Quark masses and resummation in precision QCD theory

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It is shown that amplitude-based, exact resummation tames the uncanceled IR divergences at $\mathcal{O}(\alpha_s^2)$ in initial state radiation in QCD with massive quarks. Implications for precision predictions for LHC physics are discussed.

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The era of precision QCD at the LHC, by which we mean 1% or better precision tags on the theoretical predictions, presents us with the extremely challenging task of proving that a given theoretical precision tag does in fact hold to that level. This means that all aspects of the standard formula for hadron-hadron scattering in perturbative QCD have to be examined for possible sources of uncertainty in the physical and technical precision components of any quoted total theoretical precision tag. In this connection, we note the standard practice of treating all quarks in the initial state as massless. It would be desirable to put an explicit error tag on this assumption by doing the respective calculations with the respective quark masses at their known [1] values and comparing the attendant predictions with their massless limits. This direct approach is however currently blocked by the pioneering results in Refs. [2,3], wherein it has been established that there is a lack of Bloch-Nordsieck cancellation at $\mathcal{O}(\alpha_s^2)$ in the initial state radiation in massive QCD. Hence, even the b quark has to have zero mass in the initial state radiative corrections when one works to $\mathcal{O}(\alpha_s^n)$ with $n \geq 2$.

In what follows, we revisit the results in Ref. [2] from the standpoint of recent progress [4,5] in the resummation of large IR effects in the QCD perturbation theory, where we will focus on exact resummation methods[1] with an eye toward rigorous control on any theoretical precision error budget that we may ultimately want to advocate. In this context, let us recall already the master formula that we have derived in Refs. [4]: using a $2 \to 2 + X$ hard process with multiple gluon (G) emission, $q(p_1) + q'(q_1) \to q''(p_2) + q'''(q_2) + n(G) + X(p_X)$ in an obvious 4-momentum assignment notation, we have the differential cross section

$$d\hat{\sigma}_{\rm exp} = e^{{\rm SUM}_{\rm IR}({\rm QCD})} \sum_{n=0}^{\infty} \frac{1}{n!} \int \prod_{j=1}^{n} \frac{d^3 k_j}{k_j} \int \frac{d^4 y}{(2\pi)^4} e^{iy\cdot(p_1+q_1-p_2-q_2-p_X-\sum k_j)+D_{\rm QCD}} * \tilde{\bar{\beta}}_n(k_1,\ldots,k_n) \times ((d^3 p_2)/p_2^0)((d^3 q_2)/q_2^0)((d^3 p_X)/p_X^0), \tag{1}$$

where the hard gluon residuals $\tilde{\bar{\beta}}_n(k_1,\ldots,k_n)$ and the infrared functions ${\rm SUM}_{\rm IR}({\rm QCD}), D_{\rm QCD}$ are defined in Ref. [4] and we stress that the $\tilde{\bar{\beta}}_n(k_1,\ldots,k_n)$ are free of all infrared divergences to all orders in $\alpha_s(Q)$. (See especially Ref. [5] for explicit application of (1) to a real bremsstrahlung process.) Note that the hard gluon residuals $\tilde{\bar{\beta}}_n(k_1,\ldots,k_n)$ have the structure [4]

$$\tilde{\bar{\beta}}_n(k_1,\ldots,k_n) = [{\rm tr}\mathfrak{h}\mathcal{M}^{(n)\dagger}\mathcal{M}^{(n)}]|_{\rm IR\text{-}subtracted}$$

where the IR-subtraction is as given in Ref. [4] and $\mathfrak{h}$ is the initial-state color-spin density matrix so that the full quantum mechanical color effects are included in (1). For our initial state radiation (ISR) analysis, we take q and q' to be massive quarks of mass m_q, we take X to be a (QCD singlet) electroweak gauge boson to match the problem studied in Ref. [2] and we only compute ISR radiative effects in QCD. Let us then recall the pioneering result in Ref. [2]: working in the eikonal approximation and discussing the part of the cross section proportional to the color structure (here, H corresponds to the attendant hard subprocess)

$$F_1 = C_2(G) H_{ab}^{\alpha\beta}(T_i)^{\beta\alpha}(T_i)_{ba} \tag{2}$$

for the process $q^\alpha q^a \to V^{(*)} + X'$ where $V^{(*)}$ is our (off-shell) electroweak gauge boson and α, a are the colors of the quarks, the authors in Ref. [2] find the IR divergent result

$${\rm flux}\ \frac{d\sigma}{d^3Q} = \frac{-g^4\bar{H}}{(d-4)32\pi^2}\left(\frac{1-\beta}{\beta}\right)\left(\frac{1}{\beta}\ln\left(\frac{1+\beta}{1-\beta}\right)-2\right), \tag{3}$$

where g is the QCD coupling constant, $\bar{H}$ is the attendant hard subprocess factor dressed as in the color structure F_1,

