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Moduli evolution in the presence of thermal corrections

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We study the effect of thermal corrections on the evolution of moduli in effective supergravity models. This is motivated by previous results in the literature suggesting that these corrections could alter and even erase the presence of a minimum in the zero temperature potential, something that would have disastrous consequences in these particular models. We show that, in a representative sample of flux compactification constructions, this need not be the case, although we find that the inclusion of thermal corrections can dramatically decrease the region of initial conditions for which the moduli are stabilized. Moreover, the bounds on the reheating temperature coming from demanding that the full, finite temperature potential, has a minimum can be considerably relaxed given the slow pace at which the evolution proceeds.

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I. INTRODUCTION

The study of moduli evolution is an active field of research in the context of the phenomenology and cosmology of string models. At the level of the $D = 4$ effective theory, which we normally assume to be $\mathcal{N} = 1$ supergravity (SUGRA), moduli are complex scalar fields, some of which physically parametrize the size and shape of the six or seven original string dimensions that have been compactified. It is therefore mandatory that any realistic model provides, at the end of the day, moduli with a non trivial vacuum expectation value at the right scale (which is the Planck mass, M_P for conventional small extra dimensions), and that the minimum corresponds to almost Minkowski space and supersymmetry broken, in order to connect with the standard model. Throughout the past 20 years there has been steady progress in our understanding of the dynamics of moduli, with two outstanding problems having to be addressed: the first one is the recurrence of a negative (i.e. anti-de Sitter, AdS) vacuum energy for every model in which moduli were successfully stabilized, while the second one is the fact that these potentials are so steep along certain directions that, from the dynamical point of view, it looked impossible that given any initial conditions away from their minimum, the moduli would end up in it. This issue was first pointed out by Brustein and Steinhardt [1] and is commonly known as the problem of the "runaway dilaton."

Concerning the first problem, namely, the recurrence of AdS solutions within all successful attempts at stabilizing moduli, recent developments in the context of flux compactifications in type IIB string theory [2] have opened up new ways of trying to achieve either Minkowski or de Sitter (dS) vacua. In particular, the mechanism presented by Kachru *et al.* [3] (KKLT from now on) realized this by adding D-terms to a SUSY-preserving, AdS F-term vacuum. Although not entirely correct in the context of supergravity (see [4,5] for some criticism, and [6,7] for proposed solutions), its main features have triggered an enormous amount of interesting work, and subsequent progress in this field through the past years (in particular, for explicit string realizations, see [8,9]).

Also on the topic of runaway moduli, in general, substantial developments have taken place in the past decade. For a range of initial conditions the problem can be alleviated considerably by considering a background perfect fluid which decelerates the fields and prevents them from passing the barrier dividing the physical vacuum from the runaway one [10–16]. Thermal corrections, however, are potentially dangerous as they modify the shape of the potential and, at high temperatures, the physical vacuum is entirely lost [17,18]. However, there is a potential limit to the extent to which this argument can be used. In the very early Universe as the value of the Hubble parameter, becomes close to the Planck scale, scattering processes

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are unable to establish thermal equilibrium because they do not have sufficient time compared to the expansion rate of the Universe [19]. In this era thermal corrections arising out of these scatterings would not be present, and so the physical vacuum would not be destabilized. In the context of the standard model Enqvist and Sirkka considered the thermalization of a hot QCD gas in the early universe, and calculated the critical temperature above which the Universe can not thermalize to be $T_{\rm crit} = 3 \times 10^{14}$ GeV [20]. In Ref. [21] the authors used this argument in considering racetrack inflation and assisted moduli stabilization.

In this work we present a detailed quantitative analysis of the effect of these thermal corrections on the region of initial conditions leading to stabilization of the moduli, for a given SUGRA model. Given that we are working in regimes beyond the standard model, where we do not have a proper handle on the conditions under which we will move out of thermal equilibrium, we adopt two approaches. The more theoretical is to accept that the Universe may have been in a period of thermal equilibrium close to the Planck scale and investigate the impact of thermal corrections in those regimes. The second is to take seriously the QCD bound, and investigate the impact of the corrections in these lower temperature regimes. In both cases we will conclude that thermal corrections do decrease the area of the region of stabilization by a similar relative amount. This effect from the thermal coupling will act on top of the previous results, where the absolute size of the stabilization region decreases for a smaller initial density of the background fluid $\rho_r^{\rm init}$ [16].

