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Geometric phase for a neutral particle in the presence of a topological defect

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In this paper we study the quantum dynamics of a neutral particle in the presence of a topological defect. We investigate the appearance of a geometric phase in the relativistic quantum dynamics of a neutral particle which possesses permanent magnetic and electric dipole moments in the presence of an electromagnetic field in this curved space-time. The nonrelativistic quantum dynamics are investigated using the Foldy-Wouthuysen expansion. The gravitational Aharonov-Casher and He-McKellar-Wilkens effects are investigated for a series of electric and magnetic field configurations.

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I. INTRODUCTION

Topological defects are predicted in some unified theories of fundamental interactions. They may have been formed at phase transitions in the earliest history of the Universe [1]. Examples of such topological defects are the domain wall [2], the cosmic string [2,3], and the global monopole [4]. In particular, cosmic strings provide a bridge between the physical descriptions of microscopic and macroscopic scales.

The appearance of topological phases in the quantum dynamics of a single particle moving freely in multiply connected space-times has been studied in a variety of physical systems. The prototype of this phase is the electromagnetic Aharonov-Bohm one [5], which appears as a phase factor in the wave function of an electron that moves around a magnetic flux line. The gravitational analog of this effect has also been studied in [6–10]. Aharonov and Casher [11] demonstrated that a magnetic dipole acquires a quantum phase when encircling a linear distribution of electric charges. A classical gravitational analog of the Aharonov-Casher effect was investigated by Resnik [12]. Also, He and McKellar [13] and, independently, Wilkens [14] have demonstrated that the quantum dynamics of an electric dipole in the presence of a line of magnetic monopoles also exhibits a geometric quantum phase.

The gravitational Aharonov-Bohm phase was also investigated by Mazur [15] for the relativistic quantum dynamics of particles in the presence of rotating cosmic strings. In recent years, a series of authors investigated the geometric phase in the presence of gravitational fields. Cai and Papini [16,17] obtained a covariantly generalized form of the Berry phase and applied it to a situation involving a weak gravitational field. Corichi and Pierri [18] studied a scalar quantum particle in the presence of rotating cosmic strings and investigated the appearance of the Berry geometric phase in this dynamics. Mostafazadeh [19] also considered the relativistic Berry quantum phase in a series of problems involving scalar particles. In [20] the gravitational Berry phase was applied to the quantum dynamics of scalar quantum particles in the presence of a chiral cosmic string. Shen has carried out a series of studies concerning the Berry geometric phase in a curved space [21–23]. Recently, in [24] the nonrelativistic quantum dynamics of electric and magnetic dipoles in the presence of a cosmic string was studied. This research was motivated by the intention to investigate the quantum scattering and bound state of this dipole in the presence of an external electromagnetic field.

In this paper we analyze the relativistic quantum dynamics of electric and magnetic dipoles in the presence of a topological defect. Our intention is to investigate the influence of the gravitational field in the geometric phase of electric and magnetic dipoles in the presence of electric and magnetic fields. The relativistic geometric phase is obtained for a neutral particle. The Foldy-Wouthuysen approximation is used to investigate the gravitational Aharonov-Casher and the He-McKellar-Wilkens geometric phases.

The structure of the paper is as follows. In Sec. II the geometric aspects of conical space are presented. In Sec. III we investigate the relativistic quantum dynamics of electric and magnetic dipoles. In Sec. IV the Dirac equation in cosmic string space-time is analyzed. In Sec. V the Foldy-Wouthuysen approximation in the conical background is studied. In Sec. VI the geometric quantum phase is investigated in the nonrelativistic quantum dynamics of a neutral particle in conical background. Finally, in Sec. VII the results are discussed.

