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Comment on "Once more about the $K\bar{K}$ molecule approach to the light scalars"

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In this manuscript we comment on the criticism raised recently by Achasov and Kiselev [Phys. Rev. D **76**, 077501 (2007)] on our work on the radiative decays $\phi \to \gamma a_0/f_0$ [Eur. Phys. J. A **24**, 437 (2005)]. Specifically, we demonstrate that their criticism relies on results that violate gauge invariance and is therefore invalid.

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In a recent paper [1] we considered the radiative decay $\phi \to \gamma a_0/f_0$ in the molecular ($K\bar{K}$) model of the scalar mesons ($a_0(980)$, $f_0(980)$). In particular, we showed that there was no considerable suppression of the decay amplitude due to the molecular nature of the scalar mesons. In addition, as a more general result we demonstrated that, as soon as the vertex function of the scalar meson is treated properly, the corresponding loop integrals become very similar to those for pointlike (quarkonia) scalar mesons, provided reasonable values are chosen for the range of the interaction. We also confirmed the range of order of $10^{-3} \div 10^{-4}$ for the branching ratio obtained in Refs. [2–4] within the molecular model.

As a reaction to our work a paper appeared [5], where the authors criticize our results and claim that our paper [1] is "misleading." Specifically, they dispute our findings that the transition amplitude $\phi \to K^+K^- \to \gamma a_0/f_0$ is governed by low kaon momenta (nonrelativistic kaons) in the loop. In order to support this conjecture they present numerical results that supposedly demonstrate that "ultra-relativistic kaons determine the real part of the $\phi \to K^+K^- \to \gamma a_0/f_0$ amplitude." The dominance of such contributions of "kaon high virtualities" is then interpreted as support for a compact four-quark nature of the scalar mesons.

In this comment we want to point out a fundamental flaw in the calculations presented in Ref. [5] which, in turn, invalidates the criticism raised in that paper. Namely, in order to demonstrate that the high-momentum components determine the $\phi \to K^+K^- \to \gamma a_0/f_0$ amplitude the authors of [5] introduce a momentum cutoff in the relevant integrals. However, in doing so gauge invariance gets violated. As will be shown below, large momentum contributions appear only in this induced gauge-invariance-violating term and are therefore of no physical significance.

To keep our argument self-contained we briefly repeat the essentials of the formalism. As a consequence of gauge invariance the full matrix element for the $\phi \to \gamma S$ ($S = a_0$ or f_0) decay, M_ν, can be written as

$$M_\nu = \frac{e g_\phi g_S}{2\pi^2 i m^2} I(m_V, m_S)[\varepsilon_\nu (p\cdot q) - p_\nu (q\cdot \varepsilon)] = e g_\phi g_S \varepsilon^\mu J_{\mu\nu}, \tag{1}$$

where p and q are the momenta of the ϕ meson and the photon, respectively, m is the kaon mass, g_ϕ and g_S are the ϕK^+K^- and SK^+K^- coupling constants, and ε_ν is the polarization four-vector of the ϕ meson. The masses of the ϕ meson and the scalar are denoted by m_V and m_S, respectively. The function $I(m_V, m_S)$ has a smooth limit for $q \to 0$. As a consequence of gauge invariance the amplitude (1) is transverse, $M_\nu q^\nu = 0$, and is proportional to the photon momentum; especially it vanishes for $q \to 0$. The form (1) is well known. Details can be found, for example, in Refs. [6–10].

For pointlike scalars, only diagrams (a)–(c) of Fig. 1 contribute. If the scalars are regarded as extended objects, a

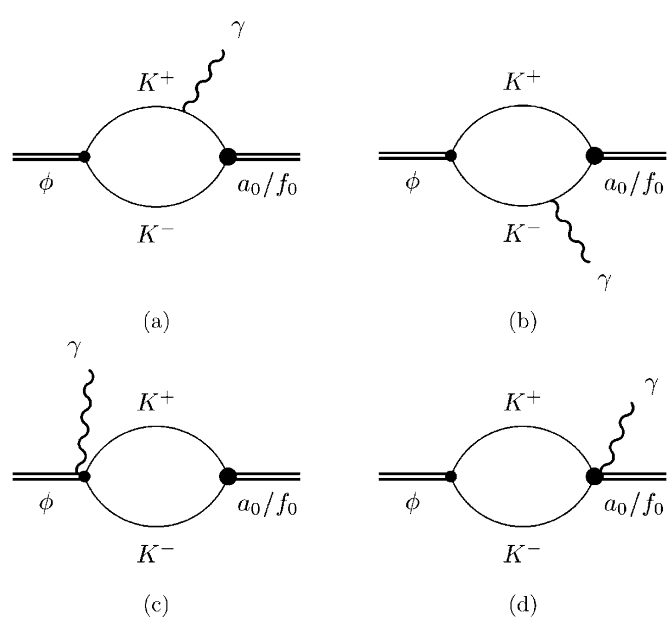

FIG. 1. Diagrams contributing to the amplitude of the radiative decay $\phi \to \gamma a_0/f_0$.

