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Implications of the anomalous top quark couplings in $B_s - \bar{B}_s$ mixing, $B \to X_s \gamma$ and top quark decays

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Combined analysis of recent measured $B_s - \bar{B}_s$ mixing and $B \to X_s \gamma$ decays provides constraints on the anomalous $\bar{t}sW$ couplings. We discuss the perspectives to examine the anomalous $\bar{t}sW$ couplings through Cabibbo-Kobayashi-Maskawa quark-mixing matrix-suppressed $t \to sW$ decays at the LHC.

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I. INTRODUCTION

The standard model (SM) has been demonstrated to be remarkably successful in describing present data. Most parameters of the SM has been directly measured with high accuracy at various experiments. The only unobserved ingredient of the SM is the Higgs boson responsible for the electroweak symmetry breaking and a few top quark couplings are not measured directly. However, it is not believed that the SM is the final theory of our universe since there are still many theoretical and experimental problems that could not be explained in the SM framwork. It is natural to expect that the hint of the new physics beyond the SM would be found at the unexamined part of the SM.

The top quark has been discovered at the Tevatron and its mass and production cross section are measured [1]. We will be able to study the top quark couplings with more than 10^8 top quark pairs per year produced at the CERN Large Hadron Collider (LHC) [2,3]. The dominant channel of the top quark decay is the $t \to bW$ channel in the SM and the $\bar{t}bW$ coupling will be measured at LHC with high precision to be directly tested. Other channels are highly suppressed by small mixing angles. The subdominant channel in the SM is the Cabibbo-Kobayashi-Maskawa (CKM) nondiagonal $t \to sW$ decay of which branching ratio is estimated as

$$\mathrm{Br}\,(t \to sW) \sim 1.6 \times 10^{-3}, \tag{1}$$

when $|V_{ts}| = 0.04$ is assumed in the SM. Although the branching ratio of this channel is rather small, the $t \to sW$ process may be detectable at the LHC due to the large number of top quark production and the $\bar{t}sW$ coupling be measured to provide a clue to new physics beyond the SM. Therefore the anomalous $\bar{t}sW$ coupling is worth examining at present. We do not specify the underlying model here but present an effective Lagrangian to describe the new effects on the top quark couplings by introducing two parameters for each flavor. The relevant couplings are parametrized by the effective Lagrangian as

$$\mathcal{L} = -\frac{g}{\sqrt{2}} \sum_{q=d,s,b} V_{tq}^{\mathrm{eff}} \bar{t} \gamma^\mu (P_L + \xi_q P_R) q W_\mu^+ + \mathrm{H.c.}, \tag{2}$$

where ξ_q are complex parameters measuring effects of the anomalous right-handed couplings while V_{tq}^{eff} measures the SM-like left-handed couplings. Effects of the anomalous top quark couplings have been studied in direct and indirect ways [4–13].

Particularly interesting is $b \to s$ transition in search of the anomalous top quark couplings. The radiative decay $B \to X_s \gamma$ is the first observation of $b \to s$ transition and provides strict constraints on the anomalous top quark couplings [4,5] Since no CP phase is involved in V_{ts} and V_{tb} in the SM, a large direct CP violation in $b \to s$ is evidence of the new physics beyond the SM [6,11]. Recently the first observation of the $B_s - \bar{B}_s$ mixing have been reported by the CDF [14] and D0 [15] collaborations with the results

$$\begin{aligned} \Delta M_s &= (17.77 \pm 0.10 \pm 0.07)\ \mathrm{ps}^{-1} \quad (\mathrm{CDF}) \\ &= (18.53 \pm 0.93 \pm 0.30)\ \mathrm{ps}^{-1} \quad (\mathrm{D0}), \end{aligned} \tag{3}$$

where the first error is statistical and the second is systematic. The $B_s - \bar{B}_s$ mixing arises through the box diagram with internal lines of W boson and u-type quarks in the SM. Since the top quark loop dominates the $B_s - \bar{B}_s$ mixing might be also a testing laboratory for the study of the $\bar{t}sW$ and $\bar{t}bW$ couplings.