II. EQUATIONS OF MOTION

In this work, we will be studying two string theory models that can be described by a four dimensional $\mathcal{N} = 1$ effective supergravity theory with action of the form

$$S = -\int d^4x\sqrt{-g}\left(\frac{1}{16\pi G}R - \mathcal{L}_\Phi + F(g, T)\right), \quad (1)$$

where

$$\mathcal{L}_\Phi = -K_{i\bar{j}}\partial_\mu\Phi^i\partial^\mu\bar{\Phi}^{\bar{j}} - V, \quad (2)$$

and $K_{i\bar{j}} = \partial^2K/\partial\Phi^i\partial\bar{\Phi}^{\bar{j}}$ is the Kähler metric; Φ^i are complex moduli scalar fields; $V(\Phi)$ is the scalar potential and G is the 4-dimensional Newton constant. The free energy $F(g, T)$ acts as a Lagrangian density of matter fields. For a SU(N_c) gauge theory with N_f multiplets at high temperature T, the free energy has a perturbative expansion in terms of the gauge coupling $g = g(\Phi_R)$, where Φ_R denotes the real part of the moduli fields Φ,

$$F(g, T) = (a_0 + a_2g^2)T^4, \quad (3)$$

and the parameters a_0 and a_2 are given by

$$a_0 = -\frac{\pi^2}{24}(N_c^2 + 2N_cN_f - 1), \quad (4)$$

$$a_2 = \tfrac{1}{64}(N_c^2 - 1)(N_c + 3N_f). \quad (5)$$

We will be treating a_0 and a_2 as variables which can be varied in order to test the results for a range of possible values of N_c and N_f. We can obtain the energy density and pressure of this thermal fluid as $p_r = -F$ and $\rho_r = -p_r + Tdp_r/dT$, hence

$$\rho_r = -3a_0(1 + rg^2)T^4, \quad (6)$$

where $r = a_2/a_0$.

In principle, a second component of relativistic particles that only interacts with the moduli fields gravitationally can also be present in the dynamics. Assuming both components of radiation to be in thermal equilibrium, the non interacting radiation will have an energy density given by $\rho_{\rm B} = \pi^2g_*T^4/30$ where g_* is the number of effective degrees of freedom at the temperature T of the thermal fluid ρ_r. In this case, the equations of motion above are still valid with a_0 being replaced by $a_0 \to a_0 - \pi^2g_*/90$. The effective $|a_0|$ can then be very large, so that $r \approx 0$, effectively washing out the effects of the thermal corrections on the moduli evolution.

The effective scalar potential for the moduli in four dimensional $\mathcal{N} = 1$ supergravity is given by

$$V = e^K(K^{i\bar{j}}D_iWD_{\bar{j}}\bar{W} - 3W\bar{W}), \quad (7)$$

where $K^{i\bar{j}}$ is the inverse Kähler metric and $D_iW = \partial_iW + \partial_iKW$ is the Kähler covariant derivative acting on the superpotential. In general, the Kähler potential K is a function of the real parts of the fields and, for most string compactifications, acquires the form

$$K = -\sum_i \ln(\Phi_i + \bar{\Phi}_i), \quad (8)$$

where the sum is understood over all moduli Φ_i. As for the superpotential, which encodes the dynamics of these fields, a combination of flux terms (normally polynomials in the different moduli) and non perturbative effects (instantons and gaugino condensation being the most well-known ones [22–25]) will provide the potential with a non trivial vacuum structure. For the purposes of showing the effects of thermal corrections, we use the toy models of KKLT [3] and Kallosh and Linde [26] (KL from now on), given that they capture the essential features of the cosmological problems usually attributed to moduli.

