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Quantum mechanics versus equivalence principle

Antonio Accioly[1,2,3,*] and Ricardo Paszko[2,+]

[1]*Laboratório de Física Experimental (LAFEX), Centro Brasileiro de Pesquisas Físicas (CBPF), Rua Dr. Xavier Sigaud 150, 22290-180, Rio de Janeiro, RJ, Brazil*

[2]*Group of Field Theory from First Principles, Laboratório de Física Experimental (LAFEX), Centro Brasileiro de Pesquisas Físicas (CBPF), Rua Dr. Xavier Sigaud 150, 22290-180, Rio de Janeiro, RJ, Brazil*

[3]*Instituto de Física Teórica (IFT), São Paulo State University (UNESP), Rua Pamplona 145, 01405-000, São Paulo, SP, Brazil*

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We consider the scattering of a photon by a weak gravitational field, treated as an external field, up to second order of the perturbation expansion. The resulting cross section is energy dependent which indicates a violation of Galileo's equivalence principle (universality of free fall) and, consequently, of the classical equivalence principle. The deflection angle θ for a photon passing by the sun is evaluated afterward and the likelihood of detecting $\frac{\Delta\theta}{\theta_E} \equiv \frac{\theta-\theta_E}{\theta_E}$ (where θ_E is the value predicted by Einstein's geometrical theory for the light bending) in the foreseeable future, is discussed.

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I. INTRODUCTION

The classical equivalence principle (CEP)—universality of free fall, or equality of inertial and gravitational masses—is the cornerstone of Newtonian gravity and has a *nonlocal* character. Actually, it is an amalgam of two principles: (i) Galileo's equivalence principle (universality of free fall), and (ii) Newton's equivalence principle (equality of inertial and gravitational masses), which are equivalent in the framework of the aforementioned gravitational theory. Both principles, of course, describe nonlocal effects.

On the other hand, Einstein's equivalence principle (EEP), which locally encompasses both the principles mentioned above, can be understood as the requirement that spacetime is Riemannian and, as a consequence, has at each point a local inertial frame [1]. Accordingly, the EEP is a statement about purely *local* effects.

We address here the issue of a possible incompatibility between quantum mechanics and the CEP by analyzing the scattering of a photon by a weak gravitational field, treated as an external field.

Nonetheless, for clarity's sake, before embarking on our main subject, we will digress a little to discuss, in passing, some works which have examined the question of the conflict between the CEP and quantum mechanics in relatively weak gravitational fields.

As far as we know, Greenberger [2] was the first to foresee the existence of mass-dependent interference effects by applying quantum mechanics to the problem of a particle bound in an external gravitational field. In this paper he also showed that the gravitational Bohr atom would allow the mass of an orbiting object to be determined from its Bohr radius, in contradiction with what is expected from Newtonian gravity and the CEP. It is worth noticing, however, that there is no conflict between this result and the EEP. Indeed, as we have already commented, the EEP is a local statement while the Bohr atom is an object extended in space. By the mid-1970s, a few years after Greenberger's seminal article, using a neutron interferometer, Colella, Overhauser, and Werner [3] observed that the quantum-mechanical shift of the neutrons caused by the interaction with Earth's gravitational field was dependent on the neutron mass. This landmark experiment (COW experiment for short) reflects probably a divergence between the CEP and quantum mechanics. Note, however, that the COW phase shift between the two neutron paths traveling at different heights in a gravitational field depends on the (macroscopic) area of the quadrilateral formed by the neutron paths, which is a nonlocal effect. Again the EEP is not violated. In the last 33 years the COW class of experiments have become more sophisticated. The latest neutron interferometry experiments report a statistically significant discrepancy between the experiment and theory, and it has been suspected by the experimenters that this discrepancy may represent a difference between the ways in which gravity acts in classical and quantum mechanics [4]. In reality, from an operational point of view one cannot claim, even in principle, that there exists, for certain quantum systems, an exact equality of gravitational and inertial masses [5,6].

After this parenthesis, we return to our main objective in this article, namely, to show that second order corrections for the scattering of a photon by a weak gravitational field is in disagreement with Galileo's equivalence and, as a result, with the CEP.

