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$I^G J^{PC} = 1^- 1^{-+}$ tetraquark states

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We study the tetraquark states with $I^G J^{PC} = 1^- 1^{-+}$ in the QCD sum rule. After exhausting all possible flavor structures, we analyze both the Shifman-Vainshtein-Zakharov (SVZ) and finite energy sum rules. Both approaches lead to a mass around 1.6 GeV for the state with the quark contents $qq\bar{q}\,\bar{q}$, and around 2.0 GeV for the state with the quark contents $qs\bar{q}\,\bar{s}$. The flavor structure $(\bar{\mathbf{3}} \otimes \bar{\mathbf{6}}) \oplus (\mathbf{6} \otimes \mathbf{3})$ is preferred. Our analysis strongly indicates that both $\pi_1(1600)$ and $\pi_1(2015)$ are also compatible with the exotic tetraquark interpretation, which are sometimes labeled as candidates of the 1^{-+} hybrid mesons. Moreover one of their dominant decay modes is a pair of axial-vector and pseudoscalar mesons such as $b_1(1235)\pi$, which is sometimes considered as the characteristic decay mode of the hybrid mesons.

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I. INTRODUCTION

Hadrons beyond the conventional quark model have been studied for more than thirty years. For example, Jaffe suggested the low-lying scalar mesons as good candidates of tetraquark states composed of strongly correlated diquarks in 1976 [1]. Especially, there may exist some low-lying exotic mesons with quantum numbers such as $(J^{PC}) = (1^{-+})$ which $\bar{q}q$ mesons cannot access [2,3]. However the hybrid mesons with explicit glue can carry such quantum numbers. The experimental establishment of these states is a direct proof of the glue degree of freedom in the low energy sector of QCD and of fundamental importance.

The mass of the nonstrange exotic hybrid meson from lattice QCD simulations includes: 2 GeV [4], 1.74 GeV [5], and 1.8 GeV [6]. The mass of its strange partner is 1.92 GeV [5] and 2 GeV [6]. The hybrid meson mass from the constituent glue model is 2 GeV [7] while the value from the flux tube model is around 1.9 GeV [8,9]. The prediction from the QCD sum rule approach is around 1.6 GeV [10,11]. However, Yang obtained a surprisingly low mass around 1.26 GeV for the 1^{-+} hybrid meson using QCD sum rule [12].

Up to now, there are several candidates of the exotic mesons with $I^G(J^{PC}) = 1^-(1^{-+})$ experimentally. They are $\pi_1(1400)$, $\pi_1(1600)$, and $\pi_1(2015)$. Their masses and widths are $(1376 \pm 17, 300 \pm 40)$ MeV, $(1653^{+18}_{-15}, 225^{+45}_{-28})$ MeV, and $(2014 \pm 20 \pm 16, 230 \pm 21 \pm 73)$ MeV, respectively [13]. $\pi_1(1400)$ was observed in the reactions $\pi^- p \to \eta\pi^0 n$ [14]; $\bar{p}p \to \pi^0\pi^0\eta$ and $\bar{p}n \to \pi^-\pi^0\eta$ [15]; $\pi^- p \to \eta\pi^- p$ [16]. $\pi_1(1600)$ was observed in the reaction $\pi^- p \to \eta'\pi^- p$ (η' decays to $\eta\pi^+\pi^-$ with a fraction 44.5%) [17]. Both $\pi_1(1600)$ and $\pi_1(2015)$ were observed in the reactions $\pi^- p \to \omega\pi^-\pi^0 p$ [18] and $\pi^- p \to \eta\pi^+\pi^-\pi^- p$ [19]. However, a more recent analysis of a higher statistics sample from E852 3π data found no evidence of $\pi_1(1600)$ [20]. All the above observations were from hadron-production experiments.

Recently, the CLAS Collaboration performed a photoproduction experiment to search for the 1^{-+} hybrid meson in the speculated 3π final state in the charge exchange reaction $\gamma p \to \pi^+\pi^+\pi^-(n)$ [21]. If $\pi_1(1600)$ was a hybrid state, it was expected to be produced with a strength near or much larger than 10% of the $a_2(1320)$ meson from the theoretical models [22]. However, $\pi_1(1600)$ was not observed with the expected strength. In fact its production rate is less than 2% of the $a_2(1320)$ meson. If the $\pi_1(1600)$ signal from the hadron-production experiments is not an artifact, the negative result of the photoproduction experiment suggests (1) either theoretical production rates are overestimated significantly or (2) $\pi_1(1600)$ is a meson with a different inner structure instead of a hybrid state.

In fact, the tetraquark states can also carry the exotic quantum numbers $I^G(J^{PC}) = 1^-(1^{-+})$. It is important to note that the gluon inside the hybrid meson can easily split into a pair of $q\bar{q}$. Therefore tetraquarks can always have the same quantum numbers as the hybrid mesons, including the exotic ones. Discovery of hadron candidates with $J^{PC} = 1^{-+}$ does not ensure that it is an exotic hybrid meson. One has to exclude the other possibilities including tetraquarks based on its mass, decay width, decay patterns, etc. This argument holds for all these claimed candidates of the hybrid meson.

Tetraquark states in general have a richer internal structure than ordinary $q\bar{q}$ states. For instance, a pair of quarks can be in channels which cannot be allowed in the ordinary hadrons. The richness of the structure introduces complication in theoretical studies. Therefore, one usually assumed one or a few particular configurations which are motivated by some intuitions.

Recently, we have developed a systematic method for the study of multiquark states in the QCD sum rule, and

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particular applications have been made for several tetraquark states [23–25]. Our method is essentially based on complete classification of independent currents. By making suitable linear combinations of the independent currents we can perform advanced analysis as compared with the analysis of using only one type of current which limits the potential of the operator product expansion (OPE), and sometimes leads to unphysical results.

In this paper, we first classify the flavor structure of the four-quark system with quantum numbers $J^{PC} = 1^{-+}$. We find that there are five isovector states. Then we construct tetraquark interpolating currents by using both diquark-antidiquark construction $[(qq)(\bar{q}\,\bar{q})]$ and quark-antiquark pairs $[(q\bar{q})(q\bar{q})]$. We verify that they are just different bases and can be related to each other. Therefore they lead to the same results. By using diquark-antidiquark currents, we perform the QCD sum rule analysis, and calculate their masses. Our results suggest that $\pi_1(1400)$ may not be explained by just using tetraquark structure, and $\pi_1(1600)$ and $\pi_1(2015)$ could be explained by the tetraquark mesons with quark contents $(qq)(\bar{q}\,\bar{q})$ and $(qs)(\bar{q}\,\bar{s})$, respectively. The diquark and antidiquark inside have a mixed flavor structure $(\bar{\mathbf{3}} \otimes \bar{\mathbf{6}}) \oplus (\mathbf{6} \otimes \mathbf{3})$.

This paper is organized as follows. In Sec. II, we construct the tetraquark currents using both diquark (qq) and antidiquark $(\bar{q}\,\bar{q})$ currents. The tetraquark currents constructed by using quark-antiquark $(\bar{q}q)$ pairs are shown in Appendix A. In Sec. III, we perform a QCD sum rule analysis by using these currents, and calculate their OPEs. In Sec. IV, the numerical result is obtained for their masses. In Sec. V, we use finite energy sum rule to calculate their masses again. We discuss the decay patterns of these 1^{-+} tetraquark states in Sec. VI. Section VII is a summary.

