PHYSICAL REVIEW D **78,** 055017 (2008)

Higgs boson mass, sparticle spectrum, and the little hierarchy problem in an extended MSSM

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(Received 31 July 2008; published 24 September 2008)

We investigate the impact of TeV-scale matter belonging to complete vectorlike multiplets of unified groups on the lightest Higgs boson in the MSSM. We find that consistent with perturbative unification and electroweak precision data the mass m_h can be as large as 160 GeV. These extended MSSM models can also render the little hierarchy problem less severe, but only for lower values of $m_h (\lesssim 125)$ GeV. We present estimates for the sparticle mass spectrum in these models.

DOI: 10.1103/PhysRevD.78.055017 PACS numbers: 12.60.Jv, 14.80.Bn, 14.80.Ly

I. INTRODUCTION

The LEP2 lower bound $m_h \geq 114.4$ GeV [1] on the standard model (SM) Higgs boson mass poses a significant challenge for the minimal supersymmetric standard model (MSSM). With the tree level upper bound of M_Z on the mass of the (lightest) SM-like Higgs boson in the MSSM, significant radiative corrections are required to lift this mass above the LEP2 bound. This situation has been further exacerbated by the most recently quoted value of 172.6 ± 1.4 GeV for the top-quark pole mass [2], significantly lower from earlier values which not so long ago were closer to 176 GeV and higher [3]. With radiative corrections proportional to the fourth power of m_t, this leads to a reduced value for m_h unless the magnitude of some MSSM parameters such as the stop mass $m_{\tilde{t}}$ (or M_S) and the soft trilinear parameter A_t are suitably increased. Values of m_h of around 123 GeV or so require stop masses as well as $|A_t|$ close to the TeV scale or higher. Such large values, in turn, lead to the so-called *little hierarchy* problem [4] because, when dealing with radiative electroweak symmetry breaking, TeV-scale quantities must conspire to yield the electroweak mass scale M_Z.

In this paper we address these two related conundrums of the MSSM by introducing TeV scale vectorlike matter superfields which reside in complete SU(5) or SO(10) multiplets. Such complete multiplets, it is well known, do not spoil unification of the MSSM gauge couplings. We illustrate this in Fig. 1, where the gauge coupling evolution is compared, using two-loop renormalization group equations (RGEs), for the case of the MSSM plus complete multiplets $10 + \overline{10}$ and $5 + \bar{5}$ of SU(5). If these vectorlike matter fields do not acquire Planck-scale masses, it appears quite plausible that they will end up order TeV masses. R symmetries, for instance, can forbid Planck-scale masses, but allow TeV-scale masses proportional to the supersymmetric (SUSY) breaking scale. The Higgs(ino) mass term (the μ parameter) for the $H_u - H_d$ superfields is an example where this happens already in the MSSM [5]. For the vectorlike matter to have any significant effect on the "upper" bounds on m_h, it is crucial that they have masses of order TeV, otherwise their effects on m_h will decouple.

In studying this possibility of extending the MSSM, we employ the perturbativity and grand unified theory (GUT) unification constraints. It turns out that perturbative unification can be maintained if we introduce either (i) one pair of $(10 + \overline{10})$, or (ii) up to four pairs of $(5 + \bar{5})$, or (iii) one set of $(10 + \overline{10} + 5 + \bar{5})$ [6], where the representations refer to SU(5). In addition, any number of SM singlet fields are also allowed. Some particles in these new supermultiplets couple to the MSSM Higgs doublet H_u, and with masses of order 0.5–1 TeV, their radiative contributions alone can lift m_h to values as high as 160 GeV. This is achieved without requiring the standard MSSM sparticles to be much heavier than their present

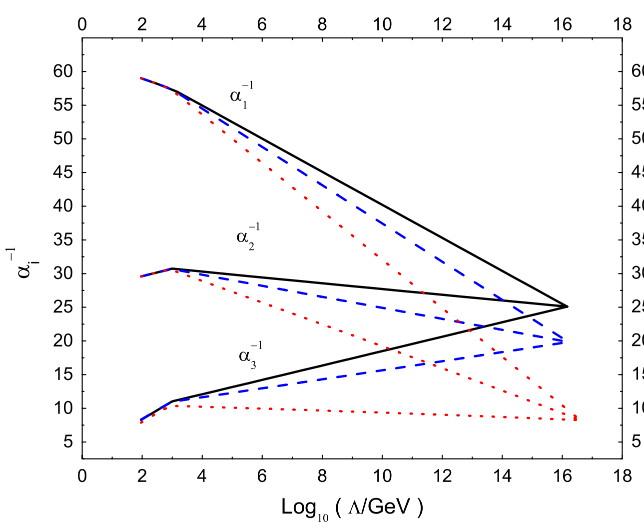

FIG. 1 (color online). Gauge coupling evolution with the effective SUSY breaking scale $M_S = 1$ TeV and $\tan\beta = 10$. Solid lines correspond to MSSM. Dashed lines correspond to MSSM + 5 + $\bar{5}$. Dotted lines are for MSSM + 10 + $\overline{10}$. The vectorlike masses for all these cases are set equal to 500 GeV.

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experimental lower bounds. We explore the impact of the additional multiplets on the MSSM parameter space and obtain the low energy sparticle spectrum in the mSUGRA framework. A comparison is presented, using semianalytic estimates, between the minimal and extended MSSM sparticle spectrum. The impact of these new particles on the little hierarchy problem is also discussed.

With the inclusion of these new vectorlike particles, the lightest Higgs boson mass, as previously noted, can be significantly increased. However, we find that resolving the little hierarchy problem is somewhat more tricky. The new Yukawa couplings of H_u to vectorlike matter, which helps in raising m_h, also has the effect of raising the soft Higgs mass parameter $m^2_{H_u}$, which could exacerbate the little hierarchy problem. If the new Yukawa couplings of H_u are relatively small, the little hierarchy problem improves relative to the MSSM, since the cumulative effect of the top Yukawa coupling y_t on $m^2_{H_u}$ becomes smaller than in MSSM. This comes about since y_t has a smaller value at the GUT scale ($y_t \sim 0.15$) compared to the MSSM case ($y_t \sim 0.5$). Thus we identify two regions of the parameter space as being of special interest: one where the little hierarchy problem becomes worse than in the MSSM, but where $m_h \sim 130$–160 GeV can be achieved, and another where the little hierarchy problem is relaxed, but where $m_h \lesssim 125$ GeV. The latter possibility appears to us to be quite interesting, as it assumes MSSM sparticle masses to be moderate, of order 200–500 GeV.

II. NEW VECTORLIKE MATTER AND PRECISION CONSTRAINTS

It is well known that one can extend the matter sector of the MSSM and still preserve the beautiful result of gauge coupling unification provided that the additional matter superfields fall into complete multiplets of any unified group which contains the SM, such as SU(5). Such extended scenarios with TeV-scale matter multiplets are well motivated. Within string theory, for instance, one often finds light (TeV-scale) multiplets in the spectrum [7], and even within the framework of GUTs one can find extra complete multiplets with masses around the TeV scale [6].

An important constraint on GUT representations and how many there can be at low ($\sim$TeV) scale comes from the perturbativity condition, which requires that the three MSSM gauge couplings remain perturbative up to M_G. One finds that there are several choices to satisfy this constraint: (i) one pair of $(10 + \overline{10})$, (ii) up to four pairs of $(5 + \bar{5})$'s, or (iii) the combination, $(5 + \bar{5} + 10 + \overline{10})$. Here all representations refer to multiplets of SU(5). In addition, any number of MSSM gauge singlets can be added without sacrificing unification or perturbativity. Option (iii), along with a pair of MSSM gauge singlets, fits nicely in SO(10) models.

Cases (i) and (iii) have been studied before in the literature. For example, the authors in [8] conclude that the mass of the lightest CP even Higgs mass could be pushed up to 180 GeV, consistent with all perturbativity constraints. When updated to account for the recent electroweak precision data, specifically the T parameter, and the current value of the top-quark mass, and improved to include two-loop RGE effects and finite corrections to the Higgs boson mass, we find that these scenarios admit m_h only as large as 160 GeV, which is significantly smaller than the bounds in [8].

It is clear that new matter will contribute at one-loop level to CP-even Higgs mass if there is direct coupling among new matter and the MSSM Higgs field. In case (i), a new coupling $10 \cdot 10 \cdot H_u$ is allowed, analogous to the top-quark Yukawa coupling, but involving the charge 2/3 quark from the 10-plet. (Here we use for simplicity SU (5) notation, but with the understanding that H_u and H_d are not complete multiplets of SU(5).) This new Yukawa coupling can modify the upper limit on m_h, which we will study in detail, taking into account perturbativity constraints. By itself, case (ii) does not allow for any new Yukawa coupling unless the new states in the $\bar{5}$ are mixed with the usual d^c-quarks and lepton doublets. Such a possibility is even more strongly constrained (by flavor violation and unitarity of the Cabibbo-Kobayashi-Maskawa (CKM) matrix, among others), and so we will forbid all such mixing. Once we add gauge singlets 1, couplings such as $\bar{5}H_u 1$ are allowed (only the leptonlike doublets from $\bar{5}$ will be involved in this Yukawa coupling.) We will analyze the effects of such couplings on m_h in detail. Case (iii) is a combination of (i) and (ii), which will also be studied in detail.

There are constraints on the couplings and masses of new matter fields. Most important are the constraints from the S and T parameters which limit the number of additional *chiral* generations. Consistent with these constraints, one should add new matter which is predominantly vectorlike.

In the limit where the vectorlike mass is much heavier than the chiral mass term (mass term arising from Yukawa coupling to the Higgs doublets), the contribution to the T parameter from a single chiral fermion is approximately [9]:

$$\delta T = \frac{N(\kappa v)^2}{10\pi \sin^2\theta_W m_W^2}\left[\left(\frac{\kappa v}{M_V}\right)^2 + O\left(\frac{\kappa v}{M_V}\right)^4\right], \qquad (1)$$

where κ is the new chiral Yukawa coupling, v is the vacuum expectation value (VEV) of the corresponding Higgs field, and N counts the additional number of SU(2) doublets. For instance, $N = 3$ when $10 + \overline{10}$ is considered at low scale, while $N = 1$ for the $5 + \bar{5}$ case. From precision electroweak data $T \leq 0.06(0.14)$ at 95% CL for $m_h = 117$ GeV (300 GeV) [10]. We will take $\delta T < 0.1$ as a realistic bound and apply it in our analysis. We then see from Eq. (1) that with M_V around 1 TeV, the Yukawa coupling κ can be $O(1)$.

