Chiral properties of baryon fields with flavor SU(3) symmetry

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We investigate chiral properties of local (nonderivative) fields of baryons consisting of three quarks with flavor $SU(3)$ symmetry. We construct explicitly independent local three-quark fields belonging to definite Lorentz and flavor representations. Chiral symmetry is spontaneously broken and therefore the baryon fields can have different chiral representations. Because of the color antisymmetric and spatially symmetric structure of the local three-quark fields, the allowed chiral representations are strongly correlated with the Lorentz group representations. We discuss some implications of the allowed chiral symmetry representations on physical quantities such as the axial coupling constants.

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I. INTRODUCTION

As the chiral symmetry of QCD is spontaneously broken, $\mathrm{SU}(N_f)_L \otimes \mathrm{SU}(N_f)_R \rightarrow \mathrm{SU}(N_f)_V$ (N_f being the number of flavors), the observed hadrons are classified by the residual symmetry group representations of $\mathrm{SU}(N_f)_V$. The full chiral symmetry may then conveniently be represented by its nonlinear realization and this broken symmetry plays a dynamical role in the presence of the Nambu-Goldstone bosons to dictate their interactions.

Yet, as pointed out by Weinberg [1], there are situations when it makes sense to consider algebraic aspects of chiral symmetry, i.e. the chiral multiplets of hadrons. Such hadrons may be classified in linear representations of the chiral symmetry group with some representation mixing. One such situation becomes realistic in the symmetry restored phase which is expected at high temperatures and/or densities [2]. If hadrons belong to certain representations of the chiral symmetry group, physical properties such as the axial coupling constants are determined by this symmetry. Therefore, the question as to what chiral representations, possibly with mixing, the hadrons belong to is of fundamental interest [3–5].

Another point of relevance is that the chiral representation can be used as a theoretical probe for the internal structure of hadrons. For instance, for a $\bar{q}q$ spin-one meson, the possible chiral representations are $(\mathbf{8}, \mathbf{1})$ and $(\mathbf{3}, \bar{\mathbf{3}})$ and their left-right conjugates for flavor octet mesons. As a matter of fact, for the multiquark hadrons, the allowed chiral representations can be more complicated/higher dimensional with increasing number of quarks and antiquarks. Hence the study of chiral representations may provide some hints to the structure of hadrons, extending possibly beyond the minimal constituent picture [6–11].

Motivated by these arguments, we have recently performed a complete classification of baryon fields constructed from three quarks in the local form with two light flavors (the so-called SU(2) sector) [12]. Such baryon fields are used as interpolators for the study of two-point correlation functions in the QCD sum rule approach and in the lattice QCD [13–18]. Strictly speaking, however, the chiral structure of an interpolator does not directly reflect that of the physical state when chiral symmetry is spontaneously broken. But the minimal configuration of three quarks provides at least a guide to the simplest expectations for baryons. Any deviation from such a simple structure may be an indication of higher Fock-space components, such as the multiquark ones [19].

Another reason for such a study of chiral classifications is related to the number of independent fields. In principle, the correlation functions, when computed exactly, should contain all information about the physical states. Practically, however, one must rely on some approximation, and it has been observed in previous studies, that the results may depend significantly on the choice of the interpolators, which are generally taken as linear combinations of the independent ones [5,15,20].

In this paper, we perform a complete classification of baryon fields written as local products (without derivatives) of three quarks according to chiral symmetry group $\mathrm{SU}(3)_L \otimes \mathrm{SU}(3)_R$. This is an extension of our previous work for the case of flavor SU(2) [12]. The SU(3) algebra introduces not only several technical complications, but also brings some physically relevant difference from the case of SU(2). For SU(2) the only allowed chiral representation for spin $1/2$ baryon is the fundamental one of $(\frac{1}{2}, \mathbf{0}) \oplus (\mathbf{0}, \frac{1}{2})$, while for SU(3), two representations $(\mathbf{8}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{8})$ and $(\mathbf{3}, \bar{\mathbf{3}}) \oplus (\bar{\mathbf{3}}, \mathbf{3})$ become possible. Indeed, they predict different F/D ratio for the axial couplings of octet

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baryons. Thus, here we attempt a systematic classification. We derive the chiral transformation rules for the baryon fields and do their classification, utilizing the Fierz transformations in order to implement the Pauli principle among the three quarks.

As in the previous paper [12], we first establish a classification under the ordinary (vector) flavor SU(3) symmetry, and then investigate the properties under the full chiral symmetry group. The method is based essentially on the tensor method for the SU(3) group representations, while the Fierz method for the Pauli principle associated with the structure in the color, flavor, and Lorentz (spin) spaces is utilized when establishing the independent fields. It turns out that for local three-quark fields, the Pauli principle puts a constraint on the structure of the Lorentz and chiral representations. This leads essentially to the same permutation symmetry structures as in the case of flavor SU(2) symmetry, with the one important difference being the existence of flavor singlets in the present case.

This paper is organized as follows. In Sec. II, we establish the independent local baryon interpolating fields, and investigate their flavor SU(3) symmetry properties. In Sec. III, we investigate the properties of the baryon fields under chiral symmetry transformations $SU(3)_L \otimes SU(3)_R$. We find that both flavor and chiral symmetry properties are related to the structure of the Lorentz group. Eventually, in Sec. IV, we find that this can be explained by the Pauli principle for the left- and right-handed quarks, which puts a constraint on permutation symmetry properties of three quarks. Some complicated formulas are shown in appendices.

II. FLAVOR SYMMETRIES OF THREE-QUARK BARYON FIELDS

Local fields for baryons consisting of three quarks can be generally written as

$$B(x) \sim \epsilon_{abc}(q_A^{aT}(x) C\Gamma_1 q_B^b(x))\Gamma_2 q_C^c(x), \qquad (1)$$

where a, b, c denote the color and A, B, C the flavor indices, $C = i\gamma_2\gamma_0$ is the charge-conjugation operator, $q_A(x) = (u(x), d(x), s(x))$ is the flavor triplet quark field at location x, and the superscript T represents the transpose of the Dirac indices only (the flavor and color SU(3) indices are *not* transposed). The antisymmetric tensor in color space ϵ_{abc}, ensures the baryons' being color singlets. For local fields, the space-time coordinate x does nothing with our studies, and we shall omit it. The matrices $\Gamma_{1,2}$ are Dirac matrices which describe the Lorentz structure. With a suitable choice of $\Gamma_{1,2}$ and taking a combination of indices of A, B and C, the baryon operators are defined so that they form an irreducible representation of the Lorentz and flavor groups, as we shall show in this section.

We employ the tensor formalism for flavor SU(3) *a la* Okubo [21–25] for the quark field q, although the explicit expressions in terms of *up*, *down* and *strange* quarks are usually employed in lattice QCD and QCD sum rule studies. We shall see that the tensor formulation simplifies the classification of baryons into flavor multiplets and leads to a straightforward, but lengthy derivation of the Fierz identities and the chiral transformations of baryon operators. This is in contrast to the $N_f = 2$ case where we explicitly included isospin/flavor into the $\Gamma_{1,2}$ matrices and thus produced isospin invariant/covariant objects [12].

A. Flavor $SU(3)_f$ decomposition for baryons

For the sake of notational completeness, we start with some definitions. The quarks of flavor SU(3) form either the contra-variant (**3**) or the covariant ($\bar{\mathbf{3}}$) fundamental representations. They are distinguished by either upper or lower index as

$$q^A \in q = \begin{pmatrix} u \\ d \\ s \end{pmatrix}, \qquad q_A \in q^\dagger = (u^*, d^*, s^*). \qquad (2)$$

The two conjugate fundamental representations transform under flavor SU(3) transformations as

$$q \to \exp\left(i\frac{\vec{\lambda}}{2}\vec{a}\right)q, \qquad q^\dagger \to q^\dagger \exp\left(-i\frac{\vec{\lambda}}{2}\vec{a}\right), \qquad (3)$$

where a_N $(N = 1, \cdots, 8)$ are the octet of $SU(3)_F$ group parameters and λ^N are the eight Gell-Mann matrices. Since the latter are Hermitian, we may replace the transposed matrices with the complex conjugate ones. The set of eight $\bar{\lambda}^N = -(\lambda^N)^T = -(\lambda^N)^*$ matrices form the generators of the irreducible $\bar{\mathbf{3}}$ representation.

Now for three quarks, we show flavor SU(3) irreducible decomposition $\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} = \mathbf{1} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{10}$ explicitly in terms of three quarks. It can be done by making suitable permutation symmetry representations of three-quark products $q_A q_B q_C$.

(1) The totally antisymmetric combination which forms the singlet,

$$\Psi_{[ABC]} = \mathcal{N}(q_A q_B q_C + q_B q_C q_A + q_C q_A q_B - q_B q_A q_C - q_A q_C q_B - q_C q_B q_A). \qquad (4)$$

The normalization constant here is $\mathcal{N} = 1/\sqrt{6}$. In the quark model this corresponds to $\Lambda(1405)$. In order to represent this totally antisymmetric combination, we can use the totally antisymmetric tensor ϵ^{ABC}. Then the flavor singlet baryon field Λ can be written as:

$$\Lambda \equiv \epsilon^{ABC}\epsilon_{abc}(q_A^{aT} C\Gamma_1 q_B^b)\Gamma_2 q_C^c. \qquad (5)$$

(2) The totally symmetric combination which forms the decuplet,

$$\Psi_{\{ABC\}} = \mathcal{N}(q_A q_B q_C + q_B q_C q_A + q_C q_A q_B + q_B q_A q_C + q_A q_C q_B + q_C q_B q_A). \quad (6)$$

The normalization constant depends on the set of quarks for baryons. For example, for $q_A, q_B, q_C = u, d, s$, $\mathcal{N} = 1/\sqrt{6}$, while it is $1/6$ for $q_A, q_B, q_C = u, u, u$. In order to represent this totally symmetric flavor structure, we introduce the totally symmetric tensor S_P^{ABC} ($P = 1, \cdots, 10$). Then the flavor decuplet baryon field Δ can be written as:

$$\Delta^P \equiv S_P^{ABC} \epsilon_{abc} (q_A^{aT} C \Gamma_1 q_B^b) \Gamma_2 q_C^c. \quad (7)$$

The nonzero components of S_P^{ABC} ($= 1$) are summarized in Table I. The rest of components are just zero, for instance, $S_1^{112} = 0$.

(3) The two mixed symmetry tensors of the ρ and λ types are defined by

$$\Psi^\rho_{[A\{B]C\}} = \mathcal{N}(2q_A q_B q_C - q_B q_C q_A - q_C q_A q_B - 2q_B q_A q_C + q_A q_C q_B + q_C q_B q_A),$$
$$\Psi^\lambda_{\{A[B\}C]} = \mathcal{N}(2q_A q_B q_C - q_B q_C q_A - q_C q_A q_B + 2q_B q_A q_C - q_A q_C q_B - q_C q_B q_A). \quad (8)$$

Here the two symbols in {} are first symmetrized and then the symbols in [] are antisymmetrized. The normalization constant depends again on the number of different kinds of terms. The correspondence of the octet fields of (8) and the physical ones can be made first by taking the following combinations

$$N^N_{8\rho} = \epsilon^{ABD} (\boldsymbol{\lambda}^N)_{DC} \Psi^\rho_{[A\{B]C\}},$$
$$N^N_{8\lambda} = \epsilon^{BCD} (\boldsymbol{\lambda}^N)_{DA} \Psi^\lambda_{\{A[B\}C]}, \quad (9)$$

where N is an octet index $N = 1, 2, \cdots, 8$. This kind of "double index" (DC for $N^N_{8\rho}$ and DA for $N^N_{8\lambda}$) notation for the baryon flavor has been used by Christos [26]. In our discussions, we shall use the following form for the flavor octet baryon field

$$N^N \equiv \epsilon^{ABD} (\boldsymbol{\lambda}^N)_{DC} \epsilon_{abc} (q_A^{aT} C \Gamma_1 q_B^b) \Gamma_2 q_C^c. \quad (10)$$

It is of the ρ type. But after using Fierz transformations to interchange the second and the third quarks, the transformed one contains λ type also, as we shall show in the following. The octet of physical baryon fields are then determined by

$$N^1 \pm iN^2 \sim \Sigma^\mp, \quad N^3 \sim \Sigma^0, \quad N^8 \sim \Lambda,$$
$$N^4 \pm iN^5 \sim \Xi^-, p, \quad N^6 \pm iN^7 \sim \Xi^0, n, \quad (11)$$

or put into the 3×3 baryon matrix

$$\mathfrak{N} = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda^8}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda^8}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda^8 \end{pmatrix}. \quad (12)$$

B. Counting the (in)dependent fields

In this section we investigate independent baryon fields for each Lorentz group representation which is formed by three quarks. The Clebsch-Gordan series for the irreducible decomposition of the direct product of three $(\frac{1}{2}, 0) \oplus (0, \frac{1}{2})$ representations of the Lorentz group (the three quark Dirac fields) is

$$((\tfrac{1}{2}, 0) \oplus (0, \tfrac{1}{2}))^3 \sim ((\tfrac{1}{2}, 0) \oplus (0, \tfrac{1}{2})) \oplus ((1, \tfrac{1}{2}) \oplus (\tfrac{1}{2}, 1)) \oplus ((\tfrac{3}{2}, 0) \oplus (0, \tfrac{3}{2})), \quad (13)$$

where we have ignored the different multiplicities of the representations on the right-hand side. The three representations $((\frac{1}{2}, 0) \oplus (0, \frac{1}{2}))$, $((1, \frac{1}{2}) \oplus (\frac{1}{2}, 1))$, $((\frac{3}{2}, 0) \oplus (0, \frac{3}{2}))$ describe the Dirac spinor field, the Rarita-Schwinger's vector-spinor field and the antisymmetric-tensor-spinor field, respectively. In order to establish independent fields we employ the Fierz transformations for the color, flavor, and Lorentz (spin) degrees of freedom, which is essentially equivalent to the Pauli principle for three quarks. Here we demonstrate the essential idea for the simplest case of the Dirac spinor, $(\frac{1}{2}, 0) \oplus (0, \frac{1}{2})$. Other cases are briefly explained in Appendices A and B.

