

## MIT bag model inspired partonic transverse momentum distribution for prompt photon production in $pp$ collisions

F. K. Diakonov, N. K. Kaplis, and X. N. Maintas

*Department of Physics, University of Athens, GR-15771 Athens, Greece*

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We consider the prompt photon production in  $pp$  collisions using, within the framework of perturbative QCD, a non-Gaussian distribution for the transverse momentum distribution of the partons inside the proton. Our description adopts the widely used in the literature factorization of the partonic momentum distribution into longitudinal and transverse components. It is argued that the non-Gaussian distribution of the intrinsic transverse momenta of the partons is dictated by the asymptotic freedom as well as the 3D confinement of the partons in the proton. To make this association more transparent we use the MIT bag model, which plainly incorporates both properties (asymptotic freedom, confinement), in order to determine in a simplified way the partonic transverse momentum distribution. A large set of data from six different experiments have been fitted with this simple description using as a single free parameter the mean partonic transverse momentum  $\langle k_T \rangle$ . Surprisingly enough, a perfect fit of the experimental data turns out to require  $\langle k_T \rangle$  values which are compatible with Heisenberg's uncertainty relation for the proton and decrease almost smoothly as a function of the scaled variable  $z = \frac{p_T}{\sqrt{s}}$ , where  $p_T$  is the transverse momentum of the final photon and  $\sqrt{s}$  is the beam energy in the center of mass frame. Our analysis indicates that asymptotic freedom and 3D confinement may influence significantly the form of the partonic transverse momentum distribution leaving an imprint on the  $pp \rightarrow \gamma + X$  cross section.

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The production of photons with large transverse momentum is an excellent probe of the dynamics in hard scattering processes [1,2]. In particular, the study of direct photon production possesses numerous and well-known advantages, both theoretical and experimental [2–9]. In the latter case the main advantage is that photons are easier to detect than jets. From the theoretical point of view, the main advantage is the simplicity of the process allowing for an accurate determination of the gluon distribution within the proton. In the lowest order ( $\mathcal{O}(\alpha_s)$ ) only two subprocesses,  $gq \rightarrow \gamma q$  (Compton) and  $q\bar{q} \rightarrow \gamma g$  (annihilation), contribute to high  $p_T$  photons. Their characteristic signature is the production of a photon isolated from the hadrons in the event, accompanied by a kinematically balancing high- $p_T$  jet appearing on the opposite site. In the next-to-leading order (NLO), the process associated with the production of a photon coming from the collinear fragmentation of a hard parton produced in a short-distance subprocess constitutes a background to the direct photon production of the same order in  $\alpha_s$  as the corresponding Born level terms [10] provided that the fragmentation scale is large enough. However, the contribution from fragmentation remains small (less than 10%) for fixed target experiments and becomes significant only in inclusive prompt photon production at higher collider energies [10]. Recently, there has been observed a systematic disagreement between theoretical NLO predictions [2,5,8,11–15] and experimental data [16,17] for prompt photon produc-

tion which cannot be globally improved modifying the gluon distribution function. Especially for fixed target experiments, NLO approximation shows a significant underestimation of the cross section for some of the measured data sets [11,16]. A similar discrepancy can be observed between NLO calculations and the experimental data of inclusive single neutral pion production:  $pp \rightarrow \pi^0 X$  in mostly the same experiments as in the photon case [18]. For the pion production the theoretical description is improved by taking into account certain large contributions to the partonic hard scattering cross section to all orders in perturbation theory using the technique of threshold resummation [18,19]. The same technique can be applied to the photon production by calculating the QCD resummation contribution to the partonic processes  $qg \rightarrow \gamma q$  and  $q\bar{q} \rightarrow \gamma g$  [20,21]. However, the result is a relatively small enhancement, not enough to compensate for the gap between the prompt photon data in fixed target experiments [11,21] and theoretical predictions. An additional improvement can be achieved by including in the theoretical treatment resummation effect to the fragmentation component succeeding in this way a good description of UA6 and R806  $pp$  data but still failing to reproduce the data of E706 [11]. The conclusion of this analysis is that resummed theoretical results present a residual shortfall in the description of photon and pion production data in fixed target experiments. One possible explanation of this effect is the existence of a nonperturbative contribution associ-

