Resonant scattering and recombination of pseudodegenerate WIMPs

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We consider the direct and indirect detection signatures of weakly interacting massive particles (WIMPs) χ^0 in kinematic regimes with a heavier, but nearly degenerate, charged state $\chi^{\pm}$. For small splittings of $\mathcal{O}(10)$ MeV, the scattering of WIMPs off nuclei may be dominated by inelastic recombination processes mediated by the formation of $(\chi^- N)$ bound states, leading to a distinct signature for direct detection. These cross sections are bound primarily by limits on the abundance of heavy isotopes, and may be considerably larger than the elastic scattering cross section in more conventional models. If the mass splitting is too large for recombination to occur, there may still be a significant resonant enhancement of loop-induced electromagnetic form factors of the WIMP, which can enhance the elastic scattering cross section. We also discuss how this regime affects the annihilation cross section and indirect detection signatures, and note the possibility of a significant mono-energetic γ signal, mediated by resonant processes near the $(\chi^+ \chi^-)$ bound-state threshold.

I. INTRODUCTION

The evidence for the existence of nonbaryonic dark matter now comes from many sources and ranges over many distance scales [1], from the rotation curves of galaxies, the dynamics of clusters, lensing data, and the characteristics of large-scale structure, to the features of the cosmic microwave background fluctuation spectrum and the success of big bang nucleosynthesis (BBN). All of these pieces of astronomical data point to a similar cosmological density of dark matter, several times that of visible baryonic matter. Moreover, recent observations of the bullet cluster [2] have for the first time pointed more directly to the existence of dark matter through spatial separation of the baryonic and nonbaryonic (or dark) components in the collision.

This situation has been, and remains, one of the primary puzzles within particle physics and also one of the strongest motivations for physics beyond the standard model (SM). It is remarkable that the theoretical shortcomings of the standard model with regard to the UV sensitivity of the Higgs mass point to new physics at or near the electroweak scale, and that a stable weakly interacting particle of this mass has just the right properties to be a dark matter candidate arising as a thermal relic from the big bang. There are also interesting links between various symmetries required to avoid large violations of approximate SM invariances, such as baryon number, and the stability required for such dark matter candidates. This apparent ''naturalness'' of weak-scale cold dark matter, or weakly interacting massive particles (WIMPs), has led to a vast literature on the subject in models such as the supersymmetric version of the standard model (MSSM) [1], models with large extra dimensions [3], etc., and also the development of ground-based direct-detection facilities searching for the recoil of large nuclei from elastic scattering with WIMPs in the galactic halo [4,5].

More recently, high-precision data from the Wilkinson microwave anisotropy probe (WMAP) [6], and the negative results of the LEP and Tevatron searches for new physics at or near the electroweak scale has put some pressure on this seemingly natural scenario. In particular, in the MSSM with unification of the soft-breaking parameters at high-energy scales (CMSSM), it is now well known that a generic point in the parameter space, consistent with collider constraints, would typically lead to a relic density for the neutralino lightest supersymmetric particle (LSP), which is orders of magnitude too high, while only a rather tuned region of parameter space remains in agreement with the data [1,7]. In fact, the simple freeze-out calculation of the neutralino WIMP relic density, which implies $\Omega_{\chi} h^2 \sim 3 \times 10^{-37}$ cm$^2/\langle \sigma_{\mathrm{ann}} v \rangle$ [8], leads to a value consistent with WMAP only in a region of the CMSSM parameter space, which is already excluded by direct search bounds on the Higgs and chargino masses. Viable regions actually rely on additional, and to a certain extent accidental, features of the spectrum in order to enhance the neutralino annihilation cross section. One such enhancement mechanism, the coannihilation [9] of the neutralino with a nearly degenerate charged slepton, allows the extension of the region of viable WIMP masses into the TeV energy domain. This issue of tuning is of course not unique to the relic density of dark matter and is now a more generic problem for models of new weak-scale physics following the absence of new discoveries at LEPII, for example. Indeed, the relic density requirement and the apparent absence of new physics right at the weak scale suggests a prominent role for mechanisms, which allow for enhanced annihilation of heavier WIMPs in coming years.

With this motivation in mind, in this paper we would like to focus on one particular enhancement mechanism, namely, coannihilation with a nearly degenerate charged state, and explore how extreme limits of this kinematic regime may alter the predictions for direct and indirect searches. We will refer to this kinematic scenario as a *pseudodegenerate* WIMP, having in mind that the WIMP χ_1^0 will be separated by a small mass gap of $\Delta m \sim \mathcal{O}(1\text{–}100)$ MeV from an excited state χ_2^0 or $\chi_2^\pm$. For a WIMP mass of $\mathcal{O}(100 \text{ GeV})$ this splitting is actually much smaller than is required for coannihilation, for which a relative splitting of less than 5% is sufficient. Whether such near degeneracies are natural or not clearly depends on the precise nature of the model at hand, but it is a particularly interesting kinematic regime in the case that χ_2 is charged for the following reason. For $\Delta m < \mathcal{O}(20 \text{ MeV})$, inelastic scattering of χ_1 with heavy nuclei becomes possible in which capture of χ_2^- occurs to form the bound state $(N\chi_2^-)$, and this process can dominate the cross section if χ_1 behaves in other respects as a conventional WIMP with a suppressed elastic scattering cross section. This process, since it involves charged exchanges, can lead to significantly different signatures in direct-detection experiments. If Δm is larger than about 20 MeV, this recombination process is no longer possible, but there can still be a significant resonant enhancement of the elastic scattering cross section. Such enhancements of elastic scattering are in some respects the trade-off for going to a kinematic regime where the annihilation cross section is similarly enhanced. For very small splittings, the annihilation cross section itself may also be dominated by resonant processes, since it is near threshold for the formation of the $(\chi^+\chi^-)$ bound state, which in turn can result in an enhanced production of γ rays. This is particularly so in the galactic environment where the characteristic velocities are quite low.

In what follows, we will explore these issues in turn. In Sec. II, we introduce the general kinematic scenario and some classes of pseudodegenerate models. In Sec. III, we consider first the recombination with nuclei and compute the simplest charged current and electromagnetic captures to the ground state, and comment on various constraints from, e.g. terrestrial heavy isotope searches. We then turn to the generic resonant enhancement of the elastic cross section. In Sec. IV, we consider the impact of resonant processes in annihilation, and we conclude in Sec. V with some additional remarks on alternative detection signatures.

II. PSEUDO-DEGENERATE WIMPS

Following the motivation outlined in the preceding section, we will consider generic WIMP scenarios in which the dark sector has some substructure, in the form of at least one excited state χ_2 nearly degenerate with the WIMP χ_1, which we also take to have a mass of $\mathcal{O}(0.1\text{–}1 \text{ TeV})$,

with the lower limit imposed by searches at LEPII. A near-degenerate neutral excited state was considered previously for different reasons in [10], but for efficient coannihilation χ_2 should have stronger interactions with the SM than χ_1 and for the present paper we will generically assume that χ_2 is electromagnetically charged $\chi_2^\pm$, although a neutral state charged under $SU(3)_c$ would also fall into a similar class. These simple requirements define the class of pseudodegenerate WIMPs to be studied in this paper.

