravity have been studied in Ref. [17], while those of Brans-Dicke gravity have been considered in Ref. [18]. The generalization of these solutions in Einstein gravity in the presence of a dilaton and Born-Infeld electromagnetic field has been done in Ref. [19]. Our aim in this paper is to construct $(n + 1)$-dimensional horizonless solutions of

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third order Lovelock gravity in the presence of a nonlinear electromagnetic field and investigate the effects of the nonlinearity of the electromagnetic field on the properties of the spacetime such as the deficit angle of the spacetime. The first reason for studying higher-dimensional solutions of gravity, which invite higher order Lovelock terms, is due to the scenario of string theory and brane cosmology. The idea of brane cosmology, which is also consistent with string theory, suggests that matter and gauge interaction (described by an open string) may be localized on a brane embedded into a higher-dimensional spacetime, and all gravitational objects are higher-dimensional [20,21]. The second reason for studying higher-dimensional solutions is the fact that four-dimensional solutions have a number of remarkable properties. It is natural to ask whether these properties are general features of the solutions or whether they crucially depend on the world being four-dimensional. Finally, we want to check whether the counterterm method introduced in Refs. [8,22] can be applied to the case of solutions in the presence of a nonlinear electromagnetic field, too.

The outline of our paper is as follows. We give a brief review of the field equations of Lovelock gravity in the presence of a Born-Infeld electromagnetic field in Sec. II. In Sec. III, we present static horizonless solutions which produce a longitudinal magnetic field and investigate the effects of the nonlinearity of the electromagnetic field and Lovelock terms on the deficit angle of the spacetime. Section IV will be devoted to the generalization of these solutions to the case of rotating solutions and use of the counterterm method to compute the conserved quantities of them. We finish our paper with some concluding remarks.

II. FIELD EQUATIONS

A natural generalization of general relativity in higher-dimensional spacetimes with the assumption of Einstein, that the left-hand side of the field equations is the most general symmetric conserved tensor containing no more than second derivatives of the metric, is Lovelock theory. Lovelock found the most general symmetric conserved tensor satisfying this property. The resultant tensor is non-linear in the Riemann tensor and differs from the Einstein tensor only if the spacetime has more than 4 dimensions. The Lovelock tensor in $(n + 1)$ dimensions may be written as [2]

$$\sum_{i=1}^{[n/2]} \alpha'_i \left[\mathcal{H}_{\mu\nu}^{(i)} - \frac{1}{2} g_{\mu\nu} \mathcal{L}^{(i)} \right] + \Lambda g_{\mu\nu} = 8\pi T_{\mu\nu}, \quad (1)$$

where $\Lambda = -n(n - 1)/2l^2$ is the cosmological constant for asymptotically anti-de Sitter (AdS) solutions, $[n/2]$ denotes the integer part of $n/2$, $\alpha'_1 = 1$, α'_i's for $i \geq 2$ are Lovelock coefficients, $T_{\mu\nu}$ is the energy-momentum tensor, and $\mathcal{H}_{\mu\nu}^{(i)}$ and $\mathcal{L}^{(i)}$ are

$$\mathcal{H}_{\mu\nu}^{(i)} = \frac{i}{2^i} \delta_{\mu\nu_2...\nu_{2i}}^{\mu_1\mu_2...\mu_{2i}} R_{\mu_1\mu_2\nu}{}^{\nu_2} R_{\mu_3\mu_4}{}^{\nu_3\nu_4} \ldots R_{\mu_{2i-1}\mu_{2i}}{}^{\nu_{2i-1}\nu_{2i}}, \quad (2)$$

$$\mathcal{L}^{(i)} = \frac{1}{2^i} \delta_{\nu_1\nu_2...\nu_{2i}}^{\mu_1\mu_2...\mu_{2i}} R_{\mu_1\mu_2}{}^{\nu_1\nu_2} \ldots R_{\mu_{2i-1}\mu_{2i}}{}^{\nu_{2i-1}\nu_{2i}}. \quad (3)$$

In this paper, we consider up to third order Lovelock terms. The first term is just the Einstein tensor, and the second and third terms are Gauss-Bonnet and third order Lovelock tensors, respectively, which are written in terms of Riemann tensor explicitly in Ref. [23]. In the presence of a Born-Infeld electromagnetic field, the energy-momentum tensor of Eq. (1) is

$$T_{\mu\nu} = \frac{1}{4\pi} \left(\frac{1}{4} g_{\mu\nu} L(F) + \frac{F_{\mu\lambda} F_\nu{}^\lambda}{\sqrt{1 + \frac{F^2}{2\beta^2}}} \right), \quad (4)$$

where $F_{\mu\nu}$ is the nonlinear electromagnetic field tensor which satisfies the BI equation

$$\partial_\mu \left(\frac{\sqrt{-g} F^{\mu\nu}}{\sqrt{1 + \frac{F^2}{2\beta^2}}} \right) = 0, \quad (5)$$

and $F^2 = F^{\mu\nu} F_{\mu\nu}$. The parameter β is called the Born-Infeld parameter which has the dimension of length^{-2}, and, as it goes to ∞, the BI equation reduces to the standard Maxwell equation.