ated with intrinsic partonic transverse momentum  $k_T$  [4,11,22]. To incorporate this effect in the conventional perturbative QCD (pQCD), one assumes a factorization ansatz based on the statistical independence between longitudinal and transverse momenta of the partons. In this treatment the distribution of intrinsic transverse momentum  $g(k_T)$  is taken to have a Gaussian form as suggested by the early work of Dalitz [23]. Using the so-called  $k_T$  smearing, one can fit most of the experimental data [1,24,25]. However, the method suffers from two serious disadvantages concerning the values of the introduced nonperturbative parameter  $\langle k_T \rangle$  needed to fit the data: (a) they are incompatible with the estimation of the proton transverse radius according to Heisenberg's uncertainty relation and (b) they do not depend smoothly on physical parameters of the process as beam energy or the transverse momentum of the produced particle. These shortcomings remain even with the inclusion of higher order contributions in perturbation theory making the  $k_T$ -smearing approach unattractive from the theoretical point of view.

Within the framework of pQCD, an alternative scenario for the description of  $pp$  collisions has been recently proposed [26], incorporating the calculation of the partonic transverse momentum distribution using a phenomenological quark potential model introduced in the past to describe baryonic spectra [27,28]. The underlying idea in this treatment is to determine the influence of bound state effects on the shape of  $g(k_T)$ . In the calculation performed in [26],  $g(k_T)$  turns out to have a non-Gaussian profile with a characteristic tail. Furthermore, when applied to the description of the  $pp \rightarrow \pi^0 + X$  process, one succeeds to overcome the disadvantages of the Gaussian  $g(k_T)$  discussed above. However, there are still unsatisfactory issues in this treatment since a connection with first principles is missing and the asymptotic freedom property in the partonic dynamics is not taken into account.

In the present work we will argue that a non-Gaussian transverse momentum distribution with a slowly vanishing tail may originate from the fundamental properties of asymptotic freedom and 3D confinement of the partonic degrees of freedom in the proton. In addition, using the direct photon production in  $pp$  collisions, it will be revealed that, within the proposed scenario, the values of  $\langle k_T \rangle$ , needed for the description of all the available experimental data, are characterized by a remarkable smooth dependence on the scaled variable  $z = \frac{p_T}{\sqrt{s}}$  where  $p_T$  is the transverse momentum of the produced photon and  $\sqrt{s}$  the beam energy in the center of mass frame. The results of our analysis clearly support the initial assumption that part of the residual shortfall in the perturbative description of  $\gamma$  or  $\pi$  production in  $pp$  collisions is associated with the *transversal confinement* of the partons in the protons.

In the next-to-leading order (NLO) of perturbation theory, the differential cross section of the single photon production in  $pp$  collision can be written as

$$E_\gamma \frac{d^3\sigma}{d^3p} (pp \rightarrow \gamma + X) = K(p_T, \sqrt{s}) \sum_{abc} \int dx_a dx_b f_{a/p}(x_a, Q^2) f_{b/p}(x_b, Q^2) \times \frac{\hat{s}}{\pi} \frac{d\sigma}{d\hat{t}} (ab \rightarrow c\gamma) \delta(\hat{s} + \hat{t} + \hat{u}), \quad (1)$$

where  $f_{i/p}$  ( $i = a, b$ ) are the MRST2006 next to next to leading order longitudinal parton distribution functions (PDF) for the colliding partons  $a$  and  $b$  as a function of longitudinal momentum fraction  $x_i$  and factorization scale  $Q$  [29].  $\frac{d\sigma}{d\hat{t}}$  is the cross section for the partonic subprocesses as a function of the Mandelstam variables  $\hat{s}$ ,  $\hat{t}$ ,  $\hat{u}$  [1]. The higher order corrections in the partonic subprocesses are effectively included in (1) through the  $K$  factor, appearing in the right-hand side, which depends on the transverse momentum of the outgoing photon and the beam energy [30]. According to our treatment, we first attempt to describe experimental data using a minimal modification of the NLO pQCD introducing partonic transverse degrees of freedom through the replacement [1,22]:

$$dx_i f_{i/p}(x_i, Q^2) \rightarrow dx_i d^2k_{T,i} g(\mathbf{k}_{T,i}) f_{i/p}(x_i, Q^2) \quad (2)$$

