

Ultrahigh energy tau neutrino flux regeneration while skimming the EarthOscar Blanch Bigas,¹ Olivier Deligny,² Kévin Payet,³ and Véronique Van Elewyck^{2,4}¹*LPNHE, CNRS/IN2P3 and Université Paris VI-VIIbb, Paris, France*²*IPN, CNRS/IN2P3 and Université Paris Sud, Orsay, France*³*LPSC, Université Joseph Fourier Grenoble 1, CNRS/IN2P3, INPG, Grenoble, France*⁴*AstroParticules et Cosmologie (UMR 7165) and Université Paris 7, Paris, France*

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The detection of Earth-skimming tau neutrinos has turned into a very promising strategy for the observation of ultra-high-energy cosmic neutrinos. The sensitivity of this channel crucially depends on the parameters of the propagation of the tau neutrinos through the terrestrial crust, which governs the flux of emerging tau leptons that can be detected. One of the characteristics of this propagation is the possibility of regeneration through multiple $\nu_\tau \leftrightarrow \tau$ conversions, which are often neglected in the standard picture. In this paper, we solve the transport equations governing the ν_τ propagation and compare the flux of emerging tau leptons obtained allowing regeneration or not. We discuss the validity of the approximation of neglecting the ν_τ regeneration using different scenarios for the neutrino-nucleon cross sections and the tau energy losses.

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I. INTRODUCTION

With the advent of a new generation of large-scale detectors of cosmic radiation, the observation of high-energy cosmic neutrinos produced in distant astrophysical sites or possibly by other, more exotic, mechanisms has become one of the major challenges of astroparticle physics. In both astrophysical and exotic models, substantial fluxes of electron and muon neutrinos are expected from the disintegration of charged pions (and kaons) produced in the interaction of accelerated particles with ambient matter and radiation, either at the source location or between the source and the observer on its way through the Universe. Given the large distances traveled by these cosmic neutrinos, approximately equal fluxes in ν_e , ν_μ , and ν_τ are expected on Earth as a result of flavor mixing and oscillations [1,2]. Important efforts are ongoing to build dedicated neutrino telescopes both in the Southern [3,4] and Northern [5–8] hemispheres, opening a window on the neutrino sky in the energy range $10^{-6} \text{ EeV} \leq E_\nu \leq 10^{-1} \text{ EeV}$. At even higher energies, other promising experiments are developing the detection of coherent radio emission produced by neutrino-induced showers in matter [9–15].

Interestingly, recent studies have shown that the new generation of ultra-high-energy (UHE) cosmic ray detectors such as the Pierre Auger Observatory [16] and the HiRes Fly’s Eye detector [17] have a comparable detection potential for UHE neutrinos in the range of energies $10^{-1} \text{ EeV} \leq E_\nu \leq 10^2 \text{ EeV}$, where neutrinos are expected to be produced in the interaction of UHE cosmic rays with the cosmic microwave background [18]. It is long known that downward-going showers induced by neutrinos that penetrate deep in the atmosphere can in principle be identified at large zenith angles ($\theta > 75^\circ$) where no back-

ground is expected from hadronic primaries [19,20]. It was also pointed out more recently that the presence of ν_τ in the cosmic neutrino flux provides another promising channel of detection for air shower detectors [21,22]. Upward-going UHE tau neutrinos that graze the Earth just below the horizon (often referred to as “Earth-skimming neutrinos”) are indeed likely to interact in the crust and produce a tau lepton which may emerge and initiate an observable air shower, provided it does not decay too far from the detector.

The sensitivity to such UHE Earth-skimming neutrinos crucially depends on the conditions of the ν_τ propagation through the terrestrial crust, and on the correct estimation of the flux of τ leptons that emerge from the Earth. This propagation problem has been widely discussed in different contexts and with different approximations [23–29]. An exhaustive treatment should account for τ and ν_τ neutral-current (NC) and charged-current (CC) interactions with nucleons, τ decay, and energy losses. However the full coupled transport equations admit no analytical solution, and even in the case of Monte Carlo calculations, simplifications are usually made such as dropping the τ weak interactions and neglecting multiple regenerations of the ν_τ .

Such approximations are defensible in the standard case where the characteristic lengths for the τ CC interaction ($\sim 600 \text{ km}$, at 1 EeV) and for the τ decay ($\sim 50 \text{ km}$) are larger than that for energy losses ($\sim 6 \text{ km}$). However, they might be challenged in other, more exotic scenarios. The knowledge of the neutrino cross section and the tau energy losses in the EeV energy range is indeed limited [30,31], and these could be significantly affected at center-of-mass energies beyond the TeV by the onset of new physics beyond the standard model. Several studies have even suggested that the comparison of the flux rates between down-

going and Earth-skimming neutrinos could actually help constrain the neutrino properties at ultrahigh energies, where no direct measurements exist [32,33].

All these considerations pinpoint the necessity of an accurate determination of the flux of emerging τ leptons. In this context, it is important to correctly describe all the contributions to the τ flux and to assess the impact of simplifications in the description of the propagation problem.

The present paper therefore focuses on understanding the effects of the ν_τ and τ regeneration while skimming the Earth on the flux of emerging τ leptons in different scenarios for the neutrino-nucleon cross section and for the τ energy losses. In Sec. II, we present the transport equations for the ν and τ propagation, and the scenarios to be studied. In Sec. III, we compare the flux of emerging taus for different conditions of propagation. Finally, we present our conclusions in Sec. IV.