[1] We do not employ here the resummation algebras that focus on the boundary of phase space [6,7], as these engender approximations which require arguments that are not opportune for our purposes.

β is the velocity of one quark in the rest frame of the other, and d is the dimension of space-time with $d > 4$ to regulate the uncanceled IR divergence. Q^2 is the invariant mass of the V^*. This divergence is clearly non-Abelian in character as it vanishes for $C_2(G) = 0$, where we define the gluon and quark representations' quadratic Casimir invariants, respectively, as usual:

$$f_{ijk}f_{ijl} = C_2(G)\delta_{kl} \qquad (T_iT_i)_{ab} = C_2(F)I_{ab} \equiv C_F I_{ab}, \tag{4}$$

where f_{ijk} are the group structure constants. The result (3) shows a clear lack of Bloch-Nordsieck cancellation at $\mathcal{O}(\alpha_s^2)$ and the standard approach is to set $m_q = 0$ so that this uncanceled IR divergence vanishes as $\beta \to 1$ as one can see from (3).

We point out that the authors in Ref. [3] have analyzed the problem studied in Ref. [2] from a coherent-state Hamiltonian approach and have corroborated the result (3) with the added understanding that, in the coherent-state approach, the single pole divergence is converted into an unfactorizable, unspecified dependence of the respective collinear singularities on the scale separating the attendant observable and unobservable gluon degrees of freedom inherent therein. This is again unacceptable and forces the use of $m_q = 0$ for initial ISR for calculations at $\mathcal{O}(\alpha_s^n)$, $n \geq 2$.

Here we propose an alternative approach. We look into the systematics of the analysis of the first paper in Ref. [2]. We see that one can represent the RHS of (3) as the leftover real IR divergence which is uncanceled by the virtual IR divergence. In the language of the diagrams analyzed by Mueller's theorem [8] in the aforementioned paper, the RHS of (3) can be identified with a fraction F_{nbn} of the contribution of the real emission from the contribution of the diagrams equivalent to the diagram contribution (q-o) in Fig. 6 in the first paper in Ref. [2], which we reproduce here for definiteness in Fig. 1. To see this, let us recall the result of this last paper for the (q-o) diagrams (see Fig. 1) contribution to the differential cross section, removing the kinematic (note in this language the hard scattering factor is kinematic to the soft interactions under study here) and color factors: from the 5th equation on page 11 of the paper, we have the result, from the equivalence of diagrams c and q and the equivalence of diagrams o and f in Fig. 6 of the paper (Fig. 1 here),

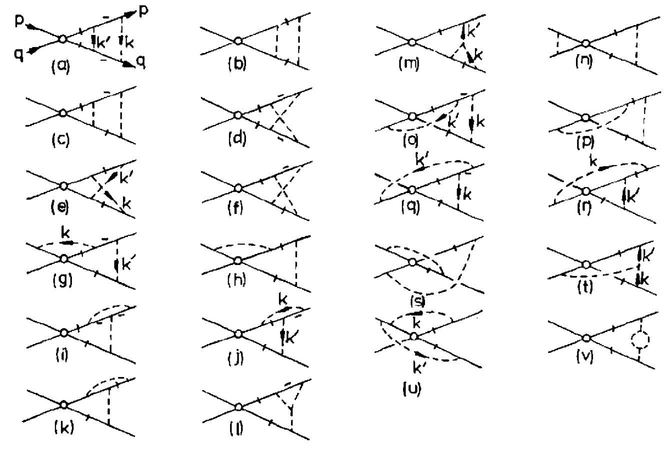

FIG. 1. Graphs evaluated in Ref. [2] (see the first paper therein, especially) in arriving at the result in (3) using Mueller's theorem for the respective cross section. Here, the usual Landau-Bjorken-Cutkosky(LBC) [20] rules obtain so that a slash puts the line on-shell and a dash changes the $i\epsilon$ prescription; and graphs that have canceled or whose contributions are implied by those in the figure are not shown explicitly.