II. TETRAQUARK CURRENTS

In order to construct proper tetraquark currents, let us start with the consideration of the charge-conjugation symmetry. The charge-conjugation transformation changes diquarks into antidiquarks, while it maintains their flavor structures. If a tetraquark state has a definite charge-conjugation parity, either positive or negative, the internal diquark (qq) and antidiquark $(\bar{q}\,\bar{q})$ must have the same flavor symmetry, which is either symmetric flavor structure $\mathbf{6_f} \otimes \bar{\mathbf{6}}_\mathbf{f}$ (**S**) or antisymmetric flavor structure $\bar{\mathbf{3}}_\mathbf{f} \otimes \mathbf{3_f}$ (**A**), and cannot have mixed flavor symmetry neither $\bar{\mathbf{3}}_\mathbf{f} \otimes \bar{\mathbf{6}}_\mathbf{f}$ nor $\mathbf{6_f} \otimes \mathbf{3_f}$ (**M**). However, combinations of $\bar{\mathbf{3}}_\mathbf{f} \otimes \bar{\mathbf{6}}_\mathbf{f}$ and $\mathbf{6_f} \otimes \mathbf{3_f}$ can have a definite charge-conjugation parity. Therefore, in order to study the tetraquark state of $I^G J^{PC} = 1^- 1^{-+}$, we need to consider the following structures of currents:

$$
\begin{aligned}
qq\bar{q}\,\bar{q}(\mathbf{S}),\, qs\bar{q}\,\bar{s}(\mathbf{S}) &\sim \mathbf{6_f} \otimes \bar{\mathbf{6}}_\mathbf{f} \quad (\mathbf{S}),\\
qs\bar{q}\,\bar{s}(\mathbf{A}) &\sim \bar{\mathbf{3}}_\mathbf{f} \otimes \mathbf{3_f} \quad (\mathbf{A}),\\
qq\bar{q}\,\bar{q}(\mathbf{M}),\, qs\bar{q}\,\bar{s}(\mathbf{M}) &\sim (\bar{\mathbf{3}}_\mathbf{f} \otimes \bar{\mathbf{6}}_\mathbf{f}) \oplus (\mathbf{6_f} \otimes \mathbf{3_f}) \quad (\mathbf{M}),
\end{aligned}
$$

where q represents an *up* or *down* quark, and s represents a *strange* quark. The flavor structures are shown in Fig. 1 in terms of $SU(3)$ weight diagrams. The quark contents indicated at vertices follow the ideal mixing scheme for inner vertices where the mixing is allowed. In the $SU(3)$ limit, the quark contents are suitable combinations of the ones shown in this figure. However, the strange quark has a significantly larger mass than up and down quarks (current quark mass), and so, the ideal mixing is expected to work well for hadrons except for pseudoscalar mesons. The flavor structure in the ideal mixing is also simpler than that in the $SU(3)$ limit. Therefore, we will use the ideal mixing in our QCD sum rule studies.

In the following subsections, we first construct currents by using diquark (qq) and antidiquark $(\bar{q}\,\bar{q})$ currents, and then we show the currents with explicit quark contents. The currents constructed by using quark-antiquark $(\bar{q}q)$ pairs can be related to these diquark currents, and are shown in the Appendix A. The tensor currents $\eta_{\mu\nu}$ $(\eta_{\mu\nu} = -\eta_{\nu\mu})$

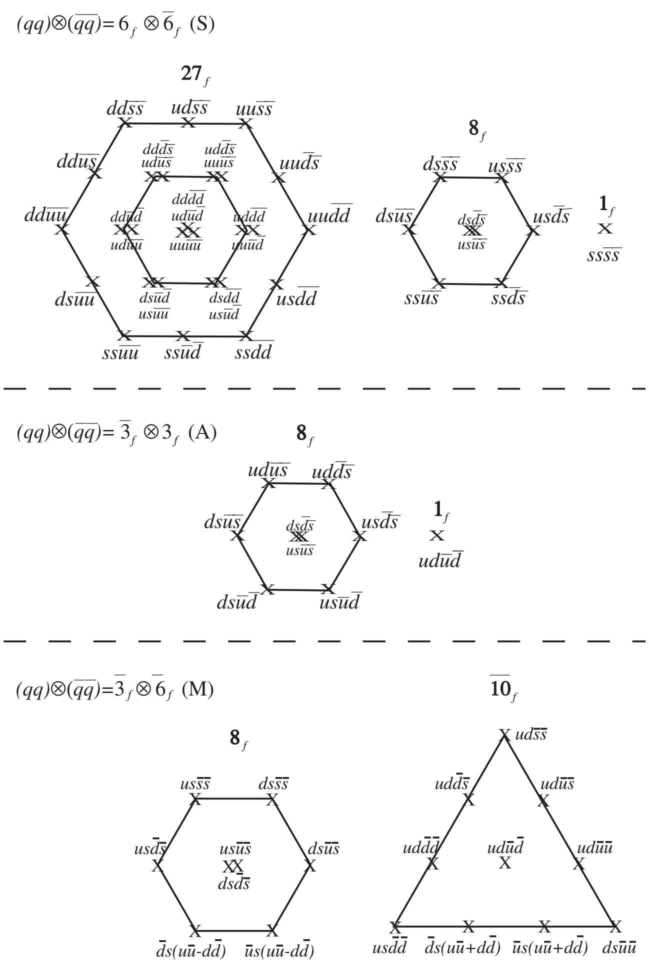

FIG. 1. Weight diagrams for $\mathbf{6_f} \otimes \bar{\mathbf{6}}_\mathbf{f}(\mathbf{S})$ (top panel), $\bar{\mathbf{3}}_\mathbf{f} \otimes \mathbf{3_f}(\mathbf{A})$ (middle panel), and $\bar{\mathbf{3}}_\mathbf{f} \otimes \bar{\mathbf{6}}_\mathbf{f}(\mathbf{M})$ (bottom panel). The weight diagram for $\mathbf{6_f} \otimes \mathbf{3_f}(\mathbf{M})$ is the charge-conjugation transformation of the bottom one.

can also have $I^G J^{PC} = 1^- 1^{-+}$. By using tensor currents, we obtain the similar results, which will be shown in our future work.

A. $(qq)(\bar{q}\,\bar{q})$ currents

We attempt to construct the tetraquark currents using diquark (qq) and antidiquark $(\bar{q}\,\bar{q})$ currents. For each state having the symmetric flavor structure $\mathbf{6_f} \otimes \bar{\mathbf{6}}_\mathbf{f}$ (**S**), there are two $(qq)(\bar{q}\,\bar{q})$ currents of $J^{PC} = 1^{-+}$, which are independent

$$\begin{aligned} \psi^S_{1\mu} &= q^T_{1a} C\gamma_5 q_{2b}(\bar{q}_{3a}\gamma_\mu\gamma_5 C\bar{q}^T_{4b} + \bar{q}_{3b}\gamma_\mu\gamma_5 C\bar{q}^T_{4a}) \\ &\quad + q^T_{1a} C\gamma_\mu\gamma_5 q_{2b}(\bar{q}_{3a}\gamma_5 C\bar{q}^T_{4b} + \bar{q}_{3b}\gamma_5 C\bar{q}^T_{4a}), \\ \psi^S_{2\mu} &= q^T_{1a} C\gamma^\nu q_{2b}(\bar{q}_{3a}\sigma_{\mu\nu} C\bar{q}^T_{4b} - \bar{q}_{3b}\sigma_{\mu\nu} C\bar{q}^T_{4a}) \\ &\quad + q^T_{1a} C\sigma_{\mu\nu} q_{2b}(\bar{q}_{3a}\gamma^\nu C\bar{q}^T_{4b} - \bar{q}_{3b}\gamma^\nu C\bar{q}^T_{4a}), \end{aligned} \tag{1}$$