in the PDF of the colliding partons ( $i = a, b$ ). To avoid singularities in the partonic subprocesses, we introduce a regularizing parton mass [25,31] with value close to the constituent quark mass  $m_q = 0.3$  GeV in the Mandelstam variables occurring in the denominator of the corresponding matrix elements [32]. Contrary to the usual treatment where  $g(\vec{k}_T)$  is taken to be a Gaussian, here we will use a different form inspired by the MIT bag model [33]. The main advantage of this model is that it incorporates the basic features of strongly interacting quark matter, namely, asymptotic freedom and 3D confinement in a very simple fashion. Despite this simplicity, a self-consistent approach to obtain partonic momentum distributions within the framework of the MIT bag model is technically very complicated [34] leading to unintegrated PDF which are in general incompatible with the factorization ansatz (2). In addition, it is not clear how to incorporate correctly such distribution functions in the pQCD scheme (1). However, since our approach is purely phenomenological, one can use the MIT bag model wave function for the ground state of a parton inside the proton:

$$\Psi(\mathbf{r}) = N \begin{pmatrix} j_0(Er) Y_{00} \left( \begin{smallmatrix} 1 \\ 0 \end{smallmatrix} \right) \\ -\frac{1}{\sqrt{3}} j_1(Er) \left( \begin{smallmatrix} -Y_{10} \\ \sqrt{2} Y_{11} \end{smallmatrix} \right) \end{pmatrix} \quad (3)$$

to obtain the partonic wave function  $\phi(|\vec{k}|)$  in momentum space through the Fourier transformation:

$$\begin{aligned}
\phi(|\mathbf{k}|) &= \int d^3r \exp[i\mathbf{k} \cdot \mathbf{r}] \Psi(\mathbf{r}) \\
&= \int d\cos\theta d\phi r^2 dr \exp[ikr \cos\theta] \Psi(\mathbf{r}) \\
&= \begin{pmatrix} 2\sqrt{\pi}\Phi_0(k) \\ 0 \\ -2i\sqrt{\pi}\Phi_1(k) \\ 0 \end{pmatrix}, \quad (4) \\
\Phi_0(k) &:= \frac{1}{k^2} \int_0^R dr j_0(Er) kr \sin kr \\
\Phi_1(k) &:= \frac{1}{k^2} \int_0^R dr j_1(Er) (kr \cos kr - \sin kr).
\end{aligned}$$

In (3)  $N$  is a normalization constant,  $j_0, j_1$  are spherical Bessel functions,  $Y_{00}, Y_{10}, Y_{11}$  are spherical harmonics, and  $E$  is the energy of the parton's ground state, while in (5)  $R$  is the radius parameter of the MIT bag model. From  $\phi(|\mathbf{k}|)$  it is straightforward to obtain a rough estimation of the partonic transverse momentum distribution through the projection:

$$\tilde{g}(|\mathbf{k}_T|) = 2\pi k_T \int_{-\infty}^{+\infty} dk_z \phi^\dagger(\sqrt{|\mathbf{k}_T|^2 + k_z^2}) \phi(\sqrt{|\mathbf{k}_T|^2 + k_z^2}). \quad (5)$$

The  $\langle k_T \rangle$  is then promoted to a free parameter by rescaling of the  $k_T$  axes, introducing the distribution:

$$g(k_T, \langle k_T \rangle) := \left( \frac{\langle \tilde{k}_T \rangle}{\langle k_T \rangle} \right)^2 \tilde{g}\left( k_T \cdot \frac{\langle \tilde{k}_T \rangle}{\langle k_T \rangle} \right) \quad (6)$$

with  $\langle \tilde{k}_T \rangle$  being the mean transverse momentum corresponding to  $\tilde{g}(\mathbf{k}_T)$  of Eq. (5) and  $\langle k_T \rangle$  being the fitting parameter corresponding to the mean transverse momentum of  $g$ . The form of the calculated distribution is presented in Fig. 1 for a particular choice of  $\langle k_T \rangle$ . At this point it should be noticed that the slowly vanishing tail of the distribution (5), which turns out to be crucial for the successful description of the experimental data, originates from the 3D character of the problem, since the involved Bessel functions are eigenfunctions of the 3D Laplacian, describing the motion of the partons inside the bag.