$$A_{q\text{-}o} = \frac{1}{\beta^2}\int \frac{d^3k d^3k' 2k_z}{(k_z + k'_z + i\epsilon)(\beta^2k_z^2 - \mathbf{k}^2)(\beta^2k_z^2 - \mathbf{k}'^2 + i\epsilon)(k_z^2 + \epsilon^2)}, \tag{5}$$

where we denote the 3-momentum by boldface letters, so that $\vec{k} = \mathbf{k}$, and where the eikonal limit has been used in (5) as it was in respective paper in Ref. [2]. Because of this approximation, there is a spurious UV divergence in (5), which does not affect the IR regime. The authors in Ref. [2] therefore regulate this UV divergence with the factor $e^{-\mathbf{k}^2/\Lambda^2}$ for each would-be 3-space integral and then use dimensional methods [9] to isolate the IR divergence of interest; one obtains in this way the UV regulated result from (5)

$$A_{q\text{-}o}|_{\text{UV-reg}} = \frac{4\pi^{n+1}(\Lambda^2)^{n-3}}{\beta^2}\left\{\frac{1}{(n-3)^2} + \frac{1}{2(n-3)} \times \ln\left(\frac{1+\beta}{1-\beta}\right)\right\}, \tag{6}$$

where the UV cutoff Λ is large compared to the soft scales in the problem, and here we have $d = n + 1$ to make contact with (3). When one adds the remaining contributions associated to the remaining graphs in Fig. 1, one sees from comparing (3) and (6) that the double pole term in (6) is canceled and that the fraction

$$F_{nbn} = \frac{(1-\beta)(\ln(\frac{1+\beta}{1-\beta}) - 2\beta)}{\ln(\frac{1+\beta}{1-\beta})} \tag{7}$$

of the single pole term is left over as the uncanceled IR divergence.

The classic Landau-Bjorken-Cutkosky (LBC) analysis then allows us to determine the relationship between the real emission in the (q-o) diagram contribution and the single pole term on the RHS of (6). Specifically, upon doing the integral on the RHS of (5) over k'_z, there are two poles in the respective complex plane below the real axis, one at $-k_z - i\epsilon$ and one at $-\sqrt{\beta^2 k_z^2 - k_\perp'^2 + i\epsilon}$, where here the energy of the k'-gluon is just $-\beta k_z$ by the LBC rules in this eikonal exercise. The contribution of the former pole does not result in on-shell k' gluons. The LBC rules tell us that the regime $\Re = \{0 \le k_\perp'^2 \le \beta^2 k_z^2\}$ represents the regime wherein the k'-gluon is actually on-shell here. Focusing on this regime, we see that we have the contribution

$$A_{q\text{-}o}|_{\Re} = \Re \frac{1}{\beta^2} \int d^3k \int_0^{\beta^2 k_z^2} \pi d(k_\perp'^2) \frac{-2\pi i}{-(-2)\sqrt{\beta^2 k_z^2 - k_\perp'^2}} \frac{1}{k_z - \sqrt{\beta^2 k_z^2 - k_\perp'^2} + i\epsilon} \frac{1}{\beta^2 k_z^2 - \mathbf{k}^2} \frac{2k_z}{k_z^2 + \epsilon^2}, \tag{8}$$

where we have written the 2-space integration measure as $\pi d(k_\perp'^2)$ by doing the respective angular integral. The integration over the latter measure can then be rewritten, using the fact that we only need the real part,