The equations of motion follow from the variation of the action (1). Considering homogeneous fields evolving in a spatially flat Friedmann-Robertson-Walker spacetime background, the equations of motion for the complex fields yield

$$\ddot{\Phi}^i + 3H\dot{\Phi}^i + \Gamma^i_{jk}\dot{\Phi}^j\dot{\Phi}^k + K^{i\bar{j}}\partial_{\bar{j}}V = \frac{r\rho_r}{3(1+rg^2)}K^{i\bar{j}}\partial_{\bar{j}}g^2, \tag{9}$$

where $\dot{\Phi}^i = \partial\Phi^i/\partial t$, $\partial_{\bar{j}}V = \partial V/\partial\bar{\Phi}^{\bar{j}}$, and the connection on the Kähler manifold has the form

$$\Gamma^i_{jk} = K^{i\bar{l}}\frac{\partial K_{j\bar{l}}}{\partial\Phi^k}. \tag{10}$$

In addition, the Hubble rate $H \equiv \dot{a}/a$, where $a(t)$ is the scale factor of the Universe, is given by the Friedmann equation

$$3H^2 = M_{\rm P}^{-2}(\rho_\Phi + \rho_r) = M_{\rm P}^{-2}(K_{i\bar{j}}\dot{\Phi}^i\dot{\bar{\Phi}}^{\bar{j}} + V + \rho_r), \tag{11}$$

with $M_{\rm P}^{-2} = 8\pi G$ (or $M_{\rm P} = 2\times 10^{18}$ GeV) and $\rho_\Phi = K_{i\bar{j}}\dot{\Phi}^i\dot{\bar{\Phi}}^{\bar{j}} + V$ and ρ_r are the energy densities of the evolving moduli fields and of the thermal fluid, respectively. In what follows we set $M_{\rm P} = 1$.

We need to understand now how the temperature T relates to the values of the fields and the scale factor of the universe. To this end we note that the equations of motion for the scalar fields can be rewritten as

$$\dot{\rho}_\Phi = -3H(\rho_\Phi + p_\Phi) + \tfrac{1}{3}\partial_i\rho_r\dot{\Phi}^i + \tfrac{1}{3}\partial_{\bar{j}}\rho_r\dot{\bar{\Phi}}^{\bar{j}}, \tag{12}$$

where the pressure of the moduli fields is defined as $p_\Phi = K_{i\bar{j}}\dot{\Phi}^i\dot{\bar{\Phi}}^{\bar{j}} - V$. By requiring conservation of the total energy density $\rho = \rho_\Phi + \rho_r$ we must have

$$\dot{\rho}_r = -4H\rho_r - \tfrac{1}{3}\partial_i\rho_r\dot{\Phi}^i - \tfrac{1}{3}\partial_{\bar{j}}\rho_r\dot{\bar{\Phi}}^{\bar{j}}, \tag{13}$$

which, upon integration, gives a solution for ρ_r of the form

$$\rho_r = \rho_r^{\rm init}\left(\frac{a_{\rm init}}{a}\right)^4\left(\frac{1+rg^2(\Phi_R^{\rm init})}{1+rg^2(\Phi_R)}\right)^{1/3}. \tag{14}$$

Comparing with Eq. (6), the evolution of the temperature can be seen to be

$$T = T_{\rm init}\frac{a_{\rm init}}{a}\left(\frac{1+rg^2(\Phi_R^{\rm init})}{1+rg^2(\Phi_R)}\right)^{1/3}. \tag{15}$$

It is worth splitting the equations of motion for the complex scalar fields into those for their real and imaginary parts

$$\ddot{\Phi}^i_R + 3H\dot{\Phi}^i_R + \Gamma^i_{jk}(\dot{\Phi}^j_R\dot{\Phi}^k_R - \dot{\Phi}^j_I\dot{\Phi}^k_I) + \frac{1}{2}K^{i\bar{j}}\partial_{j_R}V = \frac{1}{6}K^{i\bar{j}}\partial_{j_R}g^2\frac{r\rho_r}{1+rg^2}, \tag{16}$$

$$\ddot{\Phi}^i_I + 3H\dot{\Phi}^i_I + \Gamma^i_{jk}(\dot{\Phi}^j_I\dot{\Phi}^k_R + \dot{\Phi}^j_R\dot{\Phi}^k_I) + \tfrac{1}{2}K^{i\bar{j}}\partial_{j_I}V = 0, \tag{17}$$