A. Models

To provide a somewhat finer classification of pseudodegenerate scenarios, we will generally consider two classes of models, determined by the dominant interactions with gauge bosons. The WIMP χ_1, whether fermion or scalar, is required to possess no diagonal tree-level spin-independent coupling to SM gauge bosons due to constraints on direct detection. Even a very small $O(10^{-2})$ spin-independent coupling to the Z is still sufficient to produce an elastic scattering cross section with nuclei in excess of the current bound. The nullification of couplings to Z can be achieved either by a careful charge assignment in the WIMP sector, or by requiring that χ_1 and its charge conjugate field χ_1^c are the same. In other words, in order to suppress the spin-independent scattering cross section, χ_1 should be a real scalar if WIMPs have spin 0, or a Majorana fermion if χ_1 has spin $1/2$. We have defined our scenarios such that χ_2 is electromagnetically charged, and thus the primary distinction we can place on the $\chi_1 - \chi_2$ WIMP sector is on the type of charged current between these two states. Restricting our discussion to tree-level couplings, we introduce two model classes.

(i) *Type A*—The first scenario we will consider will allow for a charged-current interaction with W bosons. In other words, the pseudodegenerate WIMP sector possesses an off-diagonal vector current $J_{\chi_1\chi_2}^\mu$ so that

$$\mathcal{L}_{\text{int}} = J_{\chi_1\chi_2}^{\mu-} W_\mu^+ + \text{H.c.,} \tag{1}$$

and consequently χ_1 and χ_2 are either both bosons or fermions. A familiar MSSM model in this class would involve a neutralino χ_1 with a near-degenerate chargino $\chi_2^\pm$.

(ii) *Type B*—The second scenario will be defined by the condition that χ_1 and χ_2 are, respectively, a scalar and a fermion or vice versa. In this case, the current between χ_1 and χ_2 is fermionic and couples to the SM charged leptons ψ

$$\mathcal{L}_{\text{int}} = J_{\chi_1\chi_2}^{a-} \psi_a^+ + \text{H.c..} \tag{2}$$

A characteristic MSSM example in this case would be a neutralino χ_1 with a near-degenerate stau χ_2.

In the remainder of this section, we outline the relevant kinematic regimes of interest.

B. WIMP-nucleus binding energies

Coannihilation does not impose undue levels of tuning on the mass spectrum; a splitting between the WIMP and excited states of $\mathcal{O}(5\%)$ is generally sufficient. This does not significantly alter the expectations for interactions with baryonic matter, and we will be interested in a more extreme limit in which the splitting is in the MeV range, and thus comparable to the Coulomb binding energy of χ_2^- with nuclei. We will defer any discussion of theoretical motivations for this pseudodegeneracy, but as a benchmark point to normalize the kinematic regimes to be discussed below, it is worth noting that if χ_1^0 and $\chi_2^\pm$ were truly degenerate in the dark sector through some symmetry, then the interaction of $\chi_2^\pm$ with the SM gauge bosons would naturally imply a splitting of $\mathcal{O}(100\text{ MeV})$ [11].[1]

Denoting the mass difference as $\Delta m = m_{\chi_2} - m_{\chi_1}$, we explore the upper bound on Δm that leads to the formation of stable $(N\chi_2^-)$ bound states. In the limit $m_\chi \gg m_N$, the binding energy depends only on the nuclear mass, and is naively given by the analogue of the Rydberg energy $-E_b \sim (Z\alpha)^2 m_N/2$. More precisely, since the characteristic radius of the orbit $r_B \sim (Z\alpha m_N)^{-1}$ generally lies well within the nucleus, we obtain a better estimate by solving the Schrödinger equation with a given charge distribution inside the nucleus. This leads to the results shown in Table I for several elements that will be relevant in this paper. Two types of charge distribution with the same $\langle r_c^2 \rangle$, Gaussian and step-like, are employed that can be viewed as two extreme approximations of a more realistic nuclear charge density.

Table I reveals three distinct kinematic regimes:

TABLE I. Estimates for the binding energies of the state $(N\chi_2^-)$ assuming a Gaussian and steplike nuclear charge distribution for several relevant elements.

$(N\chi_2^-)$	Z	$-E_b$ (MeV), Gaussian	$-E_b$ (MeV), steplike
$(^1\text{H}\chi_2^-)$	1	0.025	-
$(^4\text{He}\chi_2^-)$	2	0.35	-
$(^{11}\text{B}\chi_2^-)$	5	2.2	2.1
$(^{12}\text{C}\chi_2^-)$	6	2.8	2.7
$(^{14}\text{N}\chi_2^-)$	7	3.5	3.2
$(^{16}\text{O}\chi_2^-)$	8	4.0	3.7
$(^{40}\text{Ar}\chi_2^-)$	18	9.1	8.0
$(^{74}\text{Ge}\chi_2^-)$	32	14.6	12.5
$(^{132}\text{Xe}\chi_2^-)$	54	21.7	18.4

$$[\text{(I): recombination with light elements}] \qquad 0 < \Delta m < 4 \text{ MeV}, \tag{3}$$

$$[\text{(II): recombination with heavy elements}] \qquad 4 \text{ MeV} < \Delta m \lesssim 20 \text{ MeV}, \tag{4}$$

$$[\text{(III): resonant loop enhancement}] \qquad 20 \text{ MeV} \lesssim \Delta m \lesssim 100 \text{ MeV}. \tag{5}$$

In regime (I), stable bound states with light nuclei up to oxygen may be formed. Some models in this kinematic regime are significantly constrained by searches for anomalously heavy isotopes of carbon and other light elements as discussed below. Regime (II) is equally interesting, as it opens the possibility of "recombination" processes where WIMPs may combine with the heavy elements inside the detectors used for the direct searches of dark matter. Finally, kinematic regime (III) arises when Δm is too large for recombination to occur in inelastic scattering, but still small relative to the WIMP mass, and the elastic scattering cross section can be resonantly enhanced through loop processes, with an off-shell χ_2^- appearing as an intermediate (virtual) state in scattering. The upper limit in this case is not strict, but simply gives an estimate for the level beyond which these processes are less relevant; 100 MeV is the characteristic center-of-mass momentum for interactions of a 100 GeV halo WIMP with terrestrial nuclei.

III. PSEUDODEGENERATE WIMP-NUCLEUS SCATTERING

A. WIMP-nucleus recombination

One of the more interesting processes that becomes kinematically accessible for the mass splitting in (4), is the "recombination" of WIMPs with nuclei, and indeed such inelastic scattering can dominate the cross section if χ_1 is otherwise relatively inert, as would be characteristic of a WIMP. In this section, we explore several classes of interactions.

1. Weak charged-current recombination

For models of Type A, a natural class of capture processes will proceed via weak currents, the simplest example of which is shown in Fig. 1, $\chi_1^0 + n \to \chi_2^- + p$. Depending on the value of Δm, the recombination process may occur directly to the ground state of the bound WIMP-nucleus system, or to an excited intermediate state that will

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[1]It is also worth noting that if χ_1 and χ_2 arise from the low-energy hadronization of a single relic state charged under $SU(3)_c$, e.g. as hadronized squarks [12], they can naturally be split by $\mathcal{O}(\text{MeV})$ as members of an isospin doublet. In such cases, recombination with nuclei could occur through strong rather than electromagnetic interactions, but this may be subject to more stringent constraints than the scenarios for electromagnetic capture that we consider here.
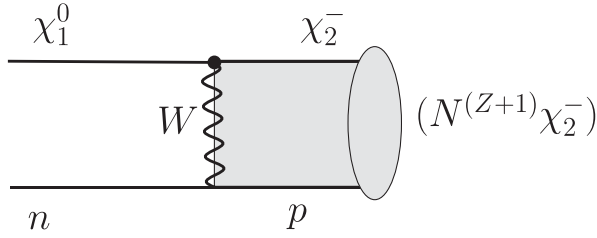
FIG. 1. The weak charged-current capture of χ_1^0 to form the bound state $(N\chi_2^-)$.

subsequently decay to the ground state through emission of γ's and/or neutrons depending on the nucleus in question,

$$\chi_1^0 + N^{(Z)} \to (N^{(Z+1)}\chi_2^-)^* \to (N^{(Z+1)}\chi_2^-) + (\gamma, n, \ldots). \tag{6}$$

This process may receive a resonant enhancement but of course is also subject to the details of nuclear binding and so may not be energetically allowed for many light nuclei, depending on the relative binding energies of $N^{(Z)}$ and $N^{(Z+1)}$. For example, the capture (6) is not possible for such abundant nuclei as ^{12}C, ^{14}N, and ^{16}O, because the $(Z + 1, A)$ nuclei are too massive. In addition to (6), there always exists a β^+-type process

$$\chi_1^0 + N^{(Z)} \to (N^{(Z)}\chi_2^-) + e^+ + \nu, \tag{7}$$

which is clearly nonresonant.