II. THE COSMIC STRING BACKGROUND

In this section we develop the structure of the curved space-time that we work out in this paper. We consider a cosmic string space-time, where the line element is given by

$$ds^2 = -dt^2 + d\rho^2 + \eta^2\rho^2 d\varphi^2 + dz^2, \tag{1}$$

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where η is called the deficit angle and is defined as $\eta = 1 - 4\mu$ where μ is the linear mass density of the cosmic string. The azimuthal angle varies in the interval $0 \leq \varphi < 2\pi$. The deficit angle can assume only values in which $\eta < 1$ (to the contrary, in [25,26], it can assume values greater than 1, which corresponds to an anticonical space-time with negative curvature). This geometry possesses a conical singularity represented by the following curvature tensor:

$$R_{\rho,\varphi}^{\rho,\varphi} = \frac{1-\eta}{4\eta}\delta_2(\vec{r}), \tag{2}$$

where $\delta_2(\vec{r})$ is the two-dimensional delta function. This behavior of the curvature tensor is called conical singularity [27]. The conical singularity gives rise to the curvature concentrated on the cosmic string axis; in all other places the curvature is null.

It is convenient to construct a frame which allows us to define the spinors in the curved space-time. We can introduce the frame using a noncoordinate basis $\hat{\theta}^a = e^a{}_\mu dx^\mu$, whose components $e^a{}_\mu(x)$ satisfy the following relation [28,29]:

$$g_{\mu\nu}(x) = e^a{}_\mu(x) e^b{}_\nu(x)\eta_{ab}. \tag{3}$$

The components of the noncoordinate basis $e^a{}_\mu(x)$ form a *tetrad* or a *vierbein*. The tetrad has an inverse defined as $dx^\mu = e^\mu{}_a\hat{\theta}^a$, where

$$e^a{}_\mu e^\mu{}_b = \delta^a{}_b, \qquad e^\mu{}_a e^a{}_\nu = \delta^\mu{}_\nu. \tag{4}$$

For the metric corresponding to a cosmic string, we choose the tetrad to be

$$e^a{}_\mu = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\varphi & -\eta\rho\sin\varphi & 0 \\ 0 & \sin\varphi & \eta\rho\cos\varphi & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{5}$$

The tetrad inverse to (5) has the following form:

$$e^\mu{}_a = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\varphi & \sin\varphi & 0 \\ 0 & -\frac{\sin\varphi}{\eta\rho} & \frac{\cos\varphi}{\eta\rho} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \tag{6}$$

which yields the correct flat space-time limit for $\eta = 1$. With the information about the choice of frame, we can obtain the one-form connection $\omega^a{}_b = \omega_\mu{}^a{}_b dx^\mu$ using the Maurer-Cartan structure equation [29]

$$d\hat{\Theta}^a + \omega^a{}_b \wedge \hat{\Theta}^b = 0. \tag{7}$$

Hence, we obtain the following nonzero one-form connections:

$$\omega_\varphi{}^1{}_2 = -\omega_\varphi{}^2{}_1 = 1 - \eta, \tag{8}$$

Witten [30] demonstrated that the field of the cosmic string is coupled to the complex scalar field and behaves like a superconducting wire. This string can develop a large current and may generate a number of interesting astrophysical effects [2]. These defects have been suggested as possible sources of ultrahigh energy cosmic rays [31]. Recently, the possibility of a superconducting cosmic string which carries both current and charge has been investigated in the literature [32–34]; these defects have been denominated vortons [35]. In this type of superconducting cosmic string, a portion can develop a charge per unit length λ_e, and the field near the topological defect is given by

$$\vec{E} = \frac{\lambda_e}{\eta\rho}\hat{\rho}. \tag{9}$$

If we admit the possibility of a magnetic charge, we can consider a phenomenological model with the possibility that a portion of the cosmic string can develop a magnetic charge per unit length λ_m, and this configuration produces a magnetic field given by

$$\vec{B} = \frac{\lambda_m}{\eta\rho}\hat{\rho}. \tag{10}$$

Notice that, in the limit where $\eta \to 1$, we obtain well-known results for electric (magnetic) fields produced by a linear density of electric (magnetic) charge in Minkowski space-time.