$$\begin{aligned}
A_{q\text{-}o}|_{\Re} &= \Re \frac{-\pi i}{\beta^2} \int d^3k \int_0^{\beta^2 k_z^2} \pi d(k_\perp'^2) \frac{1}{\sqrt{\beta^2 k_z^2 - k_\perp'^2}} \frac{k_z + i\epsilon + \sqrt{\beta^2 k_z^2 - k_\perp'^2}}{(k_z + i\epsilon)^2 - (\beta^2 k_z^2 - k_\perp'^2)} \frac{1}{\beta^2 k_z^2 - \mathbf{k}^2} \frac{2k_z}{k_z^2 + \epsilon^2} \\
&= \Re \frac{-\pi i}{\beta^2} \int d^3k \int_0^{\beta^2 k_z^2} \pi d(k_\perp'^2) \frac{1}{\sqrt{\beta^2 k_z^2 - k_\perp'^2}} \frac{k_z + i\epsilon}{(k_z + i\epsilon)^2 - (\beta^2 k_z^2 - k_\perp'^2)} \frac{1}{\beta^2 k_z^2 - \mathbf{k}^2} \frac{2k_z}{k_z^2 + \epsilon^2} \\
&= \Re \frac{-\pi i}{\beta^2} \int d^3k \int_0^{\beta^2 k_z^2} \pi d(k_\perp'^2) \frac{1}{\sqrt{\beta^2 k_z^2 - k_\perp'^2}} \frac{1}{2}\left(\frac{1}{k_z + i\epsilon - \sqrt{\beta^2 k_z^2 - k_\perp'^2}} + \frac{1}{k_z + i\epsilon + \sqrt{\beta^2 k_z^2 - k_\perp'^2}}\right) \\
&\quad \times \frac{1}{\beta^2 k_z^2 - \mathbf{k}^2} \frac{2k_z}{k_z^2 + \epsilon^2} \\
&= \Re \frac{-i\pi^2}{\beta^2} \int d^3k(-\ln(k_z + i\epsilon - \beta|k_z|) + \ln(k_z + i\epsilon + \beta|k_z|)) \frac{1}{\beta^2 k_z^2 - \mathbf{k}^2} \frac{2k_z}{k_z^2 + \epsilon^2},
\end{aligned} \tag{9}$$

where we again emphasize that the on-shell regime actually has $k'_0 = -\beta k_z < 0$ so that the real radiative contribution, by the standard LBC methods, has $k_z > 0$. If we integrate over the region $k_z > \sqrt{\epsilon}$, it is clear that the RHS of the last equation has no real part as $\epsilon \to 0$. Thus, the real emission part of (9) must arise from the regime $0 \le k_z \le \sqrt{\epsilon}$. We treat the branch cuts for the logs by joining them between $k_{z1} = -i\epsilon/(1-\beta)$ and $k_{z2} = -i\epsilon/(1+\beta)$ and then we close the contour below the real axis as shown in Fig. 2 to get the result, by Cauchy's theorem,

$$\oint_C dk_z(-\ln(k_z + i\epsilon - \beta k_z) + \ln(k_z + i\epsilon + \beta k_z)) \times \frac{1}{\beta^2 k_z^2 - \mathbf{k}^2} \frac{2k_z}{k_z^2 + \bar{\epsilon}^2} = 0, \tag{10}$$

where we use the intrinsic freedom in the Feynman $i\epsilon$-prescription to take each such infinitesimal parameter independently to 0 from above and the curve C is given in Fig. 2. We take here $k_\perp > \sqrt{\epsilon}$.[2] If we denote the integrals over the ith part of C by I_i, $i = 1, \cdots, 7$, where the labels for these parts are defined in Fig. 2, then one can readily see that (for example we may set $\bar{\epsilon} = \epsilon^{3/2}$) we have

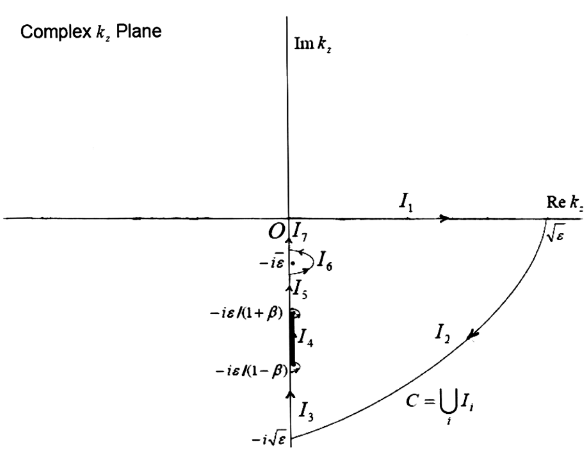

FIG. 2. The contour C used in the complex k_z-plane to evaluate the real emission part of the contribution of diagrams (q-o) in Fig. 1 to the RHS of (3). See the text for further discussion.