where the sum over repeated indices ($\mu, \nu, \cdots$ for Dirac spinor indices, and $a, b, \cdots$ for color indices) is taken. C is the charge-conjugation matrix, q_1 and q_2 represent quarks, and q_3 and q_4 represent antiquarks. For the antisymmetry flavor structure $\bar{\mathbf{3}}_\mathbf{f} \otimes \mathbf{3_f}$ (**A**), we also find that there are two independent $(qq)(\bar{q}\,\bar{q})$ currents,

$$\begin{aligned} \psi^A_{1\mu} &= q^T_{1a} C\gamma_5 q_{2b}(\bar{q}_{3a}\gamma_\mu\gamma_5 C\bar{q}^T_{4b} - \bar{q}_{3b}\gamma_\mu\gamma_5 C\bar{q}^T_{4a}) \\ &\quad + q^T_{1a} C\gamma_\mu\gamma_5 q_{2b}(\bar{q}_{3a}\gamma_5 C\bar{q}^T_{4b} - \bar{q}_{3b}\gamma_5 C\bar{q}^T_{4a}), \\ \psi^A_{2\mu} &= q^T_{1a} C\gamma^\nu q_{2b}(\bar{q}_{3a}\sigma_{\mu\nu} C\bar{q}^T_{4b} + \bar{q}_{3b}\sigma_{\mu\nu} C\bar{q}^T_{4a}) \\ &\quad + q^T_{1a} C\sigma_{\mu\nu} q_{2b}(\bar{q}_{3a}\gamma^\nu C\bar{q}^T_{4b} + \bar{q}_{3b}\gamma^\nu C\bar{q}^T_{4a}). \end{aligned} \tag{2}$$

For each state containing a diquark and antidiquark having either the flavor structure $\bar{\mathbf{3}}_\mathbf{f} \otimes \bar{\mathbf{6}}_\mathbf{f}$ or $\mathbf{6_f} \otimes \mathbf{3_f}$, there are no currents of quantum numbers $J^{PC} = 1^{-+}$. However, their combinations $(\bar{\mathbf{3}}_\mathbf{f} \otimes \bar{\mathbf{6}}_\mathbf{f}) \oplus (\mathbf{6_f} \otimes \mathbf{3_f})$ can have the quantum numbers $J^{PC} = 1^{-+}$. We first define the currents $\psi^{ML}_{i\mu}$ which belong to the flavor representation $\bar{\mathbf{3}}_\mathbf{f} \otimes \bar{\mathbf{6}}_\mathbf{f}$, and the currents $\psi^{MR}_{i\mu}$ which belong to the flavor representation $\mathbf{6_f} \otimes \mathbf{3_f}$ separately. We find the following four independent currents:

$$\begin{aligned} \psi^{ML}_{1\mu} &= q^T_{1a} C\gamma_\mu q_{2b}(\bar{q}_{3a} C\bar{q}^T_{4b} + \bar{q}_{3b} C\bar{q}^T_{4a}), \\ \psi^{ML}_{2\mu} &= q^T_{1a} C\sigma_{\mu\nu}\gamma_5 q_{2b}(\bar{q}_{3a}\gamma^\nu\gamma_5 C\bar{q}^T_{4b} + \bar{q}_{3b}\gamma^\nu\gamma_5 C\bar{q}^T_{4a}), \\ \psi^{ML}_{3\mu} &= q^T_{1a} C q_{2b}(\bar{q}_{3a}\gamma_\mu C\bar{q}^T_{4b} - \bar{q}_{3b}\gamma_\mu C\bar{q}^T_{4a}), \\ \psi^{ML}_{4\mu} &= q^T_{1a} C\gamma^\nu\gamma_5 q_{2b}(\bar{q}_{3a}\sigma_{\mu\nu}\gamma_5 C\bar{q}^T_{4b} - \bar{q}_{3b}\sigma_{\mu\nu}\gamma_5 C\bar{q}^T_{4a}), \end{aligned}$$

$$\begin{aligned} \psi^{MR}_{1\mu} &= q^T_{1a} C q_{2b}(\bar{q}_{3a}\gamma_\mu C\bar{q}^T_{4b} + \bar{q}_{3b}\gamma_\mu C\bar{q}^T_{4a}), \\ \psi^{MR}_{2\mu} &= q^T_{1a} C\gamma^\nu\gamma_5 q_{2b}(\bar{q}_{3a}\sigma_{\mu\nu}\gamma_5 C\bar{q}^T_{4b} + \bar{q}_{3b}\sigma_{\mu\nu}\gamma_5 C\bar{q}^T_{4a}), \\ \psi^{MR}_{3\mu} &= q^T_{1a} C\gamma_\mu q_{2b}(\bar{q}_{3a} C\bar{q}^T_{4b} - \bar{q}_{3b} C\bar{q}^T_{4a}), \\ \psi^{MR}_{4\mu} &= q^T_{1a} C\sigma_{\mu\nu}\gamma_5 q_{2b}(\bar{q}_{3a}\gamma^\nu\gamma_5 C\bar{q}^T_{4b} - \bar{q}_{3b}\gamma^\nu\gamma_5 C\bar{q}^T_{4a}). \end{aligned}$$

They all have quantum numbers $J^P = 1^-$ but no good charge-conjugation parity. However, their mixing can have a definite charge-conjugation parity,

$$\psi^M_{i\mu} = \psi^{ML}_{i\mu} \pm \psi^{MR}_{i\mu}, \tag{3}$$

where the $+$ and $-$ combinations correspond to the charge-conjugation parity positive and negative, respectively. In the present work, we only consider the positive one.

B. Isovector currents

For the study of the present exotic tetraquark state, we need to construct isovector ($I = 1$) currents. There are two isospin triplets belonging to the flavor representation $\mathbf{6}_f \otimes \bar{\mathbf{6}}_f$, one isospin triplet belonging to the flavor representation $\bar{\mathbf{3}}_f \otimes \mathbf{3}_f$, and two isospin triplets belonging to the flavor representation $(\bar{\mathbf{3}}_f \otimes \bar{\mathbf{6}}_f) \oplus (\mathbf{6}_f \otimes \mathbf{3}_f)$ (Fig. 1). For each state, there are several independent currents. We list them in the following.

(1) For the two isospin triplets belonging to $\mathbf{6}_f \otimes \bar{\mathbf{6}}_f$ (**S**):

$$\begin{cases} \eta^S_{1\mu} \equiv \psi^S_{1\mu}(qq\bar{q}\,\bar{q}) \sim u^T_a C\gamma_5 d_b(\bar{u}_a\gamma_\mu\gamma_5 C\bar{d}^T_b + \bar{u}_b\gamma_\mu\gamma_5 C\bar{d}^T_a) + u^T_a C\gamma_\mu\gamma_5 d_b(\bar{u}_a\gamma_5 C\bar{d}^T_b + \bar{u}_b\gamma_5 C\bar{d}^T_a), \\ \eta^S_{2\mu} \equiv \psi^S_{2\mu}(qq\bar{q}\,\bar{q}) \sim u^T_a C\gamma^\nu d_b(\bar{u}_a\sigma_{\mu\nu} C\bar{d}^T_b - \bar{u}_b\sigma_{\mu\nu} C\bar{d}^T_a) + u^T_a C\sigma_{\mu\nu} d_b(\bar{u}_a\gamma^\nu C\bar{d}^T_b - \bar{u}_b\gamma^\nu C\bar{d}^T_a), \end{cases}$$

$$\begin{cases} \eta^S_{3\mu} \equiv \psi^S_{1\mu}(qs\bar{q}\,\bar{s}) \sim u^T_a C\gamma_5 s_b(\bar{u}_a\gamma_\mu\gamma_5 C\bar{s}^T_b + \bar{u}_b\gamma_\mu\gamma_5 C\bar{s}^T_a) + u^T_a C\gamma_\mu\gamma_5 s_b(\bar{u}_a\gamma_5 C\bar{s}^T_b + \bar{u}_b\gamma_5 C\bar{s}^T_a), \\ \eta^S_{4\mu} \equiv \psi^S_{2\mu}(qs\bar{q}\,\bar{s}) \sim u^T_a C\gamma^\nu s_b(\bar{u}_a\sigma_{\mu\nu} C\bar{s}^T_b - \bar{u}_b\sigma_{\mu\nu} C\bar{s}^T_a) + u^T_a C\sigma_{\mu\nu} s_b(\bar{u}_a\gamma^\nu C\bar{s}^T_b - \bar{u}_b\gamma^\nu C\bar{s}^T_a), \end{cases}$$

where $\eta^S_{1\mu}$ and $\eta^S_{2\mu}$ are the two independent currents containing only light flavors, and $\eta^S_{3\mu}$ and $\eta^S_{4\mu}$ are the two independent ones containing one $s\bar{s}$ quark pair.