III. HIGGS MASS BOUND

A. MSSM + 10 + $\overline{10}$

The representation $10 + \overline{10}$ of SU(5) decomposes under the MSSM gauge symmetry as follows:

$$10 + \overline{10} = Q_{10}(3, 2, \tfrac{1}{6}) + \bar{Q}_{10}(\bar{3}, 2, -\tfrac{1}{6}) + U_{10}(\bar{3}, 1, -\tfrac{2}{3}) + \bar{U}_{10}(3, 1, \tfrac{2}{3}) + E_{10}(1, 1, 1) + \bar{E}_{10}(1, 1, -1). \tag{2}$$

We assume for the vectorlike matter $10 + \overline{10}$ the same R parity as the MSSM Higgs chiral superfields. So there is no mixing of this new matter with quarks, but they couple to the Higgs doublets. The contribution to the superpotential from these couplings is

$$W = \kappa_{10}Q_{10}U_{10}H_u + \kappa'_{10}\bar{Q}_{10}\bar{U}_{10}H_d + M_V(\bar{Q}_{10}Q_{10} + \bar{U}_{10}U_{10} + \bar{E}_{10}E_{10}), \tag{3}$$

where, for simplicity, we have taken a common vectorlike mass (at the GUT scale M_G). Thus the up quarklike pieces of the 10 and $\overline{10}$ get Dirac *and* vectorlike masses, while leaving the E_{10}-lepton-like pieces with only vectorlike masses. We assume that $\kappa_{10} \gg \kappa'_{10}$ because the contribution coming from the coupling κ'_{10} reduces the light Higgs mass similar to what we have with the bottom Yukawa contribution which becomes prominent for large $\tan\beta$ [11].

Employing the effective potential approach we calculate the additional contribution from the vectorlike particles to the CP-even Higgs mass at one-loop level. A similar calculation was carried out in Ref. [12].

$$[m_h^2]_{10} = -M_Z^2\cos^2 2\beta\left(\frac{3}{8\pi^2}\kappa_{10}^2 t_V\right) + \frac{3}{4\pi^2}\kappa_{10}^4 v^2 \sin^2\beta\left[t_V + \frac{1}{2}X_{\kappa_{10}}\right], \tag{4}$$

where we have assumed $M_V \gg M_D$. The corrected expression for $X_{\kappa_{10}}$ (compare the result in Ref. [8]) is given as follows:

$$X_{\kappa_{10}} = \frac{4\tilde{A}_{\kappa_{10}}^2(3M_S^2 + 2M_V^2) - \tilde{A}_{\kappa_{10}}^4 - 8M_S^2M_V^2 - 10M_S^4}{6(M_S^2 + M_V^2)^2} \tag{5}$$

and

$$t_V = \log\left(\frac{M_S^2 + M_V^2}{M_V^2}\right), \tag{6}$$

where $\tilde{A}_{\kappa_{10}} = A_{\kappa_{10}} - \mu\cot\beta$, $A_{\kappa_{10}}$ is the $Q_{10} - U_{10}$ soft mixing parameter and μ is the MSSM Higgs bilinear mixing term. $M_S \simeq \sqrt{m_{\tilde{Q}_3}m_{\tilde{U}_3^c}}$, where $m_{\tilde{Q}_3}$ and $m_{\tilde{U}_3^c}$ are the stop left and stop right soft SUSY breaking masses at low scale.

For completeness we present the leading 1- and 2- loop contributions to the CP-even Higgs boson mass in the MSSM [13,14]

$$[m_h^2]_{\rm MSSM} = M_Z^2\cos^2 2\beta\left(1 - \frac{3}{8\pi^2}\frac{m_t^2}{v^2}t\right) + \frac{3}{4\pi^2}\frac{m_t^4}{v^2}\left[t + \frac{1}{2}X_t + \frac{1}{(4\pi)^2}\left(\frac{3}{2}\frac{m_t^2}{v^2} - 32\pi\alpha_s\right)(X_t t + t^2)\right], \tag{7}$$

where

$$t = \log\left(\frac{M_S^2}{M_t^2}\right), \qquad X_t = \frac{2\tilde{A}_t^2}{M_S^2}\left(1 - \frac{\tilde{A}_t^2}{12M_S^2}\right). \tag{8}$$

Also $\tilde{A}_t = A_t - \mu\cot\beta$, where A_t denotes the stop left and stop right soft mixing parameter.

In our model for the light Higgs mass we have

$$m_h^2 = [m_h^2]_{\rm MSSM} + [m_h^2]_{10}. \tag{9}$$

From Eq. (4) we can see that the Higgs mass is very sensitive to the value of κ_{10}, which we cannot take to be arbitrarily large because the theory should be perturbative up to the GUT scale. We therefore should solve the following RGE for κ_{10} to make sure that it remains perturbative up to the GUT scale:

$$\frac{d\kappa_{10}}{dt} = \frac{\kappa_{10}}{2(4\pi)^2}\Bigg(\left(\frac{16}{3}g_3^2 + 3g_2^2 + \frac{13}{15}g_1^2 - 6\kappa_{10}^2 - 3y_t^2\right) - \frac{1}{(4\pi)^2}\Bigg(\frac{3913}{450}g_1^4 + \frac{33}{2}g_2^4 + \frac{128}{9}g_3^4 + g_1^2g_2^2 + \frac{136}{45}g_1^2g_3^2 + 8g_2^2g_3^2 + \left(\frac{2}{5}g_1^2 + 6g_2^2\right)\kappa_{10}^2 + \left(\frac{4}{5}g_1^2 + 16g_3^2\right)(y_t^2 + \kappa_{10}^2) - 9(y_t^4 + \kappa_{10}^4) - 9\kappa_{10}^2(y_t^2 + \kappa_{10}^2) - 4\kappa_{10}^4\Bigg)\Bigg), \tag{10}$$

where g_3, g_2, and g_1 are strong, weak, and hypercharge gauge couplings, respectively, and y_t denotes the top Yukawa coupling. Because the new matter couples to H_u [see Eq. (3)] there are additional contributions to the RGE for y_t at two-loop level:

$$\frac{dy_t}{dt} = \frac{y_t}{2(4\pi)^2}\Bigg(\left(\frac{16}{3}g_3^2 + 3g_2^2 + \frac{13}{15}g_1^2 - 6y_t^2 - 3\kappa_{10}^2\right) - \frac{1}{(4\pi)^2}\Bigg(\frac{3913}{450}g_1^4 + \frac{33}{2}g_2^4 + \frac{128}{9}g_3^4 + g_1^2g_2^2 + \frac{136}{45}g_1^2g_3^2 + 8g_2^2g_3^2 + \left(\frac{2}{5}g_1^2 + 6g_2^2\right)y_t^2 + \left(\frac{4}{5}g_1^2 + 16g_3^2\right)(y_t^2 + \kappa_{10}^2) - 9(y_t^4 + \kappa_{10}^4) - 9y_t^2(y_t^2 + \kappa_{10}^2) - 4y_t^4\Bigg)\Bigg). \tag{11}$$

The additional vectorlike matter fields also modify the RGEs for the MSSM gauge couplings and the corresponding beta functions can be found in [15].

In our calculation, the weak scale (M_Z) value of the gauge and top Yukawa couplings are evolved to the scale M_G via the RGE's in the $\overline{DR}$ regularization scheme, where the scale M_G is defined to be one where $g_2 = g_1$. We do not enforce an exact unification of the strong coupling $g_3 = g_2 = g_1$ at scale M_G, since a few percent deviation from the unification condition can be assigned to unknown GUT scale threshold corrections. At the scale M_G we impose $\kappa_{10} \approx 2$, in order to obtain the maximal value for κ_{10} at low scale, which is consistent with the T parameter constraints. In this case we can generate, according to Eq. (9), the maximal plausible values for the Higgs mass. Our goal is to achieve the maximal value for the CP-even Higgs mass and, as we show in Eq. (27), the Higgs mass is proportional to κ_{10}^4. The coupling κ_{10}, along with the gauge and top Yukawa couplings, are evolved back to M_Z. In the evolution of couplings, for the SUSY threshold correction we follow the effective SUSY scale approach, according to which all SUSY particles are assumed to lie at an effective scale [16]. Below $M_{\rm SUSY}$ we employ the non-SUSY RGEs. All of the couplings are iteratively run between M_Z and M_G using two-loop RGEs for both the Yukawa and gauge couplings until a stable solution is obtained. Note that M_S and $M_{\rm SUSY}$ are distinct parameters. As pointed out in Ref. [16], one can have a different set of values for stop squark masses for a given effective $M_{\rm SUSY}$ and so correspondingly one considers different values of M_S for $M_{\rm SUSY} = 200$ GeV.

Requiring $\delta T < 0.1$ with $10 + \overline{10}$ masses at $M_V = 1$ TeV, we find that $\kappa_{10}(M_V) < 1.142$ at (M_V) scale using the formula from Ref. [9]. The corresponding κ_{10} at GUT scale in this case is $\kappa_{10}(M_G) \approx 2$. We find that the S-parameter constraint is automatically satisfied once the T-parameter constraint is met.

We can see from Eqs. (4)–(8) that to maximize the CP-even Higgs boson mass we should not only take the maximal allowed value for κ_{10}, we also need to have the maximal values for the parameters X_t and $X_{\kappa_{10}}$. According to Eq. (5) we find that $X_{\kappa_{10}} = 2.95$, with $M_S = 500$ GeV and $M_V = 1$ TeV. The value for $X_{\kappa_{10}}$ increases ($X_{\kappa_{10}} = 3.42$) if we consider $M_S = 1$ TeV and $M_V = 1$ TeV, while $X_{\kappa_{10}} = 3.95$ for $M_S = 2$ TeV and $M_V = 1$ TeV.

We find that $M_V = 1$ TeV is somehow the optimum value for the vectorlike particle mass, especially because the T-parameter constraint almost disappears for this value of M_V. On the other hand Eq. (22) does not allow very low values for M_S if significant corrections are to be realized. This is the reason why we choose $M_S = 0.5$, 1, and 2 TeV for our analysis.

In Fig. 2 we present the upper bounds for the CP-even Higgs boson mass vs $\tan\beta$ with different maximal or minimal values of X_t, $X_{\kappa_{10}}$ when $M_S = 500$ GeV and $M_V = 1$ TeV, and we compare to the MSSM case. We take at scale M_G, $\kappa_{10} \approx 2$ to obtain the maximal effect for the lightest Higgs boson mass. As we see from Fig. 2, for this choice of parameters the maximal values for Higgs

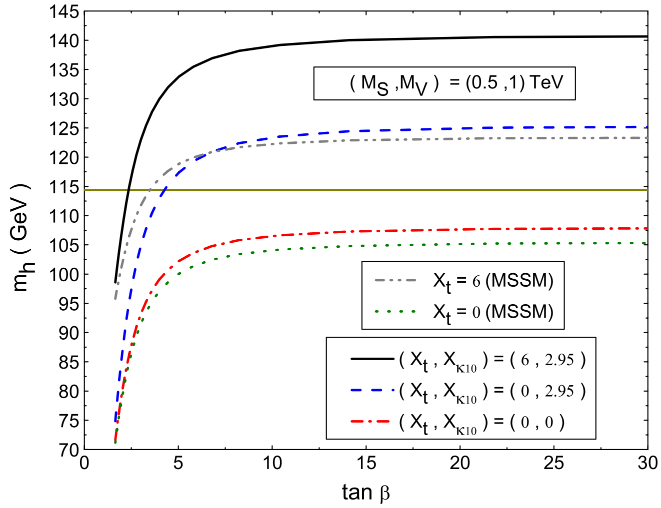

FIG. 2 (color online). Upper bounds for the lightest CP-even Higgs boson mass vs $\tan\beta$, for different maximal and minimal values of X_t, $X_{\kappa_{10}}$, with $M_S = 500$ GeV, $M_V = 1$ TeV, and $M_t = 172.6$ GeV. The dotted line corresponds to the MSSM ($X_t = 0$). The dashed-double dotted line describes the MSSM with ($X_t = 6$). The dashed-dotted curve is for MSSM + 10 + $\overline{10}$. $\kappa_{10} \approx 2$ at M_G. The dashed line shows Higgs mass with $X_{10} = 2.95$ and $X_t = 0$. The solid line corresponds to $X_{10} = 2.95$ and $X_t = 6$. The solid horizontal line denotes the LEP2 bound $m_h = 114.4$ GeV.