II. ENERGY-DEPENDENT CROSS SECTION

In the weak field approximation, i.e.,

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu},$$

*accioly@cbpf.br
+rrpaszko@ift.unesp.br

with $\eta_{\mu\nu} = \mathrm{diag}(+1, -1, -1, -1)$, and in the de Donder gauge, the general solution of the linearized Einstein's equations, having as source a point particle of mass M located at $\mathbf{r} = 0$, can be written as

$$g_{\mu\nu} = \eta_{\mu\nu} + \frac{2GM}{r}(\eta_{\mu\nu} - 2\eta_{\mu 0}\eta_{\nu 0}),$$

where $\kappa^2 = 32\pi G$.

To first order, the Feynman amplitude, $\mathcal{M}_{r,r'}$, for the scattering of a photon by a weak gravitational field, treated as an external field (see Fig. 1), is given by

$$\begin{aligned} i\mathcal{M}_{r,r'} &= i\varepsilon_r^{\mu}(\mathbf{p})\varepsilon_{r'}^{\nu}(\mathbf{p}')h^{\alpha\beta}(\mathbf{q})V_{\alpha\beta,\mu\nu}(p, p') \\ &= i\varepsilon_r^{\mu}(\mathbf{p})\varepsilon_{r'}^{\nu}(\mathbf{p}')\mathcal{M}_{\mu\nu}^{(0)}(p, p'), \end{aligned}$$

where $\varepsilon_r^{\mu}(\mathbf{p})$ $[\varepsilon_{r'}^{\nu}(\mathbf{p}')]$ is the polarization vector for the initial (final) photon,

$$h^{\alpha\beta}(\mathbf{q}) \equiv \int d^3\mathbf{r} e^{-i\mathbf{q}\cdot\mathbf{r}} h^{\alpha\beta}(\mathbf{r}) = \frac{\kappa M}{4\mathbf{q}^2}(\eta^{\alpha\beta} - 2\eta^{\alpha 0}\eta^{\beta 0})$$

is the momentum space external gravitational field, and $V_{\alpha\beta,\mu\nu}(p, p')$—the graviton-photon-photon vertex—is given by

$$\begin{aligned} -\frac{i\kappa}{2}[&(\eta_{\alpha\beta}\eta_{\mu\nu} - \eta_{\alpha\mu}\eta_{\beta\nu} - \eta_{\alpha\nu}\eta_{\beta\mu})p'\cdot p \\ &- \eta_{\alpha\beta}p'_{\mu}p_{\nu} + \eta_{\mu\beta}p'_{\alpha}p_{\nu} - \eta_{\mu\nu}p'_{\alpha}p_{\beta} + \eta_{\alpha\nu}p'_{\mu}p_{\beta} \\ &+ \eta_{\beta\nu}p'_{\mu}p_{\alpha} - \eta_{\mu\nu}p'_{\beta}p_{\alpha} + \eta_{\alpha\mu}p'_{\beta}p_{\nu}]. \end{aligned}$$

For the sake of simplicity we have multiplied the Feynman amplitude by i.

Now, taking into account that

$$\begin{aligned} \sum_{r=1}^{2}\varepsilon_r^{\mu}(\mathbf{p})\varepsilon_r^{\nu}(\mathbf{p}) = -\eta^{\mu\nu} - &\frac{1}{(p\cdot n)^2} \\ &\times[p^{\mu}p^{\nu} - p\cdot n(p^{\mu}n^{\nu} + p^{\nu}n^{\mu})], \end{aligned}$$

with $n^2 = 1$, we promptly obtain the unpolarized cross section

$$\left(\frac{d\sigma}{d\Omega}\right)_0 = \frac{1}{(4\pi)^2}\frac{1}{2}\sum_{r,r'}|i\mathcal{M}_{r,r'}|^2 = \left(\frac{GM}{\sin^2\frac{\theta}{2}}\right)^2\cos^4\frac{\theta}{2}, \qquad (1)$$

which for small angles reduces to

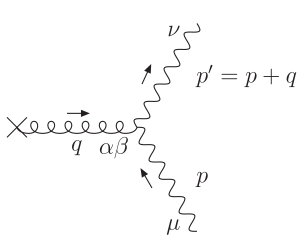

FIG. 1. Feynman graph for photon scattering by an external gravitational field; $|\mathbf{p}| = |\mathbf{p}'|$.

$$\left(\frac{d\sigma}{d\Omega}\right)_0 = \left(\frac{4GM}{\theta^2}\right)^2.$$

Accordingly, to first order the differential cross section for small gravitational deflection angles coincides with that computed using Einstein's geometrical theory—a predictable result since first order calculations are generally expected to reproduce the classical ones.