vertex function needs to be introduced at the $\bar{K}KS$ vertex. Then gauge invariance demands the inclusion of a diagram of type (d). For general kinematics a proper construction of this additional term is quite involved (see Ref. [11] where we list a few of the papers devoted to this subject) and contains some ambiguity. However, for soft photons, all the different recipes give the same result up to corrections of order $(q/\beta)^2$ that will be dropped. Here $1/\beta$ denotes the range of forces—for the case of interest one may use $\beta \sim m_\rho$, where m_ρ denotes the mass of the lightest exchange particle allowed, namely, that of the ρ meson [1]. We may then as well use the method suggested in Ref. [10] that is based on minimal substitution considerations. For more details on the issue of gauge invariance for the reaction considered here, see Ref. [12].

After this introduction let us discuss the main formula of our paper [1]. It is argued there (and confirmed by actual calculations) that, since the amplitude is finite even for the pointlike limit, the range of convergence of the integrals involved *is defined only by the kinematics of the problem*. In particular, if both masses, i.e. that of the vector and of the scalar meson, are close to the $K\bar{K}$ threshold, the integrals converge at $k_0 \sim m$, and thus for nonrelativistic values of the three-dimensional loop momentum $\vec{k}$, $|\vec{k}| \ll m$. This allows us to perform a nonrelativistic reduction of the amplitude in the rest-frame of the ϕ meson. The integrals in question for the individual graphs of Fig. 1 are (note that $J_{ik}^{(b)} = J_{ik}^{(a)}$):

$$J_{ik}^{(a)} = \frac{-i}{2m^3} \int \frac{d^3k}{(2\pi)^3} \frac{k_i K_k \Gamma(K)}{[E_V - \frac{k^2}{m} + i0][E_S - \frac{K^2}{m} + i0]},$$
$$J_{ik}^{(c)} = \frac{-i}{2m^2} \delta_{ik} \int \frac{d^3k}{(2\pi)^3} \frac{\Gamma(k)}{E_S - \frac{k^2}{m} + i0}, \tag{2}$$
$$J_{ik}^{(d)} = \frac{-i}{2m^2} \int \frac{d^3k}{(2\pi)^3} \frac{k_i k_k}{E_V - \frac{k^2}{m} + i0} \frac{1}{k} \frac{\partial \Gamma(k)}{\partial k},$$

where $E_V = m_V - 2m$, $E_S = m_S - 2m$, and $\vec{K} = \vec{k} - \frac{1}{2}\vec{q}$. The last integral can be rewritten by performing an integration by parts:

$$J_{ik}^{(d)} = \frac{i}{2m^2} \delta_{ik} \int \frac{d^3k}{(2\pi)^3} \frac{\Gamma(k)}{E_V - \frac{k^2}{m} + i0} + \frac{i}{3m^3} \delta_{ik} \times \int \frac{d^3k}{(2\pi)^3} \frac{k^2 \Gamma(k)}{[E_V - \frac{k^2}{m} + i0]^2} - \frac{i}{12\pi^2 m^2} \delta_{ik} \times \int_0^\infty dk \frac{\partial}{\partial k}\left(\frac{k^3 \Gamma(k)}{E_V - \frac{k^2}{m} + i0}\right). \tag{3}$$

Here, contrary to Ref. [1], with the last term we kept the surface integral that emerges in the calculation. In order to investigate the range of momenta relevant for the loop integrals in Ref. [5], a momentum cutoff Λ was introduced. We follow this prescription and write the full transition current as

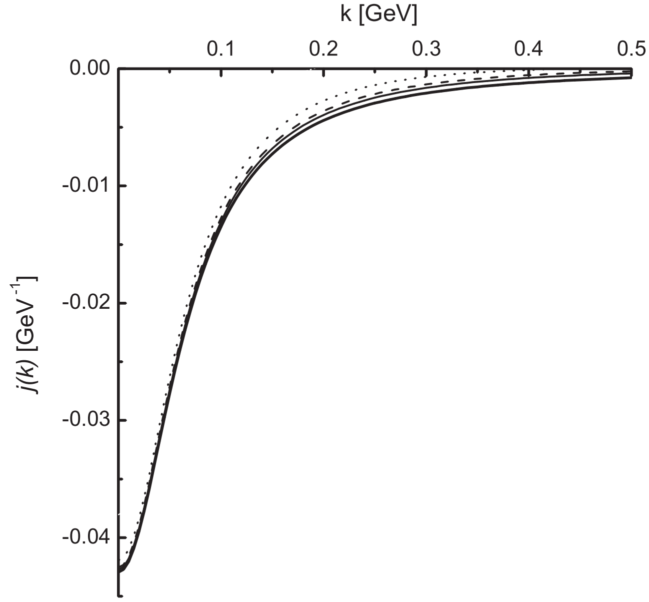

FIG. 2. The behavior of the integrand $j(k)$ of $\mathrm{Im}(\hat{J}_{ik})$, as a function of the kaon momentum floating in the loop. We chose $m_S = 0.98$ GeV, $m_V = 1.02$ GeV, $m = 0.495$ GeV, and four values of the parameter β: $\beta = 0.4$ GeV (dotted line), $\beta = 0.6$ GeV (dashed line), $\beta = 0.8$ GeV (thin solid line), and $\beta = \infty$ (thick solid line).