II. ν_τ PROPAGATION THROUGH THE EARTH: THE GENERAL PICTURE

The geometry of the propagation problem is described in Fig. 1, where an example of regeneration chain through multiple CC interactions and τ decays is sketched. Given a beam of parallel neutrinos incident on the Earth at a given angle α , the problem becomes unidimensional and the flux of τ leptons that emerge only depends on the path length traveled across the rock. In the hypothesis of a spherical Earth with a crust of constant density, this length is directly related to α , which is also the angle of the emerging tau.

A. Transport equations

We reproduce here a classical formulation in terms of the transport equations that describe the evolution of the ν_τ and τ fluxes, Φ_{ν_τ} and Φ_τ , along their path through the Earth, accounting for all possible production and absorption processes taking place within an infinitesimal dx :

$$\begin{aligned} \frac{\partial \Phi_{\nu_\tau}(E, x)}{\partial x} = & -\frac{\Phi_{\nu_\tau}(E, x)}{\lambda_{\nu_\tau}^{\text{CC}}(E)} - \frac{\Phi_{\nu_\tau}(E, x)}{\lambda_{\nu_\tau}^{\text{NC}}(E)} + \rho \mathcal{N}_A \int \frac{dy}{1-y} \Phi_{\nu_\tau}\left(\frac{E}{1-y}, x\right) \frac{d\sigma_{\nu_\tau}^{\text{NC}}(y, \frac{E}{1-y})}{dy} \\ & + \rho \mathcal{N}_A \int \frac{dy}{1-y} \Phi_\tau\left(\frac{E}{1-y}, x\right) \frac{d\sigma_\tau^{\text{CC}}(y, \frac{E}{1-y})}{dy} + \frac{1}{c} \int \frac{dy}{1-y} \Phi_\tau\left(\frac{E}{1-y}, x\right) \frac{d\Gamma_\tau(y, \frac{E}{1-y})}{dy}. \end{aligned} \quad (1)$$

Here $\lambda_{\nu_\tau}^{\text{NC}}$ and $\lambda_{\nu_\tau}^{\text{CC}}$ are the mean free paths corresponding, respectively, to NC and CC interactions of the incident ν_τ , while σ_τ^{CC} corresponds to the τ CC interaction, which regenerates a ν_τ . Γ_τ is the tau lepton lifetime.

The transport equation for the τ can be inferred in a similar way. In this case, the small inelasticity of the τ radiative interactions allows us to use the continuous energy loss approximation [34]. Defining $\gamma(E) = -dE/dx$, the equation reads:

$$\begin{aligned} \frac{\partial \Phi_\tau(E, x)}{\partial x} = & -\frac{\Phi_\tau(E, x)}{\lambda_\tau^{\text{dec}}(E)} - \frac{\Phi_\tau(E, x)}{\lambda_\tau^{\text{CC}}(E)} - \frac{\Phi_\tau(E, x)}{\lambda_\tau^{\text{NC}}(E)} + \frac{\partial}{\partial E}(\gamma(E)\Phi_\tau(E, x)) + \rho \mathcal{N}_A \int \frac{dy}{1-y} \Phi_\tau\left(\frac{E}{1-y}, x\right) \frac{d\sigma_\tau^{\text{NC}}(y, \frac{E}{1-y})}{dy} \\ & + \rho \mathcal{N}_A \int \frac{dy}{1-y} \Phi_{\nu_\tau}\left(\frac{E}{1-y}, x\right) \frac{d\sigma_{\nu_\tau}^{\text{CC}}(y, \frac{E}{1-y})}{dy} \end{aligned} \quad (2)$$

where λ_τ^{NC} and λ_τ^{CC} are now the mean free paths associated with the τ NC and CC interactions, and $\lambda_\tau^{\text{dec}}$ is the decay length corresponding to Γ_τ .

We are thus left with a system of two coupled, integro-differential equations, which cannot be solved analytically without making simplifying assumptions on the nature and the respective importance of the interactions that both the

ν_τ and the τ can undergo. To preserve the generality of the solution we use a Monte Carlo simulation that includes all the processes listed above and follows the incident particle all the way through the rock, deciding at each step on its fate according to the distributions encoded in Eqs. (1) and (2).

An iterative method that also allows us to avoid simplifications was proposed in [35], and further discussed in [27] and in [36]. Our equations differ from those in [27,36] in that we also include the two terms describing τ NC interactions in the transport equation for Φ_τ . Taking into account this generalization, we have applied the iterative method (see the Appendix for details) to crosscheck the results from the Monte Carlo simulation. In Fig. 2, we show the good agreement between both calculations for

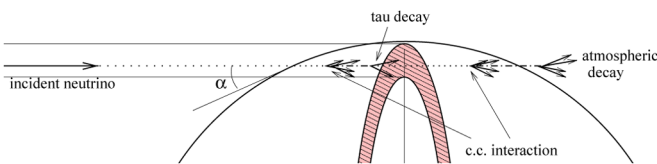


FIG. 1 (color online). Geometry of the transport problem.