where now Φ^i_R (Φ^i_I) refers to the real (imaginary) part of the scalar fields and ∂_{j_R} (∂_{j_I}) are used to denote the derivative of the potential with respect to the real (imaginary) parts of the fields, respectively. We note that, given an initial value of the energy density of the thermal fluid, $\rho_r^{\rm init}$, the equations of motion and the Friedmann equation only depend on the ratio $r = a_2/a_0$ but not on the specific values of a_0 and a_2.

Given Eqs. (9) and (16) we can define an effective scalar potential for the fields,

$$V_{\rm eff}(\Phi) = V(\Phi) - \frac{1}{3}\frac{r\rho_r g^2}{1+rg^2}, \tag{18}$$

up to a constant. With a coupling constant of the form [18]

$$g^2 = \frac{c}{\Phi}, \tag{19}$$

where c is a constant, it is clear that, at high temperature (large ρ_r), the effective potential can look very different from V and, in particular, it can be devoid of a minimum, limiting the stabilization of the moduli or the maximum temperature allowed. This is the problem raised in Ref. [18].

III. KKLT MODEL

The possibility of finding de Sitter vacua in string theory with a stabilized volume modulus, σ, was put forward in Ref. [3], and has been widely adopted in subsequent work. The key ingredient was to consider the combination of non perturbative effects and an additional flux term in the superpotential

$$W = W_0 + Ae^{-\alpha\sigma}, \tag{20}$$

which, together with the usual Kähler potential

$$K = -3\ln(\sigma + \bar{\sigma}), \tag{21}$$

defines the F-term of the SUGRA potential, see Eq. (7). It has been known for many years now that, in this context, it is possible to stabilize σ, although giving rise to an AdS vacuum. As pointed out in Ref. [3], however, if contributions from either anti-D3 or D7 branes are included, an additional D-type term of the form

$$V_D = \frac{C}{\sigma_R^3}, \tag{22}$$

is generated, where we write $\sigma = \sigma_R + i\sigma_I$. By suitably tuning the value of C one can move to a de Sitter—or even Minkowski-vacuum. The scalar potential for σ has an extremum in σ_I for $\alpha\sigma_I = n\pi$, with n an integer. Depending on the sign of $W_0\cos(\alpha\sigma_I)$ this can be either a maximum or a minimum.

In this work we are interested in studying the cosmological evolution of the field σ as it rolls towards its minimum. Previous analysis addressing the same issue were published in Refs. [14,16], however, without taking into account the effect of the thermal corrections. Here, we compare the profile and area of the region of initial positions of the field, $\sigma_{\rm init}$ that leads to stabilization when the field rolls in the presence of a background perfect fluid with the same region of stabilisation when the field evolves in the presence of thermal corrections. We illustrate that comparison for the KKLT model in Figs. 1 and 2 for two values of the initial energy density of the thermal fluid, $\rho_r^{\rm init}$. The first corresponds to the very high energy regime $\rho_r^{\rm init} = 10^{-4} M_{\rm P}^4$, where the second corresponds to a value which satisfies the condition for thermal equilibrium obtained in [20] for $|a_0| < \mathcal{O}(100)$, namely $\rho_r^{\rm init} = 10^{-13} M_{\rm P}^4$. In both cases the stabilization region is shown with thermal corrections (shaded areas) for given values of the ratio $r = a_2/a_0$ against the stabilization region with just a perfect fluid of radiation (or equivalently, with $r = 0$) for the same initial ρ_r (solid black line).

It proves convenient to work with the canonically normalized field $\phi = \sqrt{3/2}\ln\sigma_R$, instead of σ_R itself. In the lower right panel of the figures we draw the ratio between the area of the two stabilization regions—i.e. the one with thermal corrections ($A_{\rm th}$) and the one without (A)—against $|r|$.