The nonresonant contribution (7) is relatively easy to estimate, and to refine to a full calculation if needed. For our purposes it suffices to estimate the nonresonant capture cross section by considering the overlap between the scattering and bound-state wavefunctions, i.e.

$$\sigma_{\text{non-res}} \simeq \Gamma_{\chi_1 \to \chi_2} |\langle \psi_N^{\text{scat}} | \psi_N^{\text{bs}} \rangle|^2 \times F_G, \tag{8}$$

where $\Gamma_{\chi_1 \to \chi_2}$ is the "decay" width of χ_1 to χ_2 with the energy release $Q_{\text{eff}} = |E_b| - m_e - \Delta m$. Neglecting the Gamow factor F_G for now, saturating the wave-function overlap by characteristic nuclear scales, and taking $\Gamma_{\chi_1 \to \chi_2} \sim 10^{-3}$ Hz$(Q_{\text{eff}}/1 \text{ MeV})^5$, we immediately discover that the rate for this process is extremely slow,

$$\sigma_{\text{non-res}} v \sim 10^{-3} \text{ Hz} \times (2 \text{ fm})^3 \times \left(\frac{Q_{\text{eff}}}{1 \text{ MeV}}\right)^5$$
$$\sim 10^{-51} \text{ cm}^2 \left(\frac{Q_{\text{eff}}}{1 \text{ MeV}}\right)^5, \tag{9}$$

even for the largest values of Q_{eff}.

Computing the cross section for process (6) with an intermediate excited nuclear state is rather nontrivial. However, to get a reasonable estimate, it is useful to think of the excited state $(N\chi_2^-)^*$ as a resonance, for which Fig. 1 characterizes the entrance width Γ_W. Since Γ_W relates to a weak process, it is much smaller than the decay width induced by electromagnetic and strong interactions, and

we can use the Breit-Wigner formula to write

$$\sigma_{\text{res}} \sim \sum_R \frac{\pi g_*}{q_{\text{cm}}^2} \frac{\Gamma_W \Gamma}{(E - E_R)^2 + \Gamma^2/4}, \tag{10}$$

where g_* refers to the angular momentum multiplicity factor, q_{cm} is the momentum in the WIMP-nucleus center-of-mass frame, and E_R denotes the resonant levels of $(N\chi_2^-)$, which may be formed in the capture process.

The spread in kinetic energy of the colliding WIMP-nucleus system is on the order of $M_R^2(N, \chi)v^2$, where v is the relative velocity and M_R the reduced mass. Thus, for nuclear masses in the interval 10–100 GeV, the typical spread in kinetic energy is 10–100 keV, which is somewhat smaller than the characteristic spacing of bound-state energy levels of 1 MeV. This level spacing becomes even denser due to the additional Coulomb excitations in the $(N\chi_2^-)$ system, and for the purpose of obtaining an estimate it is reasonable to assume that $\Delta E \sim 0.5$ MeV. The decay width will vary quite significantly depending on Δm and the nuclei participating in capture process. For electromagnetic decay widths in bound states with light nuclei it is reasonable to expect $\Gamma \sim O(1 \text{ eV})$, while a neutron decay width in the capture of a WIMP with small Δm by a large nucleus can easily reach $O(100 \text{ keV})$, and may thus be quite comparable to the spacing of energy levels. For the latter case, we approximate the capture rate by assuming a characteristic energy denominator scale in (10) on the order of ΔE

$$\sigma_{\text{res}} v \sim \frac{1}{M_R^2 v} \times \frac{\Gamma_W}{500 \text{ keV}}. \tag{11}$$

To complete the estimate we need to evaluate the weak entrance width. Its dependence on the main parameters in the problem is captured by the following scaling:

$$\Gamma_W \sim \frac{G_F^2 M_R^2 v}{r_0^3}, \tag{12}$$

where r_0 is the characteristic distance scale entering in the evaluation of the matrix element, that for the purposes of this estimate we take to be of order the nuclear radius of 4 fm, and we have also taken the coupling of $\chi_1 \chi_2$ to W to be $O(1)$. The estimate (12) may be viewed as the maximum possible weak rate, as it supposes a high degree of coherence among nucleons. Indeed, the M_R^2 dependence suggests a coherent response of all the nucleons at characteristic momentum transfers of order $M_R v$, which might lead to an overestimate for large nuclei. One finds that a parton-level calculation of the width, which effectively ignores coherence effects completely, leads to a result that is about an order of magnitude smaller.

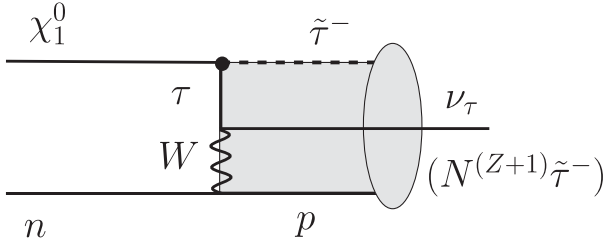Combining the two pieces together, (11) and (12), we arrive at an estimate of the capture rate given by

FIG. 2. Another charged-current capture process of χ_1^0 forming the bound state $(N\chi_2^-)$ and radiating a neutrino.

$$\sigma_{\rm res} v \sim \frac{G_F^2}{r_0^3 \times (500 \text{ keV})} \sim 10^{-38} \text{ cm}^2. \qquad (13)$$

For light nuclei where the outgoing electromagnetic width is on the order of 1 eV, the rate is expected to suffer from a further suppression by $\Gamma_{\rm em}/\Delta E$, i.e. another 5 to 6 orders of magnitude. We observe that the result is similar to the natural scale for this weak process, and thus does not appear to be significantly enhanced over more conventional elastic scattering of χ_1. However, we emphasize that this treatment of the nuclear aspects of the calculation has been very cursory, and a more complete treatment may lead to further enhancement (or suppression) factors.

The capture process above is quite distinct from a characteristic nuclear recoil event due to elastic WIMP scattering because of the large $\mathcal{O}(\text{MeV})$ scale energy release during the rapid decay of the excited bound state. Naively this makes it very difficult to observe in existing direct-detection experiments that have detectors tuned to detect a fiducial recoil energy of no more than 100 keV. For this reason, it is interesting to consider inelastic processes that would have a signature somewhat closer to a standard recoil event, and one possibility is shown in Fig. 2, for the special case in which χ_2 is a scalar partner of the τ as in the MSSM. The reverse process, a nuclear-assisted stau to neutralino decay was discussed previously in [13]. Such a capture process is analogous to (6), with the important distinction that the bound state de-excites through radiating neutrinos, which would escape the detector leaving just the recoil signal to be observed. However, this process is further suppressed and may not significantly impact the cross section purely through elastic scattering of χ_1.