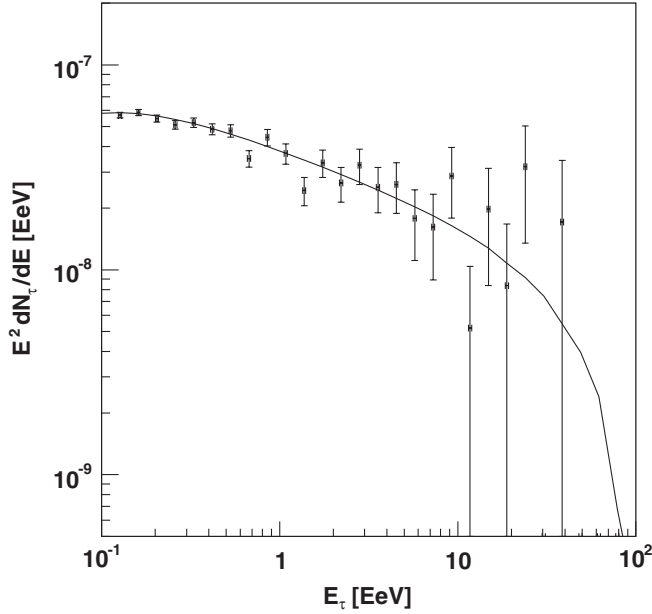


FIG. 2. Flux of emerging τ 's (scaled by E^2) obtained by solving the transport Eqs. (1) and (2), for incident tau neutrinos with an angle α up to 15° and injection flux $dN/dE = 4.6 \times 10^7 E_\nu^{-2} \text{EeV}^{-1} \text{sr}^{-1}$. The points correspond to the Monte Carlo solution, while the line is the solution obtained with the iterative method.

the flux of emerging τ leptons given an incident tau neutrino flux $dN/dE = 4.6 \times 10^7 E_\nu^{-2} \text{EeV}^{-1} \text{sr}^{-1}$.

B. Cross sections

In the framework of the standard model, the neutrino-nucleon (νN) CC and NC cross-sections, $\sigma_{\nu\tau}^{\text{CC}}$ and $\sigma_{\nu\tau}^{\text{NC}}$, describe deep-inelastic scattering processes. They are expressed in terms of the structure functions of the nucleon, which in turn depend on the individual parton distribution functions (PDFs). The PDFs are obtained from measurements at accelerators, in determinate ranges of Bjorken- x and momentum transfer Q^2 . However, the range of parameters probed by the UHE neutrinos, $x \sim 10^{-5} (1 \text{EeV}/E_\nu)$ and Q^2 up to $\sim 10^{-5} \text{EeV}$, is outside the measured domain and therefore extrapolations are needed.

We choose as a benchmark for the *standard* νN cross section a recent parametrization presented in [30], which fits to updated HERA data:

$$\sigma_{\text{std}}^{\text{CC}}(E) = 2.4\sigma_{\text{std}}^{\text{NC}}(E) = 6.04 \cdot \left(\frac{E}{10^{-9} \text{EeV}}\right)^{0.358} \text{pb.} \quad (3)$$

As pointed out in the same paper, another more speculative approach based on the color glass condensate formalism [37] has also been put forward recently to account for saturation at very low x . We will also consider this case and use the following approximate parametrization deduced from Fig. 1 of [30]:

$$\begin{aligned} \sigma_{\text{low}}^{\text{CC}}(E) &= 2.4\sigma_{\text{low}}^{\text{NC}}(E) \\ &= 3.89 \cdot \left(\frac{E}{\text{EeV}}\right)^{0.170-0.037 \cdot \log_{10}(E/\text{EeV})} 10^3 \text{pb} \quad (4) \end{aligned}$$

as an example of *low* νN cross section.

On the other hand, plenty of models using new physics predict an enhancement of the neutrino cross section [38]. As an example of *high* νN cross section, we will use

$$\sigma_{\text{high}}^{\text{CC}}(E) = 2.4\sigma_{\text{high}}^{\text{NC}}(E) = 3 \cdot \sigma_{\text{std}}^{\text{CC}}(E) \quad (5)$$

without assuming any particular model.

C. Tau energy losses

The τ energy losses are usually parametrized using the following formula:

$$\frac{dE_\tau}{dX} = -\alpha - \beta(E_\tau)E_\tau, \quad (6)$$

where α is practically constant and accounts for ionization processes, while $\beta(E_\tau)$ includes the radiative contributions from bremsstrahlung (b), pair production (pp), and photo-nuclear (pn) interactions of the τ .

For the purpose of our analysis, we adopt the same strategy as in the previous section and extract from existing computations [31,39] a panel of *ad-hoc* parametrizations of $\beta(E_\tau)$ in the relevant range of energy, representative of the values found in the literature. The set of parametrizations under study is the following (valid in the range $10^{-1} \text{EeV} \leq E_\nu \leq 10^3 \text{EeV}$):

$$\begin{aligned} \beta_{\text{std}}(E_\tau) &= \left(1.2 + 0.16 \times \ln\left(\frac{E_\tau}{10 \text{EeV}}\right)\right) 10^{-6} \text{g}^{-1} \text{cm}^2 \\ \beta_{\text{high}}(E_\tau) &= 1.36 \left(\frac{E_\tau}{\text{EeV}}\right)^{0.35} 10^{-6} \text{g}^{-1} \text{cm}^2 \\ \beta_{\text{low}}(E_\tau) &= \left(0.3 + 0.06 \times \log_{10}\left(\frac{E_\tau}{\text{EeV}}\right)\right) 10^{-6} \text{g}^{-1} \text{cm}^2 \end{aligned}$$

respectively for a standard, a high, and a low value of the energy loss parameter β .