We note that, because the energy density of the thermal fluid must be positive definite, and taking the effective 4D Yang-Mills coupling on the D7 brane $g^2 = 4\pi/\sigma_R$, we must ensure that

$$-\frac{1}{4\pi} e^{\sqrt{2/3}\phi} < r \leq 0. \tag{23}$$

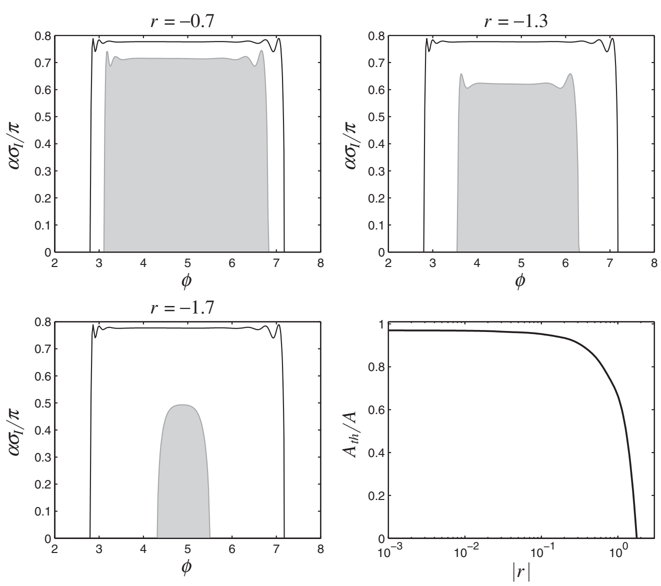

FIG. 1. The panels with the shaded regions show the regions of initial conditions (ϕ, σ_I)—in Planck units—that lead to stabilization of σ at the minimum of the potential for the KKLT model in the presence of thermal corrections. The solid black line corresponds to the region of stabilization in the presence of a perfect fluid ($\mathbf{r} = \mathbf{0}$) with the same initial energy density $\rho_r^{\rm init}$. The lower right corner shows the ratio of the areas of these regions against the ratio $r = a_2/a_0$. We have set $\rho_r^{\rm init} = 10^{-4} M_{\rm P}^4$.

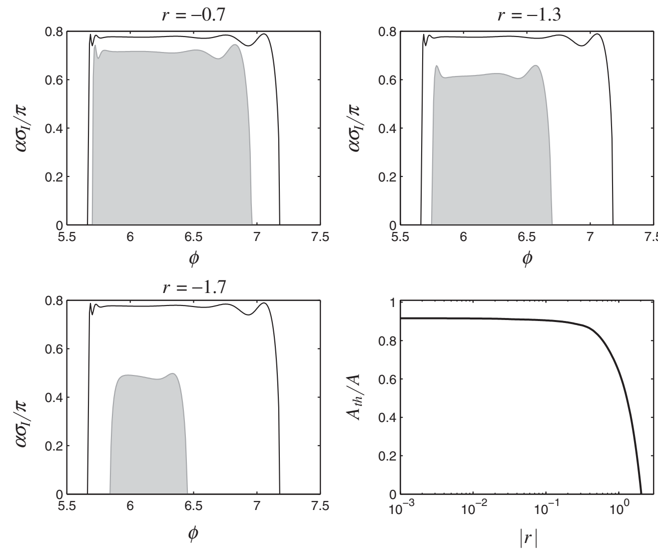

FIG. 2. Same as Fig. 1 but setting $\rho_r^{\rm init} = 10^{-13} M_{\rm P}^4$.

This means that we need to impose a larger lower bound on the initial values of ϕ as we increase $|r|$. In practice this bound is only effective for large initial $\rho_r^{\rm init}$ since the stabilization region decreases naturally for lower initial temperatures.