[2]We use standard Lebesgue integration theory to conclude that the order of integration does not matter so that fixing $k_\perp$ and integrating over k_z first, which means that the limit $\epsilon \to 0$ will always give us $k_\perp > \sqrt{\epsilon}$, followed by integration over the full range of $k_\perp$, when all integrals are finite by regularization where necessary, can not affect the final result. Alternatively, the reader can check that with the regularization we use, if one does the attendant integral over $0 \le k_\perp \le \sqrt{\epsilon}$, the respective result will vanish for $\epsilon \to 0$.

$$I_1 = \int_0^{\sqrt{\epsilon}} dk_z(-\ln(k_z + i\epsilon - \beta|k_z|) + \ln(k_z + i\epsilon + \beta|k_z|))\frac{1}{\beta^2k_z^2 - \mathbf{k}^2}\frac{2k_z}{k_z^2 + \bar{\epsilon}^2} = -\sum_{i=2}^{7} I_i. \tag{11}$$

We now treat the integrals I_i, $i = 2, \cdots, 7$ in turn.

For I_2, use the change of variable $k_z = \sqrt{\epsilon}e^{i\theta}$, for $0 \geq \theta \geq -\frac{\pi}{2}$. Then, we get

$$I_2 = \int_0^{-\pi/2} id\theta k_z(-\ln(k_z + i\epsilon - \beta k_z) + \ln(k_z + i\epsilon + \beta k_z))\frac{1}{\beta^2k_z^2 - \mathbf{k}^2}\frac{2k_z}{k_z^2 + \bar{\epsilon}^2} = 2\int_0^{-\pi/2} id\theta(-\ln(1-\beta) + \ln(1+\beta))\frac{1}{-\mathbf{k}_\perp{}^2} = -i\pi\ln\left(\frac{1+\beta}{1-\beta}\right)\frac{1}{(-\mathbf{k}_\perp{}^2)}. \tag{12}$$

For I_3 it is enough to use the change of variable $k_z = -iy$ to see that it is pure real so that it will not contribute to the imaginary part of I_1 via (10) and only this part of I_1 is needed in extracting the real emission part of the RHS of (9).

For I_4 we see from passing around the lower branch point in Fig. 2 that the respective imaginary contribution is just

$$i\Im I_4 = \int_{-i\epsilon/(1-\beta)}^{-i\epsilon/(1+\beta)} dk_z(-\pi i)\frac{1}{(-\mathbf{k}_\perp{}^2)}\frac{2k_z}{k_z^2 + \bar{\epsilon}^2} = 2\pi i\ln\left(\frac{1+\beta}{1-\beta}\right)\frac{1}{(-\mathbf{k}_\perp{}^2)}. \tag{13}$$

For I_5, we see by the change of variable $k_z = -iy$ that it is pure real and does not contribute to the imaginary part of I_1 via (10).

For I_6, we get the result

$$I_6 = \pi i \mathrm{Res}(-i\bar{\epsilon}) = 0 \tag{14}$$

since $\bar{\epsilon}/\epsilon \to 0$ when $\epsilon \to 0$.

Finally, for I_7 the change of variable $k_z = -iy$ shows that it too is pure real and does not contribute to the imaginary part of I_1 via (10).

The net result is that we arrive at

$$i\Im I_1 = -\{2\pi i - \pi i\}\frac{-1}{\mathbf{k}_\perp{}^2}\ln\left(\frac{1+\beta}{1-\beta}\right) = \frac{\pi i}{\mathbf{k}_\perp{}^2}\ln\left(\frac{1+\beta}{1-\beta}\right). \tag{15}$$

When we introduce the RHS of (15) into (9) we get the result

$$A_{q\text{-}o}|_{\Re,\text{real rad}} = \frac{2\pi^3}{\beta^2}\left(\frac{1}{2}\ln\left(\frac{1+\beta}{1-\beta}\right)\right)\int\frac{d^2k_\perp}{\mathbf{k}_\perp{}^2}, \tag{16}$$

where we explicitly indicate that this is the real emission contribution by the subscript real rad. Using the UV regulator employed in the first paper in Ref. [2], we see that the integral over $\mathbf{k}_\perp$ in (16) can be written as

$$I_{\text{UVreg}} = \int\frac{d^2k_\perp e^{-\mathbf{k}_\perp{}^2/\Lambda^2}}{\mathbf{k}_\perp{}^2} = \int\frac{d^3k\delta(k_z)e^{-\mathbf{k}^2/\Lambda^2}}{\mathbf{k}^2}. \tag{17}$$