III. STATIC MAGNETIC BRANES

Here we want to obtain the $(n + 1)$-dimensional solutions of Eqs. (1)–(5) which produce longitudinal magnetic fields in the Euclidean submanifold spans by x^i coordinates $(i = 1, \ldots, n - 2)$. We will work with the following ansatz for the metric [16]:

$$ds^2 = -\frac{\rho^2}{l^2} dt^2 + \frac{d\rho^2}{f(\rho)} + l^2 f(\rho) d\phi^2 + \frac{\rho^2}{l^2} dX^2, \quad (6)$$

where $dX^2 = \sum_{i=1}^{n-2} (dx^i)^2$ is the Euclidean metric on the $(n - 2)$-dimensional submanifold. The angular coordinate ϕ is dimensionless as usual and ranges in $[0, 2\pi]$, while x^i's range in $(-\infty, \infty)$. The motivation for this metric gauge [$g_{tt} \propto -\rho^2$ and $(g_{\rho\rho})^{-1} \propto g_{\phi\phi}$] instead of the usual Schwarzschild gauge [$(g_{\rho\rho})^{-1} \propto g_{tt}$ and $g_{\phi\phi} \propto \rho^2$] comes from the fact that we are looking for a horizonless solution.

The electromagnetic field equation (5) can be integrated immediately to give

$$F_{\phi\rho} = \frac{2q l^{n-1}}{\rho^{n-1} \sqrt{1 - \eta}}, \quad (7)$$

where q is the charge parameter of the solution and

$$\eta = \frac{4q^2 l^{2n-4}}{\beta^2 \rho^{2(n-1)}}. \quad (8)$$

Equation (7) shows that ρ should be greater than $\rho_0 = (2ql^{n-2}\beta)^{1/(n-1)}$ in order to have a real nonlinear electromagnetic field and consequently a real spacetime. To find the function $f(\rho)$, one may use any components of Eq. (1). The simplest equation is the $\rho\rho$ component of these equations which can be written as

$$(\alpha_3\rho f^2 - 2\alpha_2\rho^3 f + \rho^5)f' + \frac{n-6}{3}\alpha_3 f^3$$
$$- (n-4)\alpha_2\rho^2 f^2 + (n-2)\rho^4 f - \frac{n}{l^2}\rho^6$$
$$= \frac{\beta^2\rho^6}{2(n-1)}(1 - \sqrt{1-\eta}), \qquad (9)$$

where the prime denotes the derivative with respect to ρ and

$$\alpha_2 = (n-2)(n-3)\alpha'_2, \qquad \alpha_3 = 72\binom{n-2}{4}\alpha'_3. \quad (10)$$

The only real solution of Eq. (9) is

$$f(\rho) = \frac{\alpha_2\rho^2}{\alpha_3}\{1 - (\sqrt{\gamma + j^2(\rho)} + j(\rho))^{1/3}$$
$$+ \gamma^{1/3}(\sqrt{\gamma + j^2(\rho)} + j(\rho))^{-1/3}\}, \qquad (11)$$

where

$$\gamma = \left(\frac{\alpha_3}{\alpha_2^2} - 1\right)^3, \qquad (12)$$

$$j(\rho) = -1 + \frac{3\alpha_3}{2\alpha_2^2} - \frac{3\alpha_3^2}{2l^2\alpha_2^3}k(\rho), \qquad (13)$$

$$k(\rho) = 1 + \frac{2ml^3}{\rho^n} + \frac{\beta^2 l^2 h(\eta)}{2n(n-1)}. \qquad (14)$$

