Reply to "Comment on 'Once more about the $K\bar{K}$ molecule approach to the light scalars' "

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The need to regularize loop integrals in a manner that preserves gauge invariance, for example, using the Pauli-Villars method, requires a subtraction that in the large mass limit hides its high momentum origin. This gives rise to the illusion that only nonrelativistic kaon loop momenta are relevant, when in fact this is not the case, as we show.

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The authors of Ref. [1] adduced the following argument against our criticism [2]. Since the $\phi \to K^+K^- \to \gamma(a_0/f_0)$ amplitude vanishes for gauge invariance when the photon momentum vanishes, only those terms of the integrand have the physical sense which vanish with vanishing the photon momentum. What actually happens is that cancellation of contributions from different places of momentum (or coordinate) space is realized. It is commonplace in electrodynamics. In particular, low energy theorems are based on this. Discarding the integrand in the third term of Eq. (3), the second term in Eq. (4), and the contribution of Eq. (5), the authors of [1] distort the physical significance of the K^+K^- loop model because these contributions represent the high momentum and charge flow distributions of kaons. Below we show that the K^+K^- loop model describes the relativistic physics.

When basing the experimental investigations of the light scalar mesons production in the ϕ radiative decays $\phi \to \gamma[a_0(980)/f_0(980)] \to \gamma[(\pi^0\eta)/(\pi^0\pi^0)]$, there was suggested [3] the kaon loop model $\phi \to K^+K^- \to \gamma[a_0(980)/f_0(980)]$ with the pointlike interaction, see Fig. 1. This model is used in the data treatment and ratified by experiment. In Refs. [4,5] an analysis of mechanisms of decays under consideration was carried out, which gave the clear arguments for this kaon loop model.

Every diagram contribution in

$$T\{\phi(p) \to \gamma[a_0(q)/f_0(q)]\} = (a) + (b) + (c) \quad (1)$$

is divergent hence should be regularized in a gauge invariant manner, for example, in the Pauli-Villars one.

$$\bar{T}\{\phi(p) \to \gamma[a_0(q)/f_0(q)], M\} = \overline{(a)} + \overline{(b)} + \overline{(c)},$$
$$\bar{T}\{\phi(p) \to \gamma[a_0(q)/f_0(q)], M\} = \epsilon^\nu(\phi)\epsilon^\mu(\gamma)\bar{T}_{\nu\mu}(p,q) = \epsilon^\nu(\phi)\epsilon^\mu(\gamma)[\bar{a}_{\nu\mu}(p,q) + \bar{b}_{\nu\mu}(p,q) + \bar{c}_{\nu\mu}(p,q)], \quad (2)$$

$$\bar{a}_{\nu\mu}(p,q) = -\frac{i}{\pi^2}\int\left\{\frac{(p-2r)_\nu(p+q-2r)_\mu}{(m_K^2-r^2)[m_K^2-(p-r)^2][m_K^2-(q-r)^2]} - \frac{(p-2r)_\nu(p+q-2r)_\mu}{(M^2-r^2)[M^2-(p-r)^2][M^2-(q-r)^2]}\right\}dr, \quad (3)$$

$$\bar{b}_{\nu\mu}(p,q) = -\frac{i}{\pi^2}\int\left\{\frac{(p-2r)_\nu(p-q-2r)_\mu}{(m_K^2-r^2)[m_K^2-(p-r)^2][m_K^2-(p-q-r)^2]} - \frac{(p-2r)_\nu(p-q-2r)_\mu}{(M^2-r^2)[M^2-(p-r)^2][M^2-(p-q-r)^2]}\right\}dr, \quad (4)$$

$$\bar{c}_{\nu\mu}(p,q) = -\frac{i}{\pi^2}2g_{\nu\mu}\int dr\left\{\frac{1}{(m_K^2-r^2)[m_K^2-(q-r)^2]} - \frac{1}{(M^2-r^2)[M^2-(q-r)^2]}\right\}, \quad (5)$$

where M is the regulator field mass. $M \to \infty$ in the end

$$\bar{T}[\phi \to \gamma(a_0/f_0), M \to \infty] \to T^{\text{phys}}[\phi \to \gamma(a_0/f_0)]. \quad (6)$$