III. RELATIVISTIC QUANTUM DYNAMICS

In this section we consider the quantum dynamics of a neutral spin-1/2 particle with nonzero magnetic and electric dipole moments. We analyze the Dirac equation in a curved space-time in the presence of electric and magnetic fields. The Dirac equation with a nonminimal coupling of the spinor to the electromagnetic field embedded in a classical gravitational field is given by

$$i\gamma^\mu\nabla_\mu\psi + \frac{\mu}{2}\Sigma^{\mu\nu}F_{\mu\nu}\psi - i\frac{d}{2}\Sigma^{\mu\nu}\gamma^5 F_{\mu\nu} - m\psi = 0, \tag{11}$$

where μ is the magnetic dipole moment, d is the electric dipole moment, and

$$\nabla_\mu = \partial_\mu + \Gamma_\mu \tag{12}$$

are the components of the covariant derivative and Γ_μ is the spinor connection [28], which is given by

$$\Gamma_\mu = \tfrac{1}{8}\omega_{\mu ab}(x)[\gamma^a, \gamma^b] \tag{13}$$

$$= \tfrac{1}{8}e_{a\nu}\nabla_\mu e^\nu{}_b[\gamma^a, \gamma^b] \tag{14}$$

and

$$F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu = \{\vec{E}, \vec{B}\} \tag{15}$$

with $F_{0\alpha} = -F_{\alpha 0} = E_\alpha$, $F_{\alpha\beta} = -F_{\beta\alpha} = -\epsilon_{\alpha\beta\gamma}B^\gamma$, where $(\alpha, \beta, \gamma = \rho, \varphi, z)$ are the spatial indices of the space-time. The matrices γ^μ are generalized Dirac matrices given in terms of the flat space-time ones γ^a by the relation $\gamma^\mu = e^\mu{}_a\gamma^a$. We rewrite the Dirac equation (11) in terms of vierbeins in the following form:

$$i\gamma^a e^\mu{}_a\nabla_\mu\psi + \frac{1}{2}\mu F_{\mu\nu}e^\mu{}_a e^\nu{}_b\Sigma^{ab}\psi - \frac{i}{2}d\Sigma^{ab}e^\mu{}_a e^\nu{}_b\gamma^5 F_{\mu\nu} - m\psi = 0, \quad (16)$$

with $\Sigma^{ab} = \frac{i}{2}[\gamma^a, \gamma^b]$, where $(a, b, c = 0, 1, 2, 3)$ are the indices which indicate the local reference frame. The γ^a matrices are the Dirac matrices in flat space-time, i.e.,

$$\gamma^0 = \hat{\beta} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \qquad \gamma^i = \hat{\beta}\alpha^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}, \quad (17)$$

where σ^i are the Pauli matrices satisfying the relation $(\sigma^i\sigma^j + \sigma^j\sigma^i) = -2\eta^{ij}$, $\eta^{ab} = \mathrm{diag}(-1, 1, 1, 1)$ is the Minkowski tensor, and $(i, j, k = 1, 2, 3)$ are the spatial indices of the local reference frame. The γ^5 matrix is defined as

$$\gamma^5 = -\frac{i}{24}\epsilon_{\mu\nu\eta\lambda}\gamma^\mu\gamma^\nu\gamma^\eta\gamma^\lambda = i\gamma^0\gamma^1\gamma^2\gamma^3 = -\gamma_5 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad (18)$$

and, finally, we write $\vec{\Sigma}$ as

$$\vec{\Sigma} = \begin{pmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix}, \quad (19)$$

whose components are defined in the local reference frame.