In this work, we concentrate on $\bar{t}sW$ coupling and perform the combined analysis of $B_s - \bar{B}_s$ mixing and $B \to X_s \gamma$ to constrain the V_{ts}^{eff} and ξ_s. $B_s - \bar{B}_s$ mixing depends upon V_{ts}^{eff} and is insensitive to the right-handed couplings while both of V_{ts}^{eff} and ξ_s are important in $B \to X_s \gamma$ decays. If we measure the subdominant decay $t \to sW$ at the LHC or other future colliders, it will be the direct test of the CKM matrix element V_{ts}^{eff} to determine the $\bar{t}sW$ couplings. This paper is organized as follows: In Sec. II, the effective $\Delta B = 1$ Hamiltonian formalism with anoma-

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lous $\bar{t}sW$ couplings is given and the radiative $B \to X_s\gamma$ decays are studied. In Sec. III, the analysis on the $B_s - \bar{B}_s$ mixing with anomalous $\bar{t}sW$ couplings is presented. We discuss the top quark decays in Sec. IV. Finally we conclude in Sec. V.

II. $B \to X_s\gamma$

The $\Delta B = 1$ effective Hamiltonian for $b \to s\gamma$ process is given by

$$\mathcal{H}^{\Delta B=1}_{\rm eff} = -\frac{4G_F}{\sqrt{2}} V^*_{ts} V_{tb} \sum_{i=1}^{8} (C_i(\mu) O_i(\mu) + C'_i(\mu) O'_i(\mu)), \tag{4}$$

where the dimension 6 operators O_i constructed in the SM are given in the Ref. [16], and O'_i are their chiral conjugate operators. Matching the effective theory (5) and the Lagrangian (4) at $\mu = m_W$ scale, we have the Wilson coefficients $C_i(\mu = m_W)$ and $C'_i(\mu = m_W)$. Although we will consider the anomalous $\bar{t}sW$ couplings only, we present the full formalism including $\bar{t}sW$ and $\bar{t}bW$ couplings. In the SM, we have the Wilson coefficients

$$\begin{aligned} &C_2(m_W) = -1, \qquad C_7(m_W) = F(x_t), \\ &C_8(m_W) = G(x_t), \\ &C_i(m_W) = C'_i(m_W) = 0, \qquad \text{otherwise}, \end{aligned} \tag{5}$$

where $F(x)$ and $G(x)$ are the well-known Inami-Lim loop functions [16,17]. Let us switch on the right-handed $\bar{t}bW$ and $\bar{t}sW$ couplings. Keeping the effects of ξ_q in linear order, we obtain the modified Wilson coefficients

$$\begin{aligned} C_7 &\to C_7^{\rm SM} + \xi_b \frac{m_t}{m_b} F_R(x_t), \\ C_8 &\to C_8^{\rm SM} + \xi_b \frac{m_t}{m_b} G_R(x_t), \end{aligned} \tag{6}$$

and the new Wilson coefficients

$$C'_7 = \xi_s \frac{m_t}{m_b} F_R(x_t), \qquad C'_8 = \xi_s \frac{m_t}{m_b} G_R(x_t), \tag{7}$$

where the new loop functions

$$\begin{aligned} F_R(x) &= \frac{-20 + 31x - 5x^2}{12(x-1)^2} + \frac{x(2-3x)}{2(x-1)^3} \ln x, \\ G_R(x) &= -\frac{4 + x + x^2}{4(x-1)^2} + \frac{3x}{2(x-1)^3} \ln x, \end{aligned} \tag{8}$$

agree with those in Ref. [18].

The branching ratio of $B \to X_s\gamma$ process with the right-handed interactions at next-leading-order (NLO) is given by

$$\begin{aligned} {\rm Br}(B \to X_s\gamma) = \frac{{\rm Br}(B \to X_c e\bar{\nu})}{10.5\%} &\times [B_{22}(\delta) + B_{77}(\delta)(|r_7|^2 + |r'_7|^2) \\ &+ B_{88}(\delta)(|r_8|^2 + |r'_8|^2) + B_{27}(\delta)\,{\rm Re}(r_7) \\ &+ B_{28}(\delta)\,{\rm Re}(r_8) \\ &+ B_{78}(\delta)({\rm Re}(r_7 r_8^\star) + {\rm Re}(r'_7 r'^\star_8))], \end{aligned} \tag{9}$$

where the ratios r_i and r'_i are defined by

$$\begin{aligned} r_i &= \frac{C_i(m_W)}{C_i^{\rm SM}(m_W)} = 1 + \xi_b \frac{m_t}{m_b} \frac{F_R(x_t)}{F(x_t)}, \\ r'_i &= \xi_s \frac{m_t}{m_b} \frac{F_R(x_t)}{F(x_t)}. \end{aligned} \tag{10}$$

