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Unparticle realization through continuous mass scale invariant theories

N. G. Deshpande[1,*] and Xiao-Gang He[2,+]

[1]*Institute of Theoretical Sciences, University of Oregon, Eugene, Oregon 97401, USA*

[2]*Department of Physics and Center for Theoretical Sciences, National Taiwan University, Taipei, Taiwan, Republic of China*

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We consider scale invariant theories of continuous mass fields and show how interactions of these fields with the standard model can reproduce unparticle interactions. There is no fixed point or dimensional transmutation involved in this approach. We generalize interactions of the standard model to multiple unparticles in this formalism and explicitly work out some examples, in particular, we show that the product of two scalar unparticles behaves as a normalized scalar unparticle with a dimension equal to the sum of the two composite unparticle dimensions. Extending the formalism to scale invariant interactions of continuous mass fields, we calculate three point functions of unparticles.

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Unparticle is an interesting idea proposed by Georgi [1] and is based on a scale invariant sector weakly coupled to the standard model (SM). At lower energies the structure of the scale invariant theory is assumed to have a fixed point in the coupling at a comparatively low scale ($\sim$ TeV), below which by dimensional transmutation, operators emerge with nonintegral dimensions. As pointed out in Ref. [1], many interesting phenomena at TeV scale emerge that can be understood purely from scaling properties of the unparticle operators. Although this is completely satisfactory for phenomenology, much of the dynamics of the scale invariant sector are mysterious, and the existence of a fixed point in the coupling parameter can only be hypothesized.

In this paper we present a formulation that is based explicitly on a well-defined Lagrangian which possesses scale invariance. The Lagrangian involves continuous mass fields. One can now define unparticle like local operators that couple to the SM. The unparticle properties emerge through the choice of interactions. There is no fixed point or dimensional transmutation. The theory leads to a clear understanding of how unparticle exchange and phase space in the decay of SM particles arises.

One starting point is a free Lagrangian for a continuous mass scalar field

$$L_0 = \frac{1}{2}\int_0^\infty [\partial_\mu \phi(x, s)\partial^\mu \phi(x, s) - s\phi^2(x, s)]ds. \quad (1)$$

The field equations are given by

$$\partial_\mu \frac{\partial L_0}{\partial \partial_\mu \phi(x, s)} = \frac{\partial L_0}{\partial \phi(x, s)}. \quad (2)$$

Using functional differentiation, we obtain

$$(\partial_\mu \partial^\mu + s)\phi(x, s) = 0. \quad (3)$$

These are an infinite set of differential equations for all s from 0 to ∞.

On a historical note, we point out that such continuous mass fields were studied long back by Thirring and others [2] in the context of exactly soluble models. We also note that continuous mass fields are also discussed by several groups [3] in context of unparticles, but in a somewhat different spirit. Krasnikov in Ref. [3] has also considered continuous mass arising from a five-dimensional theory with broken Poincaré invariance. We only consider "s" as a dimension 2 mass parameter. The theory can also be obtained as the continuum limit of infinite discrete mass fields.

We now discuss the scaling property of the theory under $x \to x' = \Lambda^{-1}x$. Since Lagrangian has dimension (mass)^4, the field $\phi(x, s)$ must have dimension zero. To get its transformation property $\phi(x, s) \to \phi'(x', s)$ under scaling, we consider the scaling property of the field equation (3). We have

$$(\partial'_\mu \partial'^\mu + s)\phi' = (\Lambda^2 \partial_\mu \partial^\mu + s)\phi' = 0. \quad (4)$$

ϕ' is obviously a field of (mass)$^2 = s/\Lambda^2$. Thus taking into account that the field has dimension zero, we have under scaling,

$$\phi(x, s) \to \phi'(x', s) = \phi(x, s/\Lambda^2). \quad (5)$$

Since the mass s/Λ^2 is within the set of s from 0 to ∞, the transformed equations map on to the initial infinite set, and the theory is scale invariant. This was noted in Delgado *et al.* in Ref. [3]. To confirm scale invariance of the theory, we can explicitly see how the Lagrangian transforms under scaling. We have