The constants m and q in Eq. (14) are the mass and charge parameters, respectively, of the metric which are related to the mass and charge density of the solution, and $h(\eta)$ is given as

$$h(\eta) = 1 - \sqrt{1-\eta}$$
$$- \frac{(n-1)\eta}{(n-2)}{}_2F_1\left(\left[\frac{1}{2}, \frac{n-2}{2n-2}\right], \left[\frac{3n-4}{2n-2}\right], \eta\right), \qquad (15)$$

where ${}_2F_1([a, b], [c], z)$ is a hypergeometric function. Using the fact that ${}_2F_1([a, b], [c], z)$ has a convergent series expansion for $|z| < 1$, we can find the behavior of the metric for large ρ and β. The function $j(\rho)$ approaches the constant

$$\lambda = -1 + \frac{3\alpha_3}{2\alpha_2^2} - \frac{3\alpha_3^2}{2l^2\alpha_2^3},$$

as ρ goes to infinity, and the effective cosmological constant for the spacetime is

$$\Lambda_{\text{eff}} = -\frac{n(n-1)\alpha_2}{2\alpha_3}\{1 + (\sqrt{\gamma + \lambda^2} + \lambda)^{1/3}$$
$$- \gamma^{1/3}(\sqrt{\gamma + \lambda^2} + \lambda)^{-1/3}\}. \qquad (16)$$

In the rest of the paper, we investigate only the case of $\gamma \geq 0$, for which $f(\rho)$ is real. The function $f(\rho)$ is negative for large values of ρ, if $\Lambda_{\text{eff}} > 0$. Since $g_{\rho\rho}$ and $g_{\phi\phi}$ are related by $f(\rho) = g_{\rho\rho}^{-1} = l^{-2}g_{\phi\phi}$, when $g_{\rho\rho}$ becomes negative (which occurs for large ρ), so does $g_{\phi\phi}$. This leads to an apparent change of signature of the metric from $(n-1)+$ to $(n-2)+$ as ρ goes to infinity, which is not allowed. Thus, Λ_{eff} should be negative which occurs provided the Lovelock coefficients are assumed to be positive.

In order to study the general structure of the solution given in Eq. (11), we first look for curvature singularities. It is easy to show that the Kretschmann scalar $R_{\mu\nu\lambda\kappa}R^{\mu\nu\lambda\kappa}$ diverges at $\rho = 0$, and therefore one might think that there is a curvature singularity located at $\rho = 0$. However, as we will see below, the spacetime will never achieve $\rho = 0$. The function $f(\rho)$ is negative for $\rho < r_+$ and positive for $\rho > r_+$, where r_+ is the largest real root of $f(\rho) = 0$ which reduces to

$$k(r_+) = 1 + \frac{2ml^3}{r_+^n} + \frac{\beta^2 l^2 h(\eta_+)}{2n(n-1)} = 0. \qquad (17)$$

Again $g_{\rho\rho}$ cannot be negative (which occurs for $\rho < r_+$), because of the change of signature of the metric from $(n-1)+$ to $(n-2)+$. Thus, one cannot extend the spacetime to $\rho < r_+$. To get rid of this incorrect extension, we introduce the new radial coordinate r as

$$r^2 = \rho^2 - r_+^2 \Rightarrow d\rho^2 = \frac{r^2}{r^2 + r_+^2}dr^2. \qquad (18)$$

With this new coordinate, the metric (6) is

$$ds^2 = -\frac{r^2 + r_+^2}{l^2}dt^2 + \frac{r^2}{(r^2 + r_+^2)f(r)}dr^2 + l^2 f(r)d\phi^2$$
$$+ \frac{r^2 + r_+^2}{l^2}dX^2, \qquad (19)$$

where the coordinates r and ϕ assume the values $0 \leq r < \infty$ and $0 \leq \phi < 2\pi$, respectively. The function $f(r)$ is now given as

$$f(r) = \frac{\alpha_2(r^2 + r_+^2)}{\alpha_3}\{1 - (\sqrt{\gamma + j^2(r)} + j(r))^{1/3}$$
$$+ \gamma^{1/3}(\sqrt{\gamma + j^2(r)} + j(r))^{-1/3}\}, \qquad (20)$$

where

$$\eta = \frac{4q^2 l^{2n-4}}{\beta^2(r^2 + r_+^2)^{(n-1)}}, \qquad (21)$$

$$j(r) = -1 + \frac{3\alpha_3}{2\alpha_2^2} - \frac{3\alpha_3^2}{2l^2\alpha_2^3}k(r), \qquad (22)$$

$$k(r) = 1 + \frac{ml^3}{(r^2 + r_+^2)^{n/2}} + \frac{\beta^2 l^2 h(\eta)}{4\pi n(n-1)}. \qquad (23)$$

The electromagnetic field equation in the new coordinate is

$$F_{\phi r} = \frac{2ql^{n-1}}{(r^2 + r_+^2)^{(n-1)/2}\sqrt{1-\eta}}. \qquad (24)$$