III. THE FLUX OF EMERGING TAUS: RESULTS AND COMPARISONS

The Earth-skimming technique has been recently used to search for ν_τ with the HiRes telescopes [17] and the Pierre Auger Observatory [16]. While both detectors have a similar energy threshold ($\sim 0.1 \text{EeV}$), they are sensitive to a different range in the emerging angle α of the τ leptons. The latter detects the particles from the air shower initiated by the τ decay that reach the ground and, hence, the detector will only be triggered by τ leptons slightly up-going ($\alpha < 3^\circ$). The former is designed to record the faint ultraviolet light emitted by nitrogen molecules that are excited as the shower traverses the atmosphere; it is therefore sensitive to τ leptons emerging with larger angles.

In Fig. 3, we present the results obtained with the Monte Carlo approach for the flux of emerging τ leptons with an angle α up to 3° and 15° produced by an incident tau neutrino flux $dN/dE = 4.6 \times 10^7 E_\nu^{-2} \text{EeV}^{-1} \text{sr}^{-1}$. All cross-sections and energy loss parameters are here set to *standard* values, as defined in Secs. II B and II C. The effect of the regeneration is negligible if only almost horizontal emerging τ leptons ($\alpha < 3^\circ$) are taken into account. On the other hand, if the detector is sensitive to larger angles and energies down to a fraction of EeV, neglecting the regeneration becomes important already within this *standard* picture (30% less τ leptons at 0.3 EeV for an incident neutrino flux $dN/dE \propto E_\nu^{-2}$).

The approximation of neglecting the regeneration seems thus to be safe in the standard picture for detectors only sensitive to almost horizontal showers. However, we will now show that this approximation is questionable even for them as soon as one considers non-*standard* scenarios. We have investigated the effect of the regeneration for those detectors using the set of parametrizations defined in Secs. II B and II C. These expressions do not constitute an exhaustive list of all the alternative and exotic models that exist in the literature, but rather a selection of a few examples that allow us to point out the relevance of this process. The combinations of cross sections and tau energy losses should be chosen on the basis of coherent PDFs, but this coherency is hard to assert even inside the frame of the standard model. Therefore, we chose to investigate the regeneration effects for the $3 \otimes 3$ possible combinations, which is sufficient to provide a qualitative answer to the question raised in this paper.

As a starting point, one should however notice that the regeneration requires the τ to be converted back into a ν_τ before losing too much energy, and then this ν_τ to undergo another CC weak interaction to produce a τ again. Hence,

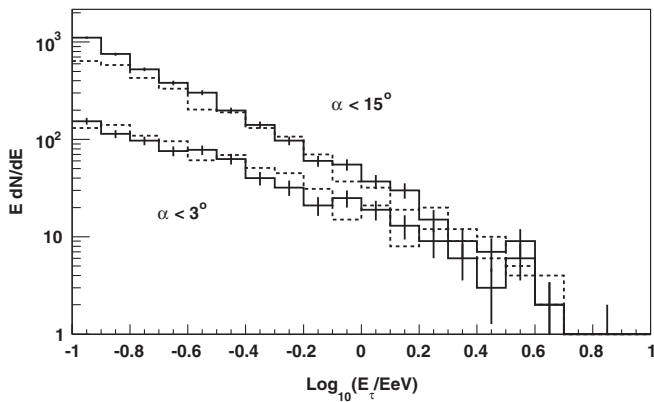


FIG. 3. Flux of emerging taus corresponding to incident tau neutrinos with an angle α up to 3° or 15° and $dN/dE = 4.6 \times 10^7 E_\nu^{-2} \text{EeV}^{-1} \text{sr}^{-1}$ using the standard combination of parametrizations $\sigma_{\text{std}} \otimes \beta_{\text{std}}$ as defined in Secs. II B and II C. The regeneration is neglected in the dashed histograms and included in the solid ones.

one expects that a higher energy loss or a lower cross section will reduce the effect of the regeneration, while, on the other hand, a higher cross section or a lower energy loss will enhance it. This is indeed what we find. In the following, we focus on the particular combinations for which the regeneration is not negligible anymore, and compare them to the standard case, essentially on the basis of monoenergetic incident beams of ν_τ 's. This is because the E^{-2}