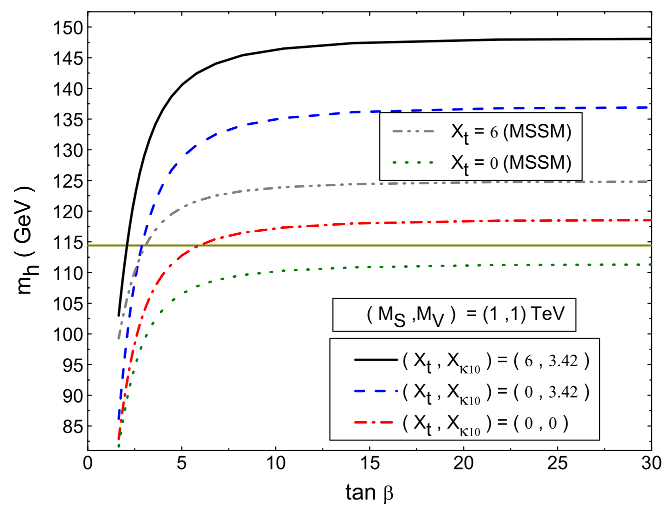

FIG. 3 (color online). Upper bounds for the lightest CP-even Higgs boson mass vs $\tan\beta$ for different maximal and minimal values of X_t, $X_{\kappa_{10}}$, with $M_S = 1$ TeV, $M_V = 1$ TeV, and $M_t = 172.6$ GeV. The dotted line corresponds to the MSSM ($X_t = 0$). The dashed-double dotted line describes the MSSM with ($X_t = 6$). The dashed-dotted curve is for MSSM + 10 + $\overline{10}$. $\kappa_{10} \approx 2$ at M_G. The dashed line shows Higgs mass with $X_{\kappa_{10}} = 2.95$ and $X_t = 0$. The solid line corresponds to $X_{\kappa_{10}} = 2.95$ and $X_t = 6$.

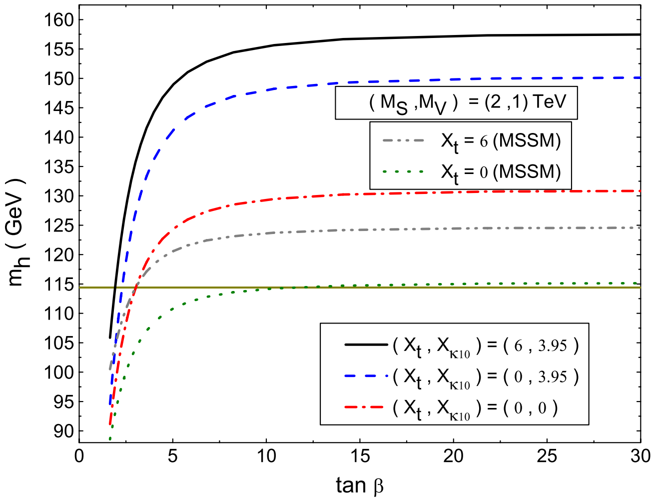

FIG. 4 (color online). Upper bounds for the lightest *CP*-even Higgs boson mass vs $\tan\beta$ for different maximal and minimal values of X_t, $X_{\kappa_{10}}$, with $M_S = 2$ TeV, $M_V = 1$ TeV, and $M_t = 172.6$ GeV. The dotted line corresponds to the MSSM ($X_t = 0$). The dashed-double dotted line describes the MSSM with ($X_t = 6$). The dashed-dotted curve is for MSSM + 10 + $\overline{10}$. $\kappa_{10} \approx 2$ at M_G. The dashed line shows Higgs mass with $X_{\kappa_{10}} = 3.95$ and $X_t = 0$. The solid line corresponds to $X_{\kappa_{10}} = 3.95$ and $X_t = 6$.

mass is 141 GeV. In Fig. 3 we present the results for the case in which the mass for vectorlike matter is $M_V = 1$ TeV and $M_S = 1$ TeV too. In this case the *CP*-even Higgs mass can be as large as 148 GeV. Finally in Fig. 4 we consider the $M_V = 1$ TeV and $M_S = 2$ TeV case and obtain the maximal value of 158 GeV for the Higgs mass.

B. MSSM + 5 + $\bar{5}$

In this subsection we consider the case in which at the TeV scale we have extra matter which belongs to the five-dimensional representation of SU(5). This decomposes under the MSSM gauge symmetry as follows:

$$5 + \bar{5} = L_5(1, 2, \tfrac{1}{2}) + \bar{L}_5(1, 2, -\tfrac{1}{2}) + D_5(\bar{3}, 1, \tfrac{1}{3}) + \bar{D}_5(3, 1, -\tfrac{1}{3}). \tag{12}$$

Our goal is to generate new trilinear couplings of this extra matter with the MSSM Higgs fields. The $5 + \bar{5}$ itself cannot generate this kind of coupling. However, if we introduce an MSSM singlet S, then Yukawa couplings of the form (in SU(5) notation) $\bar{5} \cdot S \cdot H_u$ and $5 \cdot S \cdot H_d$ are permitted. In this case the MSSM superpotential has the following additional contribution:

$$W = \kappa_5 L_5 S H_u + \kappa_5' \bar{L}_5 S H_d + M_V(S\bar{S} + \bar{L}_5 L_5 + \bar{D}_5 D_5). \tag{13}$$

We take $\kappa_5 \gg \kappa_5'$, for the same reason mentioned in the previous section. We also assume that there is an additional symmetry which forbids the mixing of the vectorlike particle with the MSSM matter fields. With this assumption the singlet field S cannot be identified with the right-handed neutrino.

Using the effective potential approach we calculate the additional contribution to the *CP*-even Higgs mass at one-loop level. [A similar calculation was done in Ref. [12].]

$$[m_h^2]_5 = -M_Z^2 \cos^2 2\beta\left(\frac{1}{8\pi^2}\kappa_5^2 t_V\right) + \frac{1}{4\pi^2}\kappa_5^4 v^2 \sin^2\beta\left[t_V + \frac{1}{2}X_{\kappa_5}\right], \tag{14}$$

where we have assumed $M_V \gg M_D$ and

$$X_{\kappa_5} = \frac{4\tilde{A}_{\kappa_5}^2(3M_S^2 + 2M_V^2) - \tilde{A}_{\kappa_5}^4 - 8M_S^2 M_V^2 - 10M_S^4}{6(M_S^2 + M_V^2)^2} \tag{15}$$

and

$$t_V = \log\left(\frac{M_S^2 + M_V^2}{M_V^2}\right). \tag{16}$$

Here $\tilde{A}_{\kappa_5} = A_{\kappa_5} - \mu \cot\beta$, A_{κ_5} is the $L_5 - S$ soft mixing parameter, and μ is the MSSM Higgs bilinear mixing term.

The RGE for κ_5 has the following form:

$$\frac{d\kappa_5}{dt} = \frac{\kappa_5}{2(4\pi)^2}\left(\left(3g_2^2 + \frac{3}{5}g_1^2 - 4\kappa_5^2 - 3y_t^2\right) - \frac{1}{(4\pi)^2}\left(\frac{237}{50}g_1^4 + \frac{21}{2}g_2^4 + \frac{9}{5}g_1^2 g_2^2 + \left(\frac{6}{5}g_1^2 + 6g_2^2\right)\kappa_5^2 + \left(\frac{4}{5}g_1^2 + 16g_3^2\right)y_t^2 - 3(3y_t^4 + \kappa_5^4) - 3\kappa_5^2(3y_t^2 + \kappa_5^2) - 4\kappa_5^4\right)\right). \tag{17}$$

Because the new matter fields couple to H_u [see Eq. (13)], there are additional contributions to the RGE for y_t which to two-loop level is given by

$$\frac{dy_t}{dt} = \frac{y_t}{2(4\pi)^2}\left(\left(\frac{16}{3}g_3^2 + 3g_2^2 + \frac{13}{15}g_1^2 - 6y_t^2 - \kappa_5^2\right) - \frac{1}{(4\pi)^2}\left(\frac{3133}{450}g_1^4 + \frac{21}{2}g_2^4 + \frac{32}{9}g_3^4 + g_1^2 g_2^2 + \frac{136}{45}g_1^2 g_3^2 + 8g_2^2 g_3^2 + \left(\frac{2}{5}g_1^2 + 6g_2^2\right)y_t^2 + \left(\frac{4}{5}g_1^2 + 16g_3^2\right)y_t^2 - 3(3y_t^4 + \kappa_5^4) - 3y_t^2(3y_t^2 + \kappa_5^2) - 4y_t^4\right)\right). \tag{18}$$

We can see from Eq. (17) that κ_5 cannot be as large at M_Z scale as κ_{10} was. The reason for this is that in the RGE

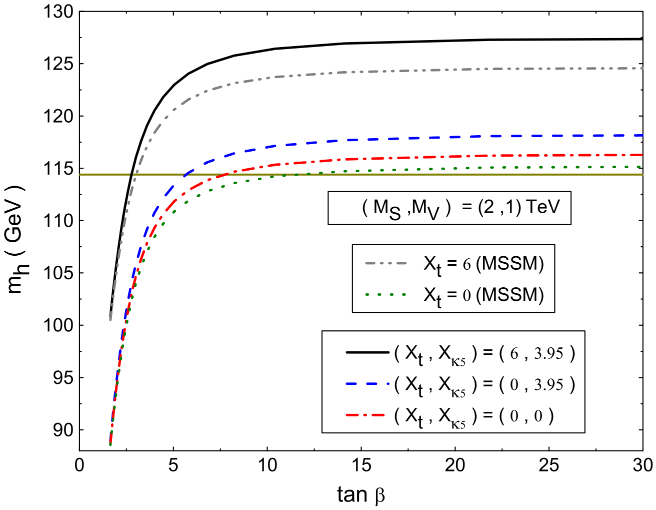

FIG. 5 (color online). Upper bounds for the lightest CP-even Higgs boson mass vs $\tan\beta$ for different maximal and minimal values of X_t, X_{κ_5}, with $M_S = 2$ TeV, $M_V = 1$ TeV, and $M_t = 172.6$ GeV. The dotted line corresponds to the MSSM with ($X_t = 0$). The dashed-double dotted line describes the MSSM with ($X_t = 6$). The dashed-dotted curve is for MSSM + 5 + $\bar{5}$. $\kappa_5 \approx 2$ at M_G. The dashed line shows Higgs mass with $X_{\kappa_5} = 3.95$ and $X_t = 0$. The solid line corresponds to $X_{\kappa_5} = 3.95$ and $X_t = 6$.

for κ_5, in contrast to the case for κ_{10}, the strong gauge coupling does not participate at one-loop level. Because of this we find that $\kappa_5(M_Z) = 0.74$ for $M_S = 2$ TeV and $M_V = 1$ TeV.