(2) For the isospin triplet belonging to $\bar{\mathbf{3}}_f \otimes \mathbf{3}_f$ (**A**):

$$\begin{cases} \eta^A_{1\mu} \equiv \psi^A_{1\mu}(qs\bar{q}\,\bar{s}) \sim u^T_a C\gamma_5 s_b(\bar{u}_a\gamma_\mu\gamma_5 C\bar{s}^T_b - \bar{u}_b\gamma_\mu\gamma_5 C\bar{s}^T_a) + u^T_a C\gamma_\mu\gamma_5 s_b(\bar{u}_a\gamma_5 C\bar{s}^T_b - \bar{u}_b\gamma_5 C\bar{s}^T_a), \\ \eta^A_{2\mu} \equiv \psi^A_{2\mu}(qs\bar{q}\,\bar{s}) \sim u^T_a C\gamma^\nu s_b(\bar{u}_a\sigma_{\mu\nu} C\bar{s}^T_b + \bar{u}_b\sigma_{\mu\nu} C\bar{s}^T_a) + u^T_a C\sigma_{\mu\nu} s_b(\bar{u}_a\gamma^\nu C\bar{s}^T_b + \bar{u}_b\gamma^\nu C\bar{s}^T_a), \end{cases}$$

where $\eta^A_{1\mu}$ and $\eta^A_{2\mu}$ are the two independent currents.

(3) For the two isospin triplets belonging to $(\bar{\mathbf{3}}_f \otimes \bar{\mathbf{6}}_f) \oplus (\mathbf{6}_f \otimes \mathbf{3}_f)$ (**M**):

$$\begin{cases} \eta^M_{1\mu} \equiv \psi^M_{1\mu}(qq\bar q\bar q) \sim u_a^T C\gamma_\mu d_b(\bar u_a C\bar d_b^T + \bar u_b C\bar d_a^T) + u_a^T C d_b(\bar u_a \gamma_\mu C\bar d_b^T + \bar u_b\gamma_\mu C\bar d_a^T), \\ \eta^M_{2\mu} \equiv \psi^M_{2\mu}(qq\bar q\bar q) \sim u_a^T C\sigma_{\mu\nu}\gamma_5 d_b(\bar u_a\gamma^\nu\gamma_5 C\bar d_b^T + \bar u_b\gamma^\nu\gamma_5 C\bar d_a^T) + u_a^T C\gamma^\nu\gamma_5 d_b(\bar u_a\sigma_{\mu\nu}\gamma_5 C\bar d_b^T + \bar u_b\sigma_{\mu\nu}\gamma_5 C\bar d_a^T), \\ \eta^M_{3\mu} \equiv \psi^M_{3\mu}(qq\bar q\bar q) \sim u_a^T C d_b(\bar u_a\gamma_\mu C\bar d_b^T - \bar u_b\gamma_\mu C\bar d_a^T) + u_a^T C\gamma_\mu d_b(\bar u_a C\bar d_b^T - \bar u_b C\bar d_a^T), \\ \eta^M_{4\mu} \equiv \psi^M_{4\mu}(qq\bar q\bar q) \sim u_a^T C\gamma^\nu\gamma_5 d_b(\bar u_a\sigma_{\mu\nu}\gamma_5 C\bar d_b^T - \bar u_b\sigma_{\mu\nu}\gamma_5 C\bar d_a^T) + u_a^T C\sigma_{\mu\nu}\gamma_5 d_b(\bar u_a\gamma^\nu\gamma_5 C\bar d_b^T - \bar u_b\gamma^\nu\gamma_5 C\bar d_a^T), \end{cases}$$

$$\begin{cases} \eta^M_{5\mu} \equiv \psi^M_{1\mu}(qs\bar q\bar s) \sim u_a^T C\gamma_\mu s_b(\bar u_a C\bar s_b^T + \bar u_b C\bar s_a^T) + u_a^T C s_b(\bar u_a \gamma_\mu C\bar s_b^T + \bar u_b\gamma_\mu C\bar s_a^T), \\ \eta^M_{6\mu} \equiv \psi^M_{2\mu}(qs\bar q\bar s) \sim u_a^T C\sigma_{\mu\nu}\gamma_5 s_b(\bar u_a\gamma^\nu\gamma_5 C\bar s_b^T + \bar u_b\gamma^\nu\gamma_5 C\bar s_a^T) + u_a^T C\gamma^\nu\gamma_5 s_b(\bar u_a\sigma_{\mu\nu}\gamma_5 C\bar s_b^T + \bar u_b\sigma_{\mu\nu}\gamma_5 C\bar s_a^T), \\ \eta^M_{7\mu} \equiv \psi^M_{3\mu}(qs\bar q\bar s) \sim u_a^T C s_b(\bar u_a\gamma_\mu C\bar s_b^T - \bar u_b\gamma_\mu C\bar s_a^T) + u_a^T C\gamma_\mu s_b(\bar u_a C\bar s_b^T - \bar u_b C\bar s_a^T), \\ \eta^M_{8\mu} \equiv \psi^M_{4\mu}(qs\bar q\bar s) \sim u_a^T C\gamma^\nu\gamma_5 s_b(\bar u_a\sigma_{\mu\nu}\gamma_5 C\bar s_b^T - \bar u_b\sigma_{\mu\nu}\gamma_5 C\bar s_a^T) + u_a^T C\sigma_{\mu\nu}\gamma_5 s_b(\bar u_a\gamma^\nu\gamma_5 C\bar s_b^T - \bar u_b\gamma^\nu\gamma_5 C\bar s_a^T), \end{cases}$$

where $\eta^M_{1,2,3,4}$ are the four independent currents containing only light flavors, and $\eta^M_{1,2,3,4}$ are the four independent ones containing one $s\bar s$ quark pair.

We use $\sim$ to make clear that the quark contents here are not exactly correct. For instance, in the current $\eta^A_{1\mu}$, the state $us\bar u\bar s$ does not have an isospin one. The correct quark contents should be ($us\bar u\bar s - ds\bar d\bar s$). However, in the following QCD sum rule analysis, we shall not include the mass of up and down quarks and choose the same value for $\langle\bar u u\rangle$ and $\langle\bar d d\rangle$. Therefore, the QCD sum rule results for η^A_1 with quark contents $us\bar u\bar s$ and ($us\bar u\bar s - ds\bar d\bar s$) are the same.