2. Electromagnetic recombination

For models of Type B, which lack a weak charged current, alternative recombination channels are open if the charged states represent bosons and the neutral states fermions, or vice versa. In particular, the electromagnetic processes shown in Fig. 3 become possible. In the early universe, the abundance of charged states is then rapidly depleted via processes such as $\chi_2^\pm \to \chi_1^0 + e^\pm$, while in the current epoch χ_2^- may be regenerated via recombination in the form
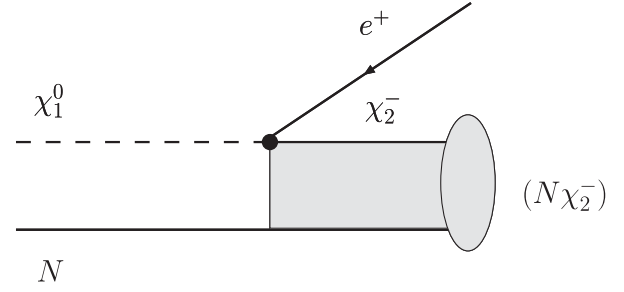

FIG. 3. The electromagnetic capture of χ_1^0 to form the bound state $(N\chi_2^-)$ and radiating a positron.

$$\chi_1^0 + N \to (N\chi_2^-) + e^+, \qquad (14)$$

which may occur given the appropriate kinematics (3) and (4), where N is again a generic nucleus, and $(N\chi_2^-)$ a stable bound state. Process (14) may appear to violate lepton number, and thus be suppressed. However, this need not be the case if either χ_1 or χ_2 carry lepton number as happens, e.g. in models where χ_1 is a sneutrino, and χ_2 a chargino, or when χ_1 is a neutralino, and χ_2 a charged slepton. In the widely discussed neutralino-stau scenario, the coupling to the positron would actually represent a flavor-changing process in the lepton sector, which may be somewhat suppressed but does not have to be vanishingly small.

Process (14) is an interesting variant of standard radiative recombination of a nucleus with an electron, in that Coulomb interactions are present in the final rather than the initial state, and since the positron has to tunnel out of the nucleus the cross section will be Gamow suppressed, particularly when the positron is nonrelativistic. To be concrete, we will consider the situation where χ_1 is a scalar and χ_2 a Dirac fermion, with the effective vertex

$$\mathcal{L} = g\chi_1 \bar{e}\chi_2^- + \text{H.c.} \qquad (15)$$

The free decay width Γ_{χ_2} of χ_2 in the early universe is then sufficiently fast to avoid problems with BBN provided $g^2 > 10^{-17}$ [14].

We estimate the capture rate by considering the overlap between the scattering and bound-state wavefunctions as in (8), i.e. $\sigma \simeq \Gamma_{\chi_1 \to \chi_2} |\langle \psi_N^{\rm scat} | \psi_N^{\rm bs} \rangle|^2 \times F_G$ where $\Gamma_{\chi_1 \to \chi_2}$ is the free decay width (for $\Delta m < 0$), and F_G is the Gamow factor associated with the interaction of the radiated positron with the nuclear potential. In the center-of-mass frame of χ_1 and N, the positron energy is $E_{e^+} \approx |E_b| + |E_{\rm kin}| - \Delta m + m_e \sim 5\text{–}15$ MeV, while the nuclear barrier is characteristically of $\mathcal{O}(Z\alpha/R_N)$, which may reach 20 MeV for large nuclei. In that case, the positron will have to tunnel out of the Coulomb barrier leading to a suppression in the cross section, i.e. $F_G \ll 1$. We will discuss this point further below.

In what follows, we consider the capture to the ground state that maximizes the kinetic energy of the outgoing

positron and thus minimizes the Gamow factors. The bound-state wavefunction can be taken as that of a 3D harmonic oscillator, with the inner non-Coulombic part of the nuclear potential given by $V(r) = (Z\alpha)/(2R_N) \times ((r/R_N)^2 - 3)$. Computing the overlap and converting to nuclear parameters, we obtain

$$\langle \sigma_{\text{rec}} v \rangle = \frac{g^2}{2\pi} \frac{(E_{e^+} + m_e)|p_{e^+}|}{M_\chi} R_N^3 \left(\frac{a_B}{R_N}\right)^{3/4} 8\pi^{3/2}$$
$$\times \exp(-(\mu v)^2 R_N^{3/2} a_B^{1/2}) \times F_G, \quad (16)$$

where $\mu = M_R(N, \chi)$ is the reduced mass of N and χ_1, $a_B = 1/(Z\alpha\mu)$ is the "Bohr radius," and R_N the nuclear radius.

If we suppress the detailed dependence on the bound-state wavefunction, then in the limit of a relativistic positron and assuming $a_B \sim R_N$, this reduces to

$$\langle \sigma_{\text{rec}} v \rangle \sim g^2 a_B^3 \frac{E_{e^+}^2}{M_\chi}, \quad (17)$$

which exhibits the inverse scaling of the inelastic cross section with the WIMP momentum. The most significant numerical correction to (17) arises from the fact that in practice $a_B \ll R_N$ for the nuclei of interest. The exponential factor in (16), associated with the ground state wavefunction, provides a correction of $\mathcal{O}(20\%)$ for large nuclei. Evaluating the estimate (17) for WIMP capture on light nuclei, such as carbon or nitrogen, we take $a_B \sim 1$ fm, and $E_{e^+} \sim 1$ MeV, to obtain the recombination rate

$$\langle \sigma_{\text{rec}} v \rangle_{\text{light nuclei}} \sim 10^{-34} \text{ cm}^2 \times \left(\frac{g^2}{4\pi\alpha_w}\right) \times \left(\frac{1 \text{ TeV}}{M_\chi}\right). \quad (18)$$

For heavier nuclei, the Gamow factor F_G can no longer be ignored and leads to significant suppression if the positron energy is well below the nuclear potential barrier. The present situation differs slightly from conventional nonrelativistic tunneling, where $F_G \sim e^{-G}$ with $G = \int_C dr|k| \sim 2\pi Z\alpha/v$, since for generic values of Δm the positron will be relativistic even for energies well below the barrier. Consequently, $Z\alpha/v$ will not be too large and we can ignore Coulomb resummation effects after tunneling has occurred and simply account for the tunneling factor. This we estimate as $G^{\text{rel}} = \text{Re} \int_C dr|k_b|$ using the relativistic momentum k_b under the (wide) nuclear potential, which leads to $G^{\text{rel}} \sim 2\pi Z\alpha E/k$ provided the positron is only mildly relativistic. The latter constraint is necessary to be in the "wide barrier" regime, which restricts the positron to have $\gamma \lesssim \mathcal{O}(5)$. For the values of Z relevant here, the cross section is still somewhat suppressed for generic γ factors of the emitted positron, but far less so than in the nonrelativistic regime. The impact is most significant for our maximal choice of Δm as the positron is then barely relativistic even with capture to the ground

state, and Coulomb effects may also become important. For a radiated positron with $\gamma \sim 1$–2, we find $F_G \sim 10^{-3}$–10^{-1}.

Inserting characteristic numerical scales for heavy nuclei into (16), taking $E_{e^+} \sim 10$ MeV and setting $F_G \sim 10^{-2}$, we can convert this into an effective cross section per nucleon, the characteristic quantity quoted in direct WIMP searches,

$$\sigma_{p,n} = \frac{M_R^2(p, \chi)}{M_R^2(N, \chi)} \frac{1}{A^2} \frac{\langle \sigma_{\text{rec}} v \rangle}{\langle v \rangle} \sim 10^{-39} \left(\frac{g^2}{4\pi\alpha_w}\right) \text{ cm}^2, \quad (19)$$

where M_R denotes the reduced mass, and which for $g^2/(4\pi) \sim \alpha_w$ is interestingly a few orders of magnitude above the current direct-detection constraint on elastic scattering, e.g. with Ge or Xe [4,5].