The function $f(r)$ given in Eq. (20) is positive in the whole spacetime and is zero at $r = 0$. One can easily show that the Kretschmann scalar does not diverge in the range $0 \le r < \infty$. However, the spacetime has a conic geometry and has a conical singularity at $r = 0$, since

$$\lim_{r \to 0} \frac{1}{r}\sqrt{\frac{g_{\phi\phi}}{g_{rr}}} \neq 1. \qquad (25)$$

That is, as the radius r tends to zero, the limit of the ratio "circumference/radius" is not 2π, and therefore the spacetime has a conical singularity at $r = 0$. The canonical singularity can be removed if one identifies the coordinate ϕ with the period

$$\text{Period}_\phi = 2\pi\left(\lim_{r \to 0} \frac{1}{r}\sqrt{\frac{g_{\phi\phi}}{g_{rr}}}\right)^{-1} = 2\pi(1 - 4\tau), \qquad (26)$$

where τ is given by

$$\tau = \frac{1}{4}\left[1 - \frac{2}{lr_+ f_0''}\right]. \qquad (27)$$

In Eq. (27), f_0'' is the value of the second derivative of $f(r)$ at $r = 0$, which can be calculated as

$$f_0'' = 2r_+^2 k_0'', \qquad (28)$$

where $k_0'' = k''(r = 0)$. By the above analysis, one concludes that near the origin $r = 0$ the metric (19) may be written as

$$\frac{r_+^2}{l^2}(-dt^2 + dX^2) + \frac{1}{r_+^4 k_0''}[dr^2 + (lr_+^3 k_0'')^2 r^2 d\phi^2]. \qquad (29)$$

This metric describes a spacetime that is locally flat but has a conical singularity at $r = 0$ with a deficit angle $\delta = 8\pi\tau$, which is proportional to the brane tension at $r = 0$ [24]. Of course, one may ask for the completeness of the spacetime with $r \ge 0$ (or $\rho \ge r_+$). It is easy to see that the spacetime described by Eq. (19) is both null and timelike geodesically complete as in the case of four-dimensional solutions [16,25]. In fact, one can show that every null or timelike geodesic starting from an arbitrary point can either extend to infinite values of the affine parameter along the geodesic or end on a singularity at $r = 0$ [9].

Now we investigate the effects of different parameters of the solution on the deficit angle of the spacetime. First, we investigate the effect of the nonlinearity of the magnetic field on the deficit angle. In order to do this, we plot δ versus the parameter β. This is shown in Fig. 1, which shows that as β decreases (the nonlinearity of the electromagnetic field becomes more evident) the deficit angle increases. Since for fixed values of mass, charge, and the cosmological constant the parameter β has a minimum value which can be evaluated by the fact that $\eta(r = 0) < 1$, the deficit angle starts from a maximum value $\delta_{\max}$ for $\beta_{\min}$ and goes to its minimum value as β goes to infinity.

Second, we consider the effect of Lovelock terms on the deficit angle in the following subsection.

A. Static magnetic branes in lower order Lovelock gravity

As two special cases, we introduce the static magnetic solutions of Einstein and Gauss-Bonnet gravity in the presence of a Born-Infeld electromagnetic field. The first special case is when both of the Lovelock coefficients are zero. Solving Eq. (9) in the case of $\alpha_2 = \alpha_3 = 0$ and performing the transformation (18), one obtains

$$f_{\text{Ein}}(r) = \frac{r^2 + r_+^2}{l^2}k(r), \qquad (30)$$

where $k(r)$ is given by Eq. (23) and r_+ is again the largest real root of $k(r = 0) = 0$. Of course, one may also obtain the above solution by calculating the limit of $f(r)$ of third order Lovelock gravity given in Eq. (20) as α_2 and α_3 go to zero.

As the second special case, we introduce the magnetic solution in Gauss-Bonnet gravity which may be obtained by solving Eq. (9) in the case of $\alpha_3 = 0$ and performing transformation (18). In this case, one obtains

$$f_{\text{GB}}(r) = \frac{r^2 + r_+^2}{2\alpha_2}\left(1 - \sqrt{1 - \frac{4\alpha_2}{l^2}k(r)}\right), \qquad (31)$$