In Fig. 5 we present the upper bounds for the CP-even Higgs boson mass vs $\tan\beta$ with different maximal or minimal values of X_t, X_{κ_5} with $M_S = 2$ TeV and $M_V = 1$ TeV and which we compare with the MSSM case. We take $\kappa_5 \approx 2$ at scale M_G as before. For this choice of parameters the maximal value for the Higgs mass is 127.5 GeV.

C. MSSM + 5 + $\bar{5}$ + 10 + $\overline{10}$

In this section we will consider extra vectorlike matter belonging to the representation $5 + \bar{5} + 10 + \overline{10}$ of SU(5). There are two choices to consider here, namely, with or without two SM singlet fields. This does not affect the perturbativity condition, but the presence of the singlets suggests an underlying SO(10) gauge symmetry.

Case I. Without the singlet the MSSM superpotential acquires the following additional contribution:

$$\begin{aligned} W = {} & \kappa_1 Q_{10} U_{10} H_u + \kappa_2 \bar{Q}_{10} \bar{D}_5 H_u + \kappa_3 \bar{Q}_{10} \bar{U}_{10} H_d \\ & + \kappa_4 Q_{10} D_5 H_d + M_V(\bar{Q}_{10} Q_{10} + \bar{U}_{10} U_{10} + \bar{E}_{10} E_{10} \\ & + \bar{L}_5 L_5 + \bar{D}_5 D_5). \end{aligned} \tag{19}$$

The new interaction yields the following additional contribution to the MSSM CP-even Higgs boson mass:

$$\begin{aligned} [m_h^2]_1 = {} & -M_Z^2 \cos^2 2\beta \left(\frac{3}{8\pi^2} \kappa_1^2 t_V \right) \\ & + \frac{3}{4\pi^2} \kappa_1^4 v^2 \sin^2\beta \left[t_V + \frac{1}{2} X_{\kappa_1} \right], \\ & - M_Z^2 \cos^2 2\beta \left(\frac{3}{8\pi^2} \kappa_2^2 t_V \right) \\ & + \frac{3}{4\pi^2} \kappa_2^4 v^2 \sin^2\beta \left[t_V + \frac{1}{2} X_{\kappa_2} \right], \end{aligned} \tag{20}$$

where we have assumed $M_V \gg M_D$, and we defined

$$X_{\kappa_i} = \frac{4\tilde{A}_{\kappa_i}^2 (3M_S^2 + 2M_V^2) - \tilde{A}_{\kappa_i}^4 - 8M_S^2 M_V^2 - 10 M_S^4}{6(M_S^2 + M_V^2)^2}, \tag{21}$$

and

$$t_V = \log\left(\frac{M_S^2 + M_V^2}{M_V^2} \right), \tag{22}$$

where $i = 1, 2$, $\tilde{A}_{\kappa_i} = A_{\kappa_i} - \mu \cot\beta$ and A_{κ_i} is the soft mixing parameter.

In this case the lightest CP-even Higgs mass is

$$m_h^2 = [m_h^2]_{\rm MSSM} + [m_h^2]_1, \tag{23}$$

where the expression for $[m_h^2]_{\rm MSSM}$ is given in Eq. (7)

The RGEs for κ_1 and κ_2 are given to one-loop order by

$$\begin{aligned} \frac{d\kappa_1}{dt} &= \frac{\kappa_1}{2(4\pi)^2} \left(\frac{16}{3} g_3^2 + 3g_2^2 + \frac{13}{15} g_1^2 - 6\kappa_1^2 - 3\kappa_2^2 - 3y_t^2 \right), \\ \frac{d\kappa_2}{dt} &= \frac{\kappa_2}{2(4\pi)^2} \left(\frac{16}{3} g_3^2 + 3g_2^2 + \frac{7}{15} g_1^2 - 6\kappa_2^2 - 3\kappa_2^2 - 3y_t^2 \right). \end{aligned} \tag{24}$$

Because the new matter couples to H_u [see Eq. (19)] there is an additional contribution to the RGE for y_t at one-loop level:

$$\frac{dy_t}{dt} = \left[\frac{dy_t}{dt} \right]_{\rm MSSM} - \frac{3}{2(4\pi)^2} y_t \kappa_1^2 - \frac{3}{2(4\pi)^2} y_t \kappa_2^2. \tag{25}$$

In Fig. 6 we present the upper bounds for the CP-even Higgs boson mass vs $\tan\beta$ for different maximal or minimal values of X_t, X_{κ_1}, X_{κ_2}, with $M_S = M_V = 2.41$ TeV, and compare it with the MSSM case. We take $\kappa_i \approx 2$ at M_G as before. For the given choice of parameters the maximal value of the Higgs mass is 144.5 GeV.

Case II. Next we consider the case when at low scale we have vectorlike particles in $(16 + \overline{16})$-dimensional representation of SO(10). The MSSM superpotential for this case acquires the following additional contribution:

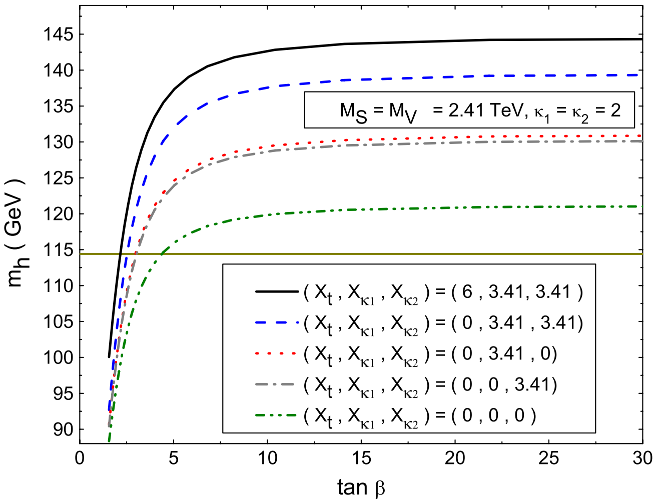

FIG. 6 (color online). Upper bounds for the lightest *CP*-even Higgs boson mass vs $\tan\beta$ for different maximal and minimal values of X_t, X_{κ_1}, X_{κ_2}, with $M_S = M_V = 2.41$ TeV, and $M_t = 172.6$ GeV. The dashed-double dotted curve describes the logarithmical correction in the model. The dotted curve corresponds to the maximum value of κ_1 or κ_2. The dashed curve corresponds to the maximum values of κ_1 and κ_2. The solid curve corresponds to the case when all corrections are taken to be maximum.

$$\begin{aligned} W = {}& \kappa_1 Q_{10} U_{10} H_u + \kappa_2 \bar{Q}_{10} \bar{D}_5 H_u + \kappa_3 L_5 S H_u \\ &+ \kappa_4 Q_{10} D_5 H_d + \kappa_5 \bar{Q}_{10} \bar{U}_{10} H_d + \kappa_6 \bar{L}_5 H_d \bar{S} \\ &+ M_V(\bar{Q}_{10} Q_{10} + \bar{U}_{10} U_{10} + \bar{E}_{10} E_{10} + \bar{L}_5 L_5 \\ &+ \bar{D}_5 D_5 + \bar{S} S). \end{aligned} \tag{26}$$

The new interactions provide the following additional contribution to the MSSM *CP*-even Higgs boson mass

$$\begin{aligned} [m_h^2]_2 = {}& -M_Z^2 \cos^2 2\beta \left(\frac{3}{8\pi^2} \kappa_1^2 t_V\right) \\ &+ \frac{3}{4\pi^2} \kappa_1^4 v^2 \sin^2\beta \left[t_V + \frac{1}{2} X_{\kappa_1} \right] \\ &- M_Z^2 \cos^2 2\beta \left(\frac{3}{8\pi^2} \kappa_2^2 t_V\right) \\ &+ \frac{3}{4\pi^2} \kappa_2^4 v^2 \sin^2\beta \left[t_V + \frac{1}{2} X_{\kappa_2} \right] \\ &- M_Z^2 \cos^2 2\beta \left(\frac{1}{8\pi^2} \kappa_3^2 t_V\right) \\ &+ \frac{1}{4\pi^2} \kappa_3^4 v^2 \sin^2\beta \left[t_V + \frac{1}{2} X_{\kappa_3} \right], \end{aligned} \tag{27}$$

where we have assumed $M_V \gg M_D$, and defined

$$X_{\kappa_i} = \frac{4\tilde{A}_{\kappa_i}^2 (3M_S^2 + 2M_V^2) - \tilde{A}_{\kappa_i}^4 - 8M_S^2 M_V^2 - 10M_S^4}{6(M_S^2 + M_V^2)^2}, \tag{28}$$

and

$$t_V = \log\left(\frac{M_S^2 + M_V^2}{M_V^2}\right). \tag{29}$$

Here $i = 1, 2, 3$ and $\tilde{A}_{\kappa_i} = A_{\kappa_i} - \mu \cot\beta$. The lightest *CP*-even Higgs mass is

$$m_h^2 = [m_h^2]_{\rm MSSM} + [m_h^2]_2, \tag{30}$$

where the expression for $[m_h^2]_{\rm MSSM}$ is given in Eq. (7).

The RGEs for κ_i are given by

$$\begin{aligned} \frac{d\kappa_1}{dt} &= \frac{\kappa_1}{2(4\pi)^2}\left(\frac{16}{3} g_3^2 + 3g_2^2 + \frac{13}{15} g_1^2 - 6\kappa_1^2 - 3\kappa_2^2 - \kappa_3^2 - 3y_t^2\right), \\ \frac{d\kappa_2}{dt} &= \frac{\kappa_2}{2(4\pi)^2}\left(\frac{16}{3} g_3^2 + 3g_2^2 + \frac{7}{15} g_1^2 - 6\kappa_2^2 - 3\kappa_2^2 - \kappa_3^2 - 3y_t^2\right), \\ \frac{d\kappa_3}{dt} &= \frac{\kappa_3}{2(4\pi)^2}\left(3g_2^2 + \frac{3}{5} g_1^2 - 4\kappa_3^2 - 3\kappa_1^2 - 3\kappa_2^2 - 3y_t^2\right). \end{aligned} \tag{31}$$

The RGE for y_t is modified as follows:

$$\frac{dy_t}{dt} = \left[\frac{dy_t}{dt}\right]_{\rm MSSM} - \frac{3}{2(4\pi)^2} y_t \kappa_1^2 - \frac{3}{2(4\pi)^2} y_t \kappa_2^2 - \frac{1}{2(4\pi)^2} y_t \kappa_3^2. \tag{32}$$

In Fig. 7 we present the upper bounds for the *CP*-even Higgs boson mass vs $\tan\beta$ for different maximal and minimal values of X_t, X_{κ_1}, X_{κ_2}, X_{κ_3}, with $M_S = M_V = 2.41$ TeV, and compare it with the MSSM case. We take

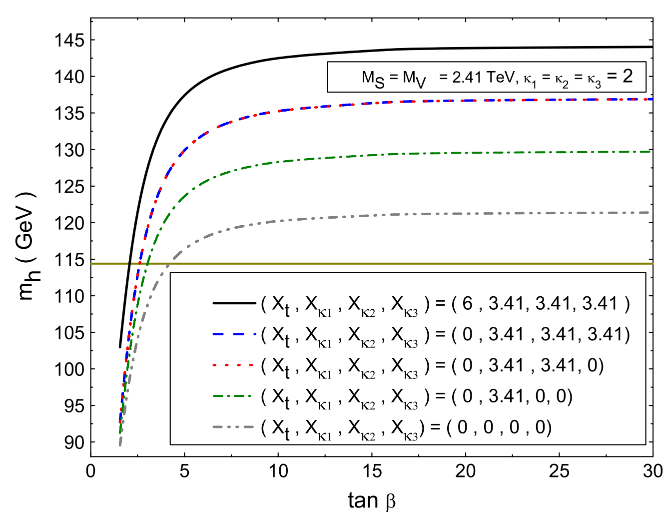

FIG. 7 (color online). Upper bounds for the lightest *CP*-even Higgs boson mass vs $\tan\beta$ for different maximal and minimal values of X_t, X_{κ_1}, X_{κ_2}, X_{κ_3}, with $M_S = M_V = 2.41$ TeV and $M_t = 172.6$ GeV.