*desh@uoregon.edu
+hexg@phys.ntu.edu.tw

$$L_0 \to \frac{1}{2}\int_0^\infty ds[\Lambda^2\partial_\mu\phi(x, s/\Lambda^2)\partial^\mu\phi(x, s/\Lambda^2) - s\phi^2(x, s/\Lambda^2)] = \frac{1}{2}\Lambda^4\int_0^\infty\left[\partial_\mu\phi(x, s/\Lambda^2)\partial^\mu\phi(x, s/\Lambda^2) - \frac{s}{\Lambda^2}\phi^2(x, s/\Lambda^2)\right]d\frac{s}{\Lambda^2}. \quad (6)$$

Changing the integration variable to $s' = s/\Lambda^2$, we have $L_0 \to \Lambda^4 L_0$ and the action $S = \int d^4x L_0$ is invariant. Continuous mass thus restores the scale invariance that is broken by a theory with a discrete nonzero mass.

The field $\phi(x, s)$ in many ways is similar to a usual scalar field, except that it is also labeled by a continuous mass parameter s. We write a real $\phi(x, s)$ in its Fourier representation as

$$\phi(x, s) = \int\frac{d^4p}{(2\pi)^4}2\pi\delta(p^2 - s)\theta(p_0)[a(p, s)e^{-ipx} + a^\dagger(p, s)e^{ipx}]. \quad (7)$$

Because of the fact that the $\phi(x, s)$ has a continuous mass parameter s, the quantization rules for the creation and annihilation operators $a(k, s)$ and $a^\dagger(p, s)$ will be different from that for a usual scalar filed. Appropriate generalization is the following

$$[a(p, s), a^\dagger(k, s')] = (2\pi)^3 2p_0\delta^3(\vec{p} - \vec{k})\delta(s - s'). \quad (8)$$

Note that the dimension for a and $a^\dagger$ is -2.

With the above quantization rules, we have

$$\langle 0|\phi(x, s)\phi(0, s')|0\rangle = \int\frac{d^4p}{(2\pi)^4}e^{-ipx}2\pi\delta(p^2 - s) \times \delta(s - s'), \quad (9)$$

and also the propagator is

$$\int d^4x e^{ipx}\langle 0|T\phi(x, s)\phi(0, s')|0\rangle = \frac{i}{p^2 - s + i\epsilon}\delta(s - s'). \quad (10)$$

A field with an arbitrary scaling dimension can now be constructed by convoluting the field $\phi(x, s)$ with a function $f(s)$ with a fixed scaling dimension to have the following form

$$\phi_{\mathcal{U}}(x) = \int_0^\infty \phi(x, s)f(s)ds. \quad (11)$$

For $f(s) = a_d(s)^{(d-2)/2}$, where a_d is an appropriately chosen normalization constant, $\phi_{\mathcal{U}}$ has the scaling dimension d as can be seen by transforming $\phi(x, s) \to \phi(x, s/\Lambda^2)$ and changing the integration variable to $s' = s/\Lambda^2$.

With the above definition, we have the following

$$\langle 0|\phi_{\mathcal{U}}(x)\phi_{\mathcal{U}}(0)|0\rangle = \int\frac{d^4p}{(2\pi)^4}e^{-ipx}2\pi f^2(p^2),$$
$$\Pi = \int d^4x e^{ipx}\langle 0|T\phi_{\mathcal{U}}(x)\phi_{\mathcal{U}}(0)|0\rangle = \int ds\frac{i}{p^2 - s + i\epsilon}f^2(s). \quad (12)$$

One can immediately identify the phase space $\rho(p^2)$ and propagator Π for $\phi_{\mathcal{U}}$ to be

$$\rho(p^2) = 2\pi f^2(p^2) = 2\pi a_d^2(p^2)^{d-2},$$
$$\Pi = \int ds\frac{i}{p^2 - s + i\epsilon}f^2(s) = \frac{(2\pi a_d^2)}{2\sin(d\pi)}\frac{i}{(-p^2)^{2-d}}. \quad (13)$$

Normalizing the constant a_d as

$$a_d^2 = \frac{A_d}{2\pi}, \qquad A_d = \frac{16\pi^{5/2}}{(2\pi)^{2d}}\frac{\Gamma(d + 1/2)}{\Gamma(d - 1)\Gamma(2d)}, \quad (14)$$

$\phi_{\mathcal{U}}$ has the same phase space and propagator as that defined in Ref. [1], the unparticle operator. We also note that fields obeying Eq. (12) are called generalized free fields [4]. The special choice of ρ makes them transform with a unique scale dimension.