Let us then push our calculations to the next order. [We remark that these computations are rather involved and, to our knowledge, it is the first time that second order corrections for the scattering of quantum particles (with spin) by an external gravitational field are obtained. Calculations similar to the aforementioned ones, but in external electromagnetic fields, were pioneered by Dalitz [7,8].] In this case we have three graphs that contribute to the scattering process (see Fig. 2). It is noteworthy that *bremsstrahlung* and *radiative corrections* need not be taken into account because, of course, $M \gg E$, where M, as already mentioned, is the mass of the gravitational source—a macroscopic body—and E is the energy of the scattered particle, which implies that these effects are on the ratio $\frac{E}{M}$ in relation to the second order diagrams. For instance, for the solar gravitational deflection of radio waves with a frequency, say, of order 1 GHz, $\frac{E}{M} \sim 10^{-71}$.

We present in the following a condensed version of the calculations concerning the Feynman amplitudes related to the graphs displayed in Fig. 2.

A. The $\mathcal{M}_{\lambda\rho}^{(a)}$ amplitude

From Fig. 2(a), we get

$$i\mathcal{M}_{\lambda\rho}^{(a)} = \frac{i}{2}\int\frac{d^3\mathbf{k}}{(2\pi)^3}h^{\mu\nu}(\mathbf{k}-\mathbf{q})h^{\alpha\beta}(\mathbf{k})V_{\alpha\beta,\mu\nu,\lambda\rho}(p, p'),$$

where $V_{\alpha\beta,\mu\nu,\lambda\rho}(p, p')$, the graviton-graviton-photon-photon vertex, has the form

$$\begin{aligned} -\frac{i}{4}\kappa^2[&(\eta_{\alpha\beta}\eta_{\mu\nu} - 2\eta_{\alpha\mu}\eta_{\beta\nu})(\eta_{\lambda\rho}p\cdot p' - p_{\rho}p'_{\lambda}) \\ &- \eta_{\alpha\beta}(T_{\lambda\rho\mu\nu} + T_{\lambda\rho\nu\mu}) - \eta_{\mu\nu}(T_{\lambda\rho\alpha\beta} + T_{\lambda\rho\beta\alpha}) \\ &+ 2\eta_{\beta\mu}(T_{\lambda\rho\alpha\nu} + T_{\lambda\rho\nu\alpha}) + 2\eta_{\alpha\nu}(T_{\lambda\rho\mu\beta} + T_{\lambda\rho\beta\mu}) \\ &+ 2(\eta_{\mu\lambda}\eta_{\rho\nu}p_{\alpha}p'_{\beta} - \eta_{\lambda\alpha}\eta_{\rho\nu}p_{\mu}p'_{\beta} - \eta_{\lambda\mu}\eta_{\rho\beta}p_{\alpha}p'_{\nu} \\ &+ \eta_{\lambda\nu}\eta_{\rho\mu}p_{\beta}p'_{\alpha} - \eta_{\lambda\beta}\eta_{\rho\mu}p_{\nu}p'_{\alpha} - \eta_{\lambda\nu}\eta_{\rho\alpha}p_{\beta}p'_{\mu} \\ &+ \eta_{\lambda\alpha}\eta_{\rho\beta}p_{\mu}p'_{\nu} + \eta_{\lambda\beta}\eta_{\rho\alpha}p_{\nu}p'_{\mu})] \end{aligned}$$

with $T_{\alpha\beta\mu\nu} \equiv \eta_{\alpha\beta}p_{\mu}p'_{\nu} - \eta_{\alpha\mu}p_{\beta}p'_{\nu} - \eta_{\beta\nu}p_{\mu}p'_{\alpha} + \eta_{\alpha\mu}\eta_{\beta\nu}p\cdot p'$.