1. Flavor singlet baryon

Let us start with writing down five baryon fields which contain a diquark formed by five sets of Dirac matrices, 1, γ_5, γ_μ, $\gamma_\mu\gamma_5$, and $\sigma_{\mu\nu}$,

TABLE I. Nonzero components of $S_P^{ABC}(= 1)$.

P	1	2	3	4	5	6	7	8	9	10
ABC	111	112	113	122	123	133	222	223	233	333
Baryons	Δ^{++}	Δ^+	Σ^{*+}	Δ^0	Σ^{*0}	Ξ^{*0}	Δ^-	Σ^{*-}	Ξ^{*-}	Ω^-

$$\begin{aligned}\Lambda_1 &= \epsilon_{abc}\epsilon^{ABC}(q_A^{aT}Cq_B^b)\gamma_5 q_C^c,\\ \Lambda_2 &= \epsilon_{abc}\epsilon^{ABC}(q_A^{aT}C\gamma_5 q_B^b)q_C^c,\\ \Lambda_3 &= \epsilon_{abc}\epsilon^{ABC}(q_A^{aT}C\gamma_\mu\gamma_5 q_B^b)\gamma^\mu q_C^c,\\ \Lambda_4 &= \epsilon_{abc}\epsilon^{ABC}(q_A^{aT}C\gamma_\mu q_B^b)\gamma^\mu\gamma_5 q_C^c,\\ \Lambda_5 &= \epsilon_{abc}\epsilon^{ABC}(q_A^{aT}C\sigma_{\mu\nu} q_B^b)\sigma_{\mu\nu}\gamma_5 q_C^c.\end{aligned} \tag{14}$$

Among these five fields, we can show that the fourth and fifth ones vanish, $\Lambda_{4,5} = 0$. This is due to the Pauli principle between the first two quarks, and can be verified, for instance, by taking the transpose of the diquark component and compare the resulting three-quark field with the original expressions [26]. The Pauli principle can also be used between the first and the third quarks, so we construct the primed fields where the second and the third quarks are interchanged, for instance,

$$\Lambda_1' = \epsilon_{abc}\epsilon^{ABC}(q_A^{aT}Cq_C^c)\gamma_5 q_B^b.$$

Now expressing Λ_i in terms of the Fierz transformed fields Λ_i', we find the following relations (see Appendix C),

$$\begin{aligned}\Lambda_1 &= -\tfrac{1}{4}\Lambda_1' - \tfrac{1}{4}\Lambda_2' - \tfrac{1}{4}\Lambda_3',\\ \Lambda_2 &= -\tfrac{1}{4}\Lambda_1' - \tfrac{1}{4}\Lambda_2' + \tfrac{1}{4}\Lambda_3',\\ \Lambda_3 &= -\Lambda_1' + \Lambda_2' + \tfrac{1}{2}\Lambda_3'.\end{aligned}$$

On the other hand, by changing the indices B, C and b, c, for instance,

$$\begin{aligned}\Lambda_1' &= \epsilon_{acb}\epsilon^{ACB}(q_A^{aT}Cq_B^b)\gamma_5 q_C^c\\ &= \epsilon_{abc}\epsilon^{ABC}(q_A^{aT}Cq_B^b)\gamma_5 q_C^c,\end{aligned}$$

we see that the primed fields are just the corresponding unprimed ones, $\Lambda_i' = \Lambda_i$. Consequently, we obtain three homogeneous linear equations whose rank is just one, and we find the following solution

$$\Lambda_3 = 4\Lambda_2 = -4\Lambda_1, \qquad \Lambda_4 = \Lambda_5 = 0.$$

We see that there is only one nonvanishing independent field, which in the quark model corresponds to the odd-parity $\Lambda(1405)$.

2. The flavor decuplet baryons

Among the five decuplet baryon fields formed by the five different γ-matrices, only two are nonzero:

$$\begin{aligned}\Delta_4^P &= \epsilon_{abc}S_P^{ABC}(q_A^{aT}C\gamma_\mu q_B^b)\gamma^\mu\gamma_5 q_C^c,\\ \Delta_5^P &= \epsilon_{abc}S_P^{ABC}(q_A^{aT}C\sigma_{\mu\nu} q_B^b)\sigma_{\mu\nu}\gamma_5 q_C^c.\end{aligned} \tag{15}$$

Performing the Fierz transformation and with the relation $\Delta_i^{P\prime} = -\Delta_i^P$ ($\epsilon_{acb}S_P^{ACB} = -\epsilon_{abc}S_P^{ABC}$), we find that there is only a trivial (null) solution to the homogeneous linear equations. Therefore, the Dirac baryon fields (fundamental representation of the Lorentz group) formed by three quarks cannot survive the flavor decuplet.

3. The flavor octet baryon fields

Let us start once again with five fields, which have three potentially nonzero ones

$$\begin{aligned}N_1^N &= \epsilon_{abc}\epsilon^{ABD}\lambda_{DC}^N(q_A^{aT}Cq_B^b)\gamma_5 q_C^c,\\ N_2^N &= \epsilon_{abc}\epsilon^{ABD}\lambda_{DC}^N(q_A^{aT}C\gamma_5 q_B^b)q_C^c,\\ N_3^N &= \epsilon_{abc}\epsilon^{ABD}\lambda_{DC}^N(q_A^{aT}C\gamma_\mu\gamma_5 q_B^b)\gamma^\mu q_C^c,\\ N_4^N &= \epsilon_{abc}\epsilon^{ABD}\lambda_{DC}^N(q_A^{aT}C\gamma_\mu q_B^b)\gamma^\mu\gamma_5 q_C^c = 0,\\ N_5^N &= \epsilon_{abc}\epsilon^{ABD}\lambda_{DC}^N(q_A^{aT}C\sigma_{\mu\nu} q_B^b)\sigma_{\mu\nu}\gamma_5 q_C^c = 0.\end{aligned} \tag{16}$$

These octet baryon fields have been studied in Refs. [13–15], where the independent ones are clarified. As before, we perform the Fierz rearrangement to obtain five equations with the primed fields, while $N_4^{N\prime}$ and $N_5^{N\prime}$ are not zero. For the first three equations, $N_{1,2,3}$ on the left-hand side should be expressed by the primed fields. To this end, we can use the Jacobi identity

$$\epsilon^{ABD}\lambda_{DC}^N + \epsilon^{BCD}\lambda_{DA}^N + \epsilon^{CAD}\lambda_{DB}^N = 0, \tag{17}$$

which can be used to relate the original fields N_i^N and primed ones $N_i^{N\prime}$, for instance,

$$(\epsilon^{ABD}\lambda_{DC}^N + \epsilon^{BCD}\lambda_{DA}^N + \epsilon^{CAD}\lambda_{DB}^N)(q_A^{aT}Cq_B^b)\gamma_5 q_C^c = 0,$$

from which we find

$$N_1^{N\prime} = -\tfrac{1}{2}N_1^N,$$

and the same relations for $N_{2,3}^N$. There are no relations between $N_{4,5}^N$ and $N_{4,5}^{N\prime}$. Altogether, we have five equations. The equations related to N_4^N and N_5^N are also necessary because the corresponding primed ones are not zero. They can be solved to obtain the following solutions:

$$\tfrac{2}{3}N_4^{N\prime} = N_3^N = N_1^N - N_2^N, \qquad N_5^{N\prime} = -3(N_1^N + N_2^N),$$

which indicates that there are two independent octet fields, for instance, N_1^N and N_2^N. Thus we have shown the same result just as in the two-flavor case [12]. In the following sections we shall show that the difference between the two fields N_1 and N_2 lies in their chiral properties: $N_1^N - N_2^N$ together with Λ belong to $(\bar{\mathbf{3}}, \mathbf{3}) \oplus (\mathbf{3}, \bar{\mathbf{3}})$, and the other $N_1^N + N_2^N$ belongs to $(\mathbf{8}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{8})$.

There are two ways to construct the octet baryon fields. One is done already as shown in Eqs. (16), whose flavor structure is the same as the ρ type baryon field $N_{8\rho}^N$ in Eqs. (9):

$$\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} \to (\mathbf{3} \otimes \mathbf{3}) \otimes \mathbf{3} \to \bar{\mathbf{3}} \otimes \mathbf{3} \to \mathbf{8}_\rho. \tag{18}$$

The other λ type baryon field $N_{8\lambda}^N$ is complicated when used straightforwardly:

$$\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} \to (\mathbf{3} \otimes \mathbf{3}) \otimes \mathbf{3} \to \mathbf{6} \otimes \mathbf{3} \to \mathbf{8}_\lambda. \tag{19}$$

Therefore, we use another way based on

$$\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} \rightarrow \mathbf{3} \otimes (\mathbf{3} \otimes \mathbf{3}) \rightarrow \mathbf{3} \otimes \bar{\mathbf{3}} \rightarrow \mathbf{8}'_\rho. \tag{20}$$

This contains partly $\mathbf{8}_\lambda$, and it is easily to verify that (18) and (20) compose a full description of octet baryon which is also fully described by using (18) and (19). The way $\mathbf{8}_\rho$ leads to octet fields N_i^N, and the other way $\mathbf{8}'_\rho$ leads to other five ones

$$\begin{aligned}
\tilde{N}_1^N &= \epsilon_{abc}\epsilon^{ACD}\lambda_{DB}^N(q_A^{aT}Cq_B^b)\gamma_5 q_C^c, \\
\tilde{N}_2^N &= \epsilon_{abc}\epsilon^{ACD}\lambda_{DB}^N(q_A^{aT}C\gamma_5 q_B^b)q_C^c, \\
\tilde{N}_3^N &= \epsilon_{abc}\epsilon^{ACD}\lambda_{DB}^N(q_A^{aT}C\gamma_\mu\gamma_5 q_B^b)\gamma^\mu q_C^c, \\
\tilde{N}_4^N &= \epsilon_{abc}\epsilon^{ACD}\lambda_{DB}^N(q_A^{aT}C\gamma_\mu q_B^b)\gamma^\mu\gamma_5 q_C^c, \\
\tilde{N}_5^N &= \epsilon_{abc}\epsilon^{ACD}\lambda_{DB}^N(q_A^{aT}C\sigma_{\mu\nu} q_B^b)\sigma_{\mu\nu}\gamma_5 q_C^c.
\end{aligned} \tag{21}$$

However, these fields can be related to the previous ones by changing the flavor and color indices B, C and b, c:

$$\tilde{N}_i^N = -N_i^{N\prime}.$$

In nearly all the cases, the octet baryon fields from the second way can be related to the ones from the first way. Therefore, we shall omit the discussion of the second octet. One exception which concerns the chiral representation $(\bar{\mathbf{3}}, \mathbf{3}) \otimes (\mathbf{6}, \mathbf{3})$ is discussed in Appendix D.

C. A short summary of independent baryon fields

Properties of spin 3/2 baryons fields expressed by the Rarita-Schwinger fields with one Lorentz index and those of the antisymmetric tensor-spinor fields with two Lorentz indices are discussed in Appendices A and B, respectively. Here we shall make a short summary of independent baryon fields for all cases constructed from three quarks. For simplicity, here we suppress the antisymmetric tensor in color space ϵ_{abc}, since it appears in all baryon fields in the same manner. Furthermore, it is convenient to introduce a "tilde-transposed" quark field $\tilde{q}$ as follows

$$\tilde{q} = q^T C\gamma_5. \tag{22}$$

which differs from the two-flavor definition in Ref. [12] by the absence of the flavor (G-parity) matrix.