One can easily generalize the above formulation of unparticle to unparticles with different spins. We display our results for vector $A^\mu_{\mathcal{U}}$ and spinor $\psi_{\mathcal{U}}$ unparticles in the following.

For vector unparticle, we start with

$$L_0 = \int_0^\infty\left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}sA^\mu A_\mu\right]ds, \quad (15)$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. We note that presence of $(\mathrm{mass})^2 = s$ means that the vector field is not gauge invariant.

L_0 is invariant under the scaling transformation: $x \to \Lambda^{-1}x$, and $A^\mu \to A^\mu$. The vector unparticle with dimension d is defined by

$$A^\mu_{\mathcal{U}} = \int_0^\infty g(s)A^\mu(x, s)ds, \qquad g(s) = a_d(s)^{(d-2)/2}, \quad (16)$$

and the phase space and propagator are given, in the transverse gauge, by

$$\rho(p^2) = 2\pi g^2(p^2)\left(-g^{\mu\nu} + \frac{p^\mu p^\nu}{p^2}\right),$$
$$\Pi = \int_0^\infty\frac{\rho(s)}{2\pi}\frac{i}{p^2 - s + i\epsilon}ds = \frac{A_d}{2\sin(d\pi)}\frac{i}{(-p^2)^{2-d}}\left[-g^{\mu\nu} + \frac{p^\mu p^\nu}{p^2}\right]. \quad (17)$$

For the spinor unparticle $\psi_{\mathcal{U}}$, we start with

$$L_0 = \int_0^\infty [\bar{\psi} i \gamma_\mu \partial^\mu \psi - \sqrt{s}\bar{\psi}\psi] ds, \tag{18}$$

which is invariant under the transformation: $x \to \Lambda^{-1}x$, and $\psi \to \Lambda^{1/2}\psi$. The spinor unparticle with dimension $d_\psi = d + 1/2$ is given by

$$\psi_{\mathcal{U}} = \int h(s)\psi(x,s)ds, \qquad h(s) = a_d(s)^{(d-2)/2}, \tag{19}$$

and the phase space and propagator are given by

$$\rho(p^2) = 2\pi h^2(p^2)(\gamma_\mu p^\mu + \sqrt{p^2}),$$
$$\Pi = \int_0^\infty ds \frac{\rho(s)}{2\pi}\frac{i}{p^2 - s + i\epsilon} = \frac{A_d}{2\sin(d\pi)}\frac{i}{(-p^2)^{2-d}}[\gamma_\mu p^\mu - i\,\mathrm{ctg}(d\pi)\sqrt{p^2}]. \tag{20}$$

We note that if the vector field is a non-Abelian massive field, it would violate scale invariance. This is because $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$ has a mixed transformation property under scaling since derivatives transform as Λ and f^{abc} as dimension zero. We can still have vector unparticles with nontrivial transformation under a group that does not have a continuous mass description.

We can consider operators that carry nontrivial SM quantum numbers by replacing derivatives with covariant derivatives. This preserves the scale invariance of the theory for scalar and spinor unparticles since covariant derivatives have the same dimension as the usual derivatives. For spin one unparticle this is not possible because of additional self-couplings.

We now comment on interactions of unparticles. Since one can now define unparticle like local operators, the unparticle interaction with SM particles emerges through a choice of interactions. The interaction of unparticles with SM fields can be easily constructed from an effective theory point of view, using operators made of SM fields O_{SM} and the unparticles $O_{\mathcal{U}}$ which can be one of the $\phi_{\mathcal{U}}(x)$, $A^\mu_{\mathcal{U}}(x)$, or $\psi_{\mathcal{U}}(x)$ unparticle operators.

