Effects of the cosmological expansion on the bubble nucleation rate for relativistic first-order phase transitions

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I calculate the first corrections to the dynamical preexponential factor of the bubble nucleation rate for a relativistic first-order phase transition in an expanding cosmological background by estimating the effects of the Hubble expansion rate on the critical bubbles of Langer's statistical theory of metastability. I also comment on possible applications and problems that arise when one considers the field theoretical extensions of these results (the Coleman–De Luccia and Hawking-Moss instantons and decay rates).

I. INTRODUCTION

The modern nucleation theory of first-order phase transitions is based on the results of Langer in [1] and associated works. These were generalized to quantum field theory [2], at finite temperature [3], and in curved space-time [4,5]. The importance of these considerations is highlighted by the fact that the original inflationary models based on a first-order cosmological phase transition [6–9] were soon ruled out [10] and replaced by various slow-roll models with numerous fine-tuning problems. Of related recent interest are theories of the landscape and the multiverse [11,12] for which a detailed knowledge of the false vacuum decay rate in curved space-time is of vital importance and may lead to a cosmological determination of physical parameters such as the cosmological constant.

Langer's original theory led to the statistical determination of the bubble nucleation rate, Γ, that gives the number of critical sized metastable bubbles of the new phase nucleated per unit volume and per unit time,

$$\Gamma = \frac{dN}{d^3xdt} = \Omega \frac{\kappa}{2\pi} \exp(-F/T), \qquad (1)$$

where Ω and κ are kinematical and dynamical factors, respectively. Ω is proportional to the physical volume of the system, κ is the growth rate of the metastable configuration, and F its free energy. If μ is a characteristic mass scale of the theory, of the order of the temperature T, then Ω and κ are of order μ^3 and μ respectively, giving Γ the correct overall dimensionality.

In the case of a quantum field theory involving a scalar field ϕ and a Euclidean action functional

$$S(\phi) = \int d^4x \left[\frac{1}{2}(\partial\phi)^2 + U(\phi)\right], \qquad (2)$$

with a potential $U(\phi)$ that has a relative minimum (false vacuum) and an absolute minimum (true vacuum), the bubble nucleation rate or false vacuum decay rate when

the field is trapped in the false vacuum is given by [2]

$$\Gamma = \frac{dN}{d^3xdt} = A \exp(-B), \qquad (3)$$

where $B = S_0$ is the Euclidean action of the instanton that is the solution to the Euclidean equations of motion and A is a preexponential factor that is given by a functional determinant ratio. If μ is again a characteristic mass scale of the theory and λ the coupling constant, then S_0 is of order $1/\lambda$ and A of order μ^4.

If the quantum field theory is considered at finite temperature T that is much higher than the inverse critical bubble radius $1/R_*$ (generically R_* is of order $1/\lambda\mu$) then $B = S_3/T$, where S_3 is the three-dimensional action of the dimensionally reduced instanton and A is of order T^4 [3].

The question that I would like to address here is what happens to the prefactor A when gravitational effects are taken into account, as is the case in cosmological applications. It is generally true in flat space-time, and also when gravitational effects are weak, that the quantity B gives an exponentially smaller factor, hence is more important quantitatively. One can, however, very well envision a situation, in a landscape or multiverse scenario, where both A and B, although still close to their flat space values, have an intricate dependence on the model parameters such as the cosmological constant, with the net result that the rate Γ has an actual maximum at the observed values. In fact, it is not Γ *per se* that is expected to have a peak; one is rather interested in suitably defined quantities that measure the rate of conversion of physical volume to the new phase [10,13] like, for example,

$$p(t) = \exp\left[-\int_{t_0}^{t} dt' \Gamma(t') \frac{4\pi}{3} \left(\frac{R_*(t')}{R(t')} + \int_{t'}^{t} dt'' \frac{V(t'')}{R(t'')}\right)^3\right], \qquad (4)$$

which gives the probability that an arbitrary point in space remains in the false vacuum at time t. Here t_0 is the time that signifies the onset of the cosmological phase transition, $R(t)$ is the cosmological scale factor, $R_*(t)$ is the

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critical bubble radius at the time of nucleation, and $V(t)$ is the velocity with which the bubble wall is expanding. In any case Γ is one of the main inputs necessary for the calculation of these quantities.