As we have shown already, for Dirac fields without Lorentz index, there are one singlet field Λ and two octet fields N_1^N and N_2^N:

$$\begin{aligned}
\Lambda_1 &= \epsilon^{ABC}(\tilde{q}_A\gamma_5 q_B)\gamma_5 q_C, \\
N_1^N &= \epsilon^{ABD}\lambda_{DC}^N(\tilde{q}_A\gamma_5 q_B)\gamma_5 q_C, \\
N_2^N &= \epsilon^{ABD}\lambda_{DC}^N(\tilde{q}_A q_B)q_C.
\end{aligned}$$

For the Rarita-Schwinger fields with one Lorentz index, we would consider one singlet, three octet, and one decuplet fields:

$$\begin{aligned}
\Lambda_{1\mu} &= \epsilon^{ABC}(\tilde{q}_A\gamma_5 q_B)\gamma_\mu q_C, \\
N_{1\mu}^N &= \epsilon^{ABD}\lambda_{DC}^N(\tilde{q}_A\gamma_5 q_B)\gamma_\mu q_C, \\
N_{2\mu}^N &= \epsilon^{ABD}\lambda_{DC}^N(\tilde{q}_A q_B)\gamma_\mu\gamma_5 q_C, \\
N_{3\mu}^N &= -\epsilon^{ABD}\lambda_{DC}^N(\tilde{q}_A\gamma_\mu q_B)\gamma_5 q_C, \\
\Delta_{5\mu}^P &= -S_P^{ABC}(\tilde{q}_A\gamma_\mu\gamma_5 q_B)q_C.
\end{aligned}$$

However, we find that $\Lambda_{1\mu} = \gamma_\mu\gamma_5\Lambda$, $N_{1\mu}^N = \gamma_\mu\gamma_5 N_1^N$, and $N_{2\mu}^N = \gamma_\mu\gamma_5 N_2^N$. So, there are two nonvanishing independent fields: one octet field N_μ^N and one decuplet field Δ_μ. By using the projection operator:

$$P_{\mu\nu}^{3/2} = (g_{\mu\nu} - \tfrac{1}{4}\gamma_\mu\gamma_\nu), \tag{23}$$

they can be written as

$$\begin{aligned}
N_\mu^N &= P_{\mu\nu}^{3/2}N_{3\nu}^N \\
&= -(g_{\mu\nu} - \tfrac{1}{4}\gamma_\mu\gamma_\nu)\epsilon^{ABD}\lambda_{DC}^N(\tilde{q}_A\gamma_\mu q_B)\gamma_5 q_C \\
&= N_{3\mu}^N + \tfrac{1}{4}\gamma_\mu\gamma_5(N_1^N - N_2^N), \\
\Delta_\mu^P &= P_{\mu\nu}^{3/2}\Delta_{5\nu}^P = -(g_{\mu\nu} - \tfrac{1}{4}\gamma_\mu\gamma_\nu)S_P^{ABC}(\tilde{q}_A\gamma_\mu\gamma_5 q_B)q_C \\
&= \Delta_{5\mu}^P.
\end{aligned}$$

For tensor fields with two antisymmetric Lorentz indices, we would have one singlet, three octet, and two decuplet fields:

$$\begin{aligned}
\Lambda_{1\mu} &= \epsilon^{ABC}(\tilde{q}_A\gamma_5 q_B)\sigma_{\mu\nu}\gamma_5 q_C, \\
N_{3\mu\nu}^N &= -\epsilon^{ABD}\lambda_{DC}^N(\tilde{q}_A\gamma_\mu q_B)\gamma_\nu q_C + (\mu \leftrightarrow \nu), \\
N_{10\mu\nu}^N &= \epsilon^{ABD}\lambda_{DC}^N(\tilde{q}_A\gamma_5 q_B)\sigma_{\mu\nu}\gamma_5 q_C, \\
N_{11\mu\nu}^N &= \epsilon^{ABD}\lambda_{DC}^N(\tilde{q}_A q_B)\sigma_{\mu\nu}q_C, \\
\Delta_{2\mu\nu}^P &= -S_P^{ABC}(\tilde{q}_A\gamma_\mu\gamma_5 q_B)\gamma_\nu\gamma_5 q_C + (\mu \leftrightarrow \nu), \\
\Delta_{7\mu\nu}^P &= S_P^{ABC}(\tilde{q}_A\sigma_{\mu\nu}\gamma_5 q_B)\gamma_5 q_C.
\end{aligned}$$

But in this case, we can show that there is only one nonvanishing field $\Delta_{\mu\nu}$:

$$\begin{aligned}
\Delta_{\mu\nu}^P &= \Gamma^{\mu\nu\alpha\beta}\Delta_{7\mu\nu}^P = \Gamma^{\mu\nu\alpha\beta}S_P^{ABC}(\tilde{q}_A\sigma_{\mu\nu}\gamma_5 q_B)\gamma_5 q_C \\
&= \Delta_{7\mu\nu}^P - \frac{i}{2}\gamma_\mu\gamma_5\Delta_{5\nu}^P + \frac{i}{2}\gamma_\nu\gamma_5\Delta_{5\mu}^P,
\end{aligned}$$

where

$$\begin{aligned}
\Gamma^{\mu\nu\alpha\beta} = (&g^{\mu\alpha}g^{\nu\beta} - \tfrac{1}{2}g^{\nu\beta}\gamma^\mu\gamma^\alpha + \tfrac{1}{2}g^{\mu\beta}\gamma^\nu\gamma^\alpha \\
&+ \tfrac{1}{6}\sigma^{\mu\nu}\sigma^{\alpha\beta}).
\end{aligned} \tag{24}$$

III. CHIRAL TRANSFORMATIONS

In this section, we establish the chiral transformation properties of the baryon fields which we have obtained in the previous section. Technically, this leads to somewhat complicated algebraic results. However, the final result will be understood by making the left- and right-handed decomposition, which we shall perform in the next section.

Let us start with the chiral transformation properties of quarks which are given by the following equations:

$$\begin{aligned} \mathbf{U(1)_V}&: q \to \exp\left(i\frac{\lambda^0}{2}a_0\right)q = q + \delta q, \\ \mathbf{SU(3)_V}&: q \to \exp\left(i\frac{\vec{\lambda}}{2}\cdot\vec{a}\right)q = q + \delta^{\vec{a}}q, \\ \mathbf{U(1)_A}&: q \to \exp\left(i\gamma_5\frac{\lambda^0}{2}b_0\right)q = q + \delta_5 q, \\ \mathbf{SU(3)_A}&: q \to \exp\left(i\gamma_5\frac{\vec{\lambda}}{2}\cdot\vec{b}\right)q = q + \delta_5^{\vec{b}}q, \end{aligned} \tag{25}$$

where $\lambda^0 = \sqrt{2/3}\mathbf{1}$, $\vec{\lambda}$ are the eight Gell-Mann matrices and $\mathbf{1}$ is a 3×3 unit matrix. Here a^0 is an infinitesimal parameter for the $U(1)_V$ transformation, $\vec{a}$ the octet of $SU(3)_V$ group parameters, b^0 an infinitesimal parameter for the $U(1)_A$ transformation, and $\vec{b}$ the octet of the chiral transformations.

The $U(1)_V$ chiral transformation is trivial which picks up a phase factor proportional to the baryon number. The $U(1)_A$ chiral transformation is slightly less trivial, and the baryon fields are transformed as

$$\begin{aligned} \delta_5\Lambda &= -i\gamma_5\sqrt{\tfrac{1}{6}}b^0\Lambda, \\ \delta_5(N_1^N - N_2^N) &= -i\gamma_5\sqrt{\tfrac{1}{6}}b^0(N_1^N - N_2^N), \\ \delta_5(N_1^N + N_2^N) &= i\gamma_5\sqrt{\tfrac{3}{2}}b^0(N_1^N + N_2^N), \\ \delta_5 N_\mu^N &= i\gamma_5\sqrt{\tfrac{1}{6}}b^0 N_\mu^N, \\ \delta_5\Delta_\mu^P &= i\gamma_5\sqrt{\tfrac{1}{6}}b^0\Delta_\mu^P, \\ \delta_5\Delta_{\mu\nu}^P &= i\gamma_5\sqrt{\tfrac{3}{2}}b^0\Delta_{\mu\nu}^P. \end{aligned} \tag{26}$$

We note that the combinations of $N_1^N \pm N_2^N$ form different representations.

Under the vector chiral transformation, the baryon fields are transformed as

$$\begin{aligned} &\delta^{\vec{a}}\Lambda = 0, \qquad \delta^{\vec{a}}N_1^N = -a^M f^{NMO}N_1^O, \\ &\delta^{\vec{a}}N_2^N = -a^M f^{NMO}N_2^O, \qquad \delta^{\vec{a}}N_\mu^N = -a^M f^{NMO}N_\mu^N, \\ &\delta^{\vec{a}}\Delta_\mu^P = \frac{3i}{2}a^M g_7^{PMQ}\Delta_\mu^Q, \qquad \delta^{\vec{a}}\Delta_{\mu\nu}^P = \frac{3i}{2}a^M g_7^{PMQ}\Delta_{\mu\nu}^Q, \end{aligned} \tag{27}$$

where f^{ABC} is the standard antisymmetric structure constant of SU(3), and g_7^{ABC} is defined in Table II. Equations (27) show nothing but the flavor charge of the baryons. For example, we can show explicitly:

$$\delta^{a3}p = +\frac{i}{2}a_3 p, \qquad \delta^{a3}n = -\frac{i}{2}a_3 n, \qquad \delta^{a3}\Delta^{++} = \frac{3i}{2}a_3\Delta^{++}\cdots$$

The transformation rule under the axial-vector chiral transformations are rather complicated as they are no longer conserved and reflect the internal structure of baryons. To start with, we have the axial transformation of the three-quark baryon fields such as

$$\begin{aligned} \delta_5^{\vec{b}}\Lambda = \epsilon_{abc}\epsilon^{ABC}[&(q_A^{aT}Cq_B^b)\gamma_5(\delta_5^{\vec{b}}q_C^c) \\ &+ (q_A^{aT}C(\delta_5^{\vec{b}}q_B^b))\gamma_5 q_C^c + ((\delta_5^{\vec{b}}q_A^{aT})Cq_B^b)\gamma_5 q_C^c]. \end{aligned}$$

The calculation is complicated, but rather straightforward. Here, we show therefore the final result of the axial transformation:

$$\begin{aligned} \delta_5^{\vec{b}}\Lambda &= \frac{i}{2}\gamma_5 b^N(N_1^N - N_2^N), \\ \delta_5^{\vec{b}}(N_1^N - N_2^N) &= \frac{4i}{3}\gamma_5 b^N\Lambda + i\gamma_5 b^M d^{NMO}(N_1^O - N_2^O), \\ \delta_5^{\vec{b}}(N_1^N + N_2^N) &= -\gamma_5 b^M f^{NMO}(N_1^O + N_2^O), \\ \delta_5^{\vec{b}}N_\mu^N &= i\gamma_5 b^M\left(d^{MNO} - \frac{2i}{3}f^{MNO}\right)N_\mu^O \\ &\quad + i\gamma_5 b^M g_3^{MNP}\Delta_\mu^P, \\ \delta_5^{\vec{b}}\Delta_\mu^P &= -2i\gamma_5 b^M g_5^{PMO}N_\mu^O + \frac{i}{2}\gamma_5 b^M g_7^{PMQ}\Delta_\mu^Q, \\ \delta_5^{\vec{b}}\Delta_{\mu\nu}^P &= \frac{3i}{2}\gamma_5 b^M g_7^{PMQ}\Delta_{\mu\nu}^Q. \end{aligned} \tag{28}$$

The coefficients d^{ABC} are the standard symmetric structure constants of SU(3). For completeness, we show the following equation which define the d and f coefficients

$$\begin{aligned} \lambda_{AB}^N\lambda_{BC}^M = (\lambda^N\lambda^M)_{AC} &= \tfrac{1}{2}\{\lambda^N, \lambda^M\}_{AC} + \tfrac{1}{2}[\lambda^N, \lambda^M]_{AC} \\ &= \tfrac{2}{3}\delta^{NM}\delta_{AC} + (d^{NMO} + if^{NMO})\lambda_{AC}^O. \end{aligned} \tag{29}$$

Furthermore, the following formulas define the coefficients g_3, g_5 and g_7, which are proved by using MATHEMATICA, a software good at matrix calculation:

$$\begin{aligned} \epsilon^{ADE}\lambda_{DB}^N\lambda_{EC}^M &= g_1^{NMO}\epsilon^{ABD}\lambda_{DC}^O + g_2^{NMO}\epsilon^{ACD}\lambda_{DB}^O \\ &\quad + g_3^{NMP}S_P^{ABC} + g_4^{NM}\epsilon^{ABC}, \\ S_Q^{ABD}\lambda_{DC}^M &= g_5^{QMO}\epsilon^{ABD}\lambda_{DC}^O + g_6^{QMO}\epsilon^{ACD}\lambda_{DB}^O \\ &\quad + g_7^{QMP}S_P^{ABC} + g_8^{QM}\epsilon^{ABC}, \end{aligned} \tag{30}$$

where indices $A \sim E$ take values 1, 2, and 3, N, M and O $1, \cdots, 8$, and P and Q $1, \cdots, 10$. The coefficients g_3, g_5, and g_7 are listed in Table II, where we use "0" instead of "10." Other coefficients can be related to d, f, g_3, g_5, and g_7:

TABLE II. g-coefficients defined by Eqs. (30).

g_3	133, 138, 144, 146, 254, 256, 272, 279, 439, 463, 468, 573, 578, 612, 619, 636					$-1/3$
	162, 169, 313, 318, 349, 366, 414, 416, 524, 526, 643, 648, 722, 729, 753, 758					$1/3$
	154, 179, 215, 233, 246, 269, 328, 359, 376, 424, 455, 478, 516, 563, 622, 658, 712, 743, 765					$-i/3$
	125, 156, 172, 238, 244, 262, 323, 426, 473, 514, 539, 545, 568, 629, 653, 675, 719, 736, 748					$i/3$
	183, 686, 818, 835, 849	$-1/\sqrt{3}$	167, 251, 277, 411, 570, 640	-1	342, 364	$-2/3$
	188, 385, 489, 813, 866	$1/\sqrt{3}$	141, 460, 521, 617, 727, 750	1	432, 634	$2/3$
	283, 288, 589, 876	$-i/\sqrt{3}$	177, 421, 470, 511, 560, 627	$-i$	352, 374	$-2i/3$
	786, 823, 828, 859	$i/\sqrt{3}$	151, 241, 267, 650, 717, 740	i	532, 734	$2i/3$

g_5	125, 141, 227, 261, 313, 346, 357, 414, 425, 614, 625	$1/6$	318, 668, 881, 984	$1/2\sqrt{3}$
	663, 716, 727, 813, 846, 857, 927, 943, 961, 057, 064		381, 686, 818, 948	$-1/2\sqrt{3}$
	114, 152, 216, 272, 331, 364, 375, 441, 452, 636, 641	$-1/6$	382, 678, 882, 985	$i/2\sqrt{3}$
	652, 761, 772, 831, 864, 875, 916, 934, 972, 046, 075		328, 687, 828, 958	$-i/2\sqrt{3}$
	115, 124, 217, 226, 332, 347, 365, 424, 451, 615, 642	$i/6$	234, 436	$1/3$
	673, 726, 771, 823, 856, 874, 953, 962, 971, 065, 074		243, 463	$-1/3$
	142, 151, 262, 271, 323, 356, 374, 415, 442, 624, 637	$-i/6$	253, 473, 512, 554, 567	$i/3$
	651, 717, 762, 832, 847, 865, 917, 926, 935, 047, 056		235, 437, 521, 545, 576	$-i/3$
	583	$1/\sqrt{3}$	538	$-1/\sqrt{3}$

g_7	112, 143, 232, 245, 263, 315, 362, 448, 465, 619	$1/3$	214, 333, 346, 412, 513, 518	$2/3$
	636, 665, 714, 768, 815, 844, 916, 945, 046, 069		542, 549, 564, 566, 643, 869, 968	
	434, 939	$-1/3$	838	$-2/3$
	372, 675, 724, 825, 854, 926, 955, 056, 079	$i/3$	422, 523, 552, 574, 653, 978	$2i/3$
	122, 153, 255, 273, 325, 458, 475, 629, 778	$-i/3$	224, 356, 528, 559, 576, 879	$-2i/3$
	131, 211, 341, 417, 640, 867, 960	1	181, 282, 484, 787	$1/\sqrt{3}$
	737	-1	686, 989	$-1/\sqrt{3}$
	221, 351, 877	i	080	$-2/\sqrt{3}$
	427, 650, 970	$-i$		

$$
\begin{aligned}
g_1^{MNO} &= -d^{MNO} - \frac{i}{3} f^{MNO}, \\
g_2^{MNO} &= d^{MNO} - \frac{i}{3} f^{MNO}, \\
g_4^{MN} &= -\frac{1}{3}\delta^{MN}, \\
g_6^{QMO} &= -2 g_5^{QMO}, \\
g_8^{MN} &= 0.
\end{aligned}
\tag{31}
$$

Let us explain Eqs. (30) a bit more. The quantities on the left hand side have three indices A, B, and C, and therefore, they are regarded as direct products of three fundamental representations of SU(3): $\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3}$. They can be decomposed into irreducible components by applying the four kinds of operators: ϵ_{ABC}, $\epsilon^{ABD}\lambda^O_{DC}$, $\epsilon^{ACD}\lambda^O_{DB}$, and S_P^{ABC}, which correspond to **1**, **8**, **8**, and **10** of SU(3), respectively.