Returning to Figs. 1 and 2, we show the regions of initial conditions that lead to the field stabilizing in the minimum of its potential. We used the same values of the parameters for the model as in Ref. [14], namely $A = 1.0$, $\alpha = 0.1$, $C = 3 \times 10^{-26}$, and W_0 negative (with $\cos(\alpha\sigma_I) = 1$) such that the minimum at $\sigma_I = 0$ is supersymmetric. Comparing the two figures, we can see that the stabilization region for the lower initial temperature is considerably smaller, a result that was shown previously for the evolution without a thermal coupling. The effect from the thermal coupling can be more clearly seen in the lower right plots, where the ratio of the two stabilization areas is shown against $|r|$. We see that in both cases increasing the strength of the thermal coupling (that is, the relative size of a_2) can effectively eliminate the stabilization region. On the other hand, the curves for the two different choices of initial $\rho_r^{\rm init}$ are very similar, meaning that the relative effects of the thermal coupling seem to be independent of the initial value of the background. In both cases, a value of $|r| \lesssim 1$ implies that the stabilization region is decreased by less than 50%.

Using Eq. (6) with $a_0 = -100$, we can see that our choices for the initial background energy density correspond to initial temperatures of $T \sim 10^{-2} M_{\rm P}$ and $T \sim 10^{-4} M_{\rm P}$. However the dilaton potential only develops a minimum at a temperature $T_{\rm crit} \sim 10^{-8} M_{\rm P}$, so that we can have stabilization of the dilaton even when the evolution starts at a temperature where the minimum does not exist.

This behavior can be easily understood. Thermal corrections act as a background fluid in that they bring an extra contribution into the Friedmann equation. Though at high temperatures the structure of the effective potential is spoiled (that is, we have no minimum), the extra contribution reinforces the frictional term in the equation of motion and forces the field to slow down its evolution as when in the presence of a non interacting perfect fluid. Therefore, for suitable initial conditions, the field can approach the value corresponding to the minimum of the $T = 0$ potential, $\langle \phi \rangle_{T=0}$, when the thermal corrections have already become negligible, i.e. the temperature has decreased below the critical temperature for which the minimum is created. In Fig. 3, we show, in the left panel, the evolution of ρ_ϕ and ρ_r as a function of the temperature for the model used in Fig. 1, having singled out five values of the temperature. In the right panel we can see the profile of the effective potential for these five values of the temperature with the corresponding value of the field at that particular time. We can see, in position 3, at $T = 10^{-5} M_{\rm P}$, that the field is evolving in an effective potential without a minimum but that, in position 5, the effective potential has recovered the minimum and the field has been trapped in it.

We observe that, initially, the large kinetic energy forces the field to evolve to the flatter region of the potential where the field effectively freezes due to the large frictional term in the equation of motion given that $\rho_r \gg \rho_\phi$ holds. As the universe expands the thermal corrections continue to decrease in magnitude and the field reenters the steep slope of the potential which admits a scalinglike regime of evolution i.e., the field restarts rolling with a kinetic energy that is proportional to the potential energy. It is this property that prevents the field from gaining kinetic energy and going over the barrier that separates the physical minimum from the runaway one. In other words this is why the modulus does not get driven to larger values more quickly. For an analytic description of this stabilization mechanism due to scaling, see [11,13,14,16].

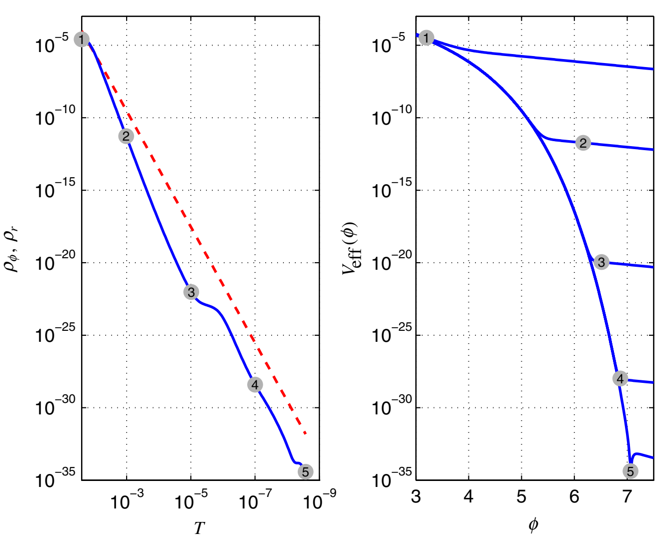

FIG. 3 (color online). In the left panel we show the evolution of the energy densities of the field, ρ_ϕ (solid line) and of the thermal fluid, ρ_r (dashed line) in the KKLT model, as a function of the temperature T (all in Planck units). In the right panel we show the profile of the effective potential at five different values of the temperature (solid lines) and the position of the field at these different stages (circles), as a function of ϕ (also in Planck units). We have set $r = -0.2$, $a_0 = -100$, $\rho_r^{\rm init} = 10^{-4}$, $\phi_{\rm init} = 3.2$, and $\sigma_I = 0$.