For one unparticle interaction with SM fields, the generic form is give by

$$L_{\mathrm{eff}} = \frac{\lambda}{\Lambda_{\mathcal{U}}^{d_{\mathrm{SM}}+d-4}} O_{\mathrm{SM}} O_{\mathcal{U}}, \tag{21}$$

where $\Lambda_{\mathcal{U}}$ is a scale for the effective interaction and λ represents a dimensionless coupling. d and d_{SM} are the dimensions of $O_{\mathcal{U}}$ and O_{SM}, respectively.

There are many ways unparticles can interact with the SM sector. A set of operators with SM operators that have dimensions less or equal to 4 have been listed [5] and many related phenomenology have been discussed [6]. We will not go into details about related applications, except to point out that since we have obtained the unparticle phase space and propagator, it is trivial to carry out calculations for various applications, such as unparticle production from colliders and decays, which go completely parallel with those that have been considered in the literature. Instead we shall consider different processes.

Since now the unparticle operator $O_{\mathcal{U}}$ is treated as a local operator, one can talk about multiunparticles coupling among themselves and also couplings to SM fields, such as interaction of the form

$$\frac{\lambda_n}{\Lambda_{\mathcal{U}}^{d_1+\cdots+d_n+d_{\mathrm{SM}}-4}} O_{\mathrm{SM}}(O^1_{\mathcal{U}} \cdots O^n_{\mathcal{U}}), \tag{22}$$

where $O^i_{\mathcal{U}}$ indicates an unparticle operator of dimension d_i. When $d_{\mathrm{SM}} = 0$, the above represents self-interactions of unparticles.

Multiunparticle interactions have some interesting properties. We give a few examples in the following. Let us first consider the propagator for the product of two scalar unparticles $\phi_{\mathcal{U}_3}(x) = \phi_{\mathcal{U}_1}(x)\phi_{\mathcal{U}_2}(x)$ where

$$\phi_{\mathcal{U}_i}(x) = \int ds f_i(s)\phi_i(x,s), \qquad f_i(s) = a_{d_i} s^{(d_i-2)/2}. \tag{23}$$

Note that the same $\phi_i(x,s) = \phi(x,s)$ can be used to construct unparticles of different dimensions by convoluting a different $f_i(s)$.

The propagator for $\phi_{\mathcal{U}_3}(x)$ is defined by $\Pi = \int d^4x e^{ipx}\langle 0|T\phi_{\mathcal{U}_3}(x)\phi_{\mathcal{U}_3}(0)|0\rangle$. We have, using the Wick contraction,

$$\langle 0|T\phi_{\mathcal{U}_3}(x)\phi_{\mathcal{U}_3}(0)|0\rangle = \langle 0|T\phi_{\mathcal{U}_1}(x)\phi_{\mathcal{U}_1}(0)|0\rangle\langle 0|T\phi_{\mathcal{U}_2}(x)\phi_{\mathcal{U}_2}(0)|0\rangle + \langle 0|T\phi_{\mathcal{U}_1}(x)\phi_{\mathcal{U}_2}(0)|0\rangle\langle 0|T\phi_{\mathcal{U}_2}(x)\phi_{\mathcal{U}_1}(0)|0\rangle$$
$$= \int \frac{d^4p_1}{(2\pi)^4} e^{-ip_1x}\int ds_1 \frac{i f_1^2(s_1)}{p_1^2 - s_1 + i\epsilon}\int \frac{d^4p_2}{(2\pi)^4} e^{-ip_2x}\int ds_2 \frac{i f_2^2(s_2)}{p_2^2 - s_2 + i\epsilon}. \tag{24}$$

Here we consider the case with $\phi_1 \neq \phi_2$ so that the cross term is zero. We will discuss the result for the same $\phi_i = \phi(x,s)$ later.