III. SVZ SUM RULE

For the past decades QCD sum rule has proven to be a very powerful and successful nonperturbative method [26,27]. In sum rule analyses, we consider two-point correlation functions:

$$\Pi_{\mu\nu}(q^2) \equiv i\int d^4x e^{iqx}\langle 0|T\eta_\mu(x)\eta_\nu^\dagger(0)|0\rangle, \tag{4}$$

where η_μ is an interpolating current for the tetraquark. The Lorentz structure can be simplified to be

$$\Pi_{\mu\nu}(q^2) = \left(\frac{q_\mu q_\nu}{q^2} - g_{\mu\nu}\right)\Pi^{(1)}(q^2) + \frac{q_\mu q_\nu}{q^2}\Pi^{(0)}(q^2). \tag{5}$$

We compute $\Pi(q^2)$ in the operator product expansion (OPE) of QCD up to a certain order in the expansion, which is then matched with a hadronic parametrization to extract information of hadron properties. At the hadron level, we express the correlation function in the form of the dispersion relation with a spectral function:

$$\Pi^{(1)}(q^2) = \int_{s_<}^\infty \frac{\rho(s)}{s - q^2 - i\varepsilon} ds, \tag{6}$$

where the integration starts from the mass square of all current quarks. The spectral density $\rho(s)$ is defined to be

$$\rho(s) \equiv \sum_n \delta(s - M_n^2)\langle 0|\eta|n\rangle\langle n|\eta^\dagger|0\rangle = f_Y^2\delta(s - M_Y^2) + \text{higher states}. \tag{7}$$

For the second equation, as usual, we adopt a parametrization of one pole dominance for the ground state Y and a continuum contribution. The sum rule analysis is then performed after the Borel transformation of the two expressions of the correlation function, (4) and (6)

$$\Pi^{(\mathrm{all})}(M_B^2) \equiv \mathcal{B}_{M_B^2}\Pi^{(1)}(p^2) = \int_{s_<}^\infty e^{-s/M_B^2}\rho(s)ds. \tag{8}$$

Assuming the contribution from the continuum states can be approximated well by the spectral density of OPE above a threshold value s_0 (duality), we arrive at the sum rule equation

$$\Pi(M_B^2) \equiv f_Y^2 e^{-M_Y^2/M_B^2} = \int_{s_<}^{s_0} e^{-s/M_B^2}\rho(s)ds. \tag{9}$$

Differentiating Eq. (9) with respect to $1/M_B^2$ and dividing it by Eq. (9), finally we obtain

$$M_Y^2 = \frac{\frac{\partial}{\partial(-1/M_B^2)}\Pi(M_B^2)}{\Pi(M_B^2)} = \frac{\int_{s_<}^{s_0} e^{-s/M_B^2}s\rho(s)ds}{\int_{s_<}^{s_0} e^{-s/M_B^2}\rho(s)ds}. \tag{10}$$

In the following, we study both Eqs. (9) and (10) as functions of the parameters such as the Borel mass M_B and the threshold value s_0 for various combinations of the tetraquark currents.

We have performed the OPE calculation up to dimension 12. Here we only show the results for currents η^M_1 and η^M_5, which have quark contents $qq\bar q\bar q$ and $qs\bar q\bar s$, respectively. Others are shown in Appendix B.

$$\begin{aligned}\Pi^M_1(M_B^2) = {} & \int_0^{s_0}\left[\frac{1}{18432\pi^6}s^4 - \frac{\langle g_s^2 GG\rangle}{18432\pi^6}s^2 + \frac{\langle\bar qq\rangle^2}{18\pi^2}s + \frac{\langle\bar qq\rangle\langle g_s\bar q\sigma Gq\rangle}{12\pi^2}\right]e^{-s/M_B^2}ds \\ & + \left(\frac{\langle g_s\bar q\sigma Gq\rangle^2}{48\pi^2} - \frac{5\langle g_s^2 GG\rangle\langle\bar qq\rangle^2}{864\pi^2}\right) \\ & + \frac{1}{M_B^2}\left(-\frac{32g_s^2\langle\bar qq\rangle^4}{81} + \frac{\langle g_s^2 GG\rangle\langle\bar qq\rangle\langle g_s\bar q\sigma Gq\rangle}{576\pi^2}\right),\end{aligned} \tag{11}$$

$$\Pi_5^M(M_B^2) = \int_{4m_s^2}^{s_0}\bigg[\frac{1}{18\,432\pi^6}s^4 - \frac{17m_s^2}{7680\pi^6}s^3 + \bigg(-\frac{\langle g_s^2GG\rangle}{18\,432\pi^6} - \frac{m_s\langle\bar{q}q\rangle}{96\pi^4} + \frac{m_s\langle\bar{s}s\rangle}{48\pi^4}\bigg)s^2 + \bigg(-\frac{\langle\bar{q}q\rangle^2}{36\pi^2} + \frac{\langle\bar{q}q\rangle\langle\bar{s}s\rangle}{9\pi^2} - \frac{\langle\bar{s}s\rangle^2}{36\pi^2} - \frac{m_s\langle g_s\bar{q}\sigma Gq\rangle}{48\pi^4} + \frac{m_s\langle g_s\bar{s}\sigma Gs\rangle}{96\pi^4} + \frac{m_s^2\langle g_s^2GG\rangle}{4608\pi^6}\bigg)s$$
$$- \frac{\langle\bar{q}q\rangle\langle g_s\bar{q}\sigma Gq\rangle}{24\pi^2} + \frac{\langle\bar{q}q\rangle\langle g_s\bar{s}\sigma Gs\rangle}{12\pi^2} + \frac{\langle\bar{s}s\rangle\langle g_s\bar{q}\sigma Gq\rangle}{12\pi^2} - \frac{\langle\bar{s}s\rangle\langle g_s\bar{s}\sigma Gs\rangle}{24\pi^2} + \frac{m_s\langle g_s^2GG\rangle\langle\bar{q}q\rangle}{256\pi^4} - \frac{m_s^2\langle\bar{q}q\rangle^2}{6\pi^2} - \frac{m_s^2\langle\bar{q}q\rangle\langle\bar{s}s\rangle}{2\pi^2} + \frac{m_s^2\langle\bar{s}s\rangle^2}{24\pi^2}\bigg]e^{-s/M_B^2}ds$$
$$+ \bigg(-\frac{\langle g_s\bar{q}\sigma Gq\rangle^2}{96\pi^2} + \frac{\langle g_s\bar{q}\sigma Gq\rangle\langle g_s\bar{s}\sigma Gs\rangle}{24\pi^2} - \frac{\langle g_s\bar{s}\sigma Gs\rangle^2}{96\pi^2} - \frac{5\langle g_s^2GG\rangle\langle\bar{q}q\rangle\langle\bar{s}s\rangle}{864\pi^2} + \frac{2m_s\langle\bar{q}q\rangle^2\langle\bar{s}s\rangle}{3} + \frac{4m_s\langle\bar{q}q\rangle\langle\bar{s}s\rangle^2}{9} + \frac{5m_s\langle g_s^2GG\rangle\langle g_s\bar{q}\sigma Gq\rangle}{4608\pi^4} - \frac{m_s^2\langle\bar{s}s\rangle\langle g_s\bar{q}\sigma Gq\rangle}{4\pi^2} - \frac{m_s^2\langle\bar{q}q\rangle\langle g_s\bar{s}\sigma Gs\rangle}{6\pi^2}\bigg)$$
$$+ \frac{1}{M_B^2}\bigg(-\frac{32g_s^2\langle\bar{q}q\rangle^2\langle\bar{s}s\rangle^2}{81} + \frac{\langle g_s^2GG\rangle\langle\bar{q}q\rangle\langle g_s\bar{s}\sigma Gs\rangle}{1152\pi^2} + \frac{\langle g_s^2GG\rangle\langle\bar{s}s\rangle\langle g_s\bar{q}\sigma Gq\rangle}{1152\pi^2} - \frac{2m_s\langle\bar{q}q\rangle^2\langle g_s\bar{s}\sigma Gs\rangle}{9} - \frac{5m_s\langle\bar{q}q\rangle\langle\bar{s}s\rangle\langle g_s\bar{q}\sigma Gq\rangle}{9} + \frac{m_s\langle\bar{q}q\rangle\langle\bar{s}s\rangle\langle g_s\bar{s}\sigma Gs\rangle}{9} + \frac{m_s\langle\bar{s}s\rangle^2\langle g_s\bar{q}\sigma Gq\rangle}{9} + \frac{m_s^2\langle g_s\bar{q}\sigma Gq\rangle^2}{24\pi^2} - \frac{m_s^2\langle g_s\bar{q}\sigma Gq\rangle\langle g_s\bar{s}\sigma Gs\rangle}{24\pi^2}\bigg). \quad (12)$$