The basic problem for the calculation of gravitational effects on A is, of course, the lack of a consistent definition of quantum gravity, so, apart from dimensional considerations, very few methods have been proposed. In [14,15] the prefactor was calculated for processes involving the creation of topological defects, and in [16] corrections to A were estimated with the use of the renormalization group. Here I would like to initiate another possibility which, although limited in scope from the outset, may be quite useful and is in fact, in principle, important, namely the generalization of Langer's original work to the case of curved space-time. Given that the results of [2,3] are the field theoretical generalizations of the statistical theory of [1], it is notable that the corresponding theory, of which the work of [4,5] is supposed to be a generalization, does not exist. The precise formulation of such a theory would involve the generalization of thermodynamics and kinetic theory in curved space-time and will be the subject of future work. Here, instead, I will consider the first gravitational corrections to the preexponential factor for the previously given description of bubble nucleation in first-order phase transitions as given in [17,18]. That is, I will be working in an approximation where the critical bubble radius is much smaller than the horizon size and neglecting the gravitational backreaction, essentially assuming that the related Hawking temperature is much smaller than the temperature and mass scales of the system. As expected, the leading effect of the cosmological expansion will be to increase the bubble nucleation rate and an estimate of the corrections is given. One may compare this with works that investigate the stability of classical and semiclassical configurations in an expanding universe [19–21] where similar results where found, the main difference here being that the bubble configuration considered is already metastable.

Although the result cannot be straightforwardly extended to the case of quantum gravity, the corrections derived suggest that a more complete treatment of the nucleation rate is needed; it is important in first-order inflationary models, landscape and multiverse scenarios, and may also be of relevance in various cases of late time and other first-order cosmological phase transitions [22–31].

In Sec. II, which is essentially the second part of the introduction, I describe the results of [4,5], the problems and the relative corrections expected. In Sec. III, I calculate the first gravitational corrections to the critical bubble configurations of Langer's statistical theory of metastability as described in [17] and I discuss the results and the approximations involved. In Sec. IV, I conclude with some comments.

II. GRAVITATIONAL EFFECTS ON VACUUM DECAY

One is interested in a quantum field theory of a scalar field ϕ in curved space-time with a metric tensor $g_{\mu\nu}$ and Euclidean action

$$S = \int \sqrt{-g}\left(\frac{1}{16\pi G}\mathcal{R} + \frac{1}{2}g_{\mu\nu}\partial^\mu\phi\partial^\nu\phi + U(\phi)\right), \quad (5)$$

where $\mathcal{R}$ is the Ricci curvature, $g = \det g_{\mu\nu}$, and for definiteness I will consider a potential $U(\phi)$ that is everywhere positive and has two minima, as before, a relative (false vacuum) and an absolute (true vacuum). I will assume again that the mass scale is μ and the coupling is λ. If the field is trapped in the false vacuum and the value of the potential there is ε, this will effectively be a cosmological constant $\Lambda = 16\pi G\varepsilon$ and space-time will be de Sitter with a Hubble expansion rate $H^2 = 8\pi G\varepsilon/3$. From this point on, one makes several assumptions using the insight from the flat space-time results described in the previous section. First of all, one assumes that the solutions to the Euclidean equations of motion will again describe tunneling and determine the exponential factor, B; that is, one expects that Euclidean quantum gravity and the associated path integral have various features similar to the flat case.

Then Γ will determine the nucleation rate of bubbles of the true vacuum, which is also de Sitter space-time, with a smaller value of the cosmological constant. In terms of the flat case critical bubble radius R_0, which is of order $1/\mu\lambda$, and the horizon radius $R_{dS} = 1/H$, the results of [4] estimate B in the thin-wall approximation as a correction to the flat case value, B_0,

$$B = \frac{B_0}{[1 + (R_0/2R_{dS})^2]^2} \quad (6)$$

and the radius of the bubble in the presence of gravity, R_*, as

$$R_* = \frac{R_0}{[1 + (R_0/2R_{dS})^2]}, \quad (7)$$

where $R_0 = 3\sigma/\varepsilon$ is the bubble radius in the absence of gravity and σ is the bubble surface tension which is the same for the bubble with gravity in the thin-wall approximation.