It is also worth noting that an additional inelastic channel is triple recombination with the use of a K-shell electron

$$\chi_1^0 + e^- + N \rightarrow (N\chi_2^-)^* \rightarrow (N\chi_2^-) + \gamma. \quad (20)$$

This process does not require tunneling through the potential barrier, but is suppressed by the small probability of finding the K-shell electron near the nucleus $(Z\alpha m_e R_N)^3$. This channel is expected to remain subdominant except for the case of very small E_{e^+}, i.e. with Δm very close to the capture threshold.

3. Constraints from BBN and heavy isotope searches

As is well known in the context of charged dark matter, there are stringent constraints on the presence of long-lived charged particles surviving from the big bang. It has recently been recognized that charged states with lifetimes longer than $\mathcal{O}(10^3$ s$)$ can also severely disrupt the successful predictions of BBN through the catalysis of interactions that, for example, would overproduce ^{6}Li and ^{9}Be by many orders of magnitude [14,15]. For this reason, we will simply assume here that the state $\chi_2^\pm$ has a sufficiently small lifetime to decay before nucleosynthesis:

$$\tau_{\chi_2} < \mathcal{O}(1) \text{ s.} \quad (21)$$

This can be a significant constraint on realizations of this scenario within the MSSM, where for example $\chi_2 = \tilde{\tau}_1$. In the absence of a flavor-changing stau-neutralino-electron coupling, the condition (21) is equivalent to requiring that the mass splitting between stau and neutralino is in excess of 100 MeV. However, even an extremely small coupling to the electron $g \sim O(10^{-8})$ would allow (21) to be satisfied without imposing significant constraints on Δm.

More direct constraints on the mass splitting Δm then arise from the terrestrial limits on the abundance of anomalous heavy isotopes. Here, the strongest constraints apply

to light elements, which are in some sense also the most reliable, arising from well-mixed gaseous or liquid media. The most stringent is the abundance constraint on heavy hydrogen (Heχ_2^-) in the form of heavy water [16], which however is only relevant here if the mass splitting is very small, less than 350 keV. There are also quite stringent constraints on ($C\chi_2^-$) (relative abundance $f_B < 10^{-14}$ per nucleon) and ($N\chi_2^-$) ($f_C < 10^{-20}$ per nucleon) [17].

To make use of these constraints, we connect the abundance of the parent nuclei that undergoes the capture process to that of the anomalously heavy daughter nucleus, a bound state with χ_2

$$\frac{n_{\text{daughter}}}{n_{\text{parent}}} = \langle \sigma_{\text{rec}} v \rangle n_{\text{DM}} \tau_{\text{exp}}. \qquad (22)$$

Here, τ_{exp} is the exposure time, and n_{DM} is some average dark matter density near the solar system, which we shall assume to be equal to the local WIMP number density $n_{\text{DM}} \sim 3 \times (\text{TeV}/m_\chi) \times 10^{-4} \text{ cm}^{-3}$. Specializing to the case of electromagnetic capture (14) on nitrogen, and using the limit on the anomalous carbon abundance $f_C < 10^{-20}$ per nucleon, we arrive at the following parameter constraint:

$$\left(\frac{1 \text{ TeV}}{m_\chi}\right)^2 \times \left(\frac{g^2}{4\pi\alpha_w^2}\right) < 10^{-9} \qquad (23)$$

for Type B models with $\Delta m \lesssim 3$ MeV.

In deriving this constraint we also assumed that carbon and nitrogen are almost equally abundant in nature, which is rather conservative: the carbon used in Ref. [17] presumably spent the majority of its time in the atmosphere, where nitrogen is significantly more abundant. Nonetheless, the constraint (23) is very significant, and, for example, implies the absence of a large coupling to the first generation of leptons in the neutralino-stau model. It clearly disfavors models of Type B with mass splittings below ~3 MeV, when the binding to nitrogen becomes energetically possible.

It is remarkable that the abundance constraints on anomalous isotopes of heavier elements are considerably weaker, while there is also greater uncertainty over whether the samples tested have a characteristic exposure time. Even interpreted in the most conservative manner, existing bounds, e.g. on heavy isotopes of Au [18], from (Hgχ_2^-), lead to relatively mild constraints that are well below the estimates for the cross sections obtained in this section. For this reason, the range quoted in (4) seems perfectly viable, although it would of course be interesting were further searches to be performed for heavy isotopes of elements with $Z > 10$.

Thus, we conclude that for models of Type B heavy isotope searches do impose a significant constraint but only in connection to binding with light elements and do not overly constrain the cross sections for Δm in the range (4). For models of Type A, where only weak charged currents

are allowed, we noted earlier that the nuclear binding energies do not permit capture by the light C, N, and O nuclei. Thus, since the capture rates are too slow to provide a significant constraint from searches for anomalous isotopes of heavy nuclei, the entire range of Δm including the smallest values down to an MeV is allowed. We note in passing that models with weak currents, $\Delta m \sim$ few MeV, and the χ_2^- lifetimes of ~2000 seconds alleviate the known problems with the primordial abundance of lithium [15].

4. Constraints from direct detection and from annihilation in the Sun

The existing limits on the elastic cross section per nucleon have reached impressive levels of order 10^{-43} cm^2 for a 100 GeV WIMP and further progress is anticipated in coming years. However, it is worth bearing in mind that the detection strategy in this case relies crucially on a characteristic recoil signal with energy of order 50 keV. In this context, it is interesting to consider if direct terrestrial dark matter searches are indeed sensitive to electromagnetic recombination processes of the type shown in (14). At first sight, it is unclear if the recoil during the capture process would be observed, due to the additional MeV-scale energy release associated with the subsequent positron annihilation in the detector. For this reason, we cannot immediately place constraints on the size of the coupling g from this source. Indeed, for the majority of detectors, events with such a large energy release would be identified with background and rejected.

Besides the direct-detection strategy that is currently focused on the search for WIMPs elastically scattering off nuclei, another traditional method to look for WIMPs is by searching for highly energetic neutrinos coming from WIMP annihilation in the solar core. The density of WIMPs in the solar core can be many orders of magnitude larger than n_{DM} due to gravitational capture through down scattering of WIMPs on nuclei inside the Sun, with an efficiency again controlled by the elastic scattering cross section. However, the pseudodegenerate WIMP regime can significantly modify the sensitivity of this method. Indeed, if Δm is less than about 10 MeV, sufficiently small to allow recombination with iron, a trapped WIMP will preferentially undergo iron capture rather than participate in direct annihilation with another WIMP. This stems from the fact that the density of iron in the solar core is many orders of magnitude larger than the density of WIMPs, and that the capture rate, (12) or (17), is quite comparable to or even larger than the annihilation rate. Simple estimates show that for a typical capture rate of $\langle \sigma v \rangle \sim 10^{-40}$ cm^2, a trapped WIMP recombines with an iron nucleus within 10^8 seconds, which is a much shorter time scale than for annihilation with another WIMP. Therefore, most of the WIMPs inside the Sun would be in bound states with iron and therefore shielded from annihilation by a large

Coulomb barrier, which would essentially cut off the flux of ultraenergetic neutrinos.