Equations (26) and (28) imply that Λ and $N_1^N - N_2^N$ are together combined into one chiral multiplet, and N_μ^N and Δ_μ^P are together combined into another chiral multiplet. While $N_1^N + N_2^N$ and $\Delta_{\mu\nu}^P$ are transformed into themselves under chiral transformation. In our following discussion, we will find that Λ and $N_1^N - N_2^N$ belong to the chiral representation $(\bar{\mathbf{3}}, \mathbf{3}) \oplus (\mathbf{3}, \bar{\mathbf{3}})$, $N_1^N + N_2^N$ belongs to the chiral representation $(\mathbf{8}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{8})$, N_μ^N and Δ_μ^P belong to the chiral representation $(\mathbf{6}, \mathbf{3}) \oplus (\mathbf{3}, \mathbf{6})$, and $\Delta_{\mu\nu}^P$ belongs to the chiral representation $(\mathbf{10}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{10})$. We show several examples of the axial-vector chiral transformation:

$$\delta_5^{b3} p_- = \frac{i}{2}\gamma_5 b_3 p_-, \qquad \delta_5^{b3} p_+ = \frac{i}{2}\gamma_5 b_3 p_+,$$
$$\delta_5^{b3} p_\mu = \frac{5i}{6}\gamma_5 b_3 p_\mu - \frac{4i}{3}\gamma_5 b_3 \Delta_\mu^+,$$

where p_- belongs to the octet baryon fields $N_1^N - N_2^N$, p_+ belongs to $N_1^N + N_2^N$, and p_μ belongs to N_μ^N [see Eqs. (11)].

IV. CHIRAL MULTIPLETS/REPRESENTATIONS

So far, we have performed classifications without explicitly taking into account the left- and right-handed components of the quark fields. However, it does not require great imagination to see that the chiral properties are also conveniently studied in that language, since chiral symmetry is defined as the symmetries upon each chiral field. Hence, we define the left- and right-handed (chiral or Weyl representation) quark fields as

$$L \equiv q_L = \frac{1-\gamma_5}{2} q, \quad \text{and} \quad R \equiv q_R = \frac{1+\gamma_5}{2} q. \tag{32}$$

They form the fundamental representations of both the Lorentz group and the chiral group,

$$L\text{: Lorentz: } \left(\frac{1}{2}, 0\right), \qquad \text{Chiral: } (3, 1),$$
$$R\text{: Lorentz: } \left(0, \frac{1}{2}\right), \qquad \text{Chiral: } (1, 3).$$

It is convenient first to note that γ-matrices are classified into two categories; chiral-even and chiral-odd classes. The chiral-even γ-matrices survive forming diquarks with identical chiralities, while the chiral-odd ones form diquarks from quarks with opposite chiralities. The chiral-even and -odd γ-matrices are

$$\text{chiral-even: } 1, \gamma_5, \sigma_{\mu\nu}, \qquad \text{chiral-odd: } \gamma_\mu, \gamma_\mu\gamma_5.$$

Therefore, we have six nonvanishing diquarks in the chiral representations,

$$\left.\begin{array}{r} L^T CL = -L^T C\gamma_5 L \\ R^T CR = +R^T C\gamma_5 R \end{array}\right\} \quad (0,0)\oplus(0,0), \quad (\bar{\mathbf{3}},\mathbf{1})\oplus(\mathbf{1},\bar{\mathbf{3}}),$$
$$\left.\begin{array}{r} L^T C\gamma_\mu\gamma_5 R = +L^T C\gamma_\mu R \\ R^T C\gamma_\mu\gamma_5 L = -R^T C\gamma_\mu L \end{array}\right\} \quad (\tfrac{1}{2},\tfrac{1}{2})\oplus(\tfrac{1}{2},\tfrac{1}{2}), \quad (\mathbf{3},\mathbf{3})\oplus(\mathbf{3},\mathbf{3}),$$
$$\left.\begin{array}{r} L^T C\sigma_{\mu\nu} L \\ R^T C\sigma_{\mu\nu} R \end{array}\right\} \quad (1,0)\oplus(0,1), \quad (\mathbf{6},\mathbf{1})\oplus(\mathbf{1},\mathbf{6}),$$

where we have indicated the Lorentz and chiral representations of the diquarks.

For three quarks, we have

$$(L+R)^3 \rightarrow \begin{cases} LLL(\tfrac{1}{2},0)\oplus(\tfrac{3}{2},0), & (\mathbf{1},\mathbf{1})\oplus(\mathbf{8},\mathbf{1})\oplus(\mathbf{8},\mathbf{1})\oplus(\mathbf{10},\mathbf{1}) \\ LLR(0,\tfrac{1}{2})\oplus(1,\tfrac{1}{2}), & (\bar{\mathbf{3}},\mathbf{3})\oplus(\mathbf{6},\mathbf{3}) \end{cases} \tag{33}$$

and together with the terms where L and R are exchanged. Now we discuss the independent fields in terms of the chiral representations. Once again, for illustration we will discuss here the case of the simplest Lorentz representation $(\frac{1}{2}, 0) \oplus (0, \frac{1}{2})$ for the Dirac fields.

A. Independent $(LL)L$ fields

The $(LL)L$ must belong to one of the following chiral representations: $(\mathbf{1},\mathbf{1})\oplus(\mathbf{8},\mathbf{1})\oplus(\mathbf{8},\mathbf{1})\oplus(\mathbf{10},\mathbf{1})$. For each chiral representation, there is one flavor representation available.

For $(\mathbf{1},\mathbf{1}) \rightarrow \mathbf{1}_f$, there are apparently two nonzero fields

$$\begin{aligned} \Lambda_{L1} &= \epsilon_{abc}\epsilon^{ABC}(L_A^{aT} C L_B^b)\gamma_5 L_C^c, \\ \Lambda_{L2} &= \epsilon_{abc}\epsilon^{ABC}(L_A^{aT} C\gamma_5 L_B^b) L_C^c, \\ \Lambda_{L3} &= \epsilon_{abc}\epsilon^{ABC}(L_A^{aT} C\gamma_\mu\gamma_5 L_B^b)\gamma^\mu L_C^c = 0, \end{aligned} \tag{34}$$

where Λ_3^L vanishes because $\gamma_\mu\gamma_5$ is chiral-odd

$$L^T C\gamma_\mu\gamma_5 L = 0.$$

After performing the Fierz transformation to relate Λ_{Li} and Λ'_{Li} as we have done before, and solving the coupled equations, we find the solution that all such fields vanish.

For $(\mathbf{10},\mathbf{1}) \rightarrow \mathbf{10_f}$, we would have again two nonzero components:

$$\begin{aligned} \Delta_{L4}^P &= \epsilon_{abc} S_P^{ABC}(L_A^{aT} C\gamma_\mu L_B^b)\gamma^\mu\gamma_5 L_C^c, \\ \Delta_{L5}^P &= \epsilon_{abc} S_P^{ABC}(L_A^{aT} C\sigma_{\mu\nu} L_B^b)\sigma^{\mu\nu}\gamma_5 L_C^c. \end{aligned} \tag{35}$$

Performing the Fierz transformation to relate Δ_{Li}^P and $\Delta_{Li}^{P\prime}$, we obtain the solution that all such $(LL)L$ fields vanish.

Finally for $(\mathbf{8},\mathbf{1}) \rightarrow \mathbf{8_f}$, we may consider once again two nonzero fields to start with

$$\begin{aligned} N_{L1}^N &= \epsilon_{abc}\epsilon^{ABD}\lambda_{DC}^N(L_A^{aT} C L_B^b)\gamma_5 L_C^c, \\ N_{L2}^N &= \epsilon_{abc}\epsilon^{ABD}\lambda_{DC}^N(L_A^{aT} C\gamma_5 L_B^b) L_C^c. \end{aligned} \tag{36}$$

Applying the Fierz transformation to relate N_{Li}^N and $N_{Li}^{N\prime}$, we obtain the solution

$$N_{L2}^N = N_{L1}^N.$$

Therefore, there is only one independent $(LL)L$ $\mathbf{8_f}$ field.

B. Independent $(LL)R$ fields

The chiral representations of $(LL)R$ are $(\bar{\mathbf{3}},\mathbf{3})\oplus(\mathbf{6},\mathbf{3})$. We will study them separately in the following.

For $(\bar{\mathbf{3}},\mathbf{3}) \rightarrow \mathbf{1_f}$, there appears to exist two nonzero components among the five fields,

$$\begin{aligned} \Lambda_{M1} &= \epsilon_{abc}\epsilon^{ABC}(L_A^{aT} C L_B^b)\gamma_5 R_C^c, \\ \Lambda_{M2} &= \epsilon_{abc}\epsilon^{ABC}(L_A^{aT} C\gamma_5 L_B^b) R_C^c, \\ \Lambda_{M3} &= \epsilon_{abc}\epsilon^{ABC}(L_A^{aT} C\gamma_\mu\gamma_5 L_B^b)\gamma^\mu R_C^c = 0, \\ \Lambda_{M4} &= \epsilon_{abc}\epsilon^{ABC}(L_A^{aT} C\gamma_\mu L_B^b)\gamma^\mu\gamma_5 R_C^c = 0, \\ \Lambda_{M5} &= \epsilon_{abc}\epsilon^{ABC}(L_A^{aT} C\sigma_{\mu\nu} L_B^b)\sigma^{\mu\nu}\gamma_5 R_C^c = 0, \end{aligned} \tag{37}$$

where M (mixed) indicates that the fields contain both left- and right-handed quarks. Performing the Fierz transformation to relate Λ_{Mi} and Λ'_{Mi}, we obtain the following relations

$$\Lambda'_{M4} = -\Lambda'_{M3} = -2\Lambda_{M2} = 2\Lambda_{M1}.$$

We may consider other ten combinations formed by (LR) and (RL) diquarks, $(LR)L$ and $(RL)L$. However, they can be related to the above ones of $(LL)R$ by a rearrangement of indices as well as the Fierz transformation, for instance,

$$\Lambda_{M6} = \epsilon_{abc}\epsilon^{ABC}(L_A^{aT} C R_B^b)\gamma_5 L_C^c = \Lambda'_{M1}. \tag{38}$$

Therefore, we have only one independent field.

For the chiral representation $(\mathbf{6}, \mathbf{3}) \rightarrow \mathbf{10_f}$, we can write five fields containing diquarks formed by five Dirac matrices. However, we can show that after performing the Fierz transformation all fields vanish. Therefore, this representation cannot support three-quark fields.

The baryon fields of chiral representations $(\bar{\mathbf{3}}, \mathbf{3}) \rightarrow \mathbf{8_f}$ can be formed

$$\begin{aligned}
N_{M1}^N &= \epsilon_{abc}\epsilon^{ABD}\lambda_{DC}^N(L_A^{aT} C L_B^b)\gamma_5 R_C^c,\\
N_{M2}^N &= \epsilon_{abc}\epsilon^{ABD}\lambda_{DC}^N(L_A^{aT} C \gamma_5 L_B^b) R_C^c,\\
N_{M3}^N &= \epsilon_{abc}\epsilon^{ABD}\lambda_{DC}^N(L_A^{aT} C \gamma_\mu \gamma_5 L_B^b)\gamma^\mu R_C^c = 0,\\
N_{M4}^N &= \epsilon_{abc}\epsilon^{ABD}\lambda_{DC}^N(L_A^{aT} C \gamma_\mu L_B^b)\gamma^\mu \gamma_5 R_C^c = 0,\\
N_{M5}^N &= \epsilon_{abc}\epsilon^{ABD}\lambda_{DC}^N(L_A^{aT} C \sigma_{\mu\nu} L_B^b)\sigma^{\mu\nu} \gamma_5 R_C^c = 0,
\end{aligned} \tag{39}$$

where we see that there are two nonzero fields. Applying the Fierz transformation, we can verify that there is only one independent field with the following relations

$$N_{M4}^{N\prime} = -N_{M3}^{N\prime} = -2N_{M2}^N = 2N_{M1}^N.$$

Another chiral representation $(\mathbf{6}, \mathbf{3}) \rightarrow \mathbf{8_f}$ can be constructed by the combinations similar to (39), for instance,

$$\begin{aligned}
N_{(6,3)1}^N = \epsilon_{abc}\epsilon^{ACD}\lambda_{DB}^N\{&(L_A^{aT} C L_B^b)\gamma_5 R_C^c\\
&+ (L_B^{aT} C L_A^b)\gamma_5 R_C^c\}.
\end{aligned} \tag{40}$$

After similar algebra we can verify that all these fields vanish.