The components $B_{ij}(\delta)$ depends on the kinematic cut δ, of which numerical values are given in the Ref. [19]. We obtain the branching ratio in terms of ξ_s and ξ_b as

$$\begin{aligned} {\rm Br}(B \to X_s\gamma) = {\rm Br}^{\rm SM}(B \to X_s\gamma)\left(\frac{|V^{\rm eff *}_{ts} V^{\rm eff}_{tb}|}{0.0404}\right)^2 \Bigg[1 + {\rm Re}(\xi_b)\frac{m_t}{m_b}\left(0.68\frac{F_R(x_t)}{F(x_t)} + 0.07\frac{G_R(x_t)}{G(x_t)}\right) + (|\xi_b|^2 + |\xi_s|^2)\frac{m_t^2}{m_b^2} \\ \times \left(0.112\frac{F_R^2(x_t)}{F^2(x_t)} + 0.002\frac{G_R^2(x_t)}{G^2(x_t)} + 0.025\frac{F_R(x_t)G_R(x_t)}{F(x_t)G(x_t)}\right)\Bigg]. \end{aligned} \tag{11}$$

The SM branching ratio is predicted to be ${\rm Br}(B \to X_s\gamma) = (3.15 \pm 0.23) \times 10^{-4}$ for $E_\gamma > 1.6$ GeV at next-to-next-to-leading order (NNLO) [20]. The current world average value of the measured branching ratio is given by [21]

$${\rm Br}\,(B \to X_s\gamma) = (3.55 \pm 0.24^{+0.09}_{-0.10} \pm 0.03) \times 10^{-4} \tag{12}$$

with the same photon energy cut. In this work, we assume $\xi_b = 0$ and focus on ξ_s. The allowed parameter sets of $(|\xi_s|, |V^{\rm eff}_{ts}|)$ are depicted in Fig. 1 by green (gray) area at 95% C.L.

III. $B_s - \bar{B}_s$ MIXING

A B_s^0 meson can oscillate into its antiparticle $\bar{B}_s^0$ via flavor-changing processes of $B_s - \bar{B}_s$ mixing. The oscillation is represented by the mass difference between the heavy and light B_s states,

$$\Delta M_s \equiv M_H^{B_s} - M_L^{B_s} = 2|M_{12}^s|, \tag{13}$$

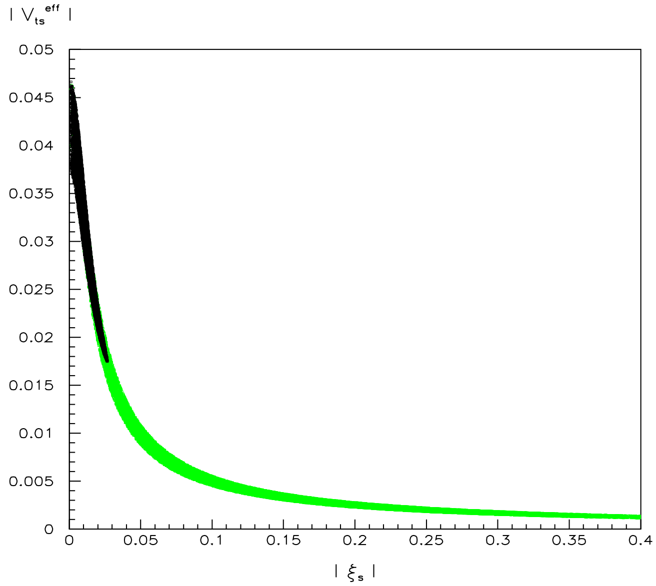

FIG. 1 (color online). Allowed parameter sets ($|\xi_s|, |V_{ts}^{\rm eff}|$) constrained by $B \to X_s\gamma$ (green) and by both ΔM_s and $B \to X_s\gamma$ (black).