B. Resonant enhancement of electromagnetic form-factors

The characteristic momentum for an $\mathcal{O}(100 \text{ GeV})$ WIMP in the galactic halo scattering with nuclei is $m_\chi v \sim 100$ MeV, which implies that even if the mass splitting Δm between χ_1 and χ_2 is too large to allow for recombination with nuclei, significant resonant enhancements from the off-shell χ_2 are possible in the elastic scattering cross section. This is the scenario we will explore in this section, having in mind a splitting in the range (5), so that χ_2 appears off shell with virtuality $q^2 \sim m\Delta m$ in the scattering process as shown schematically in Fig. 4.

The precise nature of the amplitude will depend on the mechanism via which the WIMP interacts with the nucleons. The general $\chi_1 N \to \chi_1 N$ elastic scattering amplitude will involve many contributions, but since the WIMPs in the galactic halo are nonrelativistic, one can summarize the dominant contributions through the leading low-q^2 components of form factors of the WIMP with various currents to which the nucleon couples. Imposing minimal assumptions on χ_1, namely, the absence of tree-level vector couplings to γ, Z, and gluons, there are a number of form factors that may play a role in interactions with the nuclei in the detector. To keep the discussion manageable, we will restrict our attention to electromagnetic form factors, although one should bear in mind that much of the discussion will translate directly to couplings to other SM gauge bosons, and in various scenarios gluonic or weak form factors may be of more importance. It is also important to keep in mind that these form factors are often only well defined off shell and thus in practice this is just a convenient means of keeping track of the leading parts of the scattering amplitude in an appropriate kinematic regime.

Recall that for a *neutral* state χ of arbitrary (nonzero) spin, if we impose T invariance, we can expand the matrix element of the electromagnetic current [19] as follows in powers of the momentum transfer $q = p - p'$,[2]

$$\langle p'|J_\mu|p\rangle_{T\text{even}} = e\bar{\chi}_1(p')\left[-\frac{1}{6}r_D^2 q^2 \gamma_\mu \right.$$
$$+ a(q^2\eta_{\mu\nu} - q_\mu q_\nu)\gamma_5\gamma^\nu - i\mu\Sigma_{\mu\nu}q^\nu$$
$$\left. -\frac{1}{4}Q(Sq)^2\gamma_\mu + \cdots \right]\chi_1(p), \qquad (24)$$

---

[2]For particles with no vector coupling to the photon, the anapole moment a (and also the charge radius r_D^2) are strictly defined in a gauge invariant manner only through an external current. They determine a coupling to an off-shell photon, as is relevant to nucleon scattering, and one should also include the corresponding off-shell Z coupling for gauge invariance. We will ignore this issue as in practice for the momentum scales relevant here this effect is negligible.
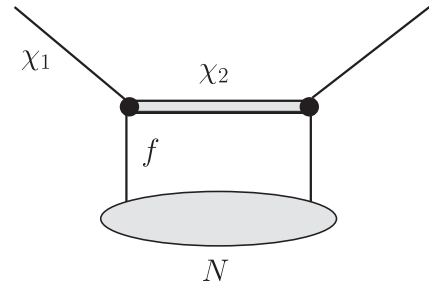

FIG. 4. A schematic representation of the resonant effects we will consider, where the shaded region denotes the interaction with the nucleus.

where $\Sigma_{\mu\nu}$ is the spinorial generator in the appropriate representation (i.e. $\sigma_{\mu\nu}$ for a spin-1/2 particle) and the corresponding spin 4-vector is $S^\mu = i\epsilon^{\mu\nu\rho\sigma}\Sigma_{\rho\sigma}q_\nu/(4m_\chi)$. The small-$q^2$ limit allows an expansion of the general form factors, and the terms on the first line contain the leading moments for a spin-1/2 particle—the charge radius r_D^2, the anapole moment a, which is P odd, and the magnetic dipole moment μ. For spin $S \geq 1$, there are also higher order multipoles and the second line contains the leading term in this limit, namely, the electric quadrupole moment Q. Many of these moments vanish given specific constraints on χ_1, while on the other hand if we relax the constraint on T invariance, there are in addition the electric dipole moment, the magnetic quadrupole moment, etc. but these will not be needed in what follows. However, particularly in the case of scalar particles, where (with the exception of an appropriately defined charge radius) these moments do not exist, we need to go beyond one-photon exchange and consider two-photon processes. We will again consider the small q^2 limit, in which case the leading constant parts of the form factors are the electric and magnetic susceptibilities η_E and η_B, and write the matrix element in position space

$$\mathcal{L}_{2\gamma} = \bar{\chi}F^{\alpha\mu}F_\mu^\beta\left[\frac{1}{2m_\chi^2}(\eta_E + \eta_B)\partial_\alpha\partial_\beta + \frac{1}{4}\eta_B\eta_{\alpha\beta}\right]\chi$$
$$+ \cdots \qquad (25)$$

Once again, if we allow for T violation, there are also mixed polarizabilities given by $\bar{\chi}F\tilde{F}\chi$, but we will drop these contributions in what follows.

At the nonrelativistic level, the relevant constant coefficients in the form factors are conveniently assembled into a Hamiltonian describing the interaction of the nonrelativistic WIMP with a slowly varying electromagnetic field. The general form of this Hamiltonian was considered previously in [20], and with the restriction to T invariance as above, takes the form

$$\mathcal{H}_{\mathrm{nr}}^{Teven} = -\mu \mathbf{B} \cdot \hat{\mathbf{S}} - a\mathbf{j} \cdot \hat{\mathbf{S}} - \frac{1}{4} Q_{ij} \partial_i E_j - \frac{1}{6} e r_D^2 \nabla \cdot \mathbf{E}$$
$$- \frac{1}{2} \eta_E E^2 - \frac{1}{2} \eta_B B^2 + \cdots \qquad (26)$$

with $Q_{ij} = Q(S_{(i}S_{j)} - (2/3)\delta_{ij}S(S+1))/(S(2S-1))$, in terms of the various multipole moments and susceptibilities above. The moments on the first line apply only for nonzero spin. If the particle is a Majorana fermion then furthermore $Q_{ij} = \mu = r_D^2 = 0$. The vanishing of the effective charge radius of a WIMP has significant consequences for WIMP-nucleus scattering, as r_D^2 contributes to spin-independent scattering enhanced by a very large nucleon coherence factor.

1. Resonant enhancement of loop processes

The cross sections for nonrelativistic scattering due to these moments are listed, for example, in [20]. The main point we wish to emphasize here is that even if these moments are zero at tree level, for pseudodegenerate WIMPs they naturally arise at loop level and indeed may be resonantly enhanced. The characteristic diagrams are shown in Fig. 5.

Let us focus first on models of Type B, where the WIMP χ_1 is a fermion with a nearly degenerate scalar partner χ_2, which appears resonantly in the scattering process. It is convenient to denote the "moments" generically as $M_{(d)}$, where d is the mass dimension of the corresponding operator, i.e. $d = 1$ for μ, $d = 2$ for r_D^2 and a, while $d = 3$ for η_E. An inspection of the loop diagrams indicates that the decoupling with the WIMP mass m_χ is generic, i.e. $M_{(d)} \sim 1/m_\chi$, given that the extra internal fermion state is light, consistent with resonance. This scaling can be straightforwardly understood by taking the nonrelativistic limit for the heavy fermions. This leads to a factor of $\sqrt{m_\chi}$ at each vertex, while the heavy scalar propagator leads to a factor of $1/m_\chi^2$. Moreover, in the resonant limit the loop is infrared divergent and thus the result is enhanced by the small mass scale Δm that sets the cutoff momentum scale, provided a light SM fermion with $m < \Delta m$ is available. Otherwise, one must replace Δm with the corresponding mass. E.g. in the case of a near neutralino-stau degeneracy, the infrared cutoff is fixed by m_τ. Thus, in the generic case, the moments scale in the following way in the resonant limit,

$$M_{(d)} \sim \frac{(\text{loop factor})}{m_\chi} \times \frac{1}{(\Delta m)^{d-1}}, \qquad (27)$$

where in fact the loop factor is also enhanced by π due to the infrared divergence. This formula is quoted for zero momentum transfer, and in practice for $q^2 \gtrsim m_\chi \Delta m > (m_\chi v)^2$ the electromagnetic form factors will decrease as a function of q^2. This scaling suggests that the enhancement is all the more impressive for the susceptibilities, but one must bear in mind that these amplitudes are $\mathcal{O}(\alpha^2)$ and more importantly involve a chirality flip and so require a double degeneracy of χ_1 with both χ_{2L} and χ_{2R}.