$\kappa_i \approx 2$ at M_G as before. For the given choice of parameters the maximal value of the Higgs mass is 143.9 GeV. It is worth noting that the resultant Higgs mass bound for *Case II* more or less coincides with what we found for *Case I* (see Fig. 6). This stems from the fact that the contribution at "low" scale from the new coupling κ_3 is small due to the absence of the strong coupling [see Eq. (31)].

IV. LITTLE HIERARCHY PROBLEM

A. MSSM

As discussed in Sec. III, in the MSSM at tree level the lightest CP-even Higgs boson mass is bounded from above by the mass of the Z boson

$$m_h^2 < M_Z^2 \cos 2\beta. \tag{33}$$

It requires large radiative corrections in order to push the lightest Higgs mass above the LEP2 limit. We can see that there are two kinds of corrections [see Eq. (7)], one proportional to $m_t^4 \log(M_S^2/m_t^2)$, where $M_S = \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$, and the second proportional to A_t. As we can see from Fig. 8, if the Higgs mass turns out to be much heavier than 114.4 GeV, we need not only a large trilinear soft SUSY breaking A_t term but also heavy stop quark masses.

On the other hand, the mass of the Z boson ($M_Z \simeq$ 91 GeV) is given from the minimization of the scalar potential as (for $\tan\beta \gtrsim 5$)

$$\tfrac{1}{2} M_Z^2 \simeq -\mu^2 - m_{H_u}^2, \tag{34}$$

and the radiative correction to the soft scalar mass squared for H_u is proportional to top squark masses

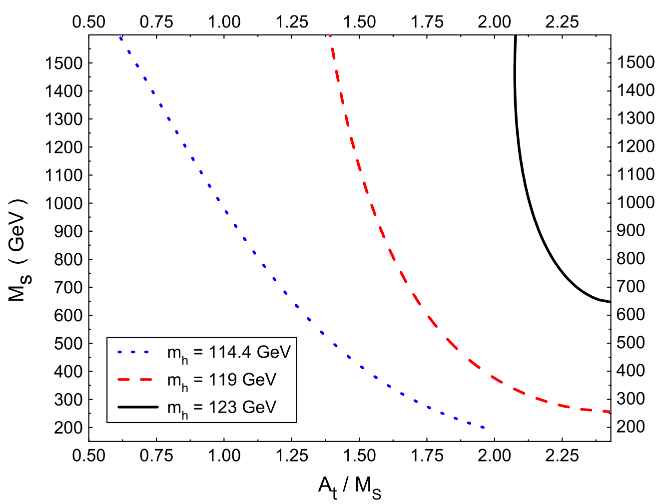

FIG. 8 (color online). M_S versus A_t/M_S for different values of the CP-even Higgs mass in MSSM. The dotted, dashed, and solid line corresponds to Higgs mass $m_h = 114.4$, 119, and 123 GeV, respectively, and $\tan\beta = 10$.

$$\Delta m_{H_u}^2(M_Z) = -\frac{3 y_t^2(M_Z)}{4\pi^2} M_S^2 \ln\frac{\Lambda}{M_S}, \tag{35}$$

where Λ is a more fundamental scale, such as M_G.

Thus, in the MSSM one needs to have heavy top squarks to generate the lightest Higgs mass above the LEP2 bound, while on the other hand from Eqs. (34) and (35) we see that some fine-tuning is needed to get the correct Z boson mass. This is known as the *little hierarchy problem*. To see how much fine-tuning is needed to satisfy Eq. (34) we performed a semianalytic calculation for the MSSM sparticle spectra with the following boundary conditions:

$$\{\alpha_G, M_G, y_t(M_G)\} = \{1/24.32, 2.0 \times 10^{16}, 0.512\}. \tag{36}$$

We express the MSSM sparticle masses at the scale M_Z in terms of the GUT/Planck-scale fundamental parameters ($m_0, m_{1/2}, A_{t_0}$) and the Higgs bilinear mixing term μ, by integrating the one-loop renormalization group equations [17]. For example, the gaugino masses at M_Z scale are

$$\{M_1(M_Z), M_2(M_Z), M_3(M_Z)\} = \{0.412, 0.822, 2.844\} m_{1/2}. \tag{37}$$

The scalar particle masses, A_t and μ at the M_Z scale are given by

$$-m_{H_u}^2(M_Z) = 2.72 m_{1/2}^2 + 0.091 m_0^2 + 0.1 A_{t_0}^2 - 0.43 m_{1/2} A_{t_0}, \tag{38}$$

$$m_{Q_t}^2(M_Z) = 5.71 m_{1/2}^2 + 0.64 m_0^2 - 0.033 A_{t_0}^2 + 0.15 m_{1/2} A_{t_0}, \tag{39}$$

$$m_{U_t}^2(M_Z) = 4.2 m_{1/2}^2 + 0.27 m_0^2 - 0.07 A_{t_0}^2 + 0.29 m_{1/2} A_{t_0}, \tag{40}$$

$$A_t(M_Z) = -2.3 m_{1/2} + 0.27 A_{t_0}, \tag{41}$$

$$m_{Q_{1,2}}^2(M_Z) = 6.79 m_{1/2}^2 + m_0^2, \tag{42}$$

$$m_{U_{1,2}}^2(M_Z) = 6.37 m_{1/2}^2 + m_0^2, \tag{43}$$

$$m_{D_{1,2,3}}^2(M_Z) = 6.32 m_{1/2}^2 + m_0^2, \tag{44}$$

$$m_{L_{1,2,3}}^2(M_Z) = 0.52 m_{1/2}^2 + m_0^2, \tag{45}$$

$$m_{E_{1,2,3}}^2(M_Z) = 0.15 m_{1/2}^2 + m_0^2, \tag{46}$$

$$\mu^2(M_Z) = 1.02 \mu_0^2, \tag{47}$$

where the subscript 1, 2, 3 are families indices and μ_0 is the value of μ at M_G.

Using Eq. (34) we can also express the dominant contribution to Z boson mass in terms of fundamental parameters:

$$M_Z^2 \simeq -2.04\mu^2 + 5.44m_{1/2}^2 + 0.183m_0^2 + 0.2A_{t_0}^2 - 0.87m_{1/2}A_{t_0}. \quad (48)$$

The magnitude of $|m_{H_U}|$ shows how much fine-tuning is needed to satisfy the minimization condition in the MSSM [see Eq. (34)]. We present in Fig. 9 $|m_{H_U}|$ versus the CP-even Higgs mass for different values of M_S. In each case A_t varies in the interval $0 < |A_t/M_S| < \sqrt{6}$. This is the reason why we find different values for the Higgs masses for a different choice of M_S.

We find that the new Yukawa couplings of H_u to the vectorlike matter, which helps in raising m_h, also has the effect of raising the soft Higgs mass parameter $m_{H_u}^2$, which tends to exacerbate the little hierarchy problem. However, when the new Yukawa couplings of H_u are relatively small, the little hierarchy problem improves relative to the MSSM, since the cumulative effect of the top Yukawa coupling y_t on the running of $m_{H_u}^2$ becomes smaller than in the MSSM. This comes about since the value of y_t is smaller at the scale M_G compared to MSSM for certain values of the new Yukawa coupling. This result is displayed in Fig. 10 for the MSSM + 10 + $\overline{10}$ case. There is also a contribution from the radiative correction involving the gluon and gluino, since, as we show in Fig. 1, introducing new vectorlike matter at low scale slows the running of the strong coupling compared to the MSSM case. As a result the cumulative effect of the strong interaction to the running of colored particle masses is smaller than in MSSM. These two effects, as we show in the next two sections, enable us to improve the little hierarchy problem compared to the MSSM.

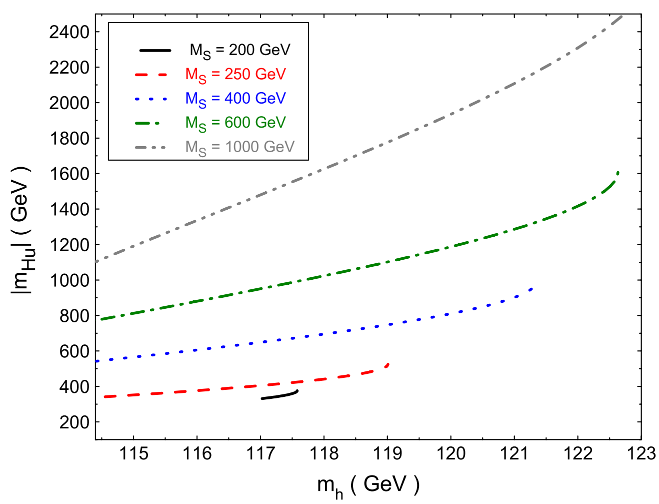

FIG. 9 (color online). $|m_{H_U}|$ versus CP-even Higgs mass in the MSSM for different values of M_S. The solid, dashed, dotted, dashed-dotted, and dashed-dotted-dotted curve corresponds to M_S = 200 GeV, 250 GeV, 400 GeV, 600 GeV, and 1000 GeV, respectively. In each case A_t varies in the interval $0 < |A_t/M_S| < \sqrt{6}$.