It is implicit in the CDL formalism that some sort of dilute instanton gas approximation must exist in de Sitter space-time in order for the instanton action to exponentiate; that is, one expects an approximation of the sort $R_* \ll R_{dS}$, in which case the gravitational corrections in (6) do not give an exponentially small correction to Γ, in fact they may be comparable to gravitational corrections that exist in the preexponential factor. The expression for B in the opposite limiting case where gravitational effects are important, when, for example, the bubble radius is comparable to the horizon size, may also be completely different than the Coleman–De Luccia (CDL) result, such as is the

case in the critical Hawking-Moss (HM) solution [5]. Thus the CDL expression has problems in its interpretation in both limiting cases. Generally, from a strict point of view, in the weak gravity limit where the gravitational corrections in the exponent are of the same order as the preexponential factor, which has not been calculated, one may say that the CDL expression, although natural and reducible to the flat case result, is not complete.

Another curious coincidence arises when one ponders the possibility of a thermal interpretation of the gravitational effects, namely, when one considers the fact that de Sitter space-time has a naturally defined Hawking temperature,

$$T_{dS} = \frac{H}{2\pi}. \qquad (8)$$

Provided that a suitable frame is chosen, and a thermal interpretation can be given [32,33], one may expect that, similarly to the transition from $A \sim \mu^4$ to $A \sim T^4$ in the limit of high temperature, $T \gg 1/R_*$, in the flat case, a similar limit may apply in the gravitational case. This would lead to the approximation $A \sim H^4$ which is expected to hold in other cases in the literature [14,15]. However, the translation of the high temperature approximation in the CDL case would read $H \gg \lambda\mu$, and it is well known [34,35] that, when relations like $H^2 > 4U''$ hold, the CDL instanton does not even exist. One then may expect that tunneling is described by the HM solution with the preexponential factor being approximated by H^4, again without much justification.

In view of the possible applications of these results (cosmological phase transitions, landscape and multiverse scenarios) one would like to have better descriptions and quantitative estimates for Γ. In principle, one expects that

$$\Gamma = \frac{dN}{d^3x dt} = \sqrt{-g} A(\mu, \lambda, \mathcal{R}, \Lambda) \exp(-B) \qquad (9)$$

is the general covariant expression for the production of bubbles of the true vacuum per unit coordinate four-volume, where $g = \det g_{\mu\nu}$, A has dimensionality μ^4 and is a function of the mass scale μ of the theory, the coupling constants λ, the Ricci scalar $\mathcal{R}$, and the cosmological constant Λ. Also in this expression, B is a dimensionless function of the same parameters, and the only constraint for both A and B is that they reduce to the flat space-time values in the limit of weak gravity. One would expect a relation such as (9) to emerge from a saddle point evaluation of a path integral and to be meaningful for the cases where the bubble radius is much smaller than the horizon; when they are comparable even an interpretation of (9) is not straightforward.

It should be noted that gravitational corrections to the preexponential factor are expected to exist and can be also estimated in a similar approximation with an entirely different method that applies the renormalization group to the CDL expression [16]. As was mentioned in the

Introduction, here I will try to obtain some more insight into this expression by calculating the first gravitational corrections to the dynamical factor κ (of dimension μ) that appears in (1), as it was calculated in [17]. The kinematical prefactor Ω in (1) gives an expression of the form $\sqrt{-g}\mu^3$ (times again a dimensionless factor that contains gravitational corrections), and similar corrections apply in the generalization of the free energy of a metastable system in curved space-time. It is assumed that a thermodynamical description of the problem can be given in the rest frame of the fluid and the cosmological expansion can be treated as a perturbation. This is expected to hold provided that a characteristic reaction time, τ, for the internal fluid interactions is much smaller than H^{-1}. I will also assume that we are in the limits of the thin-wall approximation in order to treat H as constant in the calculation, and also to compare with the CDL results. In principle, however, the equations can be solved self-consistently with a variable H.

In summary, the approximations that will be assumed throughout will be that the bubble radius is much smaller than the horizon, that one is within the limits of the thin-wall approximation, and that relations like $\tau \ll H^{-1}$ hold. Naturally also the temperature and mass scales of the theory are assumed much smaller than the Planck scale.

The physical situation considered is a metastable relativistic fluid in a spatial extend that is large enough in order to feel the cosmological expansion, yet small enough in order to treat it as a perturbation. As such, the results are not straightforwardly generalized to quantum field theory; they are intended, however, to provide some insight to similar considerations. It should be stressed, also, that what is implied is not a breakdown of the semiclassical approximations leading to the CDL result, but an interesting dependence of the preexponential factor on the cosmological expansion rate. This is expected for very small values of the cosmological constant Λ, or $R_* \ll R_{dS}$. It is suggested that the dependence of Γ on Λ (or H) is much more intricate than what is described by (6) and the associated expressions of [36].