We have studied the contributions of these enhanced amplitudes to the scattering cross section and, although significant, the results in most cases do not reach the level of the current direct search bounds. For this reason, we will not present full formulae, but simply comment on a few interesting cases.

(i) *Charge radius*—The exception to this general conclusion involves the charge radius in any scenarios in which it is present at 1 loop, arising, e.g. from the diagram on the left of Fig. 5. A straightforward calculation leads to a result of the form $r_D^2 \sim \alpha/(m_\chi \Delta m)$, where the order-one normalization depends on the precise field content. For $\Delta m \sim \mathcal{O}(100 \text{ MeV})$ this leads to a (per nucleon) elastic scattering cross section [20] on ^{72}Ge, for example, of order 10^{-38} cm^2, which is several orders of magnitude above the current bound, and close to the tree-level Z-mediated cross section. Therefore, this can provide a rather stringent constraint on scenarios which, for example, avoid a coupling to the Z by choice of the $SU(2)_L$ representation.

(ii) *Anapole moment*—For chargeless Majorana fermions, there is a single nonzero multipole moment—the anapole moment [21]—and in the scenarios considered here it can be quite sizable, consistent with the above result for the charge radius. As an explicit example, one may again have in mind a near-degenerate neutralino and stau in the MSSM. However, as the anapole is an axial moment, in elastic scattering it couples to the spatial nucleon current and so the cross section is suppressed by $\mathcal{O}(v^2) \sim 10^{-6}$ [20], which pushes it somewhat below the current level of sensitivity.
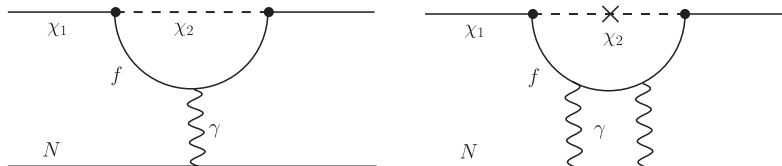

FIG. 5. A contribution to the scattering cross section of χ_1 with a nucleon, mediated by the multipole moments (on the left) or susceptibilities (on the right).
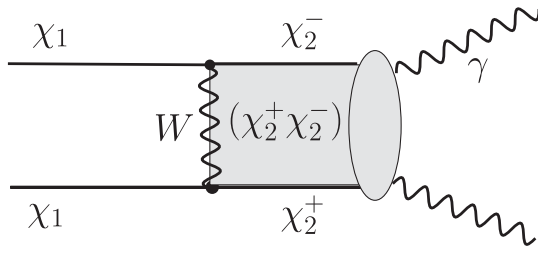
FIG. 6. A resonant contribution to the annihilation cross section of $\chi_1\chi_1 \to 2\gamma$, mediated by the $(\chi_2^+\chi_2^-)$ bound state.

(iii) *(Chromo)electric polarizability*—The velocity suppression that afflicts the cross section derived from the anapole moment does not apply to the polarizabilities. However, these operators require a chirality flip and so for resonant enhancement demand a double degeneracy of the left- and right-handed χ_2 states with χ_1. For weakly interacting states, e.g. $\tilde{\tau}_L$ and $\tilde{\tau}_R$ approximately degenerate with a neutralino (bino), the diagram on the right of Fig. 5 leads to $\chi_E \sim 10^{-2}\alpha^2/(m_\chi(\Delta m)^2)$ in accord with the scaling of (27). However, even in this degenerate regime, which resonantly enhances the amplitude, the cross section for nucleon scattering is negligible due to the small electric energy density in the nucleus $\langle N|E^2|N\rangle \sim 20$ MeV. However, there is an analogous chromoelectric polarizability η_G, replacing the photons in Fig. 5 with gluons, which is far more significant since $\langle N|\alpha_s G^2|N\rangle \sim m_N$. The corresponding contribution to the scattering cross section in the resonant regime can be as large as $\sigma_{(p,n)}(\eta_G) \sim 10^{-38}$ cm^2 for $\Delta m \sim \mathcal{O}(100$ MeV). This result is contained as an appropriate limit of the general 1-loop results of Drees and Nojiri [22] for the MSSM, but is of less interest there as it requires two squarks nearly degenerate with a neutralino LSP—a somewhat unusual spectrum.

For completeness, it is also worth remarking that if χ_1 and χ_2 are both fermionic as in models of Type A, the existence of a light boson would allow the amplitude to be independent of m_χ. This follows from the same nonrelativistic viewpoint as above, with the exception that now the heavy internal line is fermionic and so the factors of m_χ cancel out. In practice, this situation is of less interest as there are no light scalars available to provide the resonant diagram. However, this viewpoint does explain the corresponding results of [11,23], where the W-loop corrections to the scattering cross section were obtained and are explicitly independent of $m_{\rm DM}$ in the limit of large dark matter mass. This scaling is entirely consistent with the argument above, where the role of "light boson" is played by the W. We note in passing that in the model of [11] one should expect a significant difference in the scattering of Majorana- and Dirac-type dark matter. In the latter case,

the cross section is enhanced due to a significant contribution from the radiatively induced charge radius. (See also a related discussion in Ref. [24].)

IV. RESONANT WIMP ANNIHILATION

The pseudodegeneracy in the WIMP sector was motivated in part by the generic need for enhanced annihilation mechanisms for heavy WIMPs. Although we have not discussed this in detail here, the present scenario with nearly degenerate charged states clearly leads to an interesting coannihilation scenario for which the vector-mediated cross section is further enhanced by the required resummation of Coulomb effects, as discussed recently in [25]. Moreover, for small Δm this will tend to a resonant process mediated by the formation of a metastable $(\chi_2^+\chi_2^-)$ state.

We will develop this possibility further in this section considering the following process (see Fig. 6):

$$\chi_1 + \chi_1 \to (\chi_2^+\chi_2^-) \to \gamma + \gamma, \tag{28}$$

which leads to the primary emission of monochromatic gamma rays inside, e.g. the galactic center and can be effectively probed with existing and planned experiments. The Coulomb enhancements for this process in the near-threshold regime were studied previously in [26], but here we will focus on the resonant contribution.