Carrying out integrations for x and p_i for Π, Π can be written as

$$\Pi = \int_0^\infty \frac{\rho(s)}{2\pi}\frac{i}{p^2 - s + i\epsilon}, \tag{25}$$

with

$$\rho(s) = \int_0^s ds_1 \int_0^{(\sqrt{s}-\sqrt{s_1})^2} ds_2 \frac{1}{8\pi} f_1^2(s_1) f_2^2(s_2) \frac{1}{s}(s^2 - 2s(s_1+s_2) + (s_1-s_2)^2)^{1/2}$$
$$= \frac{a_{d_1}^2 a_{d_2}^2}{8\pi} s^{d_1+d_2-2} \int_0^1 dx \int_0^{(1-\sqrt{x})^2} dy x^{d_1-2} y^{d_2-2}(1-2(x+y)+(x-y)^2)^{1/2}$$
$$= \frac{a_{d_1}^2 a_{d_2}^2}{8\pi} s^{d_1+d_2-2}(d_1-1)(d_2-1)(d_1+d_2-1)\frac{\Gamma^2(d_1-1)\Gamma^2(d_2-1)}{\Gamma^2(d_1+d_2)}. \quad (26)$$

Inserting $a_d^2 = A_d/2\pi$ and using $\Gamma(2d) = \pi^{-1/2}2^{2d-1}\Gamma(d+1/2)\Gamma(d)$, the above expression can be written as

$$\rho(s) = s^{d_1+d_2-2}\frac{16\pi^{5/2}}{(2\pi)^{2(d_1+d_2)}} \times \frac{\Gamma(d_1+d_2+1/2)}{\Gamma(d_1+d_2-1)\Gamma(2(d_1+d_2))}. \quad (27)$$

This is the phase space for an unparticle of dimension $d_3 = d_1 + d_2$.

We therefore have shown that $\phi_{\mathcal{U}_3}$ is an unparticle with dimension $d_3 = d_1 + d_2$. The normalization A_d is something deeper than just convenience [1]. Had another normalization been used, the product of two scalar unparticles would not be a new unparticle with dimensions equal to the sum of the two unparticles with the correct normalization. The self-similarity of unparticle dictates the normalization.

For the case $\phi_1 = \phi_2$, the cross term will also contribute the same amount, but the total should be divided by 2! to get the right normalization, in another words, $\phi_{\mathcal{U}_3}$ should be written as $\phi_{\mathcal{U}_1}\phi_{\mathcal{U}_2}/\sqrt{2!}$. One gets $\phi_{\mathcal{U}_3}$ to be an unparticle of dimension $d_1 + d_2$. The above discussions can be easily generalized to any number of scalar unparticle products. With proper permutation normalization, the product is an unparticle with the dimension equal to the sum of the composite unparticles. Products involving spinor and vector unparticles will be more complicated, and we shall discuss them in detail in a future publication.

As a further important application of Eq. (8), we calculate the three unparticle vertex functions defined by

$$V(p_1^2, p_2^2, p_3^2) = \int d^4x e^{ip_1x} d^4y e^{ip_2y} \times \langle 0|T(\phi_{\mathcal{U}}(x)\phi_{\mathcal{U}}(y)\phi(0))|0\rangle, \quad (28)$$

where $p_3 = p_1 + p_2$.

From the general scaling argument it follows that it has dimension $3d - 8$, and is an invariant function of three variables p_1^2, p_2^2, and p_3^2. Further, it is symmetric under exchanges between p_1, p_2, and p_3. However scaling alone is not sufficient to determine this function, as we shall see.

We first evaluate the time ordered product $T_3 = \langle 0|T(\phi_{\mathcal{U}}(x)\phi_{\mathcal{U}}(y)\phi_{\mathcal{U}}(0)|0\rangle$. We have

$$T_3 = \int_0^\infty ds_1 ds_2 ds_3 f(s_1) f(s_2) f(s_3) \times \langle 0|T(\phi(x,s_1)\phi(y,s_2)\phi(0,s_3))|0\rangle. \quad (29)$$

With the Lagrangian in Eq. (1), since there are no interactions, the integral is obviously zero, and there is no three point function. We introduce scale invariant interactions of the continuous mass fields so as to have the nonvanishing T product. It is sufficient to introduce terms of the ϕ^3 type. The idea is to introduce some dynamics that are also scale invariant, but at the same time, we only consider tree level consequences of such a theory. Deeper questions like renormalizability of such a theory are beyond the scope of this paper.