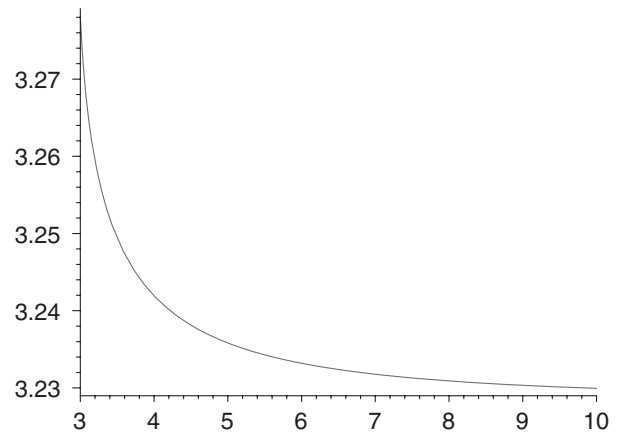where again $k(r)$ and r_+ are the same as that of Einstein or

FIG. 1. δ versus β for $n = 6$, $q = 0.2$, $r_+ = .67$, $l = 1$, $\alpha_2 = 0.2$, and $\alpha_3 = 0.1$.

Lovelock gravities. Now we investigate the effects of Lovelock terms on the properties of the spacetime such as the deficit angle. According to Eq. (29), the deficit angle depends on $k_0'' = k''(r = 0)$ and r_+, which are the same for Einstein, Gauss-Bonnet, and third order Lovelock gravities and are independent of Lovelock coefficients. Thus, one may expect that Lovelock terms of any order have no effect on the properties of our spacetime, which is a warped product of an $(n - 2)$-dimensional zero-curvature spacetime and a curved two-dimensional space. In this relation, we will give some comments in the last section.

IV. SPINNING MAGNETIC BRANES

Now we want to endow our spacetime solution (6) with a global rotation. It should be mentioned that the ($t = $ const, $r = $ const) boundary of our spacetime is curvature-free, and therefore this solution is not a counterpart of the Kerr-type solution in Lovelock gravity. We first consider the solutions with one rotation parameter. In order to add angular momentum to the spacetime, we perform the following rotation boost in the t-ϕ plane:

$$t \mapsto \Xi t - a\phi, \qquad \phi \mapsto \Xi \phi - \frac{a}{l^2} t, \qquad (32)$$

where a is the rotation parameter and $\Xi = 1 + a^2/l^2$. Substituting Eq. (32) into Eq. (6), we obtain

$$ds^2 = -\frac{r^2 + r_+^2}{l^2}(\Xi dt - a d\phi)^2 + \frac{r^2 dr^2}{(r^2 + r_+^2)f(r)}$$
$$+ l^2 f(r)\left(\frac{a}{l^2} dt - \Xi d\phi\right)^2 + \frac{r^2 + r_+^2}{l^2} dX^2, \quad (33)$$

where $f(r)$ is the same as $f(r)$ given in Eqs. (20)–(23). The nonvanishing electromagnetic field components become

$$F_{rt} = -\frac{a}{\Xi l^2} F_{r\phi} = \frac{2l^{n-3} aq}{r^{n-1}\sqrt{1 - \eta}}. \qquad (34)$$

The transformation (32) generates a new metric, because it is not a permitted global coordinate transformation. This transformation can be done locally but not globally [26]. Therefore, the metrics (6) and (33) can be locally mapped into each other but not globally, and so they are distinct. Again, this spacetime has no horizon and curvature singularity. However, it has a conical singularity at $r = 0$.

Second, we study the rotating solutions with more rotation parameters. The rotation group in $n + 1$ dimensions is $SO(n)$, and therefore the number of independent rotation parameters is $[n/2]$, where $[x]$ is the integer part of x. We now generalize the above solution given in Eq. (6) with $k \leq [n/2]$ rotation parameters. This generalized solution can be written as

$$ds^2 = -\frac{r^2 + r_+^2}{l^2}\left(\Xi dt - \sum_{i=1}^{k} a_i d\phi^i\right)^2 + f(r)\left(\sqrt{\Xi^2 - 1} dt\right.$$
$$- \frac{\Xi}{\sqrt{\Xi^2 - 1}} \sum_{i=1}^{k} a_i d\phi^i\Big)^2 + \frac{r^2}{(r^2 + r_+^2)f(r)} dr^2$$
$$+ \frac{r^2 + r_+^2}{l^2(\Xi^2 - 1)} \sum_{i<j}^{k} (a_i d\phi_j - a_j d\phi_i)^2 + \frac{(r^2 + r_+^2)}{l^2} dX^2, \quad (35)$$

where $\Xi = \sqrt{1 + \sum_i^k a_i^2/l^2}$, dX^2 is the Euclidean metric on the $(n - k - 1)$-dimensional submanifold with volume V_{n-k-1}, and $f(r)$ is the same as $f(r)$ given in Eq. (11). The nonvanishing components of the electromagnetic field tensor are

$$F_{rt} = -\frac{(\Xi^2 - 1)}{\Xi a_i} F_{r\phi^i} = \frac{2ql^{n-2}\sqrt{\Xi^2 - 1}}{r^{n-1}\sqrt{1 - \eta}}. \qquad (36)$$