where the $\Delta B = 2$ transition amplitudes given by

$$\langle B_s^0|\mathcal{H}_{\rm eff}^{\Delta B=2}|\bar{B}_s^0\rangle = M_{12}^s \tag{14}$$

is obtained by the box diagrams with internal lines of W boson and up-type quarks in the SM. The new contributions to $B_s - \bar{B}_s$ mixing with anomalous top quark couplings given in Eq. (1) would be examined with the $B_s - \bar{B}_s$ mixing data. The $B_s - \bar{B}_s$ mixing is also described by the width difference of the mass eigenstates

$$\Delta\Gamma_s \equiv \Gamma_L^s - \Gamma_H^s = 2|\Gamma_{12}^s|\cos\phi_s, \tag{15}$$

where the decay widths Γ_L and Γ_H are corresponding to the physical eigenstates B_L and B_H and the CP phase is given by $\phi_s = \arg(-M_{12}^s/\Gamma_{12}^s)$. Since the decay matrix element Γ_{12}^s is derived from the SM decays $b \to c\bar{c}s$ at tree level, it is hardly affected by the new physics. We assume that there exist the new effects of the anomalous top couplings only in M_{12}^s. Since ξ_q and $V_{ts}^{\rm eff}$ may be complex parameters, the new physics effects arise in both magnitude and phase of M_{12}^s in general. Effects of the phase and CP violation in M_{12}^s have been measured [22] and discussed in several literatures [23].

Including the odd number of right-handed couplings in the box diagram does not contribute to the transition amplitude M_{12}^s due to vanishing the loop integral of the odd number of momentum. Thus the leading contribution of the anomalous right-handed top couplings to the $B_s - \bar{B}_s$ mixing is quadratic order of ξ_q. Calculating box diagrams including the anomalous couplings, we obtain the transition amplitude and parametrize it by

$$\begin{aligned} M_{12}^s &= \frac{G_F^2 m_W^2}{12\pi} m_{B_s}\eta_B \hat{B}_{B_s} f_{B_s}^2 S_0(x_t)\left(\frac{V_{ts}^{\rm eff*}V_{tb}^{\rm eff}}{0.0404}\right)^2\left(1 + \frac{S_3(x_t)}{S_0(x_t)}\left(\frac{\xi_s^2}{4}\frac{\langle B_s^0|(\bar{b}P_Rs)(\bar{b}P_Rs)|\bar{B}_s^0\rangle}{\langle B_s^0|(\bar{b}\gamma^\mu P_Ls)(\bar{b}\gamma_\mu P_Ls)|\bar{B}_s^0\rangle}\right.\right.\\ &\quad \left.\left. + \frac{\xi_b^*\xi_s}{2}\frac{\langle B_s^0|(\bar{b}P_Ls)(\bar{b}P_Rs)|\bar{B}_s^0\rangle}{\langle B_s^0|(\bar{b}\gamma^\mu P_Ls)(\bar{b}\gamma_\mu P_Ls)|\bar{B}_s^0\rangle} + \frac{\xi_b^{*2}}{4}\frac{\langle B_s^0|(\bar{b}P_Ls)(\bar{b}P_Ls)|\bar{B}_s^0\rangle}{\langle B_s^0|(\bar{b}\gamma^\mu P_Ls)(\bar{b}\gamma_\mu P_Ls)|\bar{B}_s^0\rangle}\right)\right),\\ &\equiv M_{12}^{\rm SM,s}\cdot\Delta_s, \end{aligned} \tag{16}$$

where η_B is the perturbative QCD correction to the $B - \bar{B}$ mixing [24]. The Inami-Lim loop functions are given by

$$S_0(x) = \frac{4x - 11x^2 + x^3}{4(1-x)^2} - \frac{3x^3}{2(1-x)^3}\log x, \qquad S_3(x) = 4x^2\left(\frac{2}{(1-x)^2} + \frac{1+x}{(1-x)^3}\log x\right). \tag{17}$$

Using the vacuum insertions, we calculate

$$\begin{aligned} \frac{\langle B_s^0|(\bar{b}P_Rs)(\bar{b}P_Rs)|\bar{B}_s^0\rangle}{\langle B_s^0|(\bar{b}\gamma^\mu P_Ls)(\bar{b}\gamma_\mu P_Ls)|\bar{B}_s^0\rangle} &= \frac{\langle B_s^0|(\bar{b}P_Ls)(\bar{b}P_Ls)|\bar{B}_s^0\rangle}{\langle B_s^0|(\bar{b}\gamma^\mu P_Ls)(\bar{b}\gamma_\mu P_Ls)|\bar{B}_s^0\rangle} = \frac{5}{8}\left(\frac{m_{B_s}}{m_b + m_s}\right)^2,\\ \frac{\langle B_s^0|(\bar{b}P_Ls)(\bar{b}P_Rs)|\bar{B}_s^0\rangle}{\langle B_s^0|(\bar{b}\gamma^\mu P_Ls)(\bar{b}\gamma_\mu P_Ls)|\bar{B}_s^0\rangle} &= \frac{3}{4}\left(\frac{1}{6} - \left(\frac{m_{B_s}}{m_b + m_s}\right)^2\right), \end{aligned} \tag{18}$$