IV. DIRAC EQUATION IN COSMIC STRING BACKGROUND

The Dirac equation that describes a spin-1/2 neutral particle with nonzero magnetic and electric dipole moments moving in an external electromagnetic field is given by the expression (11) or (16). Using the expression (8), the spinorial connection has only the following nonzero component:

$$\Gamma_\varphi = \frac{1}{4}(1 - \eta)[\gamma^1, \gamma^2] = -\frac{i}{2}(1 - \eta)\Sigma^3. \quad (20)$$

Thus, the Dirac equation in curved space-time (16) has the form

$$i\gamma^t\frac{\partial\psi}{\partial t} + i\gamma^\rho\left(\partial_\rho + \frac{1}{2}\frac{(1-\eta)}{\eta\rho} + \mu E_\rho - dB_\rho\right)\psi + i\frac{\gamma^\varphi}{\eta\rho}\frac{\partial\psi}{\partial\varphi} + i\gamma^z\frac{\partial\psi}{\partial z} - \mu\vec{\Sigma}\cdot\vec{B}\psi - d\vec{\Sigma}\cdot\vec{E}\psi - m\psi = 0. \quad (21)$$

In this background the matrices $\gamma^\mu = e^\mu{}_a\gamma^a$ are given by

$$\gamma^t = e^t{}_a\gamma^a = \gamma^0; \qquad \gamma^z = e^z{}_a\gamma^a = \gamma^3; \quad (22)$$

$$\gamma^r = e^r{}_a\gamma^a = \cos\varphi\gamma^1 + \sin\varphi\gamma^2; \quad (23)$$

$$\gamma^\varphi = e^\varphi{}_a\gamma^a = -\sin\varphi\gamma^1 + \cos\varphi\gamma^2. \quad (24)$$

Notice that in the limit of $d \to 0$ Eq. (21) is the same as the one obtained in [24] for the study of a nonrelativistic scattering/bound state of a charged particle that possesses an anomalous magnetic moment. In the present article we are interested in the study of geometrical phases for neutral particles that possess electric and/or magnetic dipole moments. In this way, we investigate a generalization of a relativistic and nonrelativistic Aharonov-Casher and He-McKellar-Wilkens geometric phase in the presence of a topological defect. Now, let us discuss the relativistic geometric phase in this dynamics. We consider that the spinor ψ can be written in the following form:

$$\psi = e^{i\phi}\psi_0 \quad (25)$$

where ψ_0 is the solution of the Dirac equation in the absence of fields, and ϕ is a phase. Substituting Eq. (25) in Eq. (21), we obtain the following phase:

$$\phi = \oint\left(\frac{1}{2}(1 - \eta)\Sigma^3 - \mu\hat{\beta}(\vec{\Sigma}\times\vec{E})_\varphi + d\hat{\beta}(\vec{\Sigma}\times\vec{B})_\varphi\right)d\varphi. \quad (26)$$

Note that this phase has three contributions: the first contribution is generated by the conical geometry of cosmic string background. The other two contributions are generated by the dipole interaction and depend of the field configuration. Note that the first term in Eq. (26) can be written in the following form:

$$\phi_p = \frac{1}{4}\oint R_{\mu\nu\delta\lambda}J^{\delta\lambda}d\tau^{\mu\nu} \quad (27)$$

where $J^{\delta\lambda} = L^{\delta\lambda} + \Sigma^{\delta\lambda}$ is the total angular momentum of the particle. Substituting here the curvature tensor given by Eq. (2), we find that the expression (27) takes the following form:

$$\phi_p = \oint\frac{1}{2}(1 - \eta)\Sigma^3 d\varphi. \quad (28)$$

This phase is the relativistic Berry geometric phase proposed by Cai and Papini [16,17] for a spin-1/2 particle in a curved background using a weak field approximation. In our study, we do not use this approximation, and we find that the phase found in [16,17] is generic.

The other two terms in (26) are the contributions, due to the magnetic and electric dipole moments, to the relativistic Anandan geometric phase [36,37] in the presence of a cosmic string which is given by

$$\phi = \oint(-\mu\hat{\beta}(\vec{\Sigma}\times\vec{E})_\varphi + d\hat{\beta}(\vec{\Sigma}\times\vec{B})_\varphi)d\varphi. \quad (29)$$

In the limit $\eta \rightarrow 1$ we obtain flat space-time results for the Anandan geometric phase.