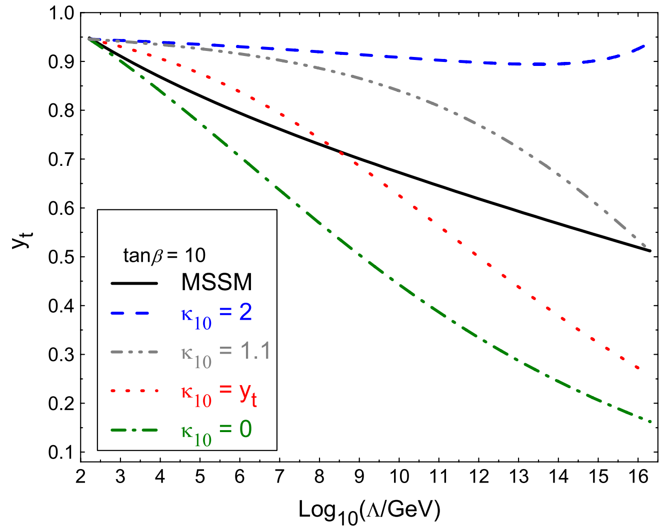

FIG. 10 (color online). Top Yukawa y_t coupling versus $\text{Log}_{10}(\Lambda/\text{GeV})$ for $\tan\beta = 10$. The dashed, dotted, dashed-dotted, and dashed-dotted-dotted lines correspond to MSSM + 10 + $\overline{10}$ with $\kappa_{10} = 2$, y_t, 0, 1.1, respectively. The solid line corresponds to the MSSM case.

B. MSSM + 10 + $\overline{10}$

Next let us consider how the little hierarchy problem can be improved in the MSSM + 10 + $\overline{10}$ case. We will consider two extreme values for the coupling κ_{10}, namely $\kappa_{10}(M_G) = 2$ and $\kappa_{10}(M_G) = 0$ to show how much little hierarchy has changed for this case.

Case I. Using the boundary conditions

$$\{\alpha_G, M_G, y_t(M_G), \kappa_{10}(M_G)\} = \{1/8.55, 2.0 \times 10^{16}, 0.94, 2\} \quad (49)$$

and RGEs from Appendix A, we obtain the sparticle spectrum. For the gaugino masses,

$$\{M_1(M_Z), M_2(M_Z), M_3(M_Z)\} = \{0.145, 0.289, 1\}m_{1/2}, \quad (50)$$

while the MSSM scalar masses along with μ^2, A_t, and $A_{\kappa_{10}}$ are given by

$$-m_{H_u}^2(M_Z) = 3.85m_{1/2}^2 + 0.95m_0^2 + 0.04A_{t_0}^2 + 0.012A_{\kappa_{10_0}}^2 - 0.12m_{1/2}A_{t_0} + 0.06m_{1/2}A_{\kappa_{10_0}} - 0.043A_{t_0}A_{\kappa_{10_0}}, \quad (51)$$

$$m_{Q_t}^2(M_Z) = 2.98m_{1/2}^2 + 0.73m_0^2 - 0.031A_{t_0}^2 - 0.002A_{\kappa_{10_0}}^2 + 0.11m_{1/2}A_{t_0} - 0.071m_{1/2}A_{\kappa_{10_0}} + 0.026A_{t_0}A_{\kappa_{10_0}}, \quad (52)$$

$$m^2_{U_t}(M_Z) = 2.04m^2_{1/2} + 0.45m^2_0 - 0.062A^2_{t_0} - 0.003A^2_{\kappa_{10_0}} + 0.23m_{1/2}A_{t_0} - 0.14m_{1/2}A_{\kappa_{10_0}} + 0.052A_{t_0}A_{\kappa_{10_0}}, \quad (53)$$

$$A_t(M_Z) = -1.02m_{1/2} + 0.2A_{t_0} - 0.13A_{\kappa_{10_0}}, \quad (54)$$

$$A_{\kappa_{10}}(M_Z) = -0.71m_{1/2} - 0.093A_{t_0} + 0.065A_{\kappa_{10_0}}, \quad (55)$$

$$m^2_{Q_{1,2}}(M_Z) = 3.63m^2_{1/2} + m^2_0, \quad (56)$$

$$m^2_{U_{1,2}}(M_Z) = 3.33m^2_{1/2} + m^2_0, \quad (57)$$

$$\mu^2(M_Z) = 0.105\mu^2_0, \quad (58)$$

$$m^2_{D_{1,2,3}}(M_Z) = m^2_0 + 3.29m^2_{1/2}, \quad (59)$$

$$m^2_{L_{1,2,3}}(M_Z) = m^2_0 + 0.37m^2_{1/2}, \quad (60)$$

$$m^2_{E_{1,2,3}}(M_Z) = m^2_0 + 0.122m^2_{1/2}. \quad (61)$$

The spectrum for the vectorlike matter is given as

$$m^2_{Q_{10}}(M_Z) = 2.86m^2_{1/2} + 0.62m^2_0 + 0.017A^2_{t_0} - 0.003A^2_{\kappa_{10_0}} - 0.073m_{1/2}A_{t_0} + 0.051m_{1/2}A_{\kappa_{10_0}} - 0.011A_{t_0}A_{\kappa_{10_0}}, \quad (62)$$

$$m^2_{U_{10}}(M_Z) = 1.81m^2_{1/2} + 0.25m^2_0 + 0.035A^2_{t_0} - 0.005A^2_{\kappa_{10_0}} - 0.15m_{1/2}A_{t_0} + 0.1m_{1/2}A_{\kappa_{10_0}} - 0.023A_{t_0}A_{\kappa_{10_0}}, \quad (63)$$

$$m^2_{E_{10}}(M_Z) = m^2_0 + 0.122m^2_{1/2}. \quad (64)$$

For this case the dominant contribution of Z-boson mass has the following expression:

$$M^2_Z \simeq -0.21\mu^2_0 + 7.7m^2_{1/2} + 1.91m^2_0 + 0.08A^2_{t_0} + 0.024A^2_{\kappa_{10_0}} - 0.24m_{1/2}A_{t_0} + 0.12m_{1/2}A_{\kappa_{10_0}} - 0.094A_{t_0}A_{\kappa_{10_0}}. \quad (65)$$

We see that the coefficient of $m^2_{1/2}$ in this expression has increased as compared to the MSSM case [see Eq. (48)], and so we expect fine-tuning to be worse in this case. This is related to the value of $\kappa_{10}(M_G)$. If we reduce $\kappa_{10}(M_G)$, $y_t(M_G)$ is reduced (see Fig. 10), and as a result the coefficient of $m^2_{1/2}$ in the M^2_Z expression is reduced. We find that for some values of $\kappa_{10}(M_G)$ the top Yukawa coupling $y_t(M_G)$ is smaller than $y_t^{\rm MSSM}(M_G)$, its value in the MSSM. This enables us to reduce the degree of fine-tuning for the case MSSM + 10 + $\overline{10}$ compared to the MSSM.

However, with smaller values of $\kappa_{10}(M_G)$, the value for the Higgs mass m_h will be lower. Thus, we need to find an optimum value of $\kappa_{10}(M_G)$, between 2 and 0, which gives a sufficiently large value for the Higgs mass, but at the same time yields a smaller value of $|m_{H_u}|$.

Case II. We next study the changes brought about by setting $\kappa_{10}(M_G) = 0$ and by applying the following boundary conditions:

$$\{\alpha_G, M_G, y_t(M_G), \kappa_{10}(M_G), A_{\kappa_{10}}(M_G)\} = \{1/8.55, 2.0 \times 10^{16}, 0.163, 0, 0\}. \quad (66)$$

For the MSSM spectrum and related quantities, we find

$$-m^2_{H_u}(M_Z) = 2.59m^2_{1/2} - 0.15m^2_0 + 0.12A^2_{t_0} - 0.76m_{1/2}A_{t_0}, \quad (67)$$

$$m^2_{Q_t}(M_Z) = 2.64m^2_{1/2} + 0.72m^2_0 - 0.041A^2_{t_0} + 0.25m_{1/2}A_{t_0}, \quad (68)$$

$$m^2_{U_t}(M_Z) = 1.35m^2_{1/2} + 0.44m^2_0 - 0.082A^2_{t_0} + 0.51m_{1/2}A_{t_0}, \quad (69)$$

$$A_t(M_Z) = -2.14m_{1/2} + 0.44A_{t_0}, \quad (70)$$

$$A_{\kappa_{10}}(M_Z) = -3.02m_{1/2} - 0.28A_{t_0} + A_{\kappa_{10_0}}, \quad (71)$$

$$m^2_{Q_{1,2}}(M_Z) = 3.63m^2_{1/2} + m^2_0, \quad (72)$$

$$m^2_{U_{1,2}}(M_Z) = 3.33m^2_{1/2} + m^2_0, \quad (73)$$

$$\mu^2(M_Z) = 1.9\mu^2_0, \quad (74)$$

$$m^2_{D_{1,2,3}}(M_Z) = m^2_0 + 3.29m^2_{1/2}, \quad (75)$$

$$m^2_{L_{1,2,3}}(M_Z) = m^2_0 + 0.374m^2_{1/2}, \quad (76)$$

$$m^2_{E_{1,2,3}}(M_Z) = m^2_0 + 0.122m^2_{1/2}, \quad (77)$$

$$m^2_{Q_{10}}(M_Z) = 3.63m^2_{1/2} + m^2_0, \quad (78)$$

$$m^2_{U_{10}}(M_Z) = 3.33m^2_{1/2} + m^2_0, \quad (79)$$

$$m^2_{E_{10}}(M_Z) = m^2_0 + 0.122m^2_{1/2}. \quad (80)$$

The dominant contribution of Z-boson mass has the following expression:

$$M^2_Z \simeq -3.78\mu^2_0 + 5.19m^2_{1/2} - 0.31m^2_0 + 0.25A^2_{t_0} - 1.52m_{1/2}A_{t_0}. \quad (81)$$

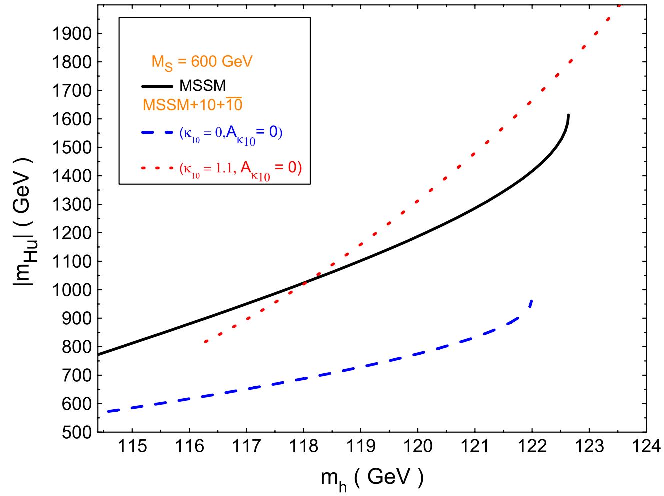

FIG. 11 (color online). $|m_{H_U}|$ versus the CP-even Higgs mass m_h, with $M_S = 600$ GeV. The solid line corresponds to the MSSM case. The dashed and dotted curves correspond to MSSM + 10 + $\overline{10}$, with $\kappa_{10} = 1.1$ and $\kappa_{10} = 0$ at M_G.

In Fig. 11 we plot $|m_{H_U}|$ versus the lightest CP-even Higgs mass m_h, with $M_S = 600$ GeV. We compare two cases, when $\kappa_{10} = 1.1$ and $\kappa_{10} = 0$ at M_G. The case $\kappa = 1.1$ is interesting in the sense that in this case the value of the top Yukawa coupling is the same as in the MSSM. We can see in Fig. 11 that for a Higgs mass less than 118 GeV, the fine-tuning responsible for the little hierarchy problem is less severe, while for larger than these values the situation becomes worse. We do not display the case $\kappa_{10} = 2$ for which $|m_{H_U}|$ exceeds 2 TeV. On the other hand one can see how the fine-tuning condition for the little hierarchy problem is improved when $\kappa = 0$ at the GUT scale.