In the above equations, $\langle\bar{s}s\rangle$ is the dimension $D = 3$ strange quark condensate; $\langle g^2GG\rangle$ is a $D = 4$ gluon condensate; $\langle g\bar{s}\sigma Gs\rangle$ is a $D = 5$ mixed condensate. There are many terms which give minor contributions, such as $\langle g^3G^3\rangle$, and we omit them. As usual, we assume the vacuum saturation for higher dimensional condensates such as $\langle 0|\bar{q}q\bar{q}q|0\rangle \sim \langle 0|\bar{q}q|0\rangle \times \langle 0|\bar{q}q|0\rangle$. To obtain these results, we keep the terms of order $O(m_q^2)$ in the propagators of a massive quark in the presence of quark and gluon condensates:

$$iS^{ab} \equiv \langle 0|T[q^a(x)q^b(0)]|0\rangle = \frac{i\delta^{ab}}{2\pi^2x^4}\hat{x} + \frac{i}{32\pi^2}\frac{\lambda_{ab}^n}{2}g_cG_{\mu\nu}^n\frac{1}{x^2}(\sigma^{\mu\nu}\hat{x} + \hat{x}\sigma^{\mu\nu}) - \frac{\delta^{ab}}{12}\langle\bar{q}q\rangle + \frac{\delta^{ab}x^2}{192}\langle g_c\bar{q}\sigma Gq\rangle - \frac{m_q\delta^{ab}}{4\pi^2x^2} + \frac{i\delta^{ab}m_q\langle\bar{q}q\rangle}{48}\hat{x} + \frac{i\delta^{ab}m_q^2}{8\pi^2x^2}\hat{x}. \quad (13)$$

IV. NUMERICAL ANALYSIS

In our numerical analysis, we use the following values for various condensates and m_s at 1 GeV and α_s at 1.7 GeV [13,28–33]:

$$\begin{aligned}\langle\bar{q}q\rangle &= -(0.240\text{ GeV})^3,\\ \langle\bar{s}s\rangle &= -(0.8 \pm 0.1) \times (0.240\text{ GeV})^3,\\ \langle g_s^2GG\rangle &= (0.48 \pm 0.14)\text{ GeV}^4,\\ \langle g_s\bar{q}\sigma Gq\rangle &= -M_0^2 \times \langle\bar{q}q\rangle,\\ M_0^2 &= (0.8 \pm 0.2)\text{ GeV}^2,\\ m_s(1\text{ GeV}) &= 125 \pm 20\text{ MeV},\\ \alpha_s(1.7\text{ GeV}) &= 0.328 \pm 0.03 \pm 0.025.\end{aligned} \quad (14)$$

There is a minus sign in the definition of the mixed condensate $\langle g_s\bar{q}\sigma Gq\rangle$, which is different from that used in some other QCD sum rule studies. This difference just comes from the definition of coupling constant g_s [28,34].

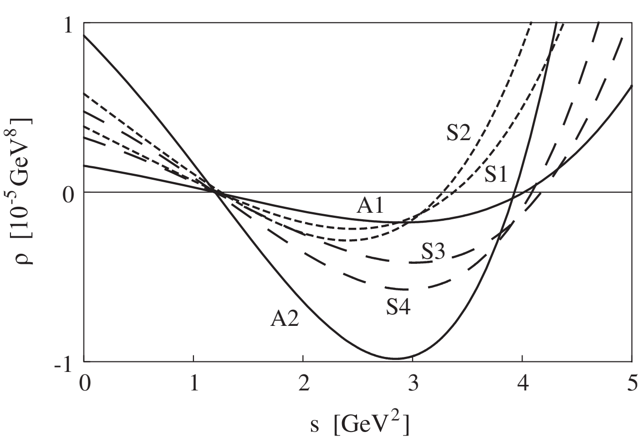

FIG. 2. Spectral densities for the current $\eta_{1\mu}^A$, $\eta_{2\mu}^A$ (solid lines), $\eta_{1\mu}^S$, $\eta_{2\mu}^S$ (short-dashed lines), $\eta_{3\mu}^S$ and $\eta_{4\mu}^S$ (long-dashed lines). The labels beside the lines indicate the flavor symmetry (S or A) and the suffix i of the current $\eta_{i\mu}^{S,A}$ ($i = 1, 2, 3, 4$).

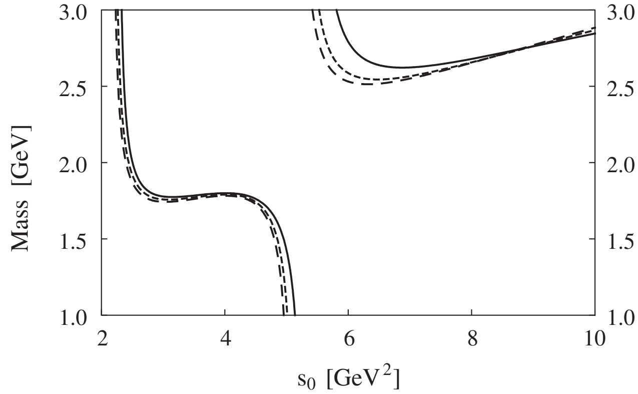

FIG. 3. The mass calculated by using the current $\eta^A_{1\mu}$, as functions of s_0 in units of GeV. The curves are obtained by setting $M_B^2 = 2$ GeV^2 (solid line), 3 GeV^2 (short-dashed line), and 4 GeV^2 (long-dashed line). The left curves (disconnected from the right part) are obtained from a negative correlation function, and have no physical meaning.

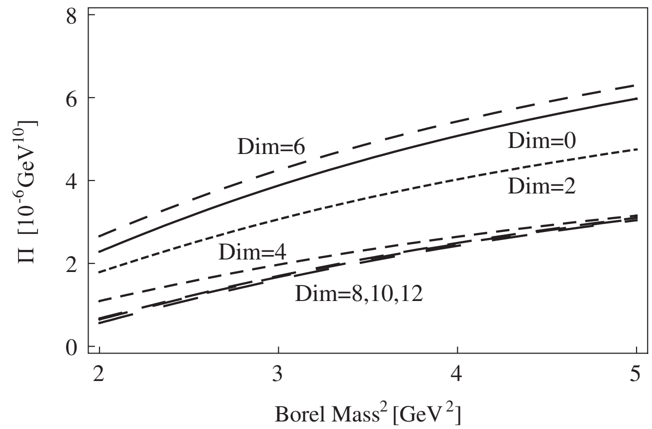

FIG. 5. The various contribution to the correlation function for the current $\eta^M_{5\mu}$ as functions of the Borel mass M_B in units of GeV^{1}0 at $s_0 = 4$ GeV^2. The labels indicate the dimension up to which the OPE terms are included.