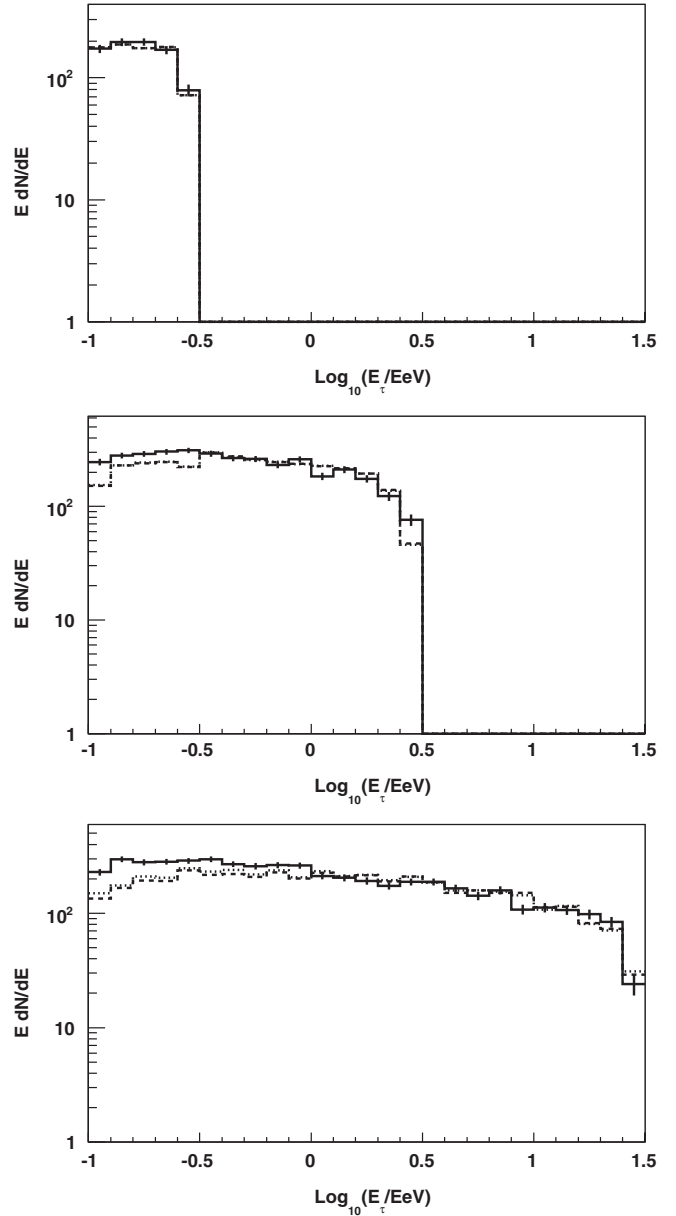


FIG. 4. Flux of emerging taus produced by incident $\Phi_{\nu_\tau}(E_\nu) = 4.6 \times 10^8 \text{sr}^{-1}$ with energy $E_\nu = 0.3, 3, \text{ and } 30 \text{ EeV}$ (from top to bottom) and an angle up to 3° . The standard combination of parametrizations $\sigma_{\text{std}} \otimes \beta_{\text{std}}$ as defined in Secs. II B and II C is used. The solid histogram takes into account all regeneration channels. The dotted histogram only accounts for the regeneration through the τ NC interaction. The dashed histogram neglects the regeneration.

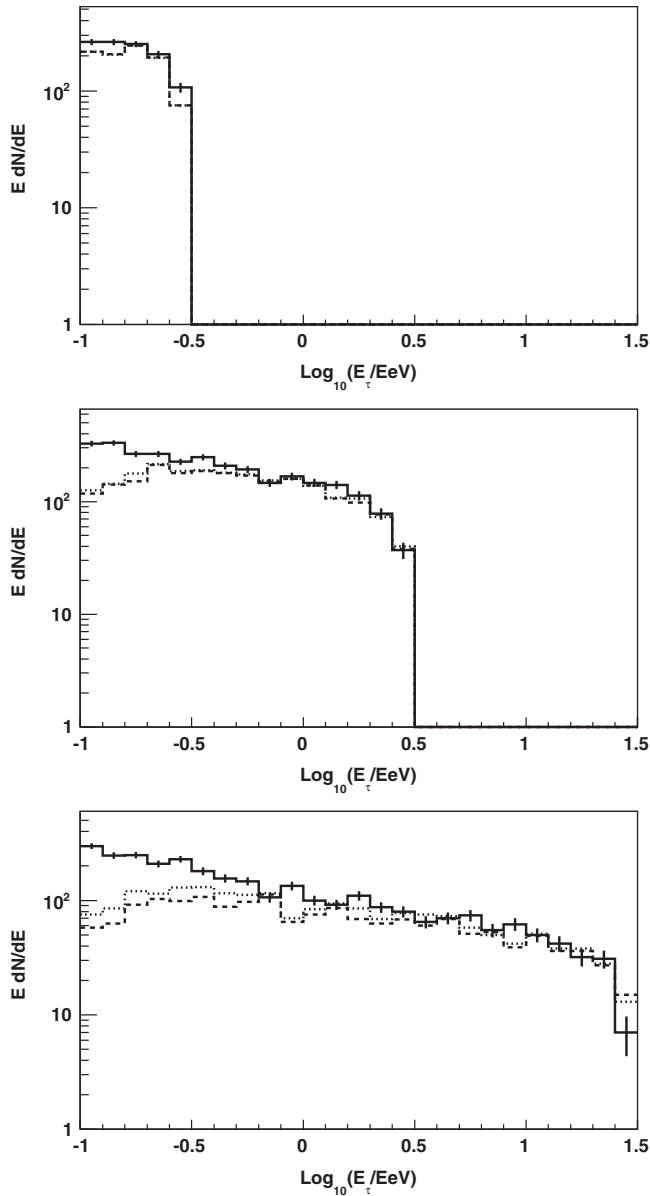


FIG. 5. Flux of emerging taus produced by incident $\Phi_{\nu_\tau}(E_\nu) = 4.6 \times 10^8 \text{ sr}^{-1}$ with energy $E_\nu = 0.3, 3,$ and 30 EeV (from top to bottom) and an angle up to 3° . The combination $\sigma_{\text{high}} \otimes \beta_{\text{std}}$ as defined in Secs. II B and II C is used (with the same histogram code as in Fig. 4).

power-law spectrum usually assumed for cosmic neutrinos, although adequate to look at the global flux of emerging τ leptons, is completely dominated by the lowest energies, while the behavior at the highest energies may be important for harder fluxes of incident neutrinos.