$$J_{ik}(\Lambda) = \hat{J}_{ik}(\Lambda) + \delta_{ik} R(\Lambda), \tag{4}$$

where

$$R(\Lambda) = -\frac{i}{12\pi^2 m^2} \frac{\Lambda^3 \Gamma(\Lambda)}{E_V - \frac{\Lambda^2}{m} + i0} \tag{5}$$

contains the above-mentioned surface term and

$$\hat{J}_{ik}(\Lambda) = -\frac{i}{m^3} \int^\Lambda \frac{d^3k}{(2\pi)^3} \times \left\{ \frac{k_i(\vec{k} - \frac{1}{2}\vec{q})_k \Gamma(\vec{k} - \frac{1}{2}\vec{q})}{[E_V - \frac{k^2}{m} + i0][E_V - q - \frac{(\vec{k} - \frac{1}{2}\vec{q})^2}{m} + i0]} + \Gamma(k)\delta_{ik}\left(\frac{m}{2} \frac{q}{[E_V - q - \frac{k^2}{m} + i0][E_V - \frac{k^2}{m} + i0]} - \frac{1}{3} \frac{k^2}{[E_V - \frac{k^2}{m} + i0]^2}\right)\right\}. \tag{6}$$

For later convenience we used energy conservation to replace E_S via $E_S = E_V - q$.[1] For $\Lambda \to \infty$ $\hat{J}_{ik}(\Lambda)$ matches to the formula used in Ref. [1] to calculate the matrix element for $\phi \to \gamma a_0/f_0$. We checked that this sum of integrals converges for nonrelativistic kaon momenta. This finding was confirmed in Ref. [5].

[1]Contrary to the claim made in Ref. [5] of Ref. [5] energy and momentum conservation are maintained in the calculations of Ref. [1].

For illustration we choose a particular form of $\Gamma(k)$, namely $\Gamma(k) = \beta^2/(k^2 + \beta^2)$, and study that part of $\hat{J}_{ik}$ proportional to the structure δ_{ik} (according to Eq. (1) exactly this structure contributes to the decay amplitude in the ϕ-meson rest-frame). In Fig. 2 we plot the behavior of the integrand $j(k)$ ($\mathrm{Im}(\hat{J}_{ik}) = \delta_{ik} \int_0^\infty j(k)dk$), as a function of k (note that the integrand in the similar integral for $\mathrm{Re}(\hat{J}_{ik})$ contains $\delta(k^2 - mE_V)$, such that $k = \sqrt{mE_V} \approx 0.12$ GeV). From Fig. 2 one can see that the integral indeed converges at nonrelativistic values of the kaon momentum, regardless of the value of the finite-range parameter β—the latter plays no role for the convergence.

In Ref. [1] the last term of Eq. (4), $R(\Lambda)$, was dropped, for it vanishes exactly for $\Lambda \to \infty$.[2] In Ref. [5], however, it is argued that this term should be kept and that it converges only for very large values of Λ, which means that the corresponding integral acquired contributions from very large momenta. The contribution of those large momentum components is then taken as a proof that only if the scalars are very compact objects, a sizable contribution from the loop can emerge. Notice that, even for finite values of Λ, $\hat{J}_{ik}(\Lambda)$ vanishes for $q \to 0$, as required by the general structure given in Eq. (1). However, since $R(\Lambda)$ is independent of the photon momentum q, it gives a nonvanishing contribution to J_{ik} even for $q = 0$ for all finite values of Λ. Therefore this term violates gauge invariance. Thus, by introducing a sharp cutoff into the problem the authors of Ref. [5] produced a term that violates gauge invariance.[3] Since the whole argument presented in Ref. [5] is based on this term, it bears no physical significance.

We therefore conclude that all results of Ref. [1] are valid. In particular, there is no strong suppression of kaon loops by the scalar wave function. Regardless of this, it should be stressed that the data for $\phi \to \gamma a_0/f_0$ [13] is very sensitive to the nature of the light scalar mesons, for it allows direct access to the effective coupling constant g_{eff} of the scalar to the kaons. As was shown in Ref. [14] this coupling is a direct measure of the molecular contribution of the scalar mesons.

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[2]For this to be true we only need to demand that $\lim_{\Lambda\to\infty} \Lambda\Gamma(\Lambda) = 0$.

[3]With sharp cutoff, gauge invariance of the amplitude can be restored by a subtraction at $q = 0$. Obviously, this procedure is equivalent to omission of the last, q-independent term in Eq. (4).

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