Using Eqs. (1) and (2), we describe the data of six different experiments using as a single fitting parameter the  $\langle k_T \rangle$  value. Within the MIT bag model such a variation can be justified as a dependence of the bag radius on  $p_T$  and  $\sqrt{s}$ . In particular, we investigate the single photon production data of the 3 CERN experiments R807 (ISR), UA6 (SPS), NA24 (SPS), as well as the FNAL experiments E704, E706 (Tevatron), and the PHENIX experiment at RHIC BNL [35]. In Fig. 2 we summarize all the considered experimental data, shown with symbols, appropriately scaled to improve the presentation. The dashed lines represent the theoretical pQCD predictions using the resummation technique according to [36]. The best fit results, using the improved pQCD with the non-Gaussian intrinsic

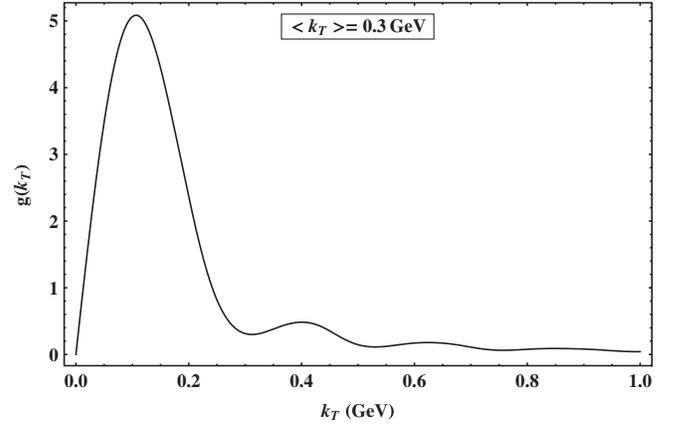


FIG. 1. The partonic transverse momentum distribution  $g(k_T)$  obtained using the MIT bag model.

transverse momentum distribution of Eqs. (5) and (6), are presented by crosses which are not easily distinguishable since they coincide with the experimental data. In all the performed calculations we use  $Q = \frac{p_T}{2}$ . It is clearly seen that with a suitable choice of  $\langle k_T \rangle$  an excellent description of the data is possible.

The significant improvement of this non-Gaussian treatment is that the optimization procedure leads to  $\langle k_T \rangle$  values which form a well defined, smooth, decreasing function of the scaled parameter  $z = \frac{p_T}{\sqrt{s}}$ . This property is illustrated in

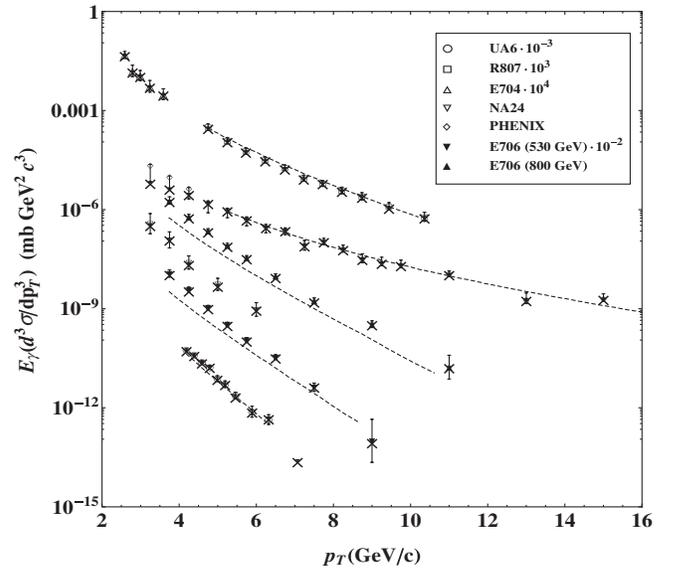


FIG. 2. The cross section of prompt photon production for six different experiments at various energies. The symbols indicate the experimental data, while the dashed line describes the theoretical results using the resummation technique. The crosses indicate the fit results using the non-Gaussian transverse momentum distribution of Fig. 1. They are not easily distinguishable since they coincide with the symbols presenting the experimental data.