In Figs. 4–7, we show the flux of τ leptons produced by incident neutrinos at given energies (0.3, 3, and 30 EeV) and with an angle α up to 3° , for the following combinations of parametrizations: $\sigma_{\text{std}} \otimes \beta_{\text{std}}$ (the same as used in Fig. 3), $\sigma_{\text{high}} \otimes \beta_{\text{std}}$, $\sigma_{\text{std}} \otimes \beta_{\text{low}}$, and $\sigma_{\text{high}} \otimes \beta_{\text{low}}$. For a *standard* choice of parametrizations, one can see from

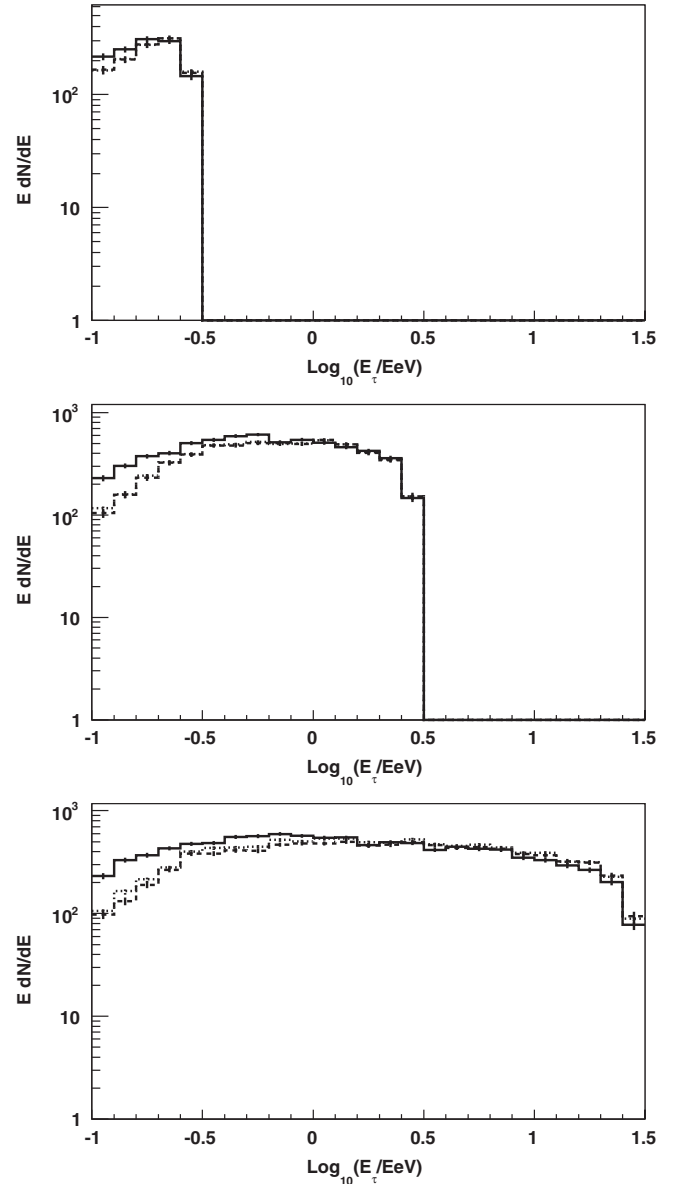


FIG. 6. Flux of emerging taus produced by incident $\Phi_{\nu_\tau}(E_\nu) = 4.6 \times 10^8 \text{ sr}^{-1}$ with energy $E_\nu = 0.3, 3$ and 30 EeV (from top to bottom) and an angle up to 3° . The combination $\sigma_{\text{std}} \otimes \beta_{\text{low}}$ as defined in sections II B and II C is used (with the same histogram code as in Fig. 4).

Fig. 4 that the impact of regeneration is completely negligible at low neutrino energies. As the ν_τ energy increases, however, the lowest-energy part of the τ spectrum starts to be significantly underestimated (15% less taus for 30 EeV incident neutrinos). This effect is washed out in the case of an incident neutrino flux $dN/dE \propto E_\nu^{-2}$, for which the contribution of the highest-energy part of the neutrino spectrum is negligible; but it could affect the spectrum of emerging τ 's if the neutrino flux were harder. For the other cases, the lowest-energy range of the τ spectrum becomes underestimated already for incident ν_τ with energy of 0.3 EeV. For the combination $\sigma_{\text{high}} \otimes \beta_{\text{low}}$, one loses about