In the remaining part of this section, we compute the conserved quantities of the solution. In general, the conserved quantities are divergent when evaluated on the solutions. A systematic method of dealing with this divergence for asymptotically AdS solutions of Einstein gravity is through the use of the counterterms method inspired by the anti-de Sitter conformal field theory correspondence [27]. For asymptotically AdS solutions of Lovelock gravity with flat boundary $\hat{R}_{abcd}(\gamma) = 0$, the finite energy-momentum tensor is [8,22]

$$T^{ab} = \frac{1}{8\pi}\left\{(K^{ab} - K\gamma^{ab}) + 2\alpha_2'(3J^{ab} - J\gamma^{ab})\right.$$
$$\left. + 3\alpha_3'(5P^{ab} - P\gamma^{ab}) + \frac{n-1}{L}\gamma^{ab}\right\}, \qquad (37)$$

where L is a constant which depends on l, α_2', and α_3' that reduces to l as the Lovelock coefficients vanish. For the special case that $\alpha_3 = \alpha_2^2$, L becomes

$$L = \frac{15l^2\sqrt{\alpha_2(1 - \Gamma)}}{5l^2 + 9\alpha_2 - l^2\Gamma^2 - 4l^2\Gamma},$$

where $\Gamma = (1 - 3\alpha_2/l^2)^{-1/3}$. In Eq. (37), K^{ab} is the extrinsic curvature of the boundary, K is its trace, γ^{ab} is the induced metric of the boundary, and J and P are traces of J^{ab} and P^{ab} given as

$$J_{ab} = \frac{1}{3}(2KK_{ac}K_b^c + K_{cd}K^{cd}K_{ab} - 2K_{ac}K^{cd}K_{db}$$
$$- K^2 K_{ab}), \qquad (38)$$

$$P_{ab} = \tfrac{1}{5}\{[K^4 - 6K^2 K^{cd}K_{cd} + 8KK_{cd}K_e^d K^{ec}$$
$$- 6K_{cd}K^{de}K_{ef}K^{fc} + 3(K_{cd}K^{cd})^2]K_{ab}$$
$$- (4K^3 - 12KK_{ed}K^{ed} + 8K_{de}K_f^e K^{fd})K_{ac}K_b^c$$
$$- 24KK_{ac}K^{cd}K_{de}K_b^e + (12K^2 - 12K_{ef}K^{ef})$$
$$\times K_{ac}K^{cd}K_{db} + 24K_{ac}K^{cd}K_{de}K^{ef}K_{bf}\}. \quad (39)$$

One may note that, when α_i's go to zero, the finite stress-energy tensor (37) reduces to that of asymptotically AdS solutions of Einstein gravity with a flat boundary.

To compute the conserved charges of the spacetime, we choose a spacelike surface $\mathcal{B}$ in $\partial \mathcal{M}$ with metric σ_{ij} and write the boundary metric in Arnowitt-Deser-Misner form:

$$\gamma_{ab}dx^a dx^a = -N^2 dt^2 + \sigma_{ij}(d\varphi^i + V^i dt)(d\varphi^j + V^j dt),$$
$$(40)$$

where the coordinates φ^i are the angular variables parametrizing the hypersurface of constant r around the origin and N and V^i are the lapse and shift functions, respectively. When there is a Killing vector field ξ on the boundary, then the quasilocal conserved quantities associated with the stress tensors of Eq. (37) can be written as

$$\mathcal{Q}(\xi) = \int_{\mathcal{B}} d^{n-1}\varphi \sqrt{\sigma} T_{ab} n^a \xi^b, \quad (41)$$

where σ is the determinant of the metric σ_{ij} and n^a is the timelike unit normal vector to the boundary $\mathcal{B}$. In the context of counterterm method, the limit in which the boundary $\mathcal{B}$ becomes infinite ($\mathcal{B}_\infty$) is taken, and the counterterm prescription ensures that the action and conserved charges are finite. For our case of horizonless rotating spacetimes, the first Killing vector is $\xi = \partial/\partial t$, and therefore its associated conserved charge of the brane is the mass per unit volume V_{n-k-1} calculated as

$$M = \int_{\mathcal{B}} d^{n-1}x \sqrt{\sigma} T_{ab} n^a \xi^b = \frac{(2\pi)^k}{4}[n(\Xi^2 - 1) + 1]m.$$
$$(42)$$