For the currents which belong to the flavor representation $\mathbf{6_f} \otimes \bar{\mathbf{6}}_\mathbf{f}$ (**S**) and $\bar{\mathbf{3}}_\mathbf{f} \otimes \mathbf{3_f}$ (**A**), the spectral densities turn out to be negative in the energy region 1 GeV $\sim$ 2 GeV as shown in Fig. 2. The spectral densities of these currents become positive in the region $s > 4$ GeV^2. They may couple to the state $\pi_1(2015)$. However, after performing the sum rule calculation, we find that the mass obtained from the currents $\eta^A_{i\mu}$ and $\eta^S_{i\mu}$ is larger than 2.5 GeV, for instance, we show the mass calculated from the current $\eta^A_{1\mu}$ in Fig. 3. The curves are obtained by setting $M_B^2 = 2$ GeV^2 (solid line), 3 GeV^2 (short-dashed line), and 4 GeV^2 (long-dashed line). The left curves (disconnected from the right part) are obtained from a negative Borel transformed correlation function, and have no physical meaning. Therefore, our QCD sum rule analysis does not support $\pi_1(1400)$, $\pi_1(1600)$, and $\pi_1(2015)$ as tetraquark states with a flavor structure either $\mathbf{6_f} \otimes \bar{\mathbf{6}}_\mathbf{f}$ or $\bar{\mathbf{3}}_\mathbf{f} \otimes \mathbf{3_f}$.

When using the currents $\eta^M_{i\mu}$, the spectral densities are positive as shown in Fig. 4. And so we shall use these currents to perform a QCD sum rule analysis. First we need to study the convergence of the OPE. The Borel transformed correlation function of the current $\eta^M_{5\mu}$ is shown in Fig. 5, when we take $s_0 = 4$ GeV^2. Besides the first term, which is the continuum piece, the $D = 6$ and $D = 8$ terms give large contributions. The $D = 6$ terms contain $\langle\bar{q}q\rangle^2$ and the $D = 8$ terms contain $\langle\bar{q}q\rangle\langle g_c\bar{q}\sigma Gq\rangle$, which are the important condensates. We find that the convergence is very good in the region of 2 GeV$^2 < M_B^2 < 5$ GeV^2. Therefore, in this region, OPEs are reliable.

The mass is calculated by using Eq. (10), and results are obtained as functions of Borel mass M_B and threshold value s_0. In Figs. 6–9, we show the mass calculated from currents $\eta^M_{1\mu}$, $\eta^M_{2\mu}$, $\eta^M_{3\mu}$, and $\eta^M_{4\mu}$, whose quark contents are $qq\bar{q}\,\bar{q}$. Although these four independent currents look much different, we find that they give a similar result. From figures on the left-hand side, we find that the dependence on Borel mass is weak. From figures on the right-hand side, where the mass is shown as functions of s_0, we find that there is a mass minimum for all curves where the stability is the best. It is 1.7 GeV, 1.6 GeV, 1.6 GeV, and 1.7 GeV for four independent currents, respectively. We find that sometimes the threshold values become smaller

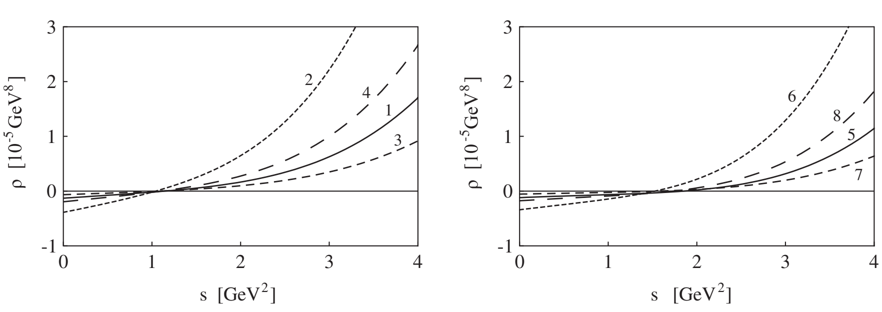

FIG. 4. Spectral densities for the current $\eta^M_{i\mu}$. The spectral densities for the currents with the quark contents $qq\bar{q}\,\bar{q}$ are shown on the left-hand side, and those with the quark contents $qs\bar{q}\,\bar{s}$ are shown on the right-hand side. The labels beside the lines indicate the suffix i of the current $\eta^M_{i\mu}$ ($i = 1, \cdots, 8$).

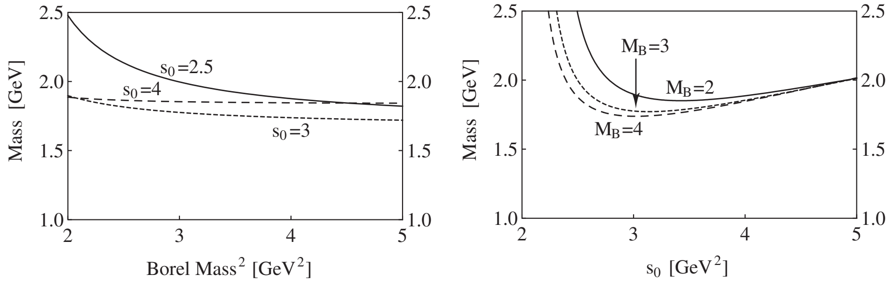

FIG. 6. The mass of the state $qq\bar{q}\,\bar{q}$ calculated by using the current $\eta^M_{1\mu}$, as functions of M_B^2 (left) and s_0 (right) in units of GeV.

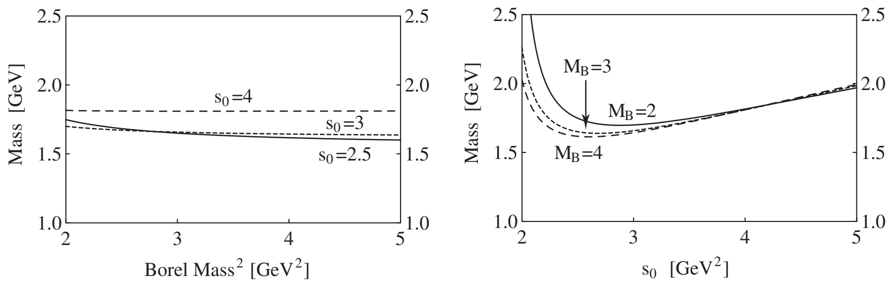

FIG. 7. The mass of the state $qq\bar{q}\,\bar{q}$ calculated by using the current $\eta^M_{2\mu}$, as functions of M_B^2 (left) and s_0 (right) in units of GeV.

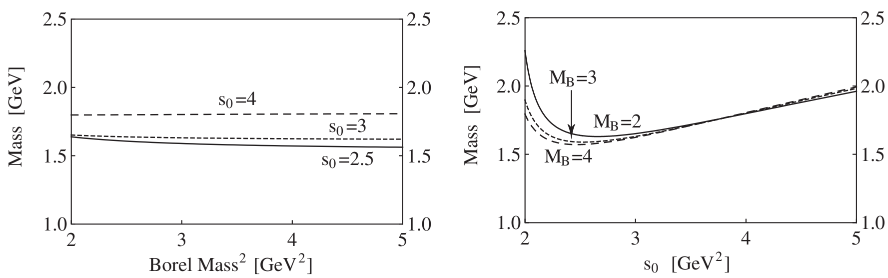

FIG. 8. The mass of the state $qq\bar{q}\,\bar{q}$ calculated by using the current $\eta^M_{3\mu}$, as functions of M_B^2 (left) and s_0 (right) in units of GeV.