Performing the computations, we obtain

$$i\mathcal{M}_{\lambda\rho}^{(a)} = \frac{4\pi^2G^2M^2}{|\mathbf{q}|}(\eta_{\lambda\rho}p\cdot p' - p'_{\lambda}p_{\rho}).$$

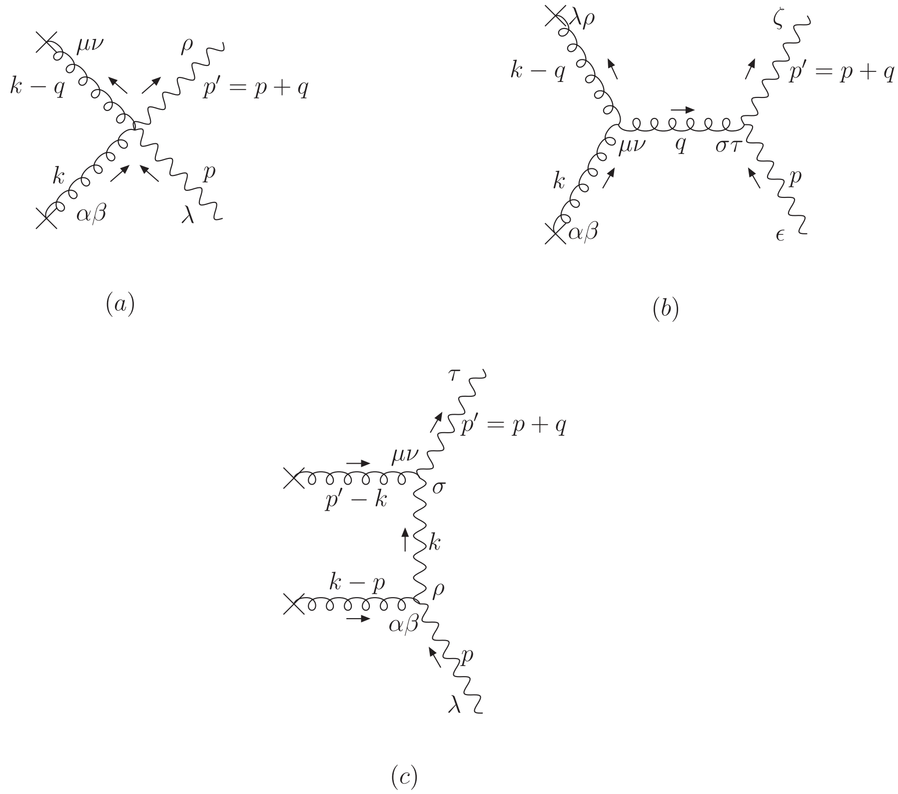

FIG. 2. Second order Feynman graphs for photon scattering by an external gravitational field; $|\mathbf{p}| = |\mathbf{p}'|$.

B. The $\mathcal{M}^{(b)}_{\epsilon\zeta}$ amplitude

A cursory glance at Fig. 2(b) allows us to conclude that

$$i\mathcal{M}^{(b)}_{\epsilon\zeta} = \frac{i}{2}\int \frac{d^3\mathbf{k}}{(2\pi)^3} h^{\lambda\rho}(\mathbf{k}-\mathbf{q})h^{\alpha\beta}(\mathbf{k})V_{\alpha\beta,\mu\nu,\lambda\rho}(k,q,k-q) \times \frac{i\mathcal{P}^{\mu\nu,\sigma\tau}}{q^2}V_{\sigma\tau,\epsilon\zeta}(p,p'),$$

where $\mathcal{P}_{\mu\nu,\sigma\tau} \equiv \frac{1}{2}(\eta_{\mu\sigma}\eta_{\nu\tau} + \eta_{\mu\tau}\eta_{\nu\sigma} - \eta_{\mu\nu}\eta_{\sigma\tau})$, and $V_{\alpha\beta,\mu\nu,\lambda\rho}(k,q,k-q)$, the graviton-graviton-graviton vertex, has the form