One possible ϕ^3 scale invariant interaction is

$$L_\lambda = \frac{\lambda}{3!}\int \frac{ds_1 ds_2 ds_3}{(s_1 s_2 s_3)^{1/3}} \phi(x,s_1)\phi(x,s_2)\phi(x,s_3). \quad (30)$$

Another possibility is

$$L_g = \frac{g}{3!}\int_0^\infty s ds \phi^3(x,s). \quad (31)$$

We note that the modifications to equations of motion from the above two interactions are, respectively,

$$\partial_\mu\partial^\mu\phi(x,s) + s\phi(x,s) = \frac{\lambda}{2s^{1/3}} \times \int \frac{ds_1 ds_2}{(s_1 s_2)^{1/3}}\phi(x,s_1)\phi(x,s_2), \quad (32)$$

and

$$(\partial_\mu\partial^\mu + s)\phi(x,s) = \frac{g}{2}s\phi^2(x,s). \quad (33)$$

Both equations under scale the transformation map within the infinite set of equations, as can be verified. The first is a integro-differential equation. Such equations have been considered previously [2], where a model is solved exactly in the case of bilinear interactions. The difference between the above two forms can be understood if one goes to the discrete limit of the theory.

We evaluate the time ordered product $t_3 = \langle 0|T(\phi(x,s_1)\phi(y,s_2)\phi(0,s_3)|0\rangle$ in the lowest order perturbation theory and find using the L_λ interaction,

$$t_3 = \lambda \int d^4z \int \frac{ds_1' ds_2' ds_3'}{(s_1' s_2' s_3')^{1/3}} \langle 0|T\phi(x, s_1)\phi(z, s_1')|0\rangle \times \langle 0|T\phi(y, s_2)\phi(z, s_2')|0\rangle \langle 0|T\phi(0, s_3)\phi(z, s_3')|0\rangle. \tag{34}$$

Using

$$\langle 0|T(\phi(x, s)\phi(z, s')|0\rangle = \int \frac{d^4q}{(2\pi)^4} e^{-iq(x-z)} \times \frac{i}{q^2 - s + i\epsilon} \delta(s - s'), \tag{35}$$

we get

$$t_3 = -i\lambda \int \frac{d^4q_1}{(2\pi)^4} \frac{d^2q_2}{(2\pi)^4} \frac{e^{-i(q_1x+q_2y)}}{(s_1s_2s_3)^{1/3}} \times \frac{1}{(q_1^2 - s_1 + i\epsilon)(q_2^2 - s_2 + i\epsilon)(q_3^2 - s_3 + i\epsilon)}, \tag{36}$$

where $q_3 = q_1 + q_2$.

We now get for the T product of unparticles

$$V(p_1^2, p_2^2, p_3^2) = -i\lambda \int \frac{f(s_1)f(s_2)f(s_3)ds_1ds_2ds_3}{(s_1s_2s_3)^{1/3}(p_1^2 - s_1 + i\epsilon)(p_2^2 - s_2 + i\epsilon)(p_3^2 - s_3 + i\epsilon)}. \tag{37}$$

Using the formula,

$$\int \frac{f(s)ds}{s^{1/3}(p^2 - s + i\epsilon)} = \int \frac{a_d s^{(d-2)/2}ds}{s^{1/3}(p^2 - s + i\epsilon)} = \frac{a_d\pi}{\sin((d/2 - 4/3)\pi)} \frac{i}{(-p^2)^{-d/2+4/3}}. \tag{38}$$

Defining a new constant $\lambda' = \lambda(a_d\pi/\sin((d/2 - 4/3)\pi))^3$, we have

$$V(p_1^2, p_2^2, p_3^2) = -i\lambda' \frac{1}{(-p_1^2)^{-d/2+4/3}(-p_2^2)^{-d/2+4/3}(-p_3^2)^{-d/2+4/3}}. \tag{39}$$

This expression has the correct dimensions and fulfills all the symmetry requirements.

A similar computation with L_g interaction gives

$$V(p_1^2, p_2^2, p_3^2) = -ig'\Big[\frac{1}{(p_1^2 - p_2^2)(p_1^2 - p_3^2)} \frac{1}{(-p_1^2)^{2-3d/2}} + \frac{1}{(p_2^2 - p_1^2)(p_2^2 - p_3^2)} \frac{1}{(-p_2^2)^{2-3d/2}} + \frac{1}{(p_3^2 - p_1^2)(p_3^2 - p_2^2)} \frac{1}{(-p_3^2)^{2-3d/2}}\Big], \tag{40}$$

where $g' = ga_d^3\pi/\sin(3d\pi/2)$. This expression also fulfills all the requirements of dimensions and symmetry.