V. NONRELATIVISTIC LIMIT

In this section we investigate the nonrelativistic limit for a spinor particle nonminimally coupled to electromagnetic fields embedded in a classical gravitational field. We will use the Foldy-Wouthuysen method [38,39]. First, let us rewrite the Dirac equation (11) in the following form:

$$i\frac{\partial \psi}{\partial t} = H\psi. \tag{30}$$

After some manipulation, we arrive at the following equation:

$$i\frac{\partial \psi}{\partial t} == m\hat{\beta}\psi + \vec{\alpha}\cdot\vec{p} - i\vec{\alpha}\cdot\vec{\xi}\psi + \mu\hat{\beta}(i\vec{\alpha}\cdot\vec{E} + \vec{B}\cdot\vec{\Sigma})\psi - d\hat{\beta}(i\vec{\alpha}\cdot\vec{B} - \vec{E}\cdot\vec{\Sigma})\psi, \tag{31}$$

where we have defined the following terms: $p_j = -ie^{\alpha}{}_{j}\partial_{\alpha}$, $E_j = e^{\alpha}{}_{j}E_{\alpha}$, $B_j = e^{\alpha}{}_{j}B_{\alpha}$, and $\xi_j = e^{\varphi}{}_{j}\Gamma_{\varphi}$, which in a cosmic string background are given by

$$\xi_j = -\frac{i}{2}(1-\eta)\Sigma^3 e^{\varphi}{}_{j}. \tag{32}$$

Thus, we can write the Dirac equation (31) in the form

$$i\frac{\partial \psi}{\partial t} = m\hat{\beta}\psi + \vec{\alpha}\cdot\vec{\pi}\psi + d\vec{E}\cdot\vec{\Sigma}\psi + \mu\hat{\beta}\vec{B}\cdot\vec{\Sigma}\psi, \tag{33}$$

where the operator $\vec{\pi}$, in the local reference frame, is defined as

$$\vec{\pi} = \vec{p} - i\mu\hat{\beta}\vec{E} + id\hat{\beta}\vec{B} - i\vec{\xi}, \tag{34}$$

and the first three terms have the same form as defined by [40] in flat space-time. The last term of (34) arises due to the topology of the space-time.

We investigate the nonrelativistic limit of the Dirac equation using the Foldy-Wouthuysen approximation [38]. In this approximation, the Hamiltonian of the system is written as the following linear combination:

$$H = \hat{\beta}m + \hat{O} + \hat{\epsilon}, \tag{35}$$

where the operators $\hat{O}$ and $\hat{\epsilon}$ should be Hermitian ones and must satisfy the relations

$$\hat{O}\,\hat{\beta} + \hat{\beta}\,\hat{O} = 0, \qquad \hat{\epsilon}\,\hat{\beta} - \hat{\beta}\,\hat{\epsilon} = 0. \tag{36}$$

The final result obtained in this approximation permits us to expand the Hamiltonian H and consider the terms up to the order of m^{-1}. So, we have

$$H''' = \hat{\beta}m + \frac{\hat{\beta}}{2m}\hat{O}^2 + \hat{\epsilon}. \tag{37}$$

Using the expression (33) we have that

$$\hat{O} = \vec{\alpha}\cdot\vec{\pi}, \tag{38}$$

$$\hat{\epsilon} = \mu\hat{\beta}\vec{B}\cdot\vec{\Sigma} + d\hat{\beta}\vec{E}\cdot\vec{\Sigma}, \tag{39}$$

and the expression for the Hamiltonian (37) becomes

$$H''' = \hat{\beta}m + \frac{\hat{\beta}}{2m}(\vec{p} + \vec{\Xi})^2 - \frac{\mu^2E^2}{2m} - \frac{d^2B^2}{2m} + \frac{\mu}{2m}\vec{\nabla}\cdot\vec{E} - \frac{d}{2m}\vec{\nabla}\cdot\vec{B} + d\hat{\beta}\vec{\Sigma}\cdot\vec{E} + \mu\hat{\beta}\vec{\Sigma}\cdot\vec{B}, \tag{40}$$

where $\vec{\nabla}$ refers to the gradient in the space-time indices and we introduce the vector, whose components are

$$\Xi_j = \mu\hat{\beta}(\vec{\Sigma}\times\vec{E})_j - d\hat{\beta}(\vec{\Sigma}\times\vec{B})_j + \tfrac{1}{2}(1-\eta)\Sigma^3 e^{\varphi}{}_{j}, \tag{41}$$

which is well defined in the local reference frame.