$$i\kappa\{-\tfrac{1}{2}(k^2 + q^2 + (k-q)^2)[I^{\sigma}_{\mu,\alpha\beta}I_{\lambda\rho,\sigma\nu} + I^{\sigma}_{\nu,\alpha\beta}I_{\lambda\rho,\sigma\mu} + \tfrac{1}{4}\eta_{\alpha\beta}\eta_{\mu\nu}\eta_{\lambda\rho} - \tfrac{1}{2}(\eta_{\alpha\beta}I_{\mu\nu,\lambda\rho} + \eta_{\mu\nu}I_{\alpha\beta,\lambda\rho} + \eta_{\lambda\rho}I_{\alpha\beta,\mu\nu})]$$
$$+ q^{\sigma}(k-q)^{\tau}[I_{\mu\nu,\lambda\rho}I_{\alpha\beta,\sigma\tau} - \tfrac{1}{2}(I_{\alpha\beta,\rho\sigma}I_{\mu\nu,\lambda\tau} + I_{\alpha\beta,\lambda\sigma}I_{\mu\nu,\rho\tau} + I_{\lambda\rho,\nu\sigma}I_{\alpha\beta,\mu\tau} + I_{\lambda\rho,\mu\sigma}I_{\alpha\beta,\nu\tau})] - k^{\sigma}(k-q)^{\tau}$$
$$\times [I_{\alpha\beta,\lambda\rho}I_{\mu\nu,\sigma\tau} - \tfrac{1}{2}(I_{\mu\nu,\rho\sigma}I_{\alpha\beta,\lambda\tau} + I_{\mu\nu,\lambda\sigma}I_{\alpha\beta,\rho\tau} + I_{\lambda\rho,\alpha\sigma}I_{\mu\nu,\beta\tau} + I_{\lambda\rho,\beta\sigma}I_{\mu\nu,\alpha\tau})] - k^{\sigma}q^{\tau}[I_{\alpha\beta,\mu\nu}I_{\lambda\rho,\sigma\tau}$$
$$- \tfrac{1}{2}(I_{\lambda\rho,\beta\tau}I_{\mu\nu,\alpha\sigma} + I_{\lambda\rho,\alpha\tau}I_{\mu\nu,\beta\sigma} + I_{\lambda\rho,\nu\sigma}I_{\alpha\beta,\mu\tau} + I_{\lambda\rho,\mu\sigma}I_{\alpha\beta,\nu\tau})]\}.$$

with $I_{\alpha\beta,\mu\nu} \equiv \frac{1}{2}(\eta_{\alpha\mu}\eta_{\beta\nu} + \eta_{\alpha\nu}\eta_{\beta\mu})$.

Consequently,

$$i\mathcal{M}^{(b)}_{\epsilon\zeta} = \frac{\pi^2G^2M^2}{2|\mathbf{q}|}[9p'_{\zeta}p_{\epsilon} + (\eta_{\epsilon\zeta} - 2\eta_{\epsilon 0}\eta_{\zeta 0})p\cdot p' - p'_{\epsilon}p_{\zeta} + 2E(p'_{\epsilon}\eta_{\zeta 0} + p_{\zeta}\eta_{\epsilon 0} - E\eta_{\epsilon\zeta})],$$

where E is the energy of the incident photon.

C. The $\mathcal{M}^{(c)}_{\lambda\tau}$ amplitude

Figure 2(c) tells us that

$$i\mathcal{M}^{(c)}_{\lambda\tau} = i\int \frac{d^3\mathbf{k}}{(2\pi)^3} h^{\mu\nu}(\mathbf{p}'-\mathbf{k})V_{\mu\nu,\sigma\tau}(p',k) \times \frac{-i\eta^{\sigma\rho}}{k^2 + i\epsilon}V_{\alpha\beta,\lambda\rho}(k,p)h^{\alpha\beta}(\mathbf{k}-\mathbf{p}).$$