We regulate the infrared divergence by analytic continuation to n dimensions to get

$$I_{\text{UVreg,IRreg}} = \int\frac{d^nk\delta(k_z)e^{-\mathbf{k}^2/\Lambda^2}}{\mathbf{k}^2} = \int_0^\infty d\rho\int d^nk\delta(k_z)e^{-\mathbf{k}^2/\Lambda^2-\rho\mathbf{k}^2} = \frac{2\pi^{(n-1)/2}}{n-3}(\Lambda^2)^{(n-3)/2}. \tag{18}$$

Introducing this last result into (16), we get

$$A_{q\text{-}o}|_{\Re,\text{real rad,UVreg}} = \frac{4\pi^4(\pi\Lambda^2)^{(n-3)/2}}{\beta^2}\times\left(\frac{1}{2(n-3)}\ln\left(\frac{1+\beta}{1-\beta}\right)\right), \tag{19}$$

which shows that the real emission part of $A_{q\text{-}o}$ saturates its single IR pole contribution.

Isolating the divergent single pole IR term in (19) we may now rewrite the pioneering result of Ref. [2] as follows: the uncanceled IR singular contribution to the respective differential cross section is

$$\text{flux}\,\frac{d\sigma}{d^3Q} = \frac{-g^4\bar{H}}{64\pi^6}F_{nbn}A_{q\text{-}o}|_{\Re,\text{real rad,IRpole part}}, \tag{20}$$

where from (19) we have

$$A_{q\text{-}o}|_{\Re,\text{real rad,IRpole part}} = \frac{4\pi^4}{\beta^2}\left(\frac{1}{2(n-3)}\ln\left(\frac{1+\beta}{1-\beta}\right)\right). \tag{21}$$

We note that the result in (19) agrees with the single pole term in (6) and with (21) up to finite terms.

From the result (20) we can now see how the theory of exact, amplitude resummation may impact the conclusions of Ref. [2]. We apply the formula in (1) to the real emission process in $A_{q\text{-}o}|_\Re$, following, for example, the steps given in Ref. [5]. We stress that we apply the resummation only to that fraction, F_{nbn}, of the real emission that has the uncanceled IR singularity in (20). The remaining $1 - F_{nbn}$ is not resummed because it is canceled by the sum of the remaining contributions associated with the diagrams in Fig. 1. We get, in this way, the result

$$F_{nbn}A_{q-o}|_{\Re,\text{real rad,resummed}} = F_{nbn}\Re\frac{-i\pi^2}{\beta^2}\int d^2k_\perp\int_0^{\sqrt{\epsilon}} dk_z F_{YFS}(\gamma_q)e^{\delta_q/2}(\beta k_z)^{\gamma_q} \times(-\ln(k_z+i\epsilon-\beta k_z)+\ln(k_z+i\epsilon+\beta k_z)) \times\frac{1}{\beta^2k_z^2-\mathbf{k}^2}\frac{2k_z}{k_z^2+\epsilon^2}, \tag{22}$$

where we have defined the resummation functions, from Ref. [5],

$$\gamma_q = 2C_F\frac{\alpha_s(Q^2)}{\pi}(\ln(s/m^2)-1) \tag{23}$$

$$\delta_q = \frac{\gamma_q}{2}+\frac{2\alpha_s C_F}{\pi}\left(\frac{\pi^2}{3}-\frac{1}{2}\right) \tag{24}$$

and

$$F_{\text{YFS}}(\gamma_q) = \frac{e^{-C_E\gamma_q}}{\Gamma(1+\gamma_q)}. \tag{25}$$

Here, C_F is the quark representation quadratic Casimir invariant already defined in (4), $s=(p_1+q_1)^2$ in our process in (1) specialized to $X=V^{(*)}$ with $Q^2=p_X^2$,

$$C_E = .5772\ldots$$

is Euler's constant and $\Gamma(w)$ is Euler's gamma function. The function $F_{\text{YFS}}(z)$ was already introduced by Yennie, Frautschi, and Suura [10] in their analysis of the IR behavior of QED. Using the substitution $k_z=\sqrt{\epsilon}\bar{k}_z$, we have