C. A short summary of chiral representations

To summarize this section, we find that possible chiral representations for Dirac spinor baryon fields without Lorentz index are

$$\begin{aligned}
\Lambda &= \epsilon_{abc}\epsilon^{ABC}(L_A^{aT} C L_B^b)\gamma_5 R_C^c + \epsilon_{abc}\epsilon^{ABC}(R_A^{aT} C R_B^b)\gamma_5 L_C^c\\
&= \Lambda_{M1} + (L \leftrightarrow R),
\end{aligned} \tag{41}$$

$$\begin{aligned}
N_1^N - N_2^N &= 2\epsilon_{abc}\epsilon^{ABD}\lambda_{DC}^N(L_A^{aT} C L_B^b)\gamma_5 R_C^c\\
&\quad + 2\epsilon_{abc}\epsilon^{ABD}\lambda_{DC}^N(R_A^{aT} C R_B^b)\gamma_5 L_C^c\\
&= 2N_{M1}^N + (L \leftrightarrow R),
\end{aligned} \tag{42}$$

$$\begin{aligned}
N_1^N + N_2^N &= 2\epsilon_{abc}\epsilon^{ABD}\lambda_{DC}^N(L_A^{aT} C L_B^b)\gamma_5 L_C^c\\
&\quad + 2\epsilon_{abc}\epsilon^{ABD}\lambda_{DC}^N(R_A^{aT} C R_B^b)\gamma_5 R_C^c\\
&= 2N_{L1}^N + (L \leftrightarrow R).
\end{aligned} \tag{43}$$

So we can see that the fields Λ and $N_1^N - N_2^N$ have a type of $LLR \oplus RRL$, and belong to the chiral representation $(\bar{\mathbf{3}}, \mathbf{3}) \oplus (\mathbf{3}, \bar{\mathbf{3}})$; while the field $N_1^N + N_2^N$ has a type of $LLL \oplus RRR$, and belongs to the chiral representation $(\mathbf{8}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{8})$.

The chiral properties of Rarita-Schwinger fields (Lorentz rep.$(\mathbf{1}, \frac{1}{2}) \oplus (\frac{1}{2}, \mathbf{1})$) are listed in Appendix D. We summarize the results here:

$$\begin{aligned}
N_\mu^N &= 2\epsilon_{abc}\epsilon^{ABD}\lambda_{DC}^N(L_A^{aT} C \gamma_\mu \gamma_5 R_B^b)\gamma_5 L_C^c\\
&\quad + 2\epsilon_{abc}\epsilon^{ABD}\lambda_{DC}^N(R_A^{aT} C \gamma_\mu \gamma_5 L_B^b)\gamma_5 R_C^c\\
&\quad + \tfrac{1}{2}\epsilon_{abc}\epsilon^{ABD}\lambda_{DC}^N(L_A^{aT} C L_B^b)\gamma_\mu R_C^c\\
&\quad + \tfrac{1}{2}\epsilon_{abc}\epsilon^{ABD}\lambda_{DC}^N(R_A^{aT} C R_B^b)\gamma_\mu L_C^c,
\end{aligned} \tag{44}$$

$$\begin{aligned}
\Delta_\mu^P &= 2\epsilon_{abc} S_P^{ABC}(L_A^{aT} C \gamma_\mu R_B^b) L_C^c\\
&\quad + 2\epsilon_{abc} S_P^{ABC}(R_A^{aT} C \gamma_\mu L_B^b) R_C^c.
\end{aligned} \tag{45}$$

So we see that N_μ^N and Δ_μ^P are of the type $LLR \oplus RRL$, and belong to the chiral representation $(\mathbf{6}, \mathbf{3}) \oplus (\mathbf{3}, \mathbf{6})$. The (similar) results for $\Delta_{\mu\nu}^P$, which is of the type $LLL \oplus RRR$, and belongs to the chiral representation $(\mathbf{10}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{10})$, are omitted here.

V. AXIAL COUPLING CONSTANTS

As a simple application of the present mathematical formalism, we can extract the (diagonal) axial coupling constants g_A for these baryons. All information is contained in Eqs. (26) and (28), from which one can extract the Abelian $U(1)_A$ axial coupling constant g_A^0 and the non-Abelian $SU(3)_V \times SU(3)_A$ diagonal axial coupling constants, g_A^3 and g_A^8. The latter two can be extracted from the δ_5^{b3} and δ_5^{b8} subset of chiral transformations Eqs. (28), respectively.

In general, the diagonal elements of the SU(3) g_A's can be decomposed into so-called F and D components, which are defined by the axial-vector current A_μ^a $(a = 0, 1, \ldots 8)$

$$A_\mu^a = g_A^F \operatorname{tr}\left(\bar{\mathfrak{N}}\gamma_\mu\gamma_5\left[\frac{\lambda_a}{2}, \mathfrak{N}\right]\right) + g_A^D \operatorname{tr}\left(\bar{\mathfrak{N}}\gamma_\mu\gamma_5\left\{\frac{\lambda_a}{2}, \mathfrak{N}\right\}\right), \tag{46}$$

where $\mathfrak{N}$ is the 3×3 baryon octet matrix, Eq. (12). Therefore, we have

$$\begin{aligned}
A_\mu^3 &= (g_A^F + g_A^D)(p^+ p - n^+ n)\\
&\quad + 2g_A^F((\Sigma^+)^+\Sigma^+ - (\Sigma^-)^+\Sigma^-)\\
&\quad + (g_A^F - g_A^D)((\Xi^0)^+\Xi^0 - (\Xi^-)^+\Xi^-),
\end{aligned} \tag{47}$$

$$A^8_\mu = \left(\sqrt{3}g_A^F - \frac{g_A^D}{\sqrt{3}}\right)(p^+p + n^+n) + \frac{2g_A^D}{\sqrt{3}}((\Sigma^+)^+\Sigma^+ + (\Sigma^-)^+\Sigma^-) + \left(-\sqrt{3}g_A^F - \frac{g_A^D}{\sqrt{3}}\right)((\Xi^0)^+\Xi^0 + (\Xi^-)^+\Xi^-) - \frac{2g_A^D}{\sqrt{3}}(\Lambda^8)^+\Lambda^8, \tag{48}$$

where we omit the Lorentz indices. In other words,

$$g_A^3(N) \sim (g_A^F + g_A^D)\mathbf{I_z}, \qquad g_A^3(\Sigma) \sim 2g_A^D\mathbf{I_z},$$
$$g_A^3(\Xi) \sim (g_A^F - g_A^D)\mathbf{I_z}, \qquad g_A^8(N) \sim \sqrt{3}g_A^F - \frac{g_A^D}{\sqrt{3}},$$
$$g_A^8(\Sigma) \sim \frac{2g_A^D}{\sqrt{3}}, \qquad g_A^8(\Xi) \sim -\sqrt{3}g_A^F - \frac{g_A^D}{\sqrt{3}}, \tag{49}$$
$$g_A^8(\Lambda) \sim -\frac{2g_A^D}{\sqrt{3}},$$

for the octet parts. The operator $\mathbf{I_z}$ is the third component of isospin, whereas the SU(3) singlet term g_A^0 contains only the D term and is therefore trivial.

For the decuplet baryons, the SU(3) coupling constants contain only one SU(3) irreducible term because the SU(3) Clebsch-Gordan series for $\bar{\mathbf{10}} \otimes \mathbf{10} \otimes \mathbf{8}$ contains only one singlet. In order to extract the coupling constants, we first rewrite Eqs. (26) and (28) in the following form, for all the singlet, octet and decuplet baryon fields:

(1) The Abelian g_A^0 basically counts the difference between the numbers of left- and right-handed quarks in a baryon of definite/positive chirality (helicity). Several definitions of g_A^0 can be found in the literature. No matter what convention we adopt, we must make sure that it is consistent with the definition of the SU(3) singlet vector current that counts the baryon-, or the quark number. So, either we normalize g_A^0 to the baryon number, or to the quark number. Of course, the difference is just a multiplicative factor (3), but inconsistent definitions will lead to confusion later on when one constructs chirally invariant interactions. At this time we shall adopt the latter (quark number) normalization.
Because $\lambda^0_{11} = \lambda^0_{22} = \lambda^0_{33}$ for g_A^0, the chiral transformations δ_5 are identical for all baryon fields within the same chiral representation, so we may define g_A^0 by

$$\delta_5 B = i\gamma_5 \frac{\lambda^0_{11} b_0}{2} g_A^0 B = \frac{i\gamma_5 b_0}{\sqrt{6}} g_A^0 B, \tag{50}$$

where B represents the baryon field, such as Λ and $N_1^N - N_2^N$ etc. This convention is based on the quark number, implying that the SU(3) singlet vector charge of a nucleon is three (+ 3).

(2) For g_A^3, because $\lambda^3_{11} = -\lambda^3_{22}$, the chiral transformation δ_5^{b3} is proportional to the isospin value of $\mathbf{I_z}$, which is factored out from the definition of g_A^3

$$\delta_5^{b3} B = i\gamma_5 b_3 g_A^3 \mathbf{I}_z B + \cdots, \tag{51}$$

where the ellipsis $\cdots$ on the right-hand side denote the off-diagonal terms.

(3) For g_A^8, because $\lambda^8_{11} = \lambda^8_{22}$, the chiral transformations δ_5^{b8} is the same for the baryon fields belonging to one isospin multiplet. We define it to be

$$\delta_5^{b8} B = i\gamma_5 \frac{\boldsymbol{\lambda}^8_{11} b_8}{2} g_A^8 B + \cdots = \frac{i\gamma_5 b_8}{2\sqrt{3}} g_A^8 B + \cdots. \tag{52}$$

The resulting axial coupling constants g_A^0, g_A^3, and g_A^8 are shown in Table III, where Λ is the (only) singlet field Λ; then N_-, Σ_-, Ξ_- and Λ_- are the octet fields of the type $N_1^N - N_2^N$; the N_+, Σ_+, Ξ_+ and Λ_+ are the octet fields of the type $N_1^N + N_2^N$; the N_μ, Σ_μ, Ξ_μ and Λ_μ are the octet fields N_μ^N; the Δ_μ, Σ^*_μ, Ξ^*_μ and Ω_μ are the decuplet fields Δ^P_μ; $\Delta_{\mu\nu}$, $\Sigma^*_{\mu\nu}$, $\Xi^*_{\mu\nu}$ and $\Omega_{\mu\nu}$ are the decuplet fields $\Delta^P_{\mu\nu}$.

From the values in Table III, one can compute the F and D couplings easily. The resulting F/D ratio,

$$\alpha = \frac{g_A^D}{g_A^F + g_A^D}, \tag{53}$$

TABLE III. Axial coupling constants g_A^0, g_A^3, and g_A^8. In the last column $\alpha = g_A^D/(g_A^F + g_A^D)$.

$SU(3)_L \otimes SU(3)_R$	$SU(3)_F$		g_A^0	g_A^3	g_A^8	α
	1	Λ	−1	—	—	—
$(\bar{\mathbf{3}}, \mathbf{3}) \oplus (\mathbf{3}, \bar{\mathbf{3}})$	**8**	N_-	−1	1	−1	1
		Σ_-	−1	0	2	
		Ξ_-	−1	−1	−1	
		Λ_-	−1	—	−2	
$(\mathbf{8}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{8})$	**8**	N_+	3	1	3	0
		Σ_+	3	1	0	
		Ξ_+	3	1	−3	
		Λ_+	3	—	0	
$(\mathbf{3}, \mathbf{6}) \oplus (\mathbf{6}, \mathbf{3})$	**8**	N_μ	1	5/3	1	3/5
		Σ_μ	1	2/3	2	
		Ξ_μ	1	−1/3	−3	
		Λ_μ	1	—	−2	
	10	Δ_μ	1	1/3	1	—
		Σ^*_μ	1	1/3	0	
		Ξ^*_μ	1	1/3	−1	
		Ω_μ	1	—	−2	
$(\mathbf{10}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{10})$	**10**	$\Delta_{\mu\nu}$	3	1	3	—
		$\Sigma^*_{\mu\nu}$	3	1	0	
		$\Xi^*_{\mu\nu}$	3	1	−3	
		$\Omega_{\mu\nu}$	3	—	−6	

is also tabulated in the last column of Table III. Empirically, $\alpha \sim 0.6$, which is fairly close to the SU(6) quark model value. In the present formalism we see that only the $(\mathbf{3}, \mathbf{6}) \oplus (\mathbf{6}, \mathbf{3})$ chiral multiplet/representation reproduces this value. Previous works have shown that this value is physically related to the coupling of the nucleon to the $\Delta(1232)$, as demonstrated in the Adler-Weisberger sum rule [27,28]. This was also shown algebraically by Weinberg [1]. In both cases, saturation of the pion (axial-vector) induced transition from the nucleon to the $\Delta(1232)$ is essential [29]. In the present study, this is realized by the chiral representation which includes both the nucleon (isospin 1/2) and delta (isospin 3/2) states.

It is also interesting that Table III shows that $g_A^3(N) = 5/3$, $g_A^0(N) = 1$ for $(\mathbf{3}, \mathbf{6}) \oplus (\mathbf{6}, \mathbf{3})$, while $g_A^3(N) = 1$, $g_A^0(N) = -1$ for $(\bar{\mathbf{3}}, \mathbf{3}) \oplus (\mathbf{3}, \bar{\mathbf{3}})$.