Regularizing the infrared divergence of this graph by regarding the Newtonian potential $\frac{GM}{r}$ as the limit $\mu \to 0$

of the Yukawa potential $GMe^{-\mu r}/r$, yields

$$i\mathcal{M}^{(c)}_{\lambda\tau} = \frac{GM^2}{4\pi^2}\int d^3\mathbf{k}\,\frac{(\eta^{\mu\nu} - 2\eta^{\mu 0}\eta^{\nu 0})V_{\mu\nu,\sigma\tau}(p', k)\eta^{\sigma\rho}}{[(\mathbf{p}' - \mathbf{k})^2 + \mu^2][(\mathbf{p} - \mathbf{k})^2 + \mu^2]} \times \frac{(\eta^{\alpha\beta} - 2\eta^{\alpha 0}\eta^{\beta 0})V_{\alpha\beta,\lambda\rho}(k, p)}{(\mathbf{p}^2 - \mathbf{k}^2 + i\epsilon)}. \qquad (2)$$

Note, however, that the numerator of the above integral is equal to

$$\begin{aligned} &-\kappa^2\{4E^2(E^2\eta_{\lambda\tau} - Ep'_\lambda\eta_{\tau 0} - Ep_\tau\eta_{\lambda 0} + p'\cdot p\,\eta_{\lambda 0}\eta_{\tau 0}) \\ &\quad + k_\tau(2E^2p'_\lambda - 2E\eta_{\lambda 0}p'\cdot p) + k_\lambda(2E^2p_\tau - 2E\eta_{\tau 0}p'\cdot p) \\ &\quad + k^\alpha[-2E^2\eta_{\lambda\tau}(p'_\alpha + p_\alpha) + 2E(p'_\alpha\eta_{\lambda 0}p_\tau + p_\alpha\eta_{\tau 0}p'_\lambda)] \\ &\quad + k^\alpha k^\beta(p'_\alpha p_\beta\eta_{\lambda\tau} - \eta_{\alpha\lambda}p'_\beta p_\tau - \eta_{\alpha\tau}p_\beta p'_\lambda \\ &\quad + \eta_{\alpha\lambda}\eta_{\beta\tau}p'\cdot p)\} \end{aligned}$$

implying that the evaluation of the integral appearing in Eq. (2) requires the knowledge of a series of integrals of the form

$$(I, I^\alpha, I^{\alpha\beta}) = \int\frac{(1, k^\alpha, k^\alpha k^\beta)d^3\mathbf{k}}{[(\mathbf{p}' - \mathbf{k})^2 + \mu^2][(\mathbf{p} - \mathbf{k})^2 + \mu^2]} \times \frac{1}{(\mathbf{p}^2 - \mathbf{k}^2 + i\epsilon)}. \qquad (3)$$

Instead of displaying the final expression for $\mathcal{M}^{(c)}_{\lambda\tau}$ which, incidentally, is quite long besides being not so illuminating, we exhibit the real part of the results given by the integration of the Eq. (3): the imaginary part of the alluded results gives no contribution to the evaluation of the cross section. We remark that $k^0 = E$.

$$I = 0, \qquad I^\alpha = \left(0, \frac{-\pi^3(\mathbf{p}' + \mathbf{p})^i}{8|\mathbf{p}|^3\sin\frac{\theta}{2}(1 + \sin\frac{\theta}{2})}\right), \qquad I^{00} = 0,$$

$$I^{0i} = I^{i0} = EI^i,$$

$$\begin{aligned} I^{ij} = I^{ji} &= \frac{-\pi^3}{4|\mathbf{p}|(1 + \sin\frac{\theta}{2})}\left[-\eta^{ij} + \frac{\mathbf{q}^i\mathbf{q}^j}{4\mathbf{p}^2\sin\frac{\theta}{2}} \right. \\ &\quad \left. + \frac{(2 + \sin\frac{\theta}{2})(\mathbf{p}' + \mathbf{p})^i(\mathbf{p}' + \mathbf{p})^j}{4\mathbf{p}^2\sin\frac{\theta}{2}(1 + \sin\frac{\theta}{2})}\right]. \end{aligned}$$