$$F_{nbn}A_{q-o}|_{\Re,\text{real rad,resummed}} = F_{nbn}\Re\frac{-i\pi^2\epsilon^{\gamma_q/2}}{\beta^2}\int d^2k_\perp\int_0^1 d\bar{k}_z F_{YFS}(\gamma_q) \times e^{\delta_q/2}(\beta\bar{k}_z)^{\gamma_q}(-\ln(\bar{k}_z+i\sqrt{\epsilon}-\beta\bar{k}_z) +\ln(\bar{k}_z+i\sqrt{\epsilon}+\beta\bar{k}_z))\frac{1}{-(1-\beta^2)\epsilon\bar{k}_z^2-\mathbf{k}_\perp{}^2}\frac{2\bar{k}_z}{\bar{k}_z^2+\epsilon}. \tag{26}$$

We see that the RHS of this last equation vanishes as $\epsilon\to 0$, removing the violation of Bloch-Nordsieck cancellation in (20), and, thereby, in (3).[3]

We conclude that the result in Ref. [2] is obviated by amplitude-based exact resummation of the higher order corrections in QCD perturbation theory. Only the infrared singular term from (19) is exponentiated, so that the finite nonzero terms in the cross section are all treated on equal footing—there is then no scheme dependence introduced by our resummation. The path is open to employ the current quark masses in ISR phenomenology for the LHC. For the light quarks, their main use will be as collinear/IR regulators, as the usual factorization methods [11] will generally replace them with the scale of such factorization; for the b quark, we cannot exclude at this time that its mass may have some additional role in precision LHC theory. Indeed, in addition to current algebra constraints, we know that from the measured differences between the parton densities for s, c, and b quarks in the proton that the "heavy" quark masses cannot actually be zero. We follow Ref. [12][4] in defining parton densities for heavy quarks here. The issue then is the accuracy of the massless approximation in the ISR in the context of precision LHC physics; for, already in QED, it is known that the corresponding limit $m_e\downarrow 0$ in ISR and the condition $m_e=0$ in ISR differ in $\mathcal{O}(\alpha/\pi)$. In QCD $\alpha_s/\pi\cong 3\%$ at TeV scales and this would be unacceptable if it would occur when the precision tag is 1%, as it will be at the LHC for some processes.

We note here that there is considerable literature [13–18] on the use of quark masses in perturbative QCD phenomenology, especially for deep inelastic scattering (DIS) processes. While in the original ACOT [13] variable flavor number scheme and in Ref. [17], quark masses are retained in the initial state analysis, in most cases, following the S-ACOT [16] variable flavor number scheme and various extensions [15,18], the ISR is treated with zero quark mass in the hard scattering coefficient with possible use an appropriate rescaling variable $x(1+4m^2/Q^2)$ [14], in standard DIS notation. These analyses result in general in a better fit to the available structure function data, although for Ref. [15] the significance of the attendant improved χ^2 is within the range of uncertainty of the respective fully massless result. These efforts all speak to the need for proper treatment of quark mass effects in precision high energy QCD phenomenology.

We have discussed the theorem in Ref. [2] in which the Drell-Yan process for quark-quark scattering is considered. However, our solution for the lack of Bloch-Nordsieck cancellation only depended on the external lines in the initial state, so it will carry over to all such ISR configurations: exponentiation of real corrections will render an extra factor of $k_0^{\gamma_q}$ in the respective integral over phase-space to remove any endpoint contributions which are not already canceled by virtual corrections as required by the Bloch-Nordsieck theorem.

Further implications of the results in this paper will appear elsewhere [19].

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[3]Note that, by the mean value theorem, the RHS of (26) is equal to $\epsilon^{\gamma_q/2}F_{\text{YFS}}(\gamma_q)e^{\delta_q/2}\langle(\beta\bar{k}_z)^{\gamma_q}\rangle F_{nbn}A_{q-o}|_{\Re,\text{real rad,IRpole part}}$, where $\langle A\rangle$ denotes the respective mean value of A defined with $d>4$; thus, (26) is still a higher twist effect with a coefficient which vanishes as $\epsilon\to 0$.

[4]In the proof of factorization presented in Ref. [12] for heavy quarks, there is an implicit use of the cancellation of ISR infrared singularities; our results remove any issues concerning this use.

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[1] See, for example, W.-M. Yao *et al.* (Particle Data Group), J. Phys. G **33**, 1 (2006), and references therein.

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