The Hamiltonian given in (40) describes the behavior of the electric and magnetic dipoles in the external electric and magnetic fields with the presence of a topological defect. The influence of the topological defect (12) becomes clear due to the third term in Eq. (41). We can see that if we consider the limit $\eta \rightarrow 1$, i.e., the absence of a topological defect, we arrive at the configuration obtained in [40] for flat space-time. In the limit of $d \rightarrow 0$ we obtain the same nonrelativistic Hamiltonian discussed in [24]. The effects that arise due to the configuration of the dipoles in the presence of a topological defect with the influence of external electric and magnetic fields will be discussed in the next section.

VI. NONRELATIVISTIC GEOMETRIC QUANTUM PHASES

Now let us study the effects of the interference of the neutral particles in the presence of a topological defect, and the influence of the external electric and magnetic fields. We consider the terms which contribute to the appearance of the geometric phase in the wave function. The nonrelativistic Hamiltonian describing a neutral particle that possesses constant electric and magnetic dipole moments, in the presence of an electric field embedded in a classical gravitational field, can be written in the following way:

$$H = -\frac{1}{2m}(\vec{\nabla} - i\vec{\Xi})^2 + \Xi_0 \tag{42}$$

where $\Xi_i = \mu\hat{\beta}(\vec{\Sigma}\times\vec{E})_i - d\hat{\beta}(\vec{\Sigma}\times\vec{B})_i + \frac{1}{2}(1-\eta)\Sigma^3 e^{\varphi}{}_{i}$, and Ξ_0 is given by

$$\Xi_0 = -\frac{\mu^2E^2}{2m} - \frac{d^2B^2}{2m} + \frac{\mu}{2m}\vec{\nabla}\cdot\vec{E} - \frac{d}{2m}\vec{\nabla}\cdot\vec{B} + d\hat{\beta}\vec{\Sigma}\cdot\vec{E} + \mu\hat{\beta}\vec{\Sigma}\cdot\vec{B}. \tag{43}$$

Notice that the expression (42) is similar to a Hamiltonian of a quantum particle minimally coupled to a non-Abelian gauge field Ξ_{μ}. We can investigate the geometric phase of this system in the field configuration given by (10) and consider the dipoles oriented along the z direction. The last four terms in (43) give zero contribution to the geometrical phase for the field-dipole configuration adopted here. The terms proportional to E^2 and B^2 are local terms and do not contribute to the geometric phase [36,37,40]. So, the only terms that contribute to the geometric phase are given by the term $\vec{\Xi}$ in (42). We have the following equation:

$$-\frac{1}{2m}\left(\vec{\nabla} - i\mu\hat{\beta}\,\vec{\Sigma}\times\vec{E} + id\hat{\beta}\,\vec{\Sigma}\times\vec{B} - i\frac{1}{2}(1-\eta)\Sigma^3 e^{\varphi}\right)^2\Psi - \frac{\mu^2E^2}{2m}\Psi - \frac{d^2B^2}{2m}\Psi = E\Psi. \tag{44}$$

The quantum phase can be obtained if we consider the ansatz

$$\Psi = e^{i\Phi}\psi, \tag{45}$$

where ψ is the solution of the equation

$$-\frac{1}{2m}\nabla^2\psi - \frac{\mu^2E^2}{2m}\psi - \frac{d^2B^2}{2m}\psi = E\psi. \tag{46}$$