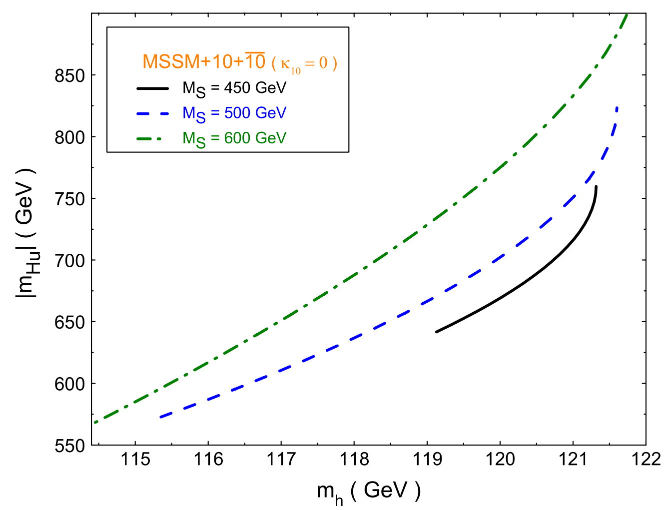

FIG. 12 (color online). $|m_{H_U}|$ versus m_h for MSSM + 10 + $\overline{10}$, with ($A_t < 0$). The solid, dashed, and dashed-dotted curves correspond to $M_S = 450$, 500, and 600 GeV, respectively. For all cases $\kappa_{10} = 0$ at M_G.

In Fig. 12 we plot $|m_{H_U}|$ versus the m_h for MSSM + 10 + $\overline{10}$, with $A_t < 0$. The solid, dashed, and dashed-dotted curves correspond to $M_S = 450$, 500, and 600 GeV, respectively. For all cases $\kappa_{10} = 0$ at M_G. We see that the fine-tuning condition is relaxed compared to the results in Fig. 9.

C. MSSM + 5 + $\bar{5}$

In this section we consider MSSM + 5 + $\bar{5}$ and perform an analysis similar to what we did for MSSM + 10 + $\overline{10}$.

Case I. Using the boundary conditions

$$\{\alpha_G, M_G, y_t(M_G), \kappa_5(M_G)\} = \{1/19.06, 2.0 \times 10^{16}, 0.57, 2\}, \tag{82}$$

we obtain the following spectrum:

$$\{M_1(M_Z), M_2(M_Z), M_3(M_Z)\} = \{0.323, 0.645, 2.23\} m_{1/2} \tag{83}$$

and

$$M_Z^2 = -1.26\mu_0^2 + 5.32 m_{1/2}^2 + 0.94 m_0^2 + 0.18 A_{t_0}^2 + 0.04 A_{\kappa_{5_0}}^2 - 0.86 m_{1/2} A_{t_0} + 0.21 m_{1/2} A_{\kappa_{5_0}} - 0.09 A_{t_0} A_{\kappa_{5_0}}, \tag{84}$$

$$-m_{H_u}^2(M_Z) = 2.66 m_{1/2}^2 + 0.47 m_0^2 + 0.09 A_{t_0}^2 + 0.02 A_{\kappa_{5_0}}^2 - 0.43 m_{1/2} A_{t_0} + 0.1 m_{1/2} A_{\kappa_{5_0}} - 0.044 A_{t_0} A_{\kappa_{5_0}}, \tag{85}$$

$$m_{Q_t}^2(M_Z) = 4.68 m_{1/2}^2 + 0.7 m_0^2 - 0.04 A_{t_0}^2 + 0.003 A_{\kappa_{5_0}}^2 + 0.16 m_{1/2} A_{t_0} - 0.04 m_{1/2} A_{\kappa_{5_0}} + 0.013 A_{t_0} A_{\kappa_{5_0}}, \tag{86}$$

$$m_{U_t}^2(M_Z) = 3.24 m_{1/2}^2 + 0.39 m_0^2 - 0.072 A_{t_0}^2 - 0.006 A_{\kappa_{5_0}}^2 + 0.32 m_{1/2} A_{t_0} - 0.08 m_{1/2} A_{\kappa_{5_0}} + 0.026 A_{t_0} A_{\kappa_{5_0}}, \tag{87}$$

$$A_t(M_Z) = -2.17 m_{1/2} + 0.28 A_{t_0} - 0.076 A_{\kappa_{5_0}}, \tag{88}$$

$$A_{\kappa_5}(M_Z) = 0.36 m_{1/2} - 0.21 A_{t_0} + 0.16 A_{\kappa_{5_0}}, \tag{89}$$

$$m_{Q_{1,2}}^2(M_Z) = 5.73 m_{1/2}^2 + m_0^2, \tag{90}$$

$$m_{U_{1,2}}^2(M_Z) = 5.36 m_{1/2}^2 + m_0^2, \tag{91}$$

$$\mu^2(M_Z) = 0.63 \mu_0^2, \tag{92}$$

$$m^2_{D_{1,2,3}}(M_Z) = m_0^2 + 5.31 m^2_{1/2}, \tag{93}$$

$$m^2_{L_{1,2,3}}(M_Z) = m_0^2 + 0.474 m^2_{1/2}, \tag{94}$$

$$m^2_{E_{1,2,3}}(M_Z) = m_0^2 + 0.141 m^2_{1/2}. \tag{95}$$

For those particles outside the MSSM, we find

$$m^2_{L_5}(M_Z) = 0.51 m^2_{1/2} + 0.45 m_0^2 + 0.02 A^2_{t_0} - 0.03 A^2_{\kappa_{5_0}} - 0.042 m_{1/2} A_{t_0} + 0.015 m_{1/2} A_{\kappa_{5_0}} + 0.005 A_{t_0} A_{\kappa_{5_0}}, \tag{96}$$

$$m^2_S(M_Z) = 0.073 m^2_{1/2} - 0.11 m_0^2 + 0.037 A^2_{t_0} - 0.06 A^2_{\kappa_{5_0}} - 0.088 m_{1/2} A_{t_0} + 0.03 m_{1/2} A_{\kappa_{5_0}} + 0.01 A_{t_0} A_{\kappa_{5_0}}, \tag{97}$$

$$m^2_{D_5}(M_Z) = m_0^2 + 5.31 m^2_{1/2}. \tag{98}$$

Case II. Employing the boundary conditions

$$\{\alpha_G, M_G, y_t(M_G), \kappa_5(M_G)\} = \{1/19.06, 2.0 \times 10^{16}, 0.39, 0\}, \tag{99}$$

we find

$$\{M_1(M_Z), M_2(M_Z), M_3(M_Z)\} = \{0.323, 0.645, 2.23\} m_{1/2} \tag{100}$$

and

$$M_Z^2 = -2.41 \mu_0^2 + 5.37 m^2_{1/2} + 0.03 m_0^2 + 0.22 A^2_{t_0} - 1.1 m_{1/2} A_{t_0}, \tag{101}$$

$$m^2_{H_u}(M_Z) = 2.68 m^2_{1/2} + 0.014 m_0^2 + 0.11 A^2_{t_0} - 0.53 m_{1/2} A_{t_0}, \tag{102}$$

$$m^2_{Q_t}(M_Z) = 4.68 m^2_{1/2} + 0.66 m_0^2 - 0.04 A^2_{t_0} + 0.18 m_{1/2} A_{t_0}, \tag{103}$$

$$m^2_{U_t}(M_Z) = 3.25 m^2_{1/2} + 0.32 m_0^2 - 0.073 A^2_{t_0} + 0.35 m_{1/2} A_{t_0}, \tag{104}$$

$$A_t(M_Z) = -2.25 m_{1/2} + 0.32 A_{t_0}, \tag{105}$$

$$A_{\kappa_5}(M_Z) = 0.23 m_{1/2} - 0.34 A_{t_0} + A_{\kappa_{5_0}}, \tag{106}$$

$$m^2_{Q_{1,2}}(M_Z) = 5.73 m^2_{1/2} + m_0^2, \tag{107}$$

$$m^2_{U_{1,2}}(M_Z) = 5.36 m^2_{1/2} + m_0^2, \tag{108}$$

$$\mu^2(M_Z) = 1.20 \mu_0^2, \tag{109}$$

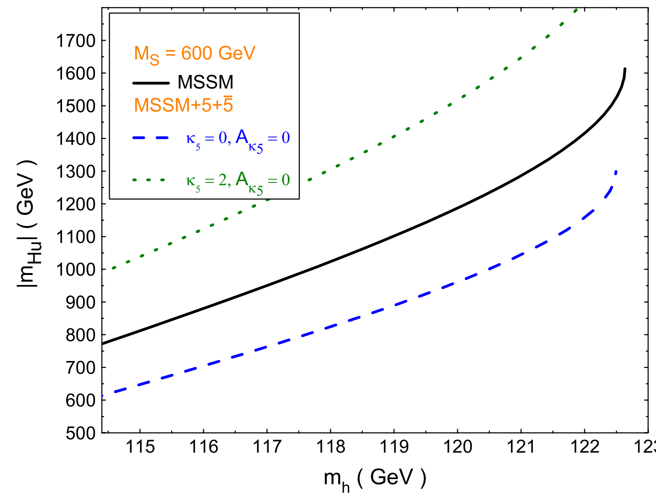

FIG. 13 (color online). $|m_{H_U}|$ versus m_h for MSSM + 5 + $\bar{5}$ ($A_t < 0$).

$$m^2_{D_{1,2,3}}(M_Z) = m_0^2 + 5.31 m^2_{1/2}, \tag{110}$$

$$m^2_{L_{1,2,3}}(M_Z) = m_0^2 + 0.474 m^2_{1/2}, \tag{111}$$

$$m^2_{E_{1,2,3}}(M_Z) = m_0^2 + 0.141 m^2_{1/2}. \tag{112}$$

Similarly,

$$m^2_{L_5}(M_Z) = 0.47 m^2_{1/2} + m_0^2, \tag{113}$$

$$m^2_S(M_Z) = m_0^2, \tag{114}$$

$$m^2_{D_5}(M_Z) = m_0^2 + 5.31 m^2_{1/2}. \tag{115}$$

In Fig. 13 we plot $|m_{H_U}|$ versus m_h, with $M_S =$ 600 GeV. We consider two cases, $\kappa_5 = 2$ and $\kappa_5 = 0$ at M_G. We can see from Fig. 13 that the fine-tuning condition for the little hierarchy problem improves relative to the

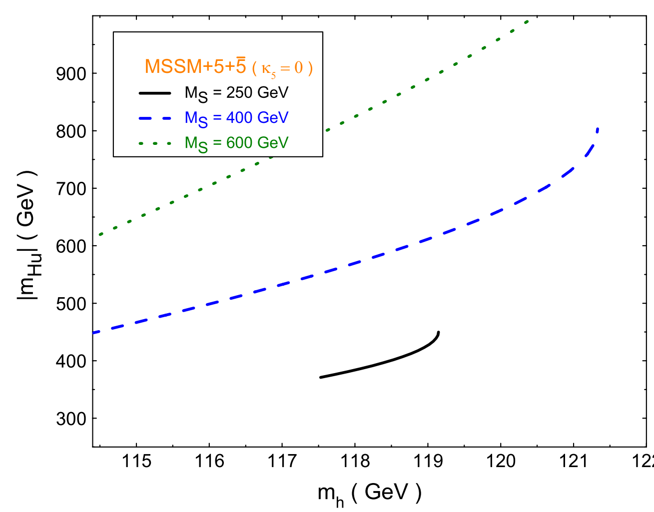

FIG. 14 (color online). $|m_{H_U}|$ versus m_h for MSSM + 5 + $\bar{5}$ ($A_t < 0$).