IV. KALLOSH-LINDE MODEL

The Kallosh-Linde model (KL) [26] generalizes the original version of the KKLT model by admitting two components in the superpotential

$$W = W_0 + Ae^{-\alpha\sigma} + Be^{-\beta\sigma}. \tag{24}$$

A particular example was investigated in a previous publication [16]. The parameters of the model were set to $A = 1$, $B = -1.5$, $C = 0$, $\alpha = 2\pi/100$, $\beta = 2\pi/99$ and W_0 such that there is a supersymmetric minimum with zero cosmological constant, i.e. W_0 is such that both W and $F_\sigma \equiv K_\sigma W + W_\sigma$ vanish at some $\sigma_R = \sigma_{\rm crit}$, $\sigma_I = 0$. Furthermore, there are a series of supersymmetric, AdS minima. In Figs. 4 and 5 we show how the stabilization region with $\star$ thermal corrections varies with r and, for comparison, we plot it against the stabilization region when a perfect fluid of radiation is present (or equivalently, when $r = 0$). As for the KKLT model, we see that for both energy regimes the profile of the stabilization regions is practically unmodified for small values of $|r|$ and that their area decreases as $|r|$ becomes larger, eventually vanishing for sufficiently high values of $|r|$. Furthermore, we see again that this effect of the thermal coupling on the stabilization region is similar for the two initial values of $\rho_r^{\rm init}$ in

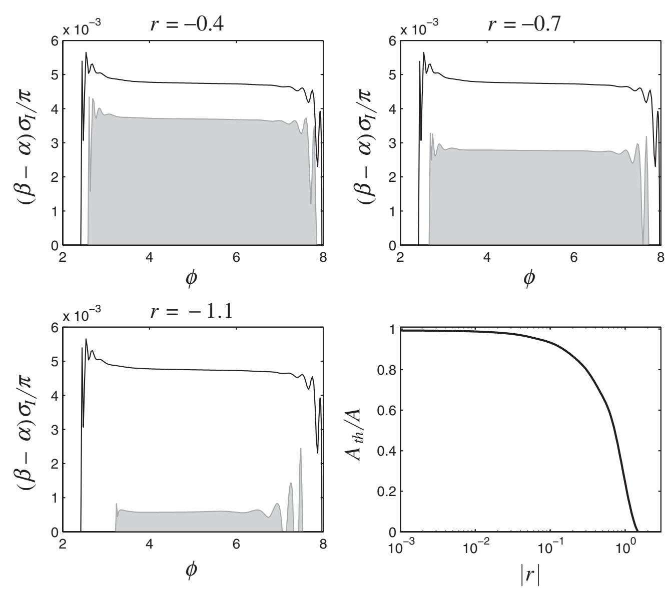

FIG. 4. Same as in Fig. 1 for the Kallosh-Linde model with $\rho_r^{\rm init} = 10^{-4} M_{\rm P}^4$.