and

$$\langle B_s^0|(\bar{b}\gamma^\mu P_Ls)(\bar{b}\gamma_\mu P_Ls)|\bar{B}_s^0\rangle = \tfrac{8}{3}m_{B_s}^2\hat{B}_{B_s}f_{B_s}^2, \tag{19}$$

where $\hat{B}_{B_s}$ is the Bag parameter and $f_{B_s}^2$ the decay constant.

We show the allowed parameter sets ($|\xi_s|$, $V_{ts}^{\rm eff}$) in Fig. 1 by the black area at 95% C.L., which are constrained by both $\mathrm{Br}(B \to X_s\gamma)$ and ΔM_s. We use the SM prediction $\Delta m_s = 19.3 \pm 6.74\ \mathrm{ps}^{-1}$ given in Ref. [25]. The conservative bounds $|\xi_s| < 0.03$ and $|V_{ts}^{\rm eff}| > 0.017$ are obtained.

Current experimental data on the phase of M_{12}^s show some deviations from the SM prediction. Figure 2 shows the scanned parameters in the complex Δ_s plane. The box is the combined experimental data given in Ref. [25]. Note that the phase of $\Delta_s = M_{12}^s/M_{12}^{\rm SM,s}$ is dominated by the phase of $V_{ts}^{\rm eff}$ in our model.

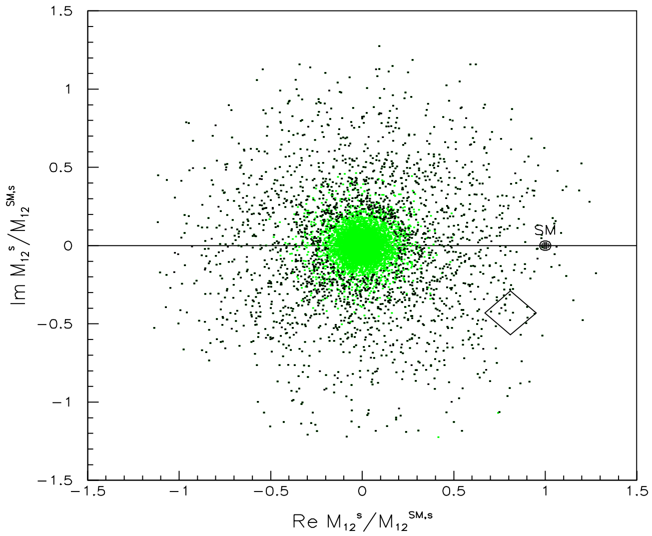

FIG. 2 (color online). Scanned results in the complex Δ_s plane from the allowed parameter sets by $B \to X_s\gamma$ (green) and by both ΔM_s and $B \to X_s\gamma$ (black). The box denotes the current experimental bounds given in Ref. [25].

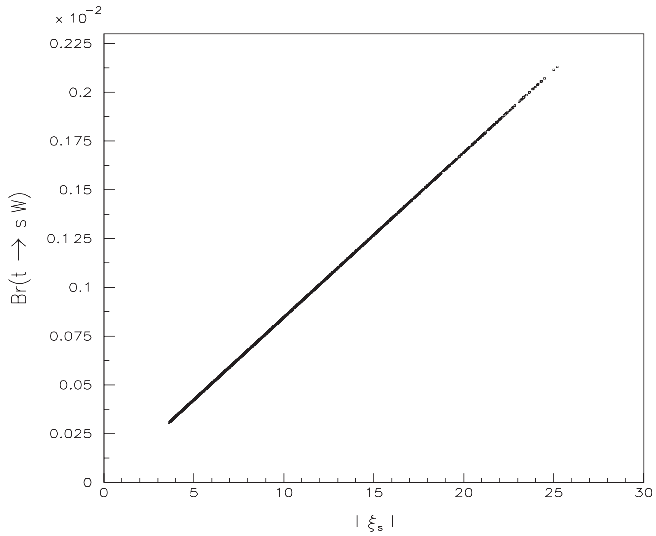

FIG. 4. Correlation of ΔM_s and Br$(t \to sW)$ with allowed values of $(|\xi_s|, |V_{ts}^{\rm eff}|)$.