Note that in Ref. [7], Feng *et al.* calculate the three point function assuming conformal invariance. Their starting point is

$$\langle 0|T(\phi_{\mathcal{U}}(x)\phi_{\mathcal{U}}(y)\phi_{\mathcal{U}}(0))|0\rangle = C\frac{1}{|x|^d} \frac{1}{|y|^d} \frac{1}{|x - y|^d}. \tag{41}$$

However this form does not seem unique if only scale invariance is imposed, for example, one can multiply this by a dimensionless function of $|x|/|y|$ and $|x - y|/|y|$. Another example for the right-hand side is $[1/|x|^{3d} + 1/|y|^{3d} + 1/|x - y|^{3d}]$. Our expressions are simple in momentum space, but complicated in coordinate space, while Ref. [7] has a complicated form in momentum space.

The simplest form for the three point function in momentum space is in Eq. (34). It is possible to probe the three point function experimentally. Feng *et al.* have discussed various signals for the three point function. When standard model particles couple to unparticles, it is possible to get signals that depend on the explicit form of the vertex. For example if we have a coupling of the type $\bar{e}e\phi_{\mathcal{U}}$, we can get events of the type $e^+e^- \to e^+e^- + e^+e^-$ where the energy distribution of e^+e^- pairs will depend on the explicit three point function. Study of such signals will be very useful, and we shall pursue it in future publications.

We can extend our analysis to three point functions involving scalar, spinor, and vector unparticles. We have to add interactions of continuous mass spinor and vector fields that preserve scale invariance. As an example, we can add $L_{fh} = \int ds\sqrt{s}[f\bar{\psi}(x,s)\psi(x,s)\phi(x,s) + h\bar{\psi}(x,s)\times \gamma_\mu\psi(x,s)A^\mu(x,s)]$ or $L_{f'h'} = \int(ds_1ds_2ds_3/\sqrt{s_1s_2s_3})\times [f'\bar{\psi}(x,s_1)\psi(x,s_2)\phi(x,s_3) + h'\bar{\psi}(x,s_1)\gamma_\mu\psi(x,s_2)A^\mu(x,s_3)]$. Consequences of such interactions will be pursued in future publications.

One can also easily generalize to unparticles with SM gauge interactions by assuming that the $\phi(x, s)$ and $\psi(x, s)$ have nontrivial SM quantum numbers. The end results are that the unparticle operators $\phi_{\mathcal{U}}$ and $\psi_{\mathcal{U}}$ have the same SM quantum numbers as $\phi(x, s)$ and $\phi(x, s)$, respectively. When taking derivatives, one should take the covariant derivative as would have to be done for usual particles. As pointed out earlier that to preserve scale invariance, the vector operator cannot be non-Abelian. Vector unparticle $A^\mu_{\mathcal{U}}$ can have a nontrivial $U(1)_Y$ quantum number, but cannot have nontrivial $SU(3)_C$ and $SU(2)_L$ quantum numbers.

Let us study a simple example, involving two unparticle operators. Consider a charged scalar S^+ decaying into a charged scalar unparticle $\phi^+_{\mathcal{U}}$ of dimension d_+ and neutral scalar unparticle $\phi^0_{\mathcal{U}}$ of dimension d_0. Under the SM gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$, S^+, $\phi^+_{\mathcal{U}}$, and $\phi^0_{\mathcal{U}}$ transform as (1,1,2), (1,1,2), and (1,1,0), respectively. The lowest dimension interaction possible is given by $L_{\rm eff} = (\lambda/\Lambda_{\mathcal{U}}^{d_+ + d_0 - 3}) S^+ \phi^-_{\mathcal{U}} \phi^0_{\mathcal{U}}$. We have decay distribution $d\Gamma$ for $S^+(p) \to \phi^+_{\mathcal{U}}(p_+)\phi^0_{\mathcal{U}}(p_0)$ given by