III. FIRST GRAVITATIONAL CORRECTIONS TO LANGER'S THEORY OF METASTABILITY

I will calculate the effects of cosmological expansion described by a metric

$$ds^2 = -dt^2 + R^2(t)[dx^2 + dy^2 + dz^2] \qquad (10)$$

on the critical bubble solution for a relativistic metastable fluid given in [17] which has been used for the description of the QCD phase transition. There the dynamical growth rate of the bubble was described by taking into account the dissipative effects of a fluid with an energy momentum tensor

$$T_{\mu\nu} = p g_{\mu\nu} + (\rho + p) u_\mu u_\nu + \tilde{T}_{\mu\nu} \qquad (11)$$

with

$$\tilde{T}_{\mu\nu} = -\eta(\partial_\mu u_\nu + \partial_\nu u_\mu + u_\mu u^\alpha \partial_\alpha u_\nu + u_\nu u^\alpha \partial_\alpha u_\mu)$$
$$- (\zeta - 2\eta/3)(\partial_\alpha u^\alpha)(g_{\mu\nu} + u_\mu u_\nu). \qquad (12)$$

Here ρ and p are the fluid energy density and pressure, respectively, and $u^\mu = (1, \vec{v})$ is the four-velocity for a relativistic fluid. As an additional phenomenological input, one needs the shear and bulk viscosities, η and ζ respectively.

The dynamical growth rate, κ, is calculated by considering a spherically symmetric, exponentially growing, perturbation

$$\gamma(\vec{r}, t) = \gamma(r)e^{\kappa t}, \qquad (13)$$

$$\vec{v}(\vec{r}, t) = v(r)e^{\kappa t}, \qquad (14)$$

of the also spherically symmetric metastable configuration of a critical bubble $\rho = \bar{\rho}(r)$, $\vec{v} = 0$. When $\rho(\vec{r}, t) = \bar{\rho}(r) + \gamma(\vec{r}, t)$ and the remaining equations are used in the equations of motion, self-consistency of the solution determines κ.

As explained before, I will calculate the first gravitational corrections by determining the effects of the Hubble drag term, $H = \dot{R}/R$, on the results of [17]. Thus, I will assume that a natural free energy description of the problem exists in the rest frame of the fluid. Although the considerations here are similar to works on stability of classical and solitonic configurations in an expanding universe [19–21], it is important to realize that we are not investigating quite the same problem; we are rather interested in the effects of cosmological expansion on the actual flat space-time growth rate, κ_0, of the already unstable (metastable) solution.

The equations of motion

$$\nabla_\mu T^{\mu\nu} = 0 \qquad (15)$$

can be solved approximately in two regions: region (I) which extends from just inside the bubble radius R_* to a few correlation lengths, ξ, outside the bubble surface, and region (II) in distances r greater than the bubble radius R_* plus a few correlation lengths. The analysis of [17] shows that the fluid velocity $v(r)$ has the following behavior: it is very close to zero from the center of the bubble up to a few ξs inside the bubble surface, then it rises abruptly until in region (I) it starts to fall like $1/r^2$ and in region (II) it falls exponentially to zero. This behavior will be modified by the Hubble drag term, and by matching the two solutions at $r \approx R_* +$ a few ξs, one obtains the corrected $\kappa = \kappa_0 + \delta\kappa$.

The notation is as follows: overbars will denote the critical bubble solution, subscripts 0 will denote the flat space-time values, subscripts I and II the respective regions, and Δ will denote the difference of a quantity between the equilibrium (subscript t) and the metastable (subscript f) phase. Also, as before, ξ will denote the correlation length and R_* the critical bubble radius in the presence of gravity.

One will also make use of the enthalpy density, $w = \rho + p$, and the bubble surface tension, σ, in terms of which the flat case growth rate has been calculated as [17]

$$\kappa_0 = \frac{4\sigma(\zeta + 4\eta/3)}{(\Delta w)^2 R_0^2} \qquad (16)$$

and the kinematical prefactor as

$$\Omega_0 = \frac{2}{3\sqrt{3}}\left(\frac{\sigma}{T}\right)^{3/2}\left(\frac{R_0}{\xi}\right)^4 \text{Vol}, \qquad (17)$$

where the volume of the system, Vol, is usually divided out as in (1).