The flavor singlet g_A^0 corresponds to the so-called nucleon spin value, as measured in polarized deep-inelastic lepton scattering. A suitable superposition of the two chiral representations may improve the nucleon axial coupling in either the isovector and/or isosinglet sectors. The importance of such mixing for the isovector axial coupling constant has been emphasized by Weinberg since the late 1960's, Ref. [1].

VI. SUMMARY

In this paper we have performed a classification of flavor vector and chiral symmetries, and established independence of several types of relativistic SU(3) baryon interpolating fields. The three-quark fields may belong to one of several different Lorentz group representations which fact imposes certain constraints on possible chiral symmetry representations. This is due to the Pauli principle and has been explicitly verified by the method of Fierz transformations.

As the present results reflect essentially the Pauli principle, they can be conveniently summarized by using the permutation symmetry group properties/representations, as shown in Table IV. This table "explains" also the previous results for the case of isospin $SU(2)_L \times SU(2)_R$ [12]. In the real world, with spontaneous breaking of chiral symmetry, physical states of pure chiral (axial) symmetry representation do not occur, but in general they can mix in a state having a definite flavor symmetry. The present results show that the three-quark structures accommodate only a few (sometimes just one) chiral representations, for instance, for the total spin 1/2 field of Dirac spinor, there are two allowed chiral representations, having the Young diagram structures ([21], −) and ([1], [11]), where — indicates the singlet. The ([21], −) Young diagram corresponds to the $(\frac{\mathbf{1}}{\mathbf{2}}, \mathbf{0})$ and $(\mathbf{8}, \mathbf{1})$ representations of SU(2) and SU(3),, respectively, whereas the ([1], [11]) Young diagram corresponds to the $(\frac{\mathbf{1}}{\mathbf{2}}, \mathbf{0})$ and $(\mathbf{3}, \bar{\mathbf{3}})$ of SU(2) and SU(3), respectively.

Note that the $N_f = 2$ chiral representations have the same form as those of the Lorentz group. In this way, the Lorentz (spin) and flavor structures are combined into a general structure with total permutation symmetry. As shown in the computation of g_A, in general, various couplings depend on the chiral representations.

We should conclude with a few historical remarks: the two-flavor baryon fields' Fierz identities have been known since the early days of QCD sum rules [13], whereas the three-flavor ones presented here seem to be the first ones. Similarly, the chiral properties of the two-flavor baryon fields' have been known at least since the work of Christos [30], but the three-flavor ones have been discussed by Christos and H.q. Zheng [26], but not systematically explored.

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APPENDIX A: RARITA-SCHWINGER FIELDS

In this appendix, we study the properties of Rarita-Schwinger fields, in the form of

$$B_\mu(x) \sim \epsilon_{abc}(q_A^{aT}(x) C \Gamma_1 q_B^b(x)) \Gamma_2 q_C^c(x), \tag{A1}$$

TABLE IV. Structure of allowed three-quark baryon fields.

Lorentz	J = Spin	Young diagram for Chiral rep.	Axial $U(1)_A$ charge g_A^0	Chiral SU(2)	Chiral SU(3)	Flavor SU(3)
$(\frac{1}{2}, 0) \oplus (0, \frac{1}{2})$	1/2	$([21], -) \oplus (-, [21])$	3	$(\frac{1}{2}, 0) \oplus (0, \frac{1}{2})$	$(8, 1) \oplus (1, 8)$	8
		$([1], [11]) \oplus ([11], [1])$	−1		$(3, \bar{3}) \oplus (\bar{3}, 3)$	1, 8
$(1, \frac{1}{2}) \oplus (\frac{1}{2}, 1)$	3/2	$([2], [1]) \oplus ([1], [2])$	1	$(\frac{1}{2}, 1) \oplus (1, \frac{1}{2})$	$(3, 6) \oplus (6, 3)$	8, 10
$(\frac{3}{2}, 0) \oplus (0, \frac{3}{2})$	3/2	$([3], -) \oplus (-, [3])$	3	$(\frac{3}{2}, 0) \oplus (0, \frac{3}{2})$	$(10, 1) \oplus (1, 10)$	10

where there are eight possible pairs of Γ_1 and Γ_2,

$$(\Gamma_1, \Gamma_2) = (\mathbf{1}, \gamma_\mu), \quad (\gamma_5, \gamma_\mu\gamma_5), \quad (\gamma_\mu\gamma_5, \gamma_5), \quad (\gamma^\nu\gamma_5, \sigma_{\mu\nu}\gamma_5), \quad (\gamma_\mu, \mathbf{1}), \quad (\gamma^\nu, \sigma_{\mu\nu}), \quad (\sigma_{\mu\nu}, \gamma^\nu), \quad (\sigma_{\mu\nu}\gamma_5, \gamma^\nu\gamma_5). \tag{A2}$$

The fields formed by these (Γ_1, Γ_2) pairs are labeled by the subscript $i = (1, \cdots, 8)$ with the ordering of Eq. (A2). The discussion is separated into singlet, decuplet and octet cases.

1. Flavor singlet baryon

For flavor singlet fields, there are four apparently nonzero fields

$$\begin{aligned}
\Lambda_{1\mu} &= \epsilon_{abc}\epsilon^{ABC}(q_A^{aT} C q_B^b)\gamma_\mu q_C^c,\\
\Lambda_{2\mu} &= \epsilon_{abc}\epsilon^{ABC}(q_A^{aT} C \gamma_5 q_B^b)\gamma_\mu\gamma_5 q_C^c,\\
\Lambda_{3\mu} &= \epsilon_{abc}\epsilon^{ABC}(q_A^{aT} C \gamma_\mu\gamma_5 q_B^b)\gamma_5 q_C^c,\\
\Lambda_{4\mu} &= \epsilon_{abc}\epsilon^{ABC}(q_A^{aT} C \gamma^\nu\gamma_5 q_B^b)\sigma_{\mu\nu}\gamma_5 q_C^c.
\end{aligned} \tag{A3}$$

As before, the Fierz transformed fields (primed fields) are just the corresponding unprimed ones, $\Lambda'_{i\mu} = \Lambda_{i\mu}$. On the other hand, by applying the Fierz rearrangement (see Appendix. C), we obtain four equations

$$\begin{aligned}
\Lambda_{1\mu} &= -\frac{1}{4}\Lambda'_{1\mu} - \frac{1}{4}\Lambda'_{2\mu} + \frac{1}{4}\Lambda'_{3\mu} - \frac{i}{4}\Lambda'_{4\mu},\\
\Lambda_{2\mu} &= -\frac{1}{4}\Lambda'_{1\mu} - \frac{1}{4}\Lambda'_{2\mu} - \frac{1}{4}\Lambda'_{3\mu} + \frac{i}{4}\Lambda'_{4\mu},\\
\Lambda_{3\mu} &= \frac{1}{4}\Lambda'_{1\mu} - \frac{1}{4}\Lambda'_{2\mu} - \frac{1}{4}\Lambda'_{3\mu} - \frac{i}{4}\Lambda'_{4\mu},\\
\Lambda_{4\mu} &= \frac{3i}{4}\Lambda'_{1\mu} - \frac{3i}{4}\Lambda'_{2\mu} + \frac{3i}{4}\Lambda'_{3\mu} + \frac{1}{4}\Lambda'_{4\mu}.
\end{aligned}$$

Thus we find the following solution

$$\begin{aligned}
&\Lambda_{1\mu} = -\Lambda_{2\mu} = \Lambda_{3\mu} = -\frac{i}{3}\Lambda_{4\mu} = \gamma_\mu\gamma_5\Lambda_1,\\
&\Lambda_{6\mu} = \Lambda_{7\mu} = \Lambda_{8\mu} = 0.
\end{aligned}$$

We see that there is only one nonvanishing independent field. However, it has a structure of $\gamma_\mu\Lambda_i$. Therefore, they are all Dirac fields, and there is no flavor singlet fields of the Rarita-Schwinger type.

2. Flavor decuplet baryon

For flavor decuplet fields, we have four potentially nonzero interpolators

$$\begin{aligned}
\Delta^P_{5\mu} &= \epsilon_{abc}S_P^{ABC}(q_A^{aT} C \gamma_\mu q_B^b)q_C^c,\\
\Delta^P_{6\mu} &= \epsilon_{abc}S_P^{ABC}(q_A^{aT} C \gamma^\nu q_B^b)\sigma_{\mu\nu} q_C^c,\\
\Delta^P_{7\mu} &= \epsilon_{abc}S_P^{ABC}(q_A^{aT} C \sigma_{\mu\nu} q_B^b)\gamma^\nu q_C^c,\\
\Delta^P_{8\mu} &= \epsilon_{abc}S_P^{ABC}(q_A^{aT} C \sigma_{\mu\nu}\gamma_5 q_B^b)\gamma^\nu\gamma_5 q_C^c.
\end{aligned} \tag{A4}$$

As before, the Fierz transformed fields can be related to the corresponding unprimed ones, $\Delta^{P\prime}_{i\mu} = -\Delta^P_{i\mu}$. On the other hand, by applying the Fierz rearrangement to relate $\Delta^N_{i\mu}$ and $\Delta^{N\prime}_{i\mu}$, we obtain the solution

$$\Delta^P_{5\mu} = i\Delta^P_{6\mu} = -i\Delta^P_{7\mu} = i\Delta^P_{8\mu}.$$

There are no Dirac decuplet fields. Therefore, we obtain one extra nonvanishing field.

3. Flavor octet baryon

To study the octet baryon fields, we start with eight baryon fields:

$$\begin{aligned}
N^N_{1\mu} &= \epsilon_{abc}\epsilon^{ABD}\lambda^N_{DC}(q_A^{aT} C q_B^b)\gamma_\mu q_C^c,\\
N^N_{2\mu} &= \epsilon_{abc}\epsilon^{ABD}\lambda^N_{DC}(q_A^{aT} C \gamma_5 q_B^b)\gamma_\mu\gamma_5 q_C^c,\\
N^N_{3\mu} &= \epsilon_{abc}\epsilon^{ABD}\lambda^N_{DC}(q_A^{aT} C \gamma_\mu\gamma_5 q_B^b)\gamma_5 q_C^c,\\
N^N_{4\mu} &= \epsilon_{abc}\epsilon^{ABD}\lambda^N_{DC}(q_A^{aT} C \gamma^\nu\gamma_5 q_B^b)\sigma_{\mu\nu}\gamma_5 q_C^c,\\
N^N_{5\mu} &= \epsilon_{abc}\epsilon^{ABD}\lambda^N_{DC}(q_A^{aT} C \gamma_\mu q_B^b)q_C^c = 0,\\
N^N_{6\mu} &= \epsilon_{abc}\epsilon^{ABD}\lambda^N_{DC}(q_A^{aT} C \gamma^\nu q_B^b)\sigma_{\mu\nu} q_C^c = 0,\\
N^N_{7\mu} &= \epsilon_{abc}\epsilon^{ABD}\lambda^N_{DC}(q_A^{aT} C \sigma_{\mu\nu} q_B^b)\gamma^\nu q_C^c = 0,\\
N^N_{8\mu} &= \epsilon_{abc}\epsilon^{ABD}\lambda^N_{DC}(q_A^{aT} C \sigma_{\mu\nu}\gamma_5 q_B^b)\gamma^\nu\gamma_5 q_C^c = 0.
\end{aligned} \tag{A5}$$

There are four zero fields, but the corresponding Fierz transformed ones are nonzero. By using the Jacobi identity in Eq. (17), we obtain

$$\begin{aligned}
N^{N\prime}_{1\mu} &= -\tfrac{1}{2}N^N_{1\mu}, & N^{N\prime}_{2\mu} &= -\tfrac{1}{2}N^N_{2\mu},\\
N^{N\prime}_{3\mu} &= -\tfrac{1}{2}N^N_{3\mu}, & N^{N\prime}_{4\mu} &= -\tfrac{1}{2}N^N_{4\mu}.
\end{aligned}$$

Similarly, performing the Fierz transformation to relate $N^N_{i\mu}$ and $N^{N\prime}_{i\mu}$, we obtain the solution

$$\begin{aligned}
N^N_{4\mu} &= -iN^N_{1\mu} + iN^N_{2\mu} - iN^N_{3\mu},\\
N^{N\prime}_{5\mu} &= -\frac{1}{2}N^N_{1\mu} + \frac{1}{2}N^N_{2\mu} - \frac{1}{2}N^N_{3\mu},\\
N^{N\prime}_{6\mu} &= -iN^N_{1\mu} + iN^N_{2\mu} + \frac{i}{2}N^N_{3\mu},\\
N^{N\prime}_{7\mu} &= iN^N_{1\mu} + \frac{i}{2}N^N_{2\mu} + iN^N_{3\mu},\\
N^{N\prime}_{8\mu} &= \frac{i}{2}N^N_{1\mu} + iN^N_{2\mu} - iN^N_{3\mu}.
\end{aligned} \tag{A6}$$

Thus we have shown that there are three different kinds of octets. However, $N^N_{1\mu}$ and $N^N_{2\mu}$ are nothing but $\gamma_\mu\gamma_5 N^N_1$ and $\gamma_\mu\gamma_5 N^N_2$ [see Eqs. (16)]. Therefore, we only obtain one extra octet baryon field. It is formed by using the projection operator:

$$P^{3/2}_{\mu\nu} = (g_{\mu\nu} - \tfrac{1}{4}\gamma_\mu\gamma_\nu),$$

as

$$N^N_\mu = P^{3/2}_{\mu\nu} N^N_{3\nu} = (g_{\mu\nu} - \tfrac{1}{4}\gamma_\mu\gamma_\nu)\epsilon_{abc}\epsilon^{ABD}\lambda^N_{DC}(q^{aT}_A C\gamma_\mu\gamma_5 q^b_B)\gamma_5 q^c_C = N^N_{3\mu} + \tfrac{1}{4}\gamma_\mu\gamma_5(N^N_1 - N^N_2).$$

APPENDIX B: TENSOR FIELDS

In this appendix, we study the antisymmetric tensor baryons fields $J_{\mu\nu}$ with $J_{\mu\nu} = -J_{\nu\mu}$. For the tensor fields, we can form nine three-quark fields where the possible pairs of Γ_1 and Γ_2 are

$$(\Gamma_1, \Gamma_2) = (\gamma_\mu, \gamma_\nu\gamma_5) - (\mu \leftrightarrow \nu), \quad (\gamma_\mu\gamma_5, \gamma_\nu) - (\mu \leftrightarrow \nu), \quad \epsilon_{\mu\nu\rho\sigma}(\gamma^\rho, \gamma^\sigma), \quad \epsilon_{\mu\nu\rho\sigma}(\gamma^\rho\gamma_5, \gamma^\sigma\gamma_5), \quad (\mathbf{1}, \sigma_{\mu\nu}\gamma_5), \quad (\gamma_5, \sigma_{\mu\nu}), \quad (\sigma_{\mu\nu}, \gamma_5), \quad (\sigma_{\mu\nu}\gamma_5, \mathbf{1}), \quad \epsilon_{\mu\nu\rho\sigma}(\sigma_{\rho l}, \sigma_{\sigma l}). \tag{B1}$$

The fields formed by these (Γ_1, Γ_2) pairs are labeled by the subscript $i = (1, \cdots, 9)$ with the ordering of Eq. (B1). The discussion is separated into singlet, decuplet and octet cases.