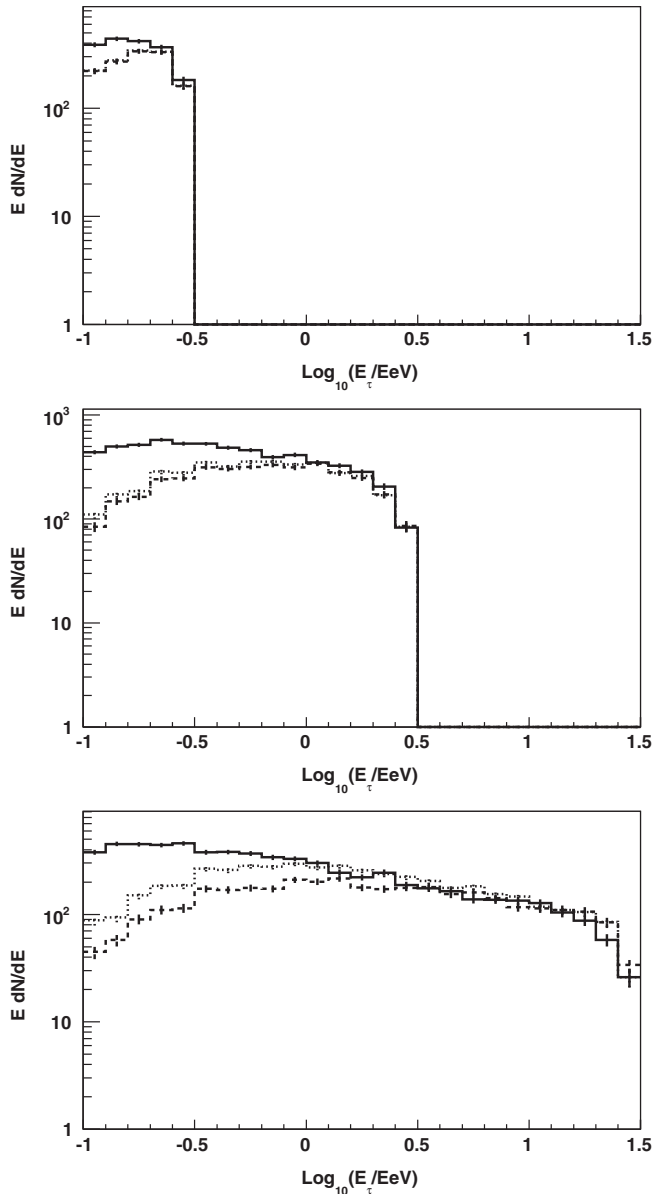


FIG. 7. Flux of emerging taus produced by incident $\Phi_{\nu_\tau}(E_\nu) = 4.6 \times 10^8 \text{ sr}^{-1}$ with energy $E_\nu = 0.3, 3,$ and 30 EeV (from top to bottom) and an angle up to 3° . The combination $\sigma_{\text{high}} \otimes \beta_{\text{low}}$ as defined in Secs. II B and II C is used (with the same histogram code as in Fig. 4).

70% of the τ leptons emerging from the Earth if the regeneration is neglected, even if assuming an incident flux of neutrinos $dN/dE \propto E_\nu^{-2}$ (Fig. 8).

The neutrino flux can be regenerated through the τ decay and the τ CC weak interaction. We present our results in a form that allows us to disentangle the two effects: the dotted histograms in Figs. 4–7, account for regeneration through the weak interaction of the τ only, while the solid ones include both effects. As can be appreciated in the figures, the CC contribution is negligible at all energies in most scenarios. It only becomes important for ν_τ 's at the

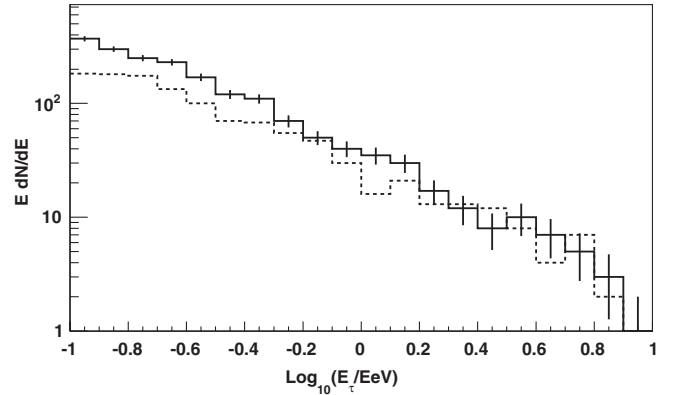


FIG. 8. Flux of emerging taus corresponding to incident tau neutrinos with an angle α up to 3° and $dN/dE = 4.6 \times 10^7 E_\nu^{-2} \text{ EeV}^{-1} \text{ sr}^{-1}$ using the combination of parametrizations $\sigma_{\text{high}} \otimes \beta_{\text{low}}$ as defined in Secs. II B and II C. The regeneration is neglected in the dashed histogram and included in the solid one.

highest energies ($\sim 30 \text{ EeV}$), in the energy range between 0.3 and 3.0 EeV of the emerging τ spectrum, if a *high* cross section is assumed (Figs. 5 and 7). There, the effect is at the level of that from the regeneration through the τ decay.

IV. CONCLUSION

In this paper, we have studied the mechanism of regeneration of the ν_τ flux while crossing the Earth to investigate its effect on the flux of emerging τ leptons. Assuming a detector with an energy threshold of 0.1 EeV and sensitive only to taus with an emerging angle below 3° , the effect is negligible for a flux of incident neutrinos $dN/dE \propto E_\nu^{-2}$ with the standard values of cross sections and energy losses. But we have shown that this is not valid for other assumptions on the detector performance, neither for less standard values of the weak cross section or tau energy losses. On the one hand, neglecting the regeneration leads to about 30% underestimation on the integrated flux for a detector sensitive up to 15° . On the other hand, the underestimation of the τ 's emerging from the Earth with an energy greater than 0.1 EeV and an emerging angle below 3° may reach 70% for the different values of σ_{ν_τ} and β_τ studied in this paper. Moreover, the error made in the computation of the flux of emerging taus can be even larger for a harder flux of incident neutrinos since the contribution of the regeneration effect increases with the energy of the incident neutrino.

The simplification of neglecting the regeneration is thus only safe for particular values of the physical properties playing a role on the propagation and specific detectors. It may lead to a significant underestimation of the flux of emerging τ 's when looking at nonstandard values of the weak cross section or tau energy losses. Therefore, it should be carefully treated and accounted for when studying the systematics due to the uncertainties on those prop-

erties or while using the Earth-skimming technique to test, for instance, higher weak cross sections. Similarly, one should carefully check the effect for the characteristics of the actual detector before neglecting the regeneration.