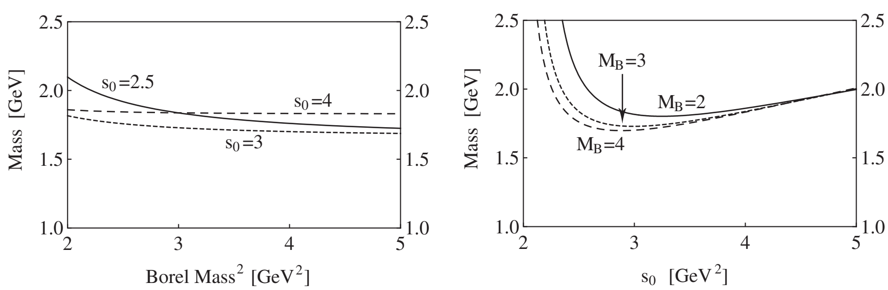

FIG. 9. The mass of the state $qq\bar{q}\,\bar{q}$ calculated by using the current $\eta^M_{4\mu}$, as functions of M_B^2 (left) and s_0 (right) in units of GeV.

than the mass obtained in the mass minimum region. This is due to the negative part of the spectral densities. We also met this in the study of $Y(2175)$. See Ref. [25] for details.

In Figs. 10–13, we show the mass calculated from currents $\eta^M_{5\mu}$, $\eta^M_{6\mu}$, $\eta^M_{7\mu}$, and $\eta^M_{8\mu}$, whose quark contents are $qs\bar{q}\,\bar{s}$. The results are similar as the previous four currents. But now the mass obtained is about 0.4 GeV larger than the previous ones. The minimum occurs at 2.1 GeV, 2.0 GeV, 1.9 GeV, and 2.0 GeV, respectively.

In a short summary, we have performed a QCD sum rule analysis for $qq\bar{q}\,\bar{q}$ and $qs\bar{q}\,\bar{s}$. The mass obtained is around 1.6 GeV and 2.0 GeV, respectively. There are four independent currents for each case, which give similar results. Their mixing would lead to a similar result, too. Compared

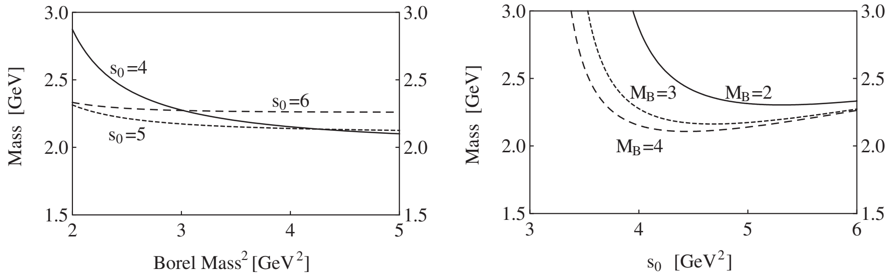

FIG. 10. The mass of the state $qs\bar{q}\,\bar{s}$ calculated by using the current $\eta^M_{5\mu}$, as functions of M_B^2 (left) and s_0 (right) in units of GeV.

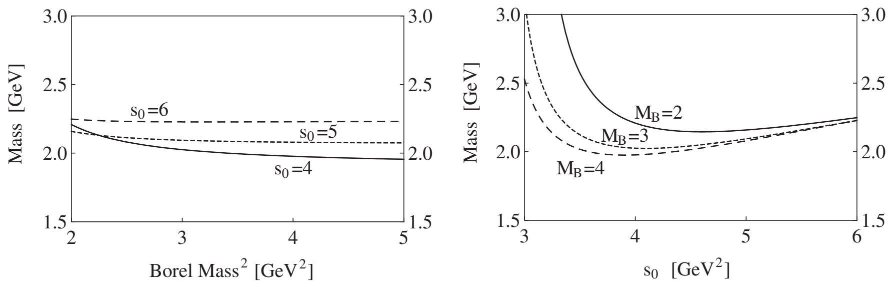

FIG. 11. The mass of the state $qs\bar{q}\,\bar{s}$ calculated by using the current $\eta^M_{6\mu}$, as functions of M_B^2 (left) and s_0 (right) in units of GeV.

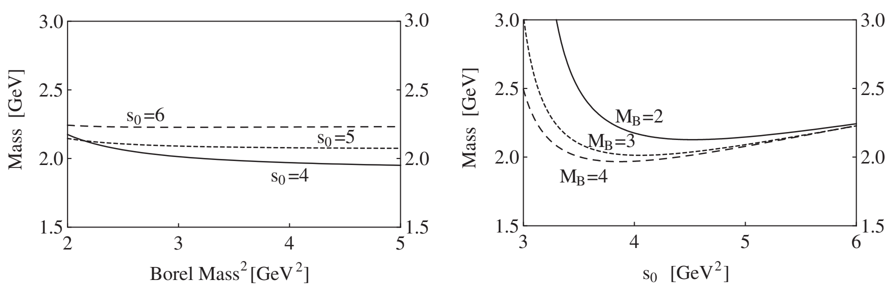

FIG. 12. The mass of the state $qs\bar{q}\,\bar{s}$ calculated by using the current $\eta^M_{7\mu}$, as functions of M_B^2 (left) and s_0 (right) in units of GeV.

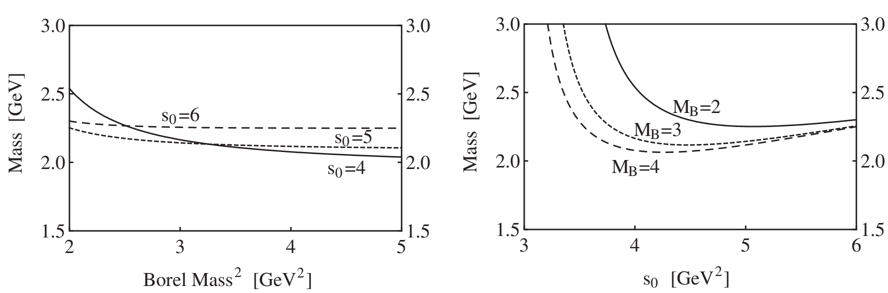

FIG. 13. The mass of the state $qs\bar{q}\,\bar{s}$ calculated by using the current $\eta^M_{8\mu}$, as functions of M_B^2 (left) and s_0 (right) in units of GeV.

with the experimental data, they can be used to interpret the states $\pi_1(1600)$ and $\pi_1(2015)$ of $I^G J^{PC} = 1^- 1^{-+}$. These analyses are very similar to our previous paper [25], where we studied the state $Y(2175)$ by using vector tetraquark currents which have quantum numbers $J^{PC} = 1^{--}$ and quark contents $ss\bar{s}\,\bar{s}$.

The pole contribution

$$\frac{\int_{s_<}^{s_0} e^{-s/M_B^2}\rho(s)ds}{\int_{s_<}^{\infty} e^{-s/M_B^2}\rho(s)ds} \tag{15}$$

is not large enough for all currents due to the high dimension nature of tetraquark currents. Another reason is that these currents have a large coupling to the continuum, which is difficult to be removed. Therefore, we arrive at a stable mass, but with a small pole. To make our analysis more reliable, we go on to use the finite energy sum rule.

V. FINITE ENERGY SUM RULE

In this section, we use the method of the finite energy sum rule (FESR). In order to calculate the mass in the FESR, we first define the nth moment by using the spectral function $\rho(s)$ in Eq. (7)

$$W(n, s_0) = \int_0^{s_0} \rho(s) s^n ds. \tag{16}$$

This integral is used for the phenomenological side, while the integral along the circular contour of radius s_0 on the q^2 complex plain should be performed for the theoretical side.

With the assumption of quark-hadron duality, we obtain

$$W(n, s_0)|_{\text{hadron}} = W(n, s_0)|_{\text{OPE}}. \tag{17}$$

The mass of the ground state can be obtained as

$$M_Y^2(n, s_0) = \frac{W(n+1, s_0)}{W(n, s_0)}. \tag{18}$$

The spectral functions $\rho_i^M(s)$ can be drawn from the Borel transformed correlation functions shown in Sec. III. The $d = 12$ terms which are proportional to $1/(q^2)^2$ do not contribute to the function $W(n, s_0)$ of Eq. (16) for