D. The unpolarized cross section

We are now ready to determine the unpolarized cross section. Noting that

$$\begin{aligned} i\mathcal{M}^{(0)}_{\mu\nu}i\mathcal{M}^{(c)\mu\nu} &= \frac{64G^3M^3}{\mathbf{q}^2}[p'\cdot p(p'\cdot p - 2E^2)I^{\mu\nu}\eta_{\mu\nu} \\ &\quad - 2p'\cdot pI^{\mu\nu}p'_\mu p_\nu - 2E^2(I^{\mu\nu}p_\mu p_\nu \\ &\quad + I^{\mu\nu}p'_\mu p'_\nu) + 8E^4(I^\mu p_\mu + I^\mu p'_\mu)] \\ &= \frac{16\pi^3G^3M^3E}{\sin^3\frac{\theta}{2}}\left(2 + \sin\frac{\theta}{2}\right)^2\left(1 - \sin\frac{\theta}{2}\right)^2, \end{aligned}$$

$$\begin{aligned} i\mathcal{M}^{(0)}_{\mu\nu}i\mathcal{M}^{(0)\mu\nu} &= \frac{32\pi^2G^2M^2}{\sin^4\frac{\theta}{2}}\cos^4\frac{\theta}{2}, \\ i\mathcal{M}^{(0)}_{\mu\nu}i\mathcal{M}^{(a)\mu\nu} &= 0, \\ i\mathcal{M}^{(0)}_{\mu\nu}i\mathcal{M}^{(b)\mu\nu} &= -\frac{4\pi^3G^3M^3E}{\sin^3\frac{\theta}{2}}\cos^4\frac{\theta}{2}, \end{aligned}$$

we find that the unpolarized cross section has the form

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_0\left[1 + \frac{\pi GME\sin\frac{\theta}{2}}{(1 + \sin\frac{\theta}{2})^2} \times\left(\frac{15 + 14\sin\frac{\theta}{2} + 3\sin^2\frac{\theta}{2}}{4}\right)\right], \qquad (4)$$

where $(\frac{d\sigma}{d\Omega})_0$ is given by Eq. (1).

III. DISCUSSION

It was shown recently that the semiclassical and effective approaches to gravity are equivalent in the limit in which one of the masses involved in the scattering process is huge [9]. In the semiclassical theory, which is utilized in this work, the particles are treated as quantum fields while the gravitation is considered as a classical source. The effective theory of gravity developed by Donoghue [10,11], on the other hand, treats gravity as a quantized field, but the nonanalytical $(\frac{1}{q^2}, \frac{1}{q}, \ln q)$ parts of the Feynman amplitude are separated from the analytical terms $(1, q, q^2, \ldots)$, where q is the exchanged momentum. This approach is only valid for large distances (or low exchanged momentum). Therefore, for a photon of energy E scattered by a particle of mass M ($M \gg E$), the effective theory of gravity leads to the same result as that obtained here [Eq. (4)]. Now, taking into account that these two powerful approaches based on quite different but correct assumptions confirm Eq. (4), we may say that this result is reliable. The computations that rely on the effective method, however, are much harder than those which appeal to the semiclassical approach, besides being exceedingly time consuming. It is also worth mentioning that to second order, the differential cross section for the scattering of a massive scalar boson by a weak gravitational field, treated as an external field, i.e.,

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_0 \left[1 + \frac{\pi GM|\mathbf{p}|\sin\frac{\theta}{2}}{1+\frac{\alpha}{2}}\left(3\alpha + \frac{15-\sin^2\frac{\theta}{2}}{4}\right)\right], \tag{5}$$

where

$$\left(\frac{d\sigma}{d\Omega}\right)_0 = \left(\frac{GM}{\sin^2\frac{\theta}{2}}\right)^2\left(1+\frac{\alpha}{2}\right)^2, \tag{6}$$

and $\alpha = \frac{1-\mathbf{v}^2}{\mathbf{v}^2}$, with $\mathbf{v}$ being the boson incident velocity, is mass dependent [12,13].