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APPENDIX: ITERATIVE SOLUTION OF THE TRANSPORT EQUATIONS

We reproduce here a numerical method proposed in [27,35], generalizing it to take into account the NC weak interaction of the τ lepton. We start from a solution of order

0 and iterate it up to get a solution that satisfies the relation $F(\Phi(E, x), \partial_x \Phi(E, x)) = 0$ as given by Eq. (1) or (2). For numerical reasons, it is better to work with numbers between 0 and 1 so that we write the equations in terms of the deviation to the solution of order 0, $\Phi_{\nu_\tau}^0$, rather than in terms of Φ_{ν_τ} itself. We therefore assume that the solution is of the form

$$\Phi_{\nu_\tau}(E, x) = \Phi_{\nu_\tau}^0(E) \exp\left[-\frac{x}{\lambda_{\nu_\tau}(E)}(1 - Z_{\nu_\tau}(E, x))\right]$$

where $\lambda_{\nu_\tau}(E)^{-1} = \lambda_{\nu_\tau}^{\text{CC}}(E)^{-1} + \lambda_{\nu_\tau}^{\text{NC}}(E)^{-1}$ accounts for the possible sources of depletion of Φ_{ν_τ} at an energy E , and $Z_{\nu_\tau}(E, x)$ is a function to be determined, which includes all contributions to the regeneration of Φ_{ν_τ} at the same energy.

$Z_{\nu_\tau}(E, x)$ can be obtained iteratively by forcing it to satisfy the transport equations, viewed as a recurrence relation with the initial condition $Z_{\nu_\tau}^0(E, x) = 0$:

$$\begin{aligned} Z_{\nu_\tau}^{n+1}(E, x) = & -x \frac{\partial Z_{\nu_\tau}^n(E, x)}{\partial x} + \rho \mathcal{N}_A \frac{\lambda_{\nu_\tau}(E)}{\Phi_{\nu_\tau}^n(E, x)} \int \frac{dy}{1-y} \Phi_{\nu_\tau}^n\left(\frac{E}{1-y}, x\right) \frac{d\sigma_{\nu_\tau}^{\text{NC}}(y, \frac{E}{1-y})}{dy} + \rho \mathcal{N}_A \frac{\lambda_{\nu_\tau}(E)}{\Phi_{\nu_\tau}^n(E, x)} \\ & \times \int \frac{dy}{1-y} \Phi_{\nu_\tau}^n\left(\frac{E}{1-y}, x\right) \frac{d\sigma_{\nu_\tau}^{\text{CC}}(y, \frac{E}{1-y})}{dy} + \frac{\lambda_{\nu_\tau}(E)}{c \Phi_{\nu_\tau}^n(E, x)} \int \frac{dy}{1-y} \Phi_{\nu_\tau}^n\left(\frac{E}{1-y}, x\right) \frac{d\Gamma(y, \frac{E}{1-y})}{dy}. \end{aligned} \quad (\text{A1})$$

As the two equations are coupled, it is necessary to know Φ_τ at a given order to proceed with the iteration at the next order. Following the same philosophy as above, we assume that the solution of Eq. (2) for Φ_τ is of the form:

$$\Phi_\tau(E, x) = \int_0^x du S(E(u-x), u) K(E, u, x)$$

where the following notations have been introduced:

$$\begin{aligned} S(E, x) &= \rho \mathcal{N}_A \int \frac{dy}{1-y} \Phi_{\nu_\tau}\left(\frac{E}{1-y}, x\right) \frac{d\sigma_{\nu_\tau}^{\text{CC}}(y, \frac{E}{1-y})}{dy} \\ K(E, u, x) &= \exp[\kappa(E, u, x) \cdot (1 - Z_\tau(E, x) \\ & \quad + Z_\tau(E(u-x), u))] \\ \kappa(E, u, x) &= \int_u^x dv \left[\frac{\partial \gamma(E(v-x))}{\partial E} - \frac{1}{\lambda_\tau(E(v-x))} \right] \end{aligned}$$

and $\lambda_\tau(E)^{-1} = \lambda_\tau^{\text{dec}}(E)^{-1} + \lambda_\tau^{\text{CC}}(E)^{-1} + \lambda_\tau^{\text{NC}}(E)^{-1}$. The equation for Z_τ^n then reads:

$$\begin{aligned} Z_\tau^{n+1}(E, x) &= \frac{1}{\Phi_\tau^n(E, x)} \int_0^x du S^{n+1}(E(u-x), u) K(E, u, x) \\ & \quad \times \left[Z_\tau^n(E(u-x), u) - \kappa(E, u, x) \right. \\ & \quad \times \left. \frac{\partial_x Z_\tau^n(E, x)}{(\partial_E \gamma(E) - \lambda_\tau^{-1}(E))} \right] \\ & \quad - \frac{\rho \mathcal{N}_A \int \frac{dy}{1-y} \Phi_{\nu_\tau}^n\left(\frac{E}{1-y}, x\right) \frac{d\sigma_{\nu_\tau}^{\text{NC}}(y, \frac{E}{1-y})}{dy}}{(\partial_E \gamma(E) - \lambda_\tau^{-1}(E)) \Phi_\tau^n(E, x)} \end{aligned}$$

again with $Z_\tau^0(E, x) = 0$. Practically, the iteration can be stopped as soon as $n = 3$.

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