$$d\Gamma(S^+ \to \phi^+_{\mathcal{U}}\phi^0_{\mathcal{U}}) = \left(\frac{\lambda}{\Lambda_{\mathcal{U}}^{d_+ + d_0 - 3}}\right)^2 \frac{1}{2m_S}(2\pi)^4 \times \delta^4(p - (p_+ + p_0)) A_{d_+}(p_+^2)^{d_+ - 2}\theta \times (p_+^2)\frac{d^4 p_+}{(2\pi)^4} A_{d_0}(p_0^2)^{d_0 - 2}\theta(p_0^2)\frac{d^4 p_0}{(2\pi)^4}, \tag{42}$$

which leads to the energy distribution for the decay,

$$\frac{d\Gamma(S^+ \to \phi^+_{\mathcal{U}}\phi^0_{\mathcal{U}})}{dE_+} = \frac{|\lambda|^2}{\Lambda_{\mathcal{U}}^{2d_+ + 2d_0 - 6}} \frac{1}{16\pi^3 m_S} A_{d_+} A_{d_0} \times E_+^{2d_+ - 1} \int_0^{x_{\max}} x^{1/2}(1 - x)^{d_+ - 2} \times \left(m_S^2 + E_+^2\left(1 - 2\frac{m_S}{E_+} - x\right)\right)^{d_0 - 2} dx. \tag{43}$$

Here $x = |\vec{p}_+|^2/E_+^2$. The limit for E_+ and x are determined by energy momentum conservation $p = p_+ + p_0$, and also $p_+^2 > 0$ and $p_0^2 > 0$. We have: $x_{\max} = 1$ for $0 < E_+ < m_S/2$, and $x_{\max} = (1 - m_S/E_+)^2$ for $m_S/2 < E_+ < m_S$.

Experimental signature would be a charged particle decay into a charge which can be detected by measuring the energy deposited in the path plus missing energy. The actual detectability depends on the scale $\Lambda_{\mathcal{U}}$ and the coupling λ. Here our emphasis is on the different features compared with other processes. If one only looks at the charged track without energy measurement, there are several other possibilities. For example: (i) a usual charged particle decays into a lighter charged particle plus a usual neutral undetected particle; or (ii) a usual charged particle decays into a neutral unparticle (particle) and a charged usual particle (unparticle). If energy distributions of the charged track are measured one can distinguish different scenarios. Possibility (i) can be easily distinguished because the daughter charged particle has a fixed energy. Possibility (ii) can also be distinguished because the charged track energy distribution is different from that for the two unparticle decays discussed above. For example consider $S^+ \to \phi^+\phi^0_{\mathcal{U}}$, here ϕ^+ is a usual charged scalar particle. The lowest dimension interaction is $(\lambda/\Lambda_{\mathcal{U}}^{d_0 - 2}) S^+ \phi^- \phi^0_{\mathcal{U}}$ which leads to a differential energy distribution of the charged scalar particle given by

$$\frac{d\Gamma(S^+ \to \phi^+\phi^0_{\mathcal{U}})}{dE_+} = \frac{|\lambda|^2}{\Lambda_{\mathcal{U}}^{2d_0 - 4}} \frac{1}{8\pi^2 m_S} A_{d_0}(m_S^2 + E_+^2 - 2m_S E_+ + m_+^2)^{d_0 - 2} \times (E_+^2 - m_+^2)^{1/2}, \tag{44}$$

where m_+ is the mass of the charged scalar particle. The range of E_+ is from m_+ to $(m_S^2 - m_+^2)/2m_S$. This differs from the two unparticle cases in Eq. (38) and is amenable to an experimental test.

In summary, we have proposed a different approach to construct unparticle operators based on scale invariant theories of continuous mass. One can define unparticles like local operators that couple to the SM. The unparticle properties emerge through choices of interactions. There is no fixed point or dimensional transmutation. The theory leads to a clear understanding of how unparticle exchange and phase space in the decay of SM particles arises. We have generalized interactions of the standard model to multiple unparticles in this formalism and have worked out some examples for illustration. We show that products of unparticles are properly normalized unparticles of dimension equal to the sum of the dimension of the individual unparticles. We have extended our formalism to calculate three point functions of unparticles. This required considering interactions of continuous mass fields.

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