The second class of conserved quantities is the angular momentum per unit volume V_{n-k-1} associated with the rotational Killing vectors $\zeta_i = \partial/\partial\phi^i$, which may be calculated as

$$J_i = \int_{\mathcal{B}} d^{n-1}x \sqrt{\sigma} T_{ab} n^a \zeta_i^b = \frac{(2\pi)^k}{4} nm\Xi a_i. \quad (43)$$

Next, we calculate the electric charge of the solutions. To determine the electric field, we should consider the projections of the electromagnetic field tensor on special hypersurfaces. The normal to such hypersurfaces for the spacetimes with a longitudinal magnetic field is

$$u^0 = \frac{1}{N}, \qquad u^r = 0, \qquad u^i = -\frac{N^i}{N},$$

and the electric field is $E^\mu = g^{\mu\rho}F_{\rho\nu}u^\nu$. Then the electric

charge per unit volume V_{n-k-2} can be found by calculating the flux of the electromagnetic field at infinity, yielding

$$Q = \frac{(2\pi)^k}{2} ql\sqrt{\Xi^2 - 1}. \quad (44)$$

Note that the electric charge is proportional to the rotation parameter and is zero for the case of static solutions. This quantity and the conserved quantities given by Eqs. (42) and (43) show that the metrics (6) and (35) cannot be mapped into each other globally.

V. CLOSING REMARKS

In this paper, we investigated the effects of the nonlinearity of the electromagnetic fields on the properties of the spacetime by finding a new class of magnetic solutions in third order Lovelock gravity in the presence of a nonlinear Born-Infeld electromagnetic field. This class of solutions yields an $(n + 1)$-dimensional spacetime with a longitudinal nonlinear magnetic field [the only nonzero component of the vector potential is $A_\phi(r)$] generated by a static magnetic brane. We found that these solutions have no curvature singularity and no horizons but have conic singularity at $r = 0$ with a deficit angle δ which is sensitive to the nonlinearity of the electromagnetic field. We found that, as the effects of the nonlinearity of the electromagnetic fields become larger, the deficit angle increases. In these static spacetimes, the electric field vanishes, and therefore the brane has no net electric charge. Next, we endow the spacetime with rotation. For the spinning brane, when the rotation parameters are nonzero, the brane has a net electric charge density which is proportional to the magnitude of the rotation parameters given by $\sqrt{\Xi^2 - 1}$. We also applied the counterterm method in order to calculate the conserved quantities of the spacetime and found that these conserved quantities do not depend on the Born-Infeld parameter β. This can be understand easily, since at the boundary at infinity the effects of the nonlinearity of the electromagnetic fields vanish.

On the effects of Lovelock terms on the properties of our spacetime, we found that the Lovelock terms have no effects on the deficit angle of the spacetime. This also happens for the mass, angular momenta, and charge given by Eqs. (41)–(43). This feature of our solution is a generic property of any kind of solutions of any order of Lovelock gravity for spacetimes which are warped products of an $(n - 2)$-dimensional zero-curvature space (spacetime) and a two-dimensional curved spacetime (space). For example, all of the thermodynamic quantities of black holes of Lovelock gravity with a flat horizon, such as the entropy, the temperature, the free energy, the mass, etc., are independent of the Lovelock coefficients [28]. Of course, one may note that the thermodynamic quantities of black holes of Lovelock gravity with a nonflat horizon depend on the Lovelock coefficients [28], and therefore one may predict that the deficit angle of a spacetime with conic geometry

which is a warped product of an $(n-2)$-dimensional nonzero-curvature spacetime and a two-dimensional curved space should depend on the Lovelock coefficients. This has been shown for Gauss-Bonnet gravity in the presence of codimension two branes [29]. Finally, we give some comments on the number of independent solutions in various order of Lovelock gravity. For static solutions of Lovelock gravity, the rr components of the field equation can be integrated in order to get an algebraic equation for $f(r)$. This algebraic equation is linear for the Einstein equation, quadratic for Gauss-Bonnet gravity, cubic for third order Lovelock gravity, quartic for fourth order Lovelock gravity, and so on. This suggests that $f(r)$ has only one branch in Einstein, two branches in Gauss-Bonnet, three branches in third order Lovelock gravity, and so on. But the point which should be regarded carefully is the number of real branches of $f(r)$. Although the number of real branches is two in Gauss-Bonnet gravity, it is one in third order Lovelock gravity.