We can shift the integration variables in the regularized amplitudes and easily check the gauge invariance condition

$$\epsilon^\nu(\phi)k^\mu\bar{T}_{\nu\mu}(p,q) = \epsilon^\nu(\phi)(p-q)^\mu\bar{T}_{\nu\mu}(p,q) = 0. \quad (7)$$

It is instructive to consider how the gauge invariance condition

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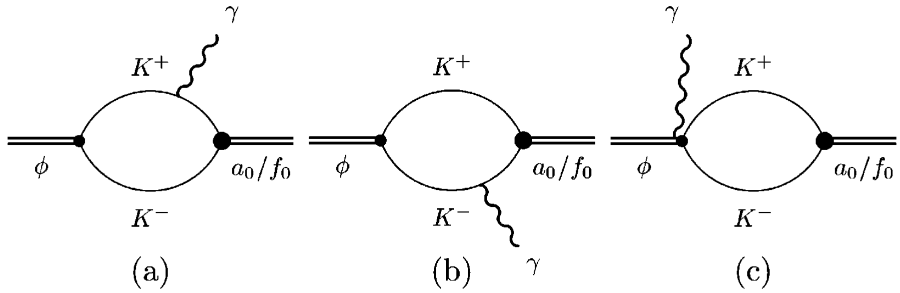

FIG. 1. Diagrams contributing to the radiative decay amplitude.

$$\epsilon^{\nu}(\phi)\epsilon^{\mu}(\gamma)\bar{T}_{\nu\mu}(p,p) = 0 \qquad (8)$$

holds true,

$$\begin{aligned}\epsilon^{\nu}(\phi)\epsilon^{\mu}(\gamma)\bar{T}_{\nu\mu}(p,p) &= \epsilon^{\nu}(\phi)\epsilon^{\mu}(\gamma)T^{m_K}_{\nu\mu}(p,p) \\ &\quad - \epsilon^{\nu}(\phi)\epsilon^{\mu}(\gamma)T^{M}_{\nu\mu}(p,p) \\ &= (\epsilon(\phi)\epsilon(\gamma))(1-1) = 0. \qquad (9)\end{aligned}$$

The superscript m_K refers to the nonregularized amplitude and the superscript M refers to the regulator field amplitude. So, the contribution of the (a), (b), and (c) diagrams does not depend on a particle mass in the loops (m_K or M) at $p = q$ [6]. But, the physical meaning of these contributions is radically different. The $\epsilon^{\nu}(\phi)\epsilon^{\mu}(\gamma)T^{m_K}_{\nu\mu}(p,p)$ contribution is caused by intermediate momenta (a few GeV) in the loops, whereas the regulator field contribution is caused fully by high momenta ($M \to \infty$) and teaches us how to allow for high K virtualities in a gauge invariant way.

Needless to say, the integrand of $\epsilon^{\nu}(\phi)\epsilon^{\mu}(\gamma)\bar{T}_{\nu\mu}(p,p)$ is not equal to 0.

It is clear that

$$\begin{aligned}&\epsilon^{\nu}(\phi)\epsilon^{\mu}(\gamma)T^{M\to\infty}_{\nu\mu}(p,q) \\ &\to \epsilon^{\nu}(\phi)\epsilon^{\mu}(\gamma)T^{M\to\infty}_{\nu\mu}(p,p) \\ &\equiv \epsilon^{\nu}(\phi)\epsilon^{\mu}(\gamma)T^{M}_{\nu\mu}(p,p) \equiv (\epsilon(\phi)\epsilon(\gamma)). \qquad (10)\end{aligned}$$

So, the regulator field contribution tends to the subtraction constant when $M \to \infty$.

The finiteness of the subtraction constant hides its high momentum origin and gives rise to an illusion of a nonrelativistic physics in the K^+K^- model with the pointlike interaction. See, for example, Ref. [7]; see Sec. 2 in this paper.

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