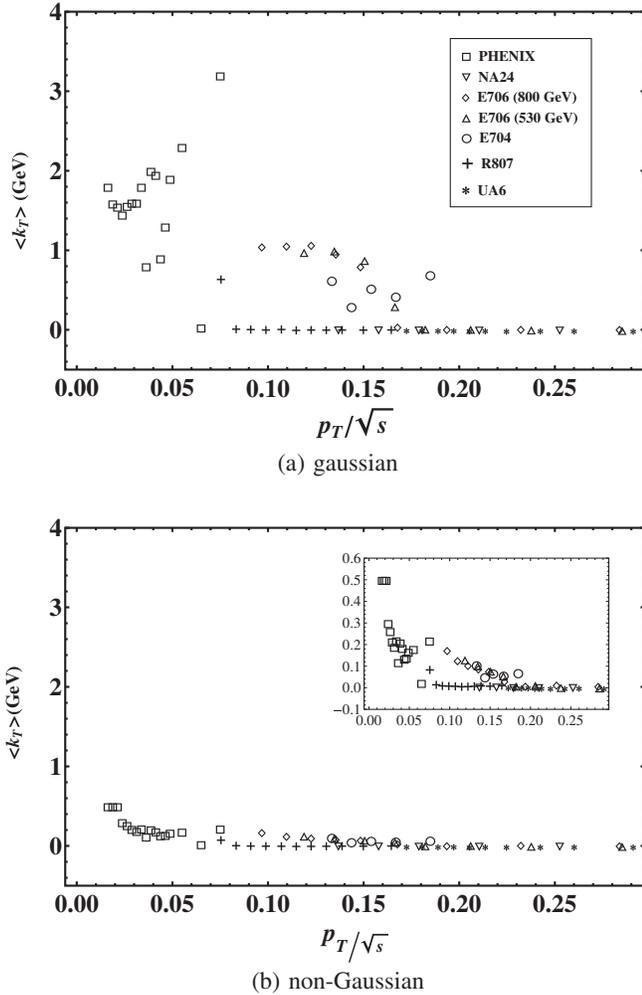


FIG. 3. The  $\langle k_T \rangle$  used in the description of six different experimental data sets for single photon production, referred in the text, (a) with a Gaussian and (b) with a non-Gaussian intrinsic transverse momentum distribution. The inset of Fig. 3(b) shows the dependence of  $\langle k_T \rangle$  on  $z$  in a refined scale.

Fig. 3 where the best fit values of  $\langle k_T \rangle$ , for the single gamma production in the experiments studied using a non-Gaussian  $k_T$  smearing [Fig. 3(b)], are compared with the corresponding results obtained using a Gaussian distribution [Fig. 3(a)]. It is observed that, in the non-Gaussian

case, for  $z \rightarrow 0$ , the resulting  $\langle k_T \rangle$  value is compatible with the transverse size of the proton according to Heisenberg's uncertainty relation. This behavior is welcome since in this limit the nonperturbative regime is entered and it is expected to trace geometrical characteristics of the proton through the considered process. In addition, for increasing  $z$ , a decrease in the contribution of the partonic transverse momenta is anticipated, in full agreement with the non-Gaussian result, since in this region the perturbative treatment becomes more and more accurate. On the contrary, as it can be seen in Fig. 3(a), the Gaussian ansatz leads to absurd behavior incompatible with physical intuition.

At this point it should be noticed that analogous calculations for  $\pi_0$  production in  $pp$  collisions have been performed leading to  $\langle k_T \rangle$  values with similar characteristics. In a more complete treatment, one should also calculate the cross sections for DIS, Drell-Yan, and  $p\bar{p}$  processes using  $\langle k_T \rangle$  values depending on  $z$  as dictated by the analysis of the  $\gamma$  production.

In conclusion, we have proposed a pQCD scheme for the description of the existing experimental data of prompt photon production in  $pp$  collisions using a non-Gaussian transverse momentum distribution for the partons inside the proton inspired by the MIT bag model. The mean transverse momentum of the partons is used as a free fitting parameter. It turns out that a perfect fit of all existing data is achieved using  $\langle k_T \rangle$  values which tend to become a smooth function of  $z = \frac{p_T}{\sqrt{s}}$ , where  $p_T$  is the transverse momentum of the outgoing photon and  $\sqrt{s}$  is the corresponding beam energy. In addition, the range of the required  $\langle k_T \rangle$  values is in accordance with the geometrical characteristics of the proton when nonperturbative effects are expected to be visible ( $z \rightarrow 0$ ). This surprisingly successful result suggests that the used partonic transverse momentum distribution captures essential characteristics (asymptotic freedom, 3D confinement) of the parton dynamics inside the proton. Thus, it looks quite promising the effort to put this purely phenomenological treatment on a more strict basis within the framework of pQCD as well as to extend it in order to describe  $pA$  and  $AA$  collisions by taking into account confinement characteristics of the involved partons. However, these challenging questions are left for a future work.

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