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[1] E. S. Fradkin and A. A. Tseytlin, Phys. Lett. **163B**, 123 (1985); E. Bergeshoeff, E. Sezgin, C. N. Pope, and P. K. Townsend, *ibid.* **188**, 70 (1987); R. R. Metsaev, M. A. Rahmanov, and A. A. Tseytlin, *ibid.* **193**, 207 (1987).

[2] D. Lovelock, J. Math. Phys. (N.Y.) **12**, 498 (1971).

[3] B. Zwiebach, Phys. Lett. **156B**, 315 (1985); B. Zumino, Phys. Rep. **137**, 109 (1986).

[4] M. Born and L. Infeld, Proc. R. Soc. A **144**, 425 (1934).

[5] D. G. Boulware and S. Deser, Phys. Rev. Lett. **55**, 2656 (1985); J. T. Wheeler, Nucl. Phys. **B268**, 737 (1986).

[6] D. L. Wiltshire, Phys. Lett. **169B**, 36 (1986).

[7] M. H. Dehghani and M. Shamirzaie, Phys. Rev. D **72**, 124015 (2005).

[8] M. H. Dehghani, N. Alinejadi, and S. H. Hendi, Phys. Rev. D **77**, 104025 (2008); S. H. Hendi and M. H. Dehghani, Phys. Lett. B **666**, 116 (2008).

[9] M. H. Dehghani and N. Bostani, Phys. Rev. D **75**, 084013 (2007).

[10] B. Hoffmann, Phys. Rev. **47**, 877 (1935).

[11] H. D. de Oliveira, Classical Quantum Gravity **11**, 1469 (1994); G. W. Gibbons and D. A. Rasheed, Nucl. Phys. **B454**, 185 (1995).

[12] R. G. Cai, D. W. Pang, and A. Wang, Phys. Rev. D **70**, 124034 (2004); T. K. Dey, Phys. Lett. B **595**, 484 (2004).

[13] T. Levi-Civita, Rend. Reale Accad. Lincei Cl. Sci. Fis. Mat. Nat. **28**, 3 (1919); L. Marder, Proc. R. Soc. A **244**, 524 (1958).

[14] A. Vilenkin, Phys. Rev. D **23**, 852 (1981); A. Banerjee, N. Banerjee, and A. A. Sen, *ibid.* **53**, 5508 (1996).

[15] B. C. Mukherji, Bull. Calcutta Math. Soc. **30**, 95 (1938); E. Witten, Nucl. Phys. **B249**, 557 (1985).

[16] O. J. C. Dias and J. P. S. Lemos, Classical Quantum Gravity **19**, 2265 (2002).

[17] O. J. C. Dias and J. P. S. Lemos, J. High Energy Phys. 01 (2002) 006.

[18] O. J. C. Dias and J. P. S. Lemos, Phys. Rev. D **66**, 024034 (2002).

[19] M. H. Dehghani, A. Sheykhi, and S. H. Hendi, Phys. Lett. B **659**, 476 (2008).

[20] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999); **83**, 4690 (1999).

[21] G. Dvali, G. Gabadadze, and M. Porrati, Phys. Lett. B **485**, 208 (2000); G. Dvali and G. Gabadadze, Phys. Rev. D **63**, 065007 (2001).

[22] M. H. Dehghani, N. Bostani, and A. Sheykhi, Phys. Rev. D **73**, 104013 (2006).

[23] F. Muller-Hoissen, Phys. Lett. **163B**, 106 (1985).

[24] S. Randjbar-Daemi and V. Rubakov, J. High Energy Phys. 10 (2004) 054; H. M. Lee and A. Papazoglou, Nucl. Phys. **B705**, 152 (2005).

[25] J. H. Horne and G. T. Horowitz, Nucl. Phys. **B368**, 444 (1992).

[26] J. Stachel, Phys. Rev. D **26**, 1281 (1982).

[27] J. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998).

[28] T. Torii and H. Maeda, Phys. Rev. D **71**, 124002 (2005); **72**, 064007 (2005); R. G. Cai, *ibid.* **65**, 084014 (2002); M. H. Dehghani and R. B. Mann, *ibid.* **73**, 104003 (2006); M. H. Dehghani, G. H. Bordbar, and M. Shamirzaie, *ibid.* **74**, 064023 (2006); M. H. Dehghani and S. H. Hendi, Int. J. Mod. Phys. D **16**, 1829 (2007).

[29] P. Wang and X. H. Meng, Phys. Rev. D **71**, 024023 (2005).