Is there any hope of measuring the energy-dependent effect predicted by Eq. (4) in the foreseeable future? It is obvious that in order to detect this possible effect we would have to know beforehand the gravitational deflection angle. Let us then determine the expression for this angle. A straightforward calculation shows that for small angles Eq. (4) reduces to

$$\left(\frac{d\sigma}{d\Omega}\right) \approx \left(\frac{4GM}{\theta^2}\right)^2\left(1 + \frac{15\pi GME\theta}{8}\right). \tag{7}$$

On the other hand, as is well known,

$$\left(\frac{d\sigma}{d\Omega}\right) \approx -\frac{b}{\theta}\frac{db}{d\theta}, \tag{8}$$

where b is the impact parameter. From Eqs. (7) and (8) we arrive at the expression for the scattering angle, i.e.,

$$\theta = \frac{4GM}{b} + \frac{60\pi^2 G^3 M^3}{\lambda b^2}, \tag{9}$$

where we have assumed $\lambda \gg GM$, with λ being the photon wavelength. Therefore,

$$\frac{\Delta\theta}{\theta_E} \equiv \frac{\theta - \theta_E}{\theta_E} = \frac{15\pi^2 G^2 M^2}{\lambda b}, \qquad \theta_E \equiv \frac{4GM}{b}.$$

Currently, the only measurements of the gravitational deflection we have at our disposal are those related to the solar deflection of photons. Now, for the sun $GM \sim 1.5$ km, which would require photons of very low energy for the measurement of the corresponding bending. Unfortunately, no measurement of gravitational deflection in this energy range is available nowadays. Consequently, the detection of the energy-dependent effect predicted by Eq. (4) seems unlikely in the immediate future.

We call attention to the fact that the second order contribution we have obtained [see Eq. (9)] requires $\lambda \gg GM$, and for this reason it does not agree with the classical calculations found in the literature [14–16]. Accordingly, if we want to recover the classical results, we need a new approach for calculating the scattering angle θ that could be applied to the scattering of particles with small wavelength. Luckily, the so-called JWKB method [17] is fitting for analyzing this type of scattering ($\frac{2\pi b}{\lambda} \gg 1$). Using this method, it can be shown that for $\frac{GM}{b} \ll 1$, the semiclassical deflection is given by [18]

$$\begin{aligned}\theta_{\text{JWKB}} = {} & \frac{4GM}{b}\left(1 - \frac{\lambda^2}{4\pi^2 b^2}\right) + \frac{15\pi G^2 M^2}{4b^2} \\ & \times \left(1 - \frac{3\lambda^2}{10\pi^2 b^2} + \frac{\lambda^4}{16\pi^4 b^4}\right) + \frac{128 G^3 M^3}{3b^3} \\ & \times \left(1 - \frac{3\lambda^2}{8\pi^2 b^2} + \frac{3\lambda^4}{32\pi^4 b^4} - \frac{\lambda^6}{64\pi^6 b^6}\right) \\ & + \cdots,\end{aligned} \tag{10}$$

which in the geometrical optics limit ($\lambda \to 0$) coincides with the result computed classically [14–16]. Note that θ_{JWKB} is also energy dependent, as it should be.

Computing

$$\frac{\Delta\theta}{\theta_E} \equiv \frac{|\theta_{\text{JWKB}} - \theta_E|}{\theta_E},$$

to first order in $\frac{GM}{b}$, yields

$$\frac{\Delta\theta}{\theta_E} = \frac{\lambda^2}{4\pi^2 b^2}. \tag{11}$$

For the sun, for instance, and for a typical frequency $\nu = 1$ GHz, $\frac{\Delta\theta}{\theta_E} \sim 10^{-20}$. Unluckily, the detection of a so tiny effect is beyond the current technology. Fortunately, all indications are that around the year of 2010 it will be possible to achieve angular resolution as fine as 300 nanoarcseconds via X-ray interferometry [19], implying consequently in the likelihood of detecting this extremely small deviation from the value predicted to first order in $\frac{GM}{b}$ by Einstein geometrical theory.

Last but not least, we remark that the second order correction for the scattering angle of a photon by a weak gravitational field, treated as an external field (which for large wavelength was computed using the semiclassical theory of gravity, whereas for small wavelength was obtained via the JWKB method), depends upon the energy (or the wavelength). Now, since any experiment carried out to test this dependence on the energy utilizes the knowledge of the gravitational deflection angle, which, of course, is an extended object, the aforementioned correction can be correctly interpreted as a violation of the CEP but not of the EEP. To conclude, we point out that the examples of violation of the CEP discussed in the Introduction, together with those we have just considered, seem to indicate that quantum mechanics and the CEP are irreconcilable.

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