The binding energy of the $(\chi_2^+\chi_2^-)$ bound state is $\alpha^2 m_\chi/4$. For TeV-scale WIMPs this is about 13 MeV, and thus can be quite comparable with the energy excess of $2\Delta m = 2(m_{\chi_2} - m_{\chi_1})$. In what follows, we analyze the regime where the $1S$ state for the $(\chi_2^+\chi_2^-)$ system is just above the energy threshold, and accessible via the kinetic energy of two WIMPs

$$0 \le 2\Delta m - \frac{m_\chi \alpha^2}{4} \lesssim m_\chi v_m^2, \tag{29}$$

where v_m is the maximal allowed WIMP velocity within the halo, on the order of 3×10^{-3}. This kinematic regime is a narrow strip on the $(m_\chi, \Delta m)$ plane

$$0 \le 1.5 \times \left(\frac{\Delta m}{10 \text{ MeV}}\right) \times \left(\frac{1 \text{ TeV}}{m_\chi}\right) - 1 \lesssim 0.7. \tag{30}$$

The spread in WIMP kinetic energy is on the order of an MeV and thus is much wider than both the capture width $\Gamma_{\rm in}$ and the outgoing width $\Gamma_{\rm out} = \Gamma_{2\gamma}$, so that the Breit-Wigner cross section is effectively a delta-function

$$\sigma = \frac{2\pi^2}{q_{\rm cm}^2} g_* \Gamma_{2\gamma} \delta(E - E_R), \tag{31}$$

where $E_R = 2\Delta m - \frac{m_\chi \alpha^2}{4}$. In Eq. (31), we took into account that the entrance width is much larger than $\Gamma_{2\gamma}$,

assuming no significant suppression of the WIMP-matter coupling, and therefore the rate is controlled by the smallest width. The annihilation rate into 2γ's of a bound state of two scalar particles is given by

$$\Gamma_{2\gamma} = \frac{\alpha^5 m_\chi}{4} \simeq 5 \times 10^{-6} \text{ MeV} \times \left(\frac{m_\chi}{1 \text{ TeV}}\right). \quad (32)$$

For the parapositronium-like annihilation of two fermionic χ_2's, the rate is twice larger, and therefore the width $\Gamma_{2\gamma}$ has rather minimal model dependence.

The capture rate is also straightforwardly calculable, but in contrast is significantly model dependent. For models of Type A, the scaling of the entrance width is

$$\Gamma_{\text{in}} \sim G_F^2 (\alpha m_\chi)^4 m_\chi \sqrt{\frac{8\Delta m}{m\alpha^2} - 1}. \quad (33)$$

This rate is heavily dependent on m_χ, but for most of the parameter space relevant for WIMPs is larger than (32). The same holds true for capture in models of Type B. It is interesting to note that for capture processes mediated by the standard model fermions, the P-wave capture may become significant. It is typically assumed that in the galactic environment, where the velocity of colliding WIMPs is on the order of 10^{-3}, p-wave annihilation is suppressed relative to the s wave by a v^2 factor, or 10^{-6}. In fact, the suppression factor is far less dramatic and for models with τ exchange it is $m_\chi^2 v^2 / m_\tau^2$, which is on the order of 10^{-2} for a 100 GeV WIMP. This allows for a sizable recombination rate of a neutralino pair with quantum numbers $L = 1$; $S = 1$; $J = 0$ into a bound state of two staus.

We are now ready to estimate the total annihilation rate by averaging (31) over the dark matter velocity distribution. If this distribution is approximated as Maxwellian, then the rate is simply given by

$$\langle \sigma v \rangle = \left(\frac{4\pi}{m_\chi T_{\text{eff}}}\right)^{3/2} g_* \Gamma_{2\gamma} \exp(-E_R/T_{\text{eff}}), \quad (34)$$

where T_{eff} must be identified with the effective "temperature" of the WIMP gas $\frac{3}{2} T_{\text{eff}} \equiv \langle E \rangle$. For the typical velocity of dark matter inside the halo, this effective temperature is approximately

$$T_{\text{eff}} \simeq 300 \text{ keV} \times \left(\frac{m_\chi}{1 \text{ TeV}}\right). \quad (35)$$

Inserting some characteristic numbers for a TeV mass WIMP, we obtain

$$\langle \sigma v \rangle \sim 5 \times 10^{-34} \text{ cm}^2 \times \left(\frac{\text{TeV}}{m_\chi}\right)^2 \exp(-E_R/T_{\text{eff}}), \quad (36)$$

which for Δm tuned so that E_R and T_{eff} are comparable,

can be very large and indeed several orders of magnitude larger than the characteristic mono-energetic γ signal [27] in generic MSSM scenarios; for example, similar enhancements were obtained in [26] via a resummation of Coulomb effects near the bound-state threshold. Such mono-energetic signals are far less sensitive to astrophysical backgrounds and so this regime is clearly promising for indirect searches. Note that such a large galactic annihilation rate (36) is not in contradiction with the required picobarn rate at freeze-out, as this resonant effect is less important in the latter regime due to the larger effective temperature, which leads to an $O(10^{-2}) - O(10^{-3})$ suppression in the thermal average. Finally, we should note that this form of enhancement can also affect the production of charged states in the annihilation process, although the rate would be more model dependent than for monochromatic photons.

V. CONCLUDING REMARKS

In this paper we have embarked on a study of some of the novel direct and indirect detection signatures of pseudodegenerate WIMP dark matter. Although this scenario lies within the characteristic WIMP sector of viable supersymmetric models and is thus far from exotic, the MeV-scale degeneracy in the spectrum does lead to some rather exotic signatures; most prominently through recombination processes with nuclei. In this concluding section, we will briefly summarize the results and comment on some further probes of this scenario.

Although we have refrained from performing detailed calculations of the capture rates, and our treatment of the nuclear physics was somewhat cursory, the estimates obtained in Sec. III are sufficient to draw some broad conclusions on direct detection to focus future studies. For models of Type A with a weak current, we found that searches for anomalous heavy isotopes in fact do not impose any significant constraints on the inelastic cross section with nuclei. However, our estimate for the rate was relatively small, at best comparable with the existing direct-detection limit on the elastic cross section. For models of Type B, the cross section is constrained by heavy isotope searches, but only in a rather small range arising from binding with light elements. For splittings in the range (4), the constraints are far weaker and our estimate for the capture rate suggests that simple models could produce rates a few orders of magnitude larger than the existing constraint on the elastic cross section. Clearly it would be very interesting to explore this possibility further and to determine whether conventional direct-detection experiments could probe capture processes of this type.

A significant motivating factor in the analysis of this scenario was that the novel direct-detection signatures may imply somewhat weaker constraints on the cross section with matter than would apply to generic WIMPs. This opens up the possibility of other interesting indirect detec-

tion possibilities in galactic astrophysics. A possible signal of this type, the observed excess of 511 keV photons from the galactic center, was considered recently within this framework [28] but seemingly required too large a cross section to be consistent with terrestrial heavy isotope bounds (in this case the mass splitting required was $\mathcal{O}(1\text{ MeV})$, which allows for binding with C, N, etc.). Here, we have explored several other signatures for indirect detection. We noted that the possibility of observing highly energetic neutrinos from DM annihilation in the Sun is significantly diminished within these scenarios, due to the far higher rate for the WIMPs to be captured by iron than to annihilate. If the mass splitting is tuned appropriately, we also observed that the annihilation rate to two mono-energetic γ's, e.g. in the galactic center, could be resonantly enhanced by several orders of magnitude through an intermediate $(\chi_2^+ \chi_2^-)$ bound state.

It would clearly be interesting to explore other indirect signatures beyond the traditional γ signal from WIMP annihilation, as may arise through the possibility for recombination processes in astrophysical environments, which may lead to new detection strategies, or alternatively more stringent constraints to be placed on these scenarios.

Finally, we should emphasize that we have focused here on direct and indirect detection of WIMPs in the galactic halo, and have avoided the question of direct production in colliders. This is primarily because direct production would at least in the near term be limited to the low mass regime, which contrasts somewhat with part of the motivation for the pseudodegenerate scenario, which was the need for heavier states. Nonetheless, this question is certainly of general interest and existing analyses of the collider physics of the WMAP strip region in the CMSSM should include the regime where the neutralino LSP is a pseudodegenerate WIMP. In this context, the charged $\chi_2^{\pm}$ states could be sufficiently long lived to either escape or become bound inside the detector.

ACKNOWLEDGMENTS

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