One expects these relations to carry over in the presence of gravity, in our approximations, by replacing R_0 by R_*, keeping the surface tension σ approximately the same in the thin-wall approximation, and the physical volume of the system giving a factor of $\sqrt{-g}$. The cosmological expansion, however, described by the Hubble drag term, will give an additional contribution to the dynamical prefactor, that can be calculated from the equations of motion.

The $\nu = 0$ equation of motion

$$\kappa\gamma(r) = -\frac{1}{r^2}\frac{d}{dr}[r^2\bar{w}v(r)] - 3H\bar{w} \qquad (18)$$

gives in region (I)

$$v_{\mathrm{I}}(r) = \frac{C}{r^2} + Hr + H\frac{R_*^3}{r^2}\frac{\Delta w}{w_f} \qquad (19)$$

and the $\nu = i$ equation of motion can be simplified in region (II) as

$$(\kappa + 3H)\bar{w}v(r) = (\zeta + 4\eta/3)\frac{d}{dr}\left(\frac{1}{r^2}\frac{d}{dr}[r^2v(r)]\right) \qquad (20)$$

to give

$$v_{\mathrm{II}}(r) = D\left(\frac{\alpha}{r} + \frac{1}{r^2}\right)e^{-\alpha r}, \qquad (21)$$

where

$$\alpha^2 = \frac{(\kappa + 3H)w}{(\zeta + 4\eta/3)}. \qquad (22)$$

The solutions depend on two constants, C and D, the matching is done for definiteness at $r = R_* + c\xi$, with c a numerical constant of order unity, and we get another condition from the fact that, for $H = 0$, the solution should be consistent with the previous result (16). The final result for the corrected $\kappa = \kappa_0 + \delta\kappa$ is

$$\delta\kappa = \kappa_0 H\sqrt{\frac{R_*}{c\xi}}\sqrt{\frac{R_*(\Delta w)}{\sigma}}\left(R_* + \frac{1}{\alpha_0}\right)\left(1 + \frac{\Delta w}{w_f}\right). \qquad (23)$$

In order to get an order-of-magnitude estimate for this result, one can assume that for sufficiently high tempera-

ture T, one can approximate η, $\zeta \sim T^3/\lambda^2$, $\sigma \sim T^3/\lambda$ to finally obtain the estimate for the corrections to the prefactor that arise from the dynamical growth rate:

$$A \sim \mu^3\left(T + \frac{H}{\lambda^{3/2}}\right). \tag{24}$$

We see that the leading effect of the cosmological expansion has been, indeed, to increase the bubble nucleation rate, and can be significant when $H \sim \lambda^{3/2}T$ while, at the same time, $H \ll \mu$, in accordance with our approximations. The surface tension was assumed unchanged in the thin-wall approximation, the correction to the bubble radius can be estimated from (7), it is also subleading, however, in the approximation used. It should be noted that this result will also be modified when gravitational corrections to Ω, the critical bubble radius, and surface tension are incorporated, when one goes beyond the limits of our approximations. In any case, even when other physical situations are considered, the corrections estimated here also exist and can be calculated, for example, by a self-consistent solution of equations like (18) and (20). What is more important, and supportive of the arguments of the previous section, however, is that the corrections estimated here are different than what is usually assumed as a naive, dimensional preexponential factor, for example μ^4 or T^4, and have an interesting dependence on Λ (or H).

IV. COMMENTS

The main purpose of this work has been to motivate the suggestion that a fuller treatment of the theory of metastability in curved space-time is needed, in order to supply the Coleman–De Luccia result with possible additional gravitational corrections that may provide valuable insight to applications in cosmological problems.

One way to approach this problem is to attempt a generalization of Langer's original theory of statistical metastability. The main difficulties of this approach stem from the fact that proper definitions of the thermodynamical quantities are needed, presumably with the use of relativistic kinetic theory in the expanding universe. It was implicitly assumed here that such an extension can be done in the fluid's rest frame and the first corrections to the flat space-time results that were presented show, indeed, the expected increase in the bubble nucleation rate due to the cosmological expansion. The fuller treatment of the relativistic thermodynamics of first-order phase transitions is expected to give additional contributions to the nucleation rate, similar to the ones presented here and generally different than the usually assumed preexponential factor.

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