1. Flavor singlet baryon

The flavor singlet baryon fields have four potentially nonzero interpolators among nine fields:

$$\begin{aligned}
\Lambda_{2\mu\nu} &= \epsilon_{abc}\epsilon^{ABC}(q^{aT}_A C\gamma_\mu\gamma_5 q^b_B)\gamma_\nu q^c_C - (\mu \leftrightarrow \nu),\\
\Lambda_{4\mu\nu} &= \epsilon_{abc}\epsilon^{ABC}\epsilon_{\mu\nu\rho\sigma}(q^{aT}_A C\gamma_\rho\gamma_5 q^b_B)\gamma_\sigma\gamma_5 q^c_C,\\
\Lambda_{5\mu\nu} &= \epsilon_{abc}\epsilon^{ABC}(q^{aT}_A C q^b_B)\sigma_{\mu\nu}\gamma_5 q^c_C,\\
\Lambda_{6\mu\nu} &= \epsilon_{abc}\epsilon^{ABC}(q^{aT}_A C\gamma_5 q^b_B)\sigma_{\mu\nu} q^c_C.
\end{aligned} \tag{B2}$$

As before, the Fierz transformed fields are just the corresponding unprimed ones, $\Lambda'_{i\mu\nu} = \Lambda_{i\mu\nu}$. On the other hand, by applying the Fierz rearrangement to relate $\Lambda_{i\mu\nu}$ and $\Lambda'_{i\mu\nu}$, we obtain the solution:

$$i\Lambda_{2\mu\nu} = \Lambda_{4\mu\nu} = 2\Lambda_{5\mu\nu} = -2\Lambda_{6\mu\nu}.$$

Therefore, there is only one independent field. However, it has a structure of $\sigma_{\mu\nu}\Lambda_i$. Therefore, there are no extra fields.

2. Flavor decuplet baryon

The flavor decuplet baryon fields have five potentially nonzero interpolators:

$$\begin{aligned}
\Delta^P_{1\mu\nu} &= \epsilon_{abc}S^{ABC}(q^{aT}_A C\gamma_\mu q^b_B)\gamma_\nu\gamma_5 q^c_C - (\mu \leftrightarrow \nu),\\
\Delta^P_{3\mu\nu} &= \epsilon_{abc}S^{ABC}\epsilon_{\mu\nu\rho\sigma}(q^{aT}_A C\gamma_\rho q^b_B)\gamma_\sigma q^c_C,\\
\Delta^P_{7\mu\nu} &= \epsilon_{abc}S^{ABC}(q^{aT}_A C\sigma_{\mu\nu} q^b_B)\gamma_5 q^c_C,\\
\Delta^P_{8\mu\nu} &= \epsilon_{abc}S^{ABC}(q^{aT}_A C\sigma_{\mu\nu}\gamma_5 q^b_B) q^c_C,\\
\Delta^P_{9\mu\nu} &= \epsilon_{abc}S^{ABC}\epsilon_{\mu\nu\rho\sigma}(q^{aT}_A C\sigma_{\rho l} q^b_B)\sigma_{\sigma l} q^c_C.
\end{aligned} \tag{B3}$$

As before, the Fierz transformed fields can be related to the corresponding unprimed ones, $\Delta^{P\prime}_{i\mu\mu} = -\Delta^P_{i\mu\mu}$. On the other hand, by applying the Fierz rearrangement to relate $\Delta^P_{i\mu\nu}$ and $\Delta^{P\prime}_{i\mu\nu}$, we obtain two independent fields: $\Delta^P_{1\mu\nu}$ and $\Delta^P_{7\mu\nu}$:

$$\Delta^P_{3\mu\nu} = -i\Delta^P_{1\mu\nu}, \qquad \Delta^P_{8\mu\nu} = i\Delta^P_{1\mu\nu} + \Delta^P_{7\mu\nu}, \qquad \Delta^P_{9\mu\nu} = -i\Delta^P_{1\mu\nu} - 2\Delta^P_{7\mu\nu}.$$

The first one $\Delta^P_{1\mu\nu}$ can be related to the Rarita-Schwinger baryon fields, but the second one $\Delta^P_{7\mu\nu}$ cannot. Therefore, we obtain one extra decuplet fields. It is formed by using the projection operator:

$$\Gamma^{\mu\nu\alpha\beta} = (g^{\mu\alpha}g^{\nu\beta} - \tfrac{1}{2}g^{\nu\beta}\gamma^\mu\gamma^\alpha + \tfrac{1}{2}g^{\mu\beta}\gamma^\nu\gamma^\alpha + \tfrac{1}{6}\sigma^{\mu\nu}\sigma^{\alpha\beta}),$$

as

$$\Delta^P_{\mu\nu} = \Gamma^{\mu\nu\alpha\beta}\Delta^P_{7\mu\nu} = \Gamma^{\mu\nu\alpha\beta}\epsilon_{abc}S^{ABC}(q^{aT}_A C\sigma_{\mu\nu} q^b_B)\gamma_5 q^c_C = \Delta^P_{7\mu\nu} - \frac{i}{2}\gamma_\mu\gamma_5\Delta^P_{5\nu} + \frac{i}{2}\gamma_\nu\gamma_5\Delta^P_{5\mu}.$$

3. Flavor octet baryon

To study the octet baryon fields, we start with nine octet baryon fields

$$\begin{aligned}
N^N_{1\mu\nu} &= \epsilon_{abc}\epsilon^{ABD}\lambda^N_{DC}(q^{aT}_A C\gamma_\mu q^b_B)\gamma_\nu\gamma_5 q^c_C - (\mu \leftrightarrow \nu) = 0,\\
N^N_{2\mu\nu} &= \epsilon_{abc}\epsilon^{ABD}\lambda^N_{DC}(q^{aT}_A C\gamma_\mu\gamma_5 q^b_B)\gamma_\nu q^c_C - (\mu \leftrightarrow \nu),\\
N^N_{3\mu\nu} &= \epsilon_{abc}\epsilon^{ABD}\lambda^N_{DC}\epsilon_{\mu\nu\rho\sigma}(q^{aT}_A C\gamma_\rho q^b_B)\gamma_\sigma q^c_C = 0,\\
N^N_{4\mu\nu} &= \epsilon_{abc}\epsilon^{ABD}\lambda^N_{DC}\epsilon_{\mu\nu\rho\sigma}(q^{aT}_A C\gamma_\rho\gamma_5 q^b_B)\gamma_\sigma\gamma_5 q^c_C,\\
N^N_{5\mu\nu} &= \epsilon_{abc}\epsilon^{ABD}\lambda^N_{DC}(q^{aT}_A C q^b_B)\sigma_{\mu\nu}\gamma_5 q^c_C,\\
N^N_{6\mu\nu} &= \epsilon_{abc}\epsilon^{ABD}\lambda^N_{DC}(q^{aT}_A C\gamma_5 q^b_B)\sigma_{\mu\nu} q^c_C,\\
N^N_{7\mu\nu} &= \epsilon_{abc}\epsilon^{ABD}\lambda^N_{DC}(q^{aT}_A C\sigma_{\mu\nu} q^b_B)\gamma_5 q^c_C = 0,\\
N^N_{8\mu\nu} &= \epsilon_{abc}\epsilon^{ABD}\lambda^N_{DC}(q^{aT}_A C\sigma_{\mu\nu}\gamma_5 q^b_B) q^c_C = 0,\\
N^N_{9\mu\nu} &= \epsilon_{abc}\epsilon^{ABD}\lambda^N_{DC}\epsilon_{\mu\nu\rho\sigma}(q^{aT}_A C\sigma_{\rho l} q^b_B)\sigma_{\sigma l} q^c_C = 0.
\end{aligned} \tag{B4}$$

There are five zero fields, but the Fierz transformed ones are nonzero. By using the Jacobi identity in Eq. (17), we obtain

$$N^{N\prime}_{2\mu\nu} = -\tfrac{1}{2}N^N_{2\mu\nu}, \qquad N^{N\prime}_{4\mu\nu} = -\tfrac{1}{2}N^N_{4\mu\nu}, \qquad N^{N\prime}_{5\mu\nu} = -\tfrac{1}{2}N^N_{5\mu\nu}, \qquad N^{N\prime}_{6\mu\nu} = -\tfrac{1}{2}N^N_{6\mu\nu}.$$

Similarly, performing the Fierz transformation to relate $N^N_{i\mu\nu}$ and $N^{N\prime}_{i\mu\nu}$, we find that there are three independent fields $N^N_{2\mu\nu}$, $N^N_{5\mu\nu}$ and $N^N_{6\mu\nu}$. Here are the relations:

$$N^N_{4\mu\nu} = -iN^N_{2\mu\nu} - N^N_{5\mu\nu} + N^N_{6\mu\nu},$$

$$N^{N\prime}_{1\mu\nu} = -\frac{1}{2}N^N_{2\mu\nu} + iN^N_{5\mu\nu} - iN^N_{6\mu\nu},$$

$$N^{N\prime}_{3\mu\nu} = \frac{i}{2}N^N_{2\mu\nu} - \frac{1}{2}N^N_{5\mu\nu} + \frac{1}{2}N^N_{6\mu\nu},$$

$$N^{N\prime}_{7\mu\nu} = -\frac{i}{2}N^N_{2\mu\nu} - \frac{1}{2}N^N_{5\mu\nu},$$

$$N^{N\prime}_{8\mu\nu} = \frac{i}{2}N^N_{2\mu\nu} - \frac{1}{2}N^N_{6\mu\nu}, \qquad N^{N\prime}_{9\mu\nu} = -N^N_{5\mu\nu} - N^N_{6\mu\nu}. \tag{B5}$$

All these three fields can be related to the Dirac spinor and Rarita-Schwinger fields. Therefore, there are no extra octet fields.

APPENDIX C: FIERZ TRANSFORMATION

In this appendix, we list the Fierz transformations used in our calculation, which are proved by using MATHEMATICA [31]. Here we would like to show only the change in the structure of Lorentz indices of direct products of two Dirac matrices under the Fierz rearrangement. Therefore, in the following equations, we do not include the minus sign which arises from the exchange of quark fields. The formulas go for the three cases corresponding to the Dirac, Rarita-Schwinger and tensor fields when applied to three-quark fields.