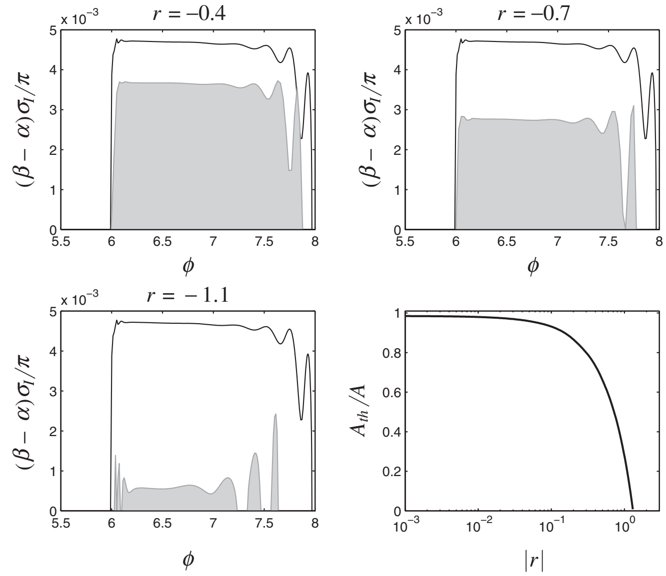

FIG. 5. Same as in Fig. 1 for the Kallosh-Linde model with $\rho_r^{\rm init} = 10^{-13} M_{\rm P}^4$.

relative terms, though a smaller initial value of the initial energy density in the background makes the stabilization areas smaller in absolute terms. The thermal coupling is slightly more effective in reducing the stabilization area in this KL model.

V. DISCUSSION

We have seen in the two models investigated that the inclusion of thermal corrections in the moduli fields evolution can have a very similar effect to a perfect fluid of non interacting radiation, in the sense that their contribution increases the friction, decelerating further the fields thus helping in their stabilization at the desired minimum. We saw that the relative effect of the thermal coupling in the stabilization area of the dilaton is nearly independent of the actual initial value of the background energy density. As we increase the value of $|r|$ (that is, we increase the relative value of a_2), the stabilization region becomes smaller, and eventually disappears. Note however, that in both models, values of $|r| \lesssim 1$ will only decrease the area for at most 50%. For reference, to have $|r| > 1$ we need $N_c \geq 17$, whereas for $|r| > 1.5$ we need $N_c \geq 26$. The values of $|r|$ obtained strictly from the thermally coupled fields only gives us a lower bound on the stabilization area. In principle, we will also have fields in the background without a thermal coupling to the dilaton in the background. In this case, we will have an effective value for $|r| \ll 1$, reducing considerably the global effect of the thermal coupling. Namely for $|r| < 10^{-2}$, the difference between the two scenarios is of less than 1%. On the other hand, it should also be clear that a larger value of the background energy density leads to a wider region of stabilisation, as the field can enter a scaling regime earlier.

The existence of a minimum in the potential is usually seen as a necessary condition for the stabilization of a moduli field, from which upper bounds to the reheating temperature can be obtained. We have seen that, if we allow for the field to evolve, the stabilization is also dependent on its initial condition; and a minimum in the potential need not be present initially. From Eq. (6) we get that for $|r| \lesssim 1$ the relation between the initial value of the temperature and energy density is approximately

$$T_{\rm init} \approx \left(\frac{\rho_r^{\rm init}}{-3a_0}\right)^{1/4}, \qquad (25)$$

which means that, for values, $a_0 \approx -100$ and $\rho_r^{\rm init} \approx 10^{-4} M_{\rm P}^4$ or $\rho_r^{\rm init} \approx 10^{-13} M_{\rm P}^4$, the initial value of the temperature can be as large as $10^{-2} M_{\rm P}$ or $10^{-4} M_{\rm P}$, respectively. These are above the usual upper limit given for stabilization, $T < 10^{-8} M_{\rm P}$ suggested in Ref. [18] obtained by requiring the minimum to appear in the effective dilaton potential.

Even though we do consider sectors different from the (MS)SM as the source for the thermal bath, the simplest scenario would be to assume that the SM fields would eventually enter thermal equilibrium at the same temperature, assuming a smooth evolution of all the background fields. Of course this does not need to be the case, and in that instance a different later background with a higher temperature that interacts with the dilaton could, in principle, destabilize the field again. Assuming the field to be already in its minimum, this would have to be at a sufficiently high temperature to effectively remove the minimum.

Though the mechanism here described seems very general, it would be interesting to evaluate its robustness for other scenarios such as the KKLT model coupled to a Polonyi field studied in Ref. [27]. One other aspect is that to large temperatures correspond large thermal fluctuations which may give rise to large spatial inhomogeneities in the moduli. A quantitative investigation of these effects and their cosmological implications deserves a complete study.

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