Now, we analyze the quantum geometric phase considering the charge densities concentrated on the symmetry axis. In this way, the fields are cylindrically symmetric and are given by Eq. (10). Hence, taking into account the local reference frame and that the charges are concentrated on the symmetry axis of the topological defect, the quantum geometric phase of this system is given by

$$\Phi = \oint \Xi_\mu dx^\mu = \oint \Xi_i e^i{}_\mu dx^\mu = \int_0^{2\pi} \Xi_i e^i{}_\varphi d\varphi = (1-\eta)\pi\sigma^3 + (\mu\lambda_e - d\lambda_m)2\pi\sigma^3. \tag{47}$$

Here we have considered only two-component spinor fields. The geometric phase (47) is a generalization of the Anandan quantum phase in the presence of a cosmic string. The contribution due to the defect can be seen from the first term in Eq. (47). If we take $\eta \to 1$, we recuperate the results obtained in [41] in the absence of a topological defect. If we consider that the particle does not possess an electric dipole moment, that is, if we apply the limit $d = 0$ in (47), we obtain the analog of the Aharonov-Casher effect in the presence of a topological defect. The quantum phase in this case becomes

$$\Phi_{\rm AC} = i\oint e^j{}_\varphi \xi_j d\varphi + \mu\hat{\beta}\oint(\vec{\Sigma}\times\vec{E})_\varphi d\varphi = (1-\eta)\pi\sigma^3 + 2\pi\mu\lambda_e\sigma^3. \tag{48}$$

Note that we have a topological contribution to the Aharonov-Casher effect due to the defect, and this term gives a topological nonvanishing contribution to the geometric phase [42]. Note that in the limit $\eta \to 1$ we obtain the well-known Aharonov-Casher geometric phase.

Our next step is to consider the limit where the magnetic moment of the particle is zero, $\mu = 0$ in Eq. (47), and we obtain the analog of the He-McKellar-Wilkens effect in the presence of a topological defect. The quantum phase in this case becomes

$$\Phi_{\rm HMW} = i\oint e^j{}_\varphi \xi_j d\varphi - d\hat{\beta}\oint(\vec{\Sigma}\times\vec{B})_\varphi d\varphi = (1-\eta)\pi\sigma^3 - 2\pi d\lambda_m\sigma^3. \tag{49}$$

Both of the results obtained above demonstrate the influence of a topological defect in the geometric phase acquired by the wave function in the dynamics of a neutral particle in the presence of a cosmic string. If we consider the absence of the external field and only the presence of the defect, we will obtain a quantum phase that depends only on the topological defect (12).

$$\Phi = (1-\eta)\pi\sigma^3. \tag{50}$$

The same contribution was obtained in [43], when the holonomy matrix was found for a spinor in a continuum model for a graphene layer with a topological defect. In that way, the general result given in the expression (47) shows us the geometric phase acquired in the dynamics of a neutral particle with permanent electric and magnetic dipole moments influenced by the presence of a topological defect.

VII. CONCLUSION

We have studied the influence of a topological defect in the geometric phases of dipoles in relativistic dynamics. We found a new contribution to the geometric phases due to the presence of a topological defect. This contribution is a nondispersive topological [44] contribution to the total geometrical phase acquired by the neutral particle. This contribution is of gravitational origin [16,17] due to curvature introduced by the defect in space-time. We have investigated the nonrelativistic quantum phase in the present paper using the Foldy-Wouthuysen approximation to obtain the nonrelativistic Hamiltonian. We saw that a topological defect introduces a new gravitational contribution to Anandan's geometric phase. We can see that when $\eta \to 1$, we recuperate the same phase obtained in [36,37] for a flat space case in the absence of a topological defect, and also the phase given in expressions (47)–(49) becomes the same as the one given in [41]. Note that the presence of a topological defect introduces a new term in the Aharonov-Casher and He-McKellar-Wilkens geometric phases. Notice that the geometric phase studied here may be investigated using neutron/atomic interferometry in a space with topological defects [45,46]. We claim that it can be interesting to investigate geometric phases in the context of a topological defect in condensed matter [47,48], where a class of linear topological defects appear, which are of the same nature as cosmic strings [25,26].

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