(1) Products of two Dirac matrices without Lorentz indices:

$$\begin{pmatrix} \mathbf{1}\otimes\gamma_5 \\ \gamma_\mu\otimes\gamma^\mu\gamma_5 \\ \sigma_{\mu\nu}\otimes\sigma^{\mu\nu}\gamma_5 \\ \gamma_\mu\gamma_5\otimes\gamma^\mu \\ \gamma_5\otimes\mathbf{1} \end{pmatrix}_{ab,cd} = \begin{pmatrix} \frac{1}{4} & -\frac{1}{4} & \frac{1}{8} & \frac{1}{4} & \frac{1}{4} \\ -1 & -\frac{1}{2} & 0 & -\frac{1}{2} & 1 \\ 3 & 0 & -\frac{1}{2} & 0 & 3 \\ 1 & -\frac{1}{2} & 0 & -\frac{1}{2} & -1 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{8} & -\frac{1}{4} & \frac{1}{4} \end{pmatrix} \begin{pmatrix} \mathbf{1}\otimes\gamma_5 \\ \gamma_\mu\otimes\gamma^\mu\gamma_5 \\ \sigma_{\mu\nu}\otimes\sigma^{\mu\nu}\gamma_5 \\ \gamma_\mu\gamma_5\otimes\gamma^\mu \\ \gamma_5\otimes\mathbf{1} \end{pmatrix}_{ad,bc} \tag{C1}$$

(2) Products of two Dirac matrices with one Lorentz index:

$$\begin{pmatrix} \mathbf{1}\otimes\gamma^\mu \\ \gamma^\mu\otimes\mathbf{1} \\ \gamma_5\otimes\gamma_\mu\gamma_5 \\ \gamma_\mu\gamma_5\otimes\gamma_5 \\ \gamma^\nu\otimes\sigma_{\mu\nu} \\ \sigma_{\mu\nu}\otimes\gamma^\nu \\ \gamma^\nu\gamma_5\otimes\sigma_{\mu\nu}\gamma_5 \\ \sigma_{\mu\nu}\gamma_5\otimes\gamma^\nu\gamma_5 \end{pmatrix}_{ab,cd} = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & -\frac{1}{4} & -\frac{i}{4} & \frac{i}{4} & \frac{i}{4} & \frac{i}{4} \\ \frac{1}{4} & \frac{1}{4} & -\frac{1}{4} & \frac{1}{4} & \frac{i}{4} & -\frac{i}{4} & \frac{i}{4} & \frac{i}{4} \\ \frac{1}{4} & -\frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{i}{4} & \frac{i}{4} & -\frac{i}{4} & \frac{i}{4} \\ -\frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{i}{4} & \frac{i}{4} & \frac{i}{4} & -\frac{i}{4} \\ \frac{3i}{4} & -\frac{3i}{4} & -\frac{3i}{4} & -\frac{3i}{4} & -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & \frac{1}{4} \\ -\frac{3i}{4} & \frac{3i}{4} & -\frac{3i}{4} & -\frac{3i}{4} & -\frac{1}{4} & -\frac{1}{4} & \frac{1}{4} & -\frac{1}{4} \\ -\frac{3i}{4} & -\frac{3i}{4} & \frac{3i}{4} & -\frac{3i}{4} & -\frac{1}{4} & \frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} \\ -\frac{3i}{4} & -\frac{3i}{4} & -\frac{3i}{4} & \frac{3i}{4} & \frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} \end{pmatrix} \begin{pmatrix} \mathbf{1}\otimes\gamma^\mu \\ \gamma^\mu\otimes\mathbf{1} \\ \gamma_5\otimes\gamma_\mu\gamma_5 \\ \gamma_\mu\gamma_5\otimes\gamma_5 \\ \gamma^\nu\otimes\sigma_{\mu\nu} \\ \sigma_{\mu\nu}\otimes\gamma^\nu \\ \gamma^\nu\gamma_5\otimes\sigma_{\mu\nu}\gamma_5 \\ \sigma_{\mu\nu}\gamma_5\otimes\gamma^\nu\gamma_5 \end{pmatrix}_{ad,bc} \tag{C2}$$

(3) Products of two Dirac matrices with two antisymmetric Lorentz indices:

$$\begin{pmatrix} \mathbf{1}\otimes\sigma_{\mu\nu}\gamma_5 \\ \gamma_5\otimes\sigma_{\mu\nu} \\ \sigma_{\mu\nu}\otimes\gamma_5 \\ \sigma_{\mu\nu}\gamma_5\otimes\mathbf{1} \\ \epsilon_{\mu\nu\rho\sigma}\sigma_{\rho l}\otimes\sigma_{\sigma l} \\ \gamma_\mu\otimes\gamma_\nu\gamma_5-(\mu\leftrightarrow\nu) \\ \gamma_\mu\gamma_5\otimes\gamma_\nu-(\mu\leftrightarrow\nu) \\ \epsilon_{\mu\nu\rho\sigma}\gamma_\rho\otimes\gamma_\sigma \\ \epsilon_{\mu\nu\rho\sigma}\gamma_\rho\gamma_5\otimes\gamma_\sigma\gamma_5 \end{pmatrix}_{ab,cd} = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{i}{4} & -\frac{i}{4} & \frac{1}{4} & -\frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & -\frac{i}{4} & \frac{i}{4} & -\frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & -\frac{1}{4} & -\frac{i}{4} & \frac{i}{4} & \frac{1}{4} & -\frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & -\frac{1}{4} & \frac{i}{4} & -\frac{i}{4} & -\frac{1}{4} & \frac{1}{4} \\ 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 \\ -\frac{i}{2} & \frac{i}{2} & \frac{i}{2} & -\frac{i}{2} & 0 & 0 & 0 & \frac{i}{2} & \frac{i}{2} \\ \frac{i}{2} & -\frac{i}{2} & -\frac{i}{2} & \frac{i}{2} & 0 & 0 & 0 & \frac{i}{2} & \frac{i}{2} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & 0 & -\frac{i}{2} & -\frac{i}{2} & 0 & 0 \\ -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & 0 & -\frac{i}{2} & -\frac{i}{2} & 0 & 0 \end{pmatrix} \begin{pmatrix} \mathbf{1}\otimes\sigma_{\mu\nu}\gamma_5 \\ \gamma_5\otimes\sigma_{\mu\nu} \\ \sigma_{\mu\nu}\otimes\gamma_5 \\ \sigma_{\mu\nu}\gamma_5\otimes\mathbf{1} \\ \epsilon_{\mu\nu\rho\sigma}\sigma_{\rho l}\otimes\sigma_{\sigma l} \\ \gamma_\mu\otimes\gamma_\nu\gamma_5-(\mu\leftrightarrow\nu) \\ \gamma_\mu\gamma_5\otimes\gamma_\nu-(\mu\leftrightarrow\nu) \\ \epsilon_{\mu\nu\rho\sigma}\gamma_\rho\otimes\gamma_\sigma \\ \epsilon_{\mu\nu\rho\sigma}\gamma_\rho\gamma_5\otimes\gamma_\sigma\gamma_5 \end{pmatrix}_{ad,bc} \tag{C3}$$

APPENDIX D: CHIRAL PROPERTIES OF RARITA-SCHWINGER FIELDS

In this appendix, we study the chiral properties of Rarita-Schwinger fields. As previously described in Sec. IV, we only need to study the properties of $(LL)L$, $(LL)R$, $(LR)L$, and $(RL)L$.

1. Chiral properties of $(LL)L$

The chiral representations of $(LL)L$ are $(\mathbf{1},\mathbf{1}) \oplus (\mathbf{8},\mathbf{1}) \oplus (\mathbf{8},\mathbf{1}) \oplus (\mathbf{10},\mathbf{1})$. We will study them separately in the following.

(1) In principle, there are eight possibilities of making the Rarita-Schwinger fields as shown in Eq. (A2). However, the chiral representation $(\mathbf{1},\mathbf{1})$ has just two nonzero fields:

$$\begin{aligned}\Lambda_{L1\mu} &= \epsilon_{abc}\epsilon^{ABC}(L_A^{aT}CL_B^b)\gamma_\mu L_C^c,\\ \Lambda_{L2\mu} &= \epsilon_{abc}\epsilon^{ABC}(L_A^{aT}C\gamma_5L_B^b)\gamma_\mu\gamma_5L_C^c.\end{aligned} \tag{D1}$$

Similarly performing the Fierz transformation to relate $\Lambda_{Li\mu}$ and $\Lambda'_{Li\mu}$, we obtain the solution that all such kind of fields vanish.

(2) The chiral representation $(\mathbf{10},\mathbf{1})$ has two nonzero fields:

$$\begin{aligned}\Delta^P_{L7\mu} &= \epsilon_{abc}S_P^{ABC}(L_A^{aT}C\sigma_{\mu\nu}L_B^b)\gamma^\nu L_C^c,\\ \Delta^P_{L8\mu} &= \epsilon_{abc}S_P^{ABC}(L_A^{aT}C\sigma_{\mu\nu}\gamma_5L_B^b)\gamma^\nu\gamma_5L_C^c.\end{aligned} \tag{D2}$$

Similarly performing the Fierz transformation to relate $\Delta^P_{Li\mu}$ and $\Delta^{P\prime}_{Li\mu}$, we obtain the solution that all such kind of fields vanish.

(3) The chiral representation $(\mathbf{8},\mathbf{1})$ has two nonzero fields:

$$\begin{aligned}N^N_{L1\mu} &= \epsilon_{abc}\epsilon^{ABD}\lambda^N_{DC}(L_A^{aT}CL_B^b)\gamma_\mu L_C^c,\\ N^N_{L2\mu} &= \epsilon_{abc}\epsilon^{ABD}\lambda^N_{DC}(L_A^{aT}C\gamma_5L_B^b)\gamma_\mu\gamma_5L_C^c.\end{aligned} \tag{D3}$$

Similarly performing the Fierz transformation to relate $N^N_{Li\mu}$ and $N^{N\prime}_{Li\mu}$, we obtain the solution

$$N^N_{L2\mu} = N^N_{L1\mu}.$$

Others are just zero. There is only one nonvanishing octet baryon field.

2. Chiral properties of $(LL)R$, $(LR)L$, and $(RL)L$

The chiral representations of $(LL)R$, $(LR)L$, and $(RL)L$ are $(\bar{\mathbf{3}},\mathbf{3}) \oplus (\mathbf{6},\mathbf{3})$. We will study them separately in the following.

(1) The chiral representation $(\bar{\mathbf{3}},\mathbf{3}) \to \mathbf{1_f}$ has two nonzero components:

$$\begin{aligned}\Lambda_{M1\mu} &= \epsilon_{abc}\epsilon^{ABC}(L_A^{aT}CL_B^b)\gamma_\mu R_C^c,\\ \Lambda_{M2\mu} &= \epsilon_{abc}\epsilon^{ABC}(L_A^{aT}C\gamma_5L_B^b)\gamma_\mu\gamma_5R_C^c.\end{aligned} \tag{D4}$$

Similarly performing the Fierz transformation to relate $\Lambda_{Mi\mu}$ and $\Lambda'_{Mi\mu}$, we obtain the solution

$$\Lambda_{M1\mu} = -\Lambda_{M2\mu}.$$

Others are just zero. There is only one nonvanishing field. Others $(LR)L$ and $(RL)L$ can be related to this one.

(2) The chiral representation $(\mathbf{6},\mathbf{3}) \to \mathbf{10_f}$ has two nonzero components:

$$\begin{aligned}\Delta^P_{M7\mu} &= \epsilon_{abc}S^{ABC}(L_A^{aT}C\sigma_{\mu\nu}L_B^b)\gamma^\nu R_C^c,\\ \Delta^P_{M8\mu} &= \epsilon_{abc}S^{ABC}(L_A^{aT}C\sigma_{\mu\nu}\gamma_5L_B^b)\gamma^\nu\gamma_5R_C^c.\end{aligned} \tag{D5}$$

Others are just zero. Similarly performing the Fierz transformation to relate $\Delta^P_{Mi\mu}$ and $\Delta^{P\prime}_{Mi\mu}$, we obtain the solution

$$\Delta^P_{M7\mu} = -\Delta^P_{M8\mu}.$$

There is only one nonvanishing field. Others $(LR)L$ and $(RL)L$ can be related to this one.

(3) The chiral representations $(\bar{\mathbf{3}},\mathbf{3}) \to \mathbf{8_f}$ has only two nonzero interpolators:

$$\begin{aligned}N^N_{M1\mu} &= \epsilon_{abc}\epsilon^{ABD}\lambda^N_{DC}(L_A^{aT}CL_B^b)\gamma_\mu R_C^c,\\ N^N_{M2\mu} &= \epsilon_{abc}\epsilon^{ABD}\lambda^N_{DC}(L_A^{aT}C\gamma_5L_B^b)\gamma_\mu\gamma_5R_C^c.\end{aligned} \tag{D6}$$

Similarly performing the Fierz transformation to relate $N^N_{Mi\mu}$ and $N^{N\prime}_{Mi\mu}$, we obtain the solution

$$N^N_{M1\mu} = -N^N_{M2\mu}.$$

In order to study the chiral representations $(\mathbf{6},\mathbf{3}) \to \mathbf{8_f}$, we need to consider the second way (see the discussion in Sec. II B 3) which leads to four nonzero fields:

$$\begin{aligned}\tilde{N}^N_{M1\mu} &= \epsilon_{abc}\epsilon^{ACD}\lambda^N_{DB}(L_A^{aT}CL_B^b)\gamma_\mu R_C^c,\\ \tilde{N}^N_{M2\mu} &= \epsilon_{abc}\epsilon^{ACD}\lambda^N_{DB}(L_A^{aT}C\gamma_5L_B^b)\gamma_\mu\gamma_5R_C^c,\\ \tilde{N}^N_{M7\mu} &= \epsilon_{abc}\epsilon^{ACD}\lambda^N_{DB}(L_A^{aT}C\sigma_{\mu\nu}L_B^b)\gamma^\nu R_C^c,\\ \tilde{N}^N_{M8\mu} &= \epsilon_{abc}\epsilon^{ACD}\lambda^N_{DB}(L_A^{aT}C\sigma_{\mu\nu}\gamma_5L_B^b)\gamma^\nu\gamma_5R_C^c.\end{aligned} \tag{D7}$$

By using the Jacobi identity in Eq. (17), we obtain:

$$\tilde{N}^N_{M1\mu} = \tfrac{1}{2}N^N_{M1\mu}, \qquad \tilde{N}^N_{M2\mu} = \tfrac{1}{2}N^N_{M2\mu}.$$

Similarly performing the Fierz transformation to relate $\tilde{N}^N_{Mi\mu}$ and $\tilde{N}^{N\prime}_{Mi\mu}$, we obtain the solution

$$\begin{aligned}\tilde{N}^N_{M2\mu} &= -\tilde{N}^N_{M1\mu} = -\tfrac{1}{2}N^N_{M1\mu},\\ \tilde{N}^N_{M8\mu} &= -\tilde{N}^N_{M7\mu}.\end{aligned}$$

All together there are two nonvanishing independent fields: $\tilde{N}^N_{M1\mu}$ and $\tilde{N}^N_{M7\mu}$. $\tilde{N}^N_{M1\mu}$ is related to $N^N_{M1\mu}$, and so belongs to the chiral representation $((\bar{\mathbf{3}},\mathbf{3}))$.

However, the other $\tilde{N}^N_{M7\mu}$ cannot be related to $N^N_{Mi\mu}$, and so contains some $(\mathbf{6}, \mathbf{3})$ component. other Others $(LR)L$ and $(RL)L$ can be related to $(LL)R$. Chiral properties of the tensor fields can be also explored in completely the same procedure explained here. Therefore, we do not show this case any more.

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