IV. TOP QUARK DECAYS

The flavur-diagonal $t \to bW$ decay dominates, Br$(t \to bW) \approx 1$. The branching ratio of the CKM-suppressed decays are given by

$$\mathrm{Br}\,(t \to sW) = |V_{ts}^{\rm eff}|^2(1 + |\xi_s|^2). \tag{20}$$

Without any enhancement factor involved, the branching ratio is insensitive to the $|\xi_s|^2$ term and solely determined by $V_{ts}^{\rm eff}$. However, the CKM factor $V_{ts}^{\rm eff}$ is constrained by $B \to X_s\gamma$ decay and ΔM_s depending on $|\xi_s|$ as shown in Fig. 1, and the branching ratio Br$(t \to sW)$ also depends on the anomalous coupling ξ_s. The predictions of Br$(t \to sW)$ are depicted in Fig. 3 with respect to $|\xi_s|$ for allowed parameter sets. We find that large deviation of Br$(t \to sW)$ from the SM prediction is possible. The correlation between ΔM_s and Br$(t \to sW)$ are shown in Fig. 4 with allowed parameters given in Fig. 1 (black area). Both observables of ΔM_s and Br$(t \to sW)$ crucially depend on $V_{ts}^{\rm eff}$ but are insensitive to ξ_s. Since the value of $V_{ts}^{\rm eff}$ will be strongly constrained by Br$(t \to sW)$, measurement of the branching ratio of $t \to sW$ at the LHC or the future colliders affects the right-handed coupling ξ_s through $B \to X_s\gamma$ decay.

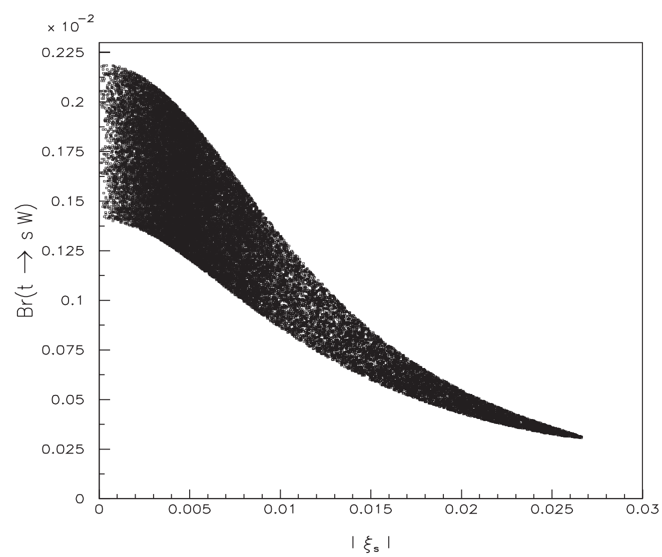

FIG. 3. Prediction of Br$(t \to sW)$ with respect to $|\xi_s|$ for allowed parameters by ΔM_s and Br$(B \to X_s\gamma)$ constraints.

V. CONCLUDING REMARKS

We consider the anomalous top quark couplings which are not direct measured yet. The $\bar{t}sW$ coupling is parametrized by $V_{ts}^{\rm eff}$ and ξ_s. Combined analysis of $B_s - \bar{B}_s$ mixing and $B \to X_s\gamma$ decay gives strong constraints on $V_{ts}^{\rm eff}$ and ξ_s. The prediction of the branching ratio of the top decay Br$(t \to sW)$ is given, and it is shown that both of ΔM_s and Br$(t \to sW)$ depend only on $V_{ts}^{\rm eff}$ but are affected by ξ_s through $B \to X_s\gamma$ decay. In conclusion, we can examine the anomalous $\bar{t}sW$ coupling through $B_s - \bar{B}_s$ mixing and $B \to X_s\gamma$ decay and suggest further study with the $t \to sW$ decay in the future colliders.

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