MSSM when $\kappa_5 = 0$, but improvement is not as significant as for MSSM + 10 + $\overline{10}$. The reason is that the values for y_t at M_G with $\kappa_5 = 0$ is large compared to y_t for MSSM + 10 + $\overline{10}$ with $\kappa_{10} = 0$.

In Fig. 14 we plot $|m_{H_U}|$ versus m_h for MSSM + 5 + $\bar{5}$, with ($A_t < 0$). The solid, dashed, and dashed-dotted curves correspond to $M_S = 250$, 400, and 600 GeV, respectively, with $\kappa_5 = 0$ at M_G. We see that the fine-tuning condition is relaxed compared to the results in Fig. 9.

V. CONCLUSION

We have shown that in an extended MSSM framework with vectorlike supermultiplets, whose masses lie in the TeV range, the mass of the lightest CP-even Higgs boson can be as high as 160 GeV. Gauge coupling unification is maintained in this approach with the three MSSM gauge couplings remaining perturbative all the way to the GUT scale M_G. As far as the little hierarchy problem is concerned, the degree of fine-tuning in this extended MSSM is severe for larger values of the Higgs mass. However, things have improved somewhat compared to the MSSM if the Higgs mass is found to be $\lesssim 125$ GeV.

ACKNOWLEDGMENTS

We thank Christopher Kolda for collaboration in the early stages of this work and for discussion. This work is supported in part by the DOE under Grant No. DE-FG02-91ER40626 (I. G. and Q. S.), No. DE-FG03-98ER-41076 (K. S. B.), and GNSF Grant No. 07_462_4-270 (I. G.).

APPENDIX A: RGES FOR MSSM + 10 + $\overline{\mathbf{10}}$

$$W = y_t Q_t \bar{U}_t H_u + \kappa_{10} Q_{10} \bar{U}_{10} H_u, \qquad \frac{d\bar{\alpha}_i}{dt} = -b_i \bar{\alpha}_i^2,$$
$$\frac{dM_i}{dt} = -b_i \bar{\alpha}_i M_i.$$

Here

$$t = \log_{10}\left(\frac{M_G^2}{Q^2}\right),$$
$$\bar{\alpha}_i = \frac{\alpha_i}{4\pi},$$
$$\frac{dm_L^2}{dt} = \left(3\bar{\alpha}_2 M_2^2 + + \frac{3}{5}\bar{\alpha}_1 M_1^2\right),$$
$$\frac{dm_E^2}{dt} = \left(\frac{12}{5}\bar{\alpha}_1 M_1^2\right),$$
$$\frac{dm_{Q_t}^2}{dt} = \left(\frac{16}{3}\bar{\alpha}_3 M_3^2 + 3\bar{\alpha}_2 M_2^2 + \frac{1}{15}\bar{\alpha}_1 M_1^2\right) - Y_t(m_{Q_t}^2 + m_{U_t}^2 + m_{H_u}^2 + A_t^2),$$
$$\frac{dm_{U_t}^2}{dt} = \left(\frac{16}{3}\bar{\alpha}_3 M_3^2 + \frac{16}{15}\bar{\alpha}_1 M_1^2\right) - 2Y_t(m_{Q_t}^2 + m_{U_t}^2 + m_{H_u}^2 + A_t^2),$$
$$\frac{d\mu^2}{dt} = 3\left(\bar{\alpha}_2 + \frac{1}{5}\bar{\alpha}_1\right)\mu^2 - 3(Y_t + K_{10})\mu^2,$$
$$\frac{dm_{H_d}^2}{dt} = 3\left(\bar{\alpha}_2 M_2^2 + \frac{1}{5}\bar{\alpha}_1 M_1^2\right),$$
$$\frac{dm_{H_u}^2}{dt} = 3\left(\bar{\alpha}_2 M_2^2 + \frac{1}{5}\bar{\alpha}_1 M_1^2\right) - 3Y_t(m_{Q_t}^2 + m_{U_t}^2 + m_{H_u}^2 + A_t^2) - 3K_{10}(m_{Q_{10}}^2 + m_{U_{10}}^2 + m_{H_u}^2 + A_{\kappa_{10}}^2),$$

$$\frac{dA_t}{dt} = -\left(\frac{16}{3}\bar{\alpha}_3 M_3 + 3\bar{\alpha}_2 M_2 + \frac{13}{15}\bar{\alpha}_1 M_1\right) - 6Y_t A_t - 3K_{10} A_{\kappa_{10}},$$
$$\frac{dY_t}{dt} = Y_t\left(\frac{16}{3}\bar{\alpha}_3 + 3\bar{\alpha}_2 + \frac{13}{15}\bar{\alpha}_1\right) - 6Y_t^2 - 3Y_t K_{10},$$
$$\frac{dm_{Q_{10}}^2}{dt} = \left(\frac{16}{3}\bar{\alpha}_3 M_3^2 + 3\bar{\alpha}_2 M_2^2 + \frac{1}{15}\bar{\alpha}_1 M_1^2\right) - K_{10}(m_{Q_{10}}^2 + m_{U_{10}}^2 + m_{H_u}^2 + A_{\kappa_{10}}^2),$$
$$\frac{dm_{U_{10}}^2}{dt} = \left(\frac{16}{3}\bar{\alpha}_3 M_3^2 + \frac{16}{15}\bar{\alpha}_1 M_1^2\right) - 2K_{10}(m_{Q_{10}}^2 + m_{U_{10}}^2 + m_{H_u}^2 + A_{\kappa_{10}}^2),$$

$$\frac{dA_{\kappa_{10}}}{dt} = -\left(\frac{16}{3}\bar{\alpha}_3 M_3 + 3\bar{\alpha}_2 M_2 + \frac{13}{15}\bar{\alpha}_1 M_1\right) - 6K_{10} A_{\kappa_{10}} - 3Y_t A_t,$$
$$\frac{dK_{10}}{dt} = K_{10}\left(\frac{16}{3}\bar{\alpha}_3 + 3\bar{\alpha}_2 + \frac{13}{15}\bar{\alpha}_1\right) - 6K_{10}^2 - 3Y_t K_{10},$$
$$\frac{dm_D^2}{dt} = \left(\frac{16}{3}\bar{\alpha}_3 M_3^2 + \frac{4}{15}\bar{\alpha}_1 M_1^2\right).$$

Here

$$Y_t = \frac{y_t^2}{(4\pi)^2}, \qquad K_{10} = \frac{\kappa_{10}^2}{(4\pi)^2},$$
$$b_i = \left\{\frac{33}{5}, 1, -3\right\} + \{3, 3, 3\}.$$

APPENDIX B: RGES FOR MSSM + 5 + $\bar{5}$

$$W = y_t Q_t \bar{U}_t H_u + \kappa_5 L_5 \bar{S} H_u,$$

$$\frac{dm_L^2}{dt} = 3\left(\bar{\alpha}_2 M_2^2 + \frac{1}{5}\bar{\alpha}_1 M_1^2\right),$$

$$\frac{dm_E^2}{dt} = \left(\frac{12}{5}\bar{\alpha}_1 M_1^2\right),$$

$$\frac{dm_{Q_t}^2}{dt} = \left(\frac{16}{3}\bar{\alpha}_3 M_3^2 + 3\bar{\alpha}_2 M_2^2 + \frac{1}{15}\bar{\alpha}_1 M_1^2\right) - Y_t(m_{Q_t}^2 + m_{U_t}^2 + m_{H_u}^2 + A_t^2),$$

$$\frac{dm_{U_t}^2}{dt} = \left(\frac{16}{3}\bar{\alpha}_3 M_3^2 + \frac{16}{15}\bar{\alpha}_1 M_1^2\right) - 2Y_t(m_{Q_t}^2 + m_{U_t}^2 + m_{H_u}^2 + A_t^2),$$

$$\frac{d\mu^2}{dt} = 3\left(\bar{\alpha}_2 + \frac{1}{5}\bar{\alpha}_1\right)\mu^2 - (3Y_t + K_5)\mu^2,$$

$$\frac{dm_{H_d}^2}{dt} = 3\left(\bar{\alpha}_2 M_2^2 + \frac{1}{5}\bar{\alpha}_1 M_1^2\right),$$

$$\frac{dm_{H_u}^2}{dt} = 3\left(\bar{\alpha}_2 M_2^2 + \frac{1}{5}\bar{\alpha}_1 M_1^2\right) - 3Y_t(m_{Q_t}^2 + m_{U_t}^2 + m_{H_u}^2 + A_t^2) - K_5(m_{L_5}^2 + m_S^2 + m_{H_u}^2 + A_{\kappa_5}^2),$$

$$\frac{dA_t}{dt} = -\left(\frac{16}{3}\bar{\alpha}_3 M_3 + 3\bar{\alpha}_2 M_2 + \frac{13}{15}\bar{\alpha}_1 M_1\right) - 6Y_t A_t - K_5 A_{\kappa_5},$$

$$\frac{dY_t}{dt} = Y_t\left(\frac{16}{3}\bar{\alpha}_3 + 3\bar{\alpha}_2 + \frac{13}{15}\bar{\alpha}_1\right) - 6Y_t^2 - Y_t K_5,$$

$$\frac{dm_{L_5}^2}{dt} = 3\left(\bar{\alpha}_2 M_2^2 + \frac{1}{5}\bar{\alpha}_1 M_1^2\right) - K_5(m_{L_5}^2 + m_S^2 + m_{H_u}^2 + A_{\kappa_5}^2),$$

$$\frac{dm_S^2}{dt} = -2K_5(m_{L_5}^2 + m_S^2 + m_{H_u}^2 + A_{\kappa_5}^2),$$

$$\frac{dA_{\kappa_5}}{dt} = -3\left(\bar{\alpha}_2 M_2 + \frac{1}{5}\bar{\alpha}_1 M_1\right) - 4K_5 A_{\kappa_5} - 3Y_t A_t,$$

$$\frac{dK_5}{dt} = 3K_5\left(\bar{\alpha}_2 + \frac{1}{5}\bar{\alpha}_1\right) - 4K_5^2 - 3Y_t K_5,$$

$$\frac{dm_D^2}{dt} = \left(\frac{16}{3}\bar{\alpha}_3 M_3^2 + + \frac{4}{15}\bar{\alpha}_1 M_1^2\right).$$

Here

$$Y_t = \frac{y_t^2}{(4\pi)^2}, \qquad K_5 = \frac{\kappa_5^2}{(4\pi)^2},$$

$$b_i = \left\{\frac{33}{5}, 1, -3\right\} + \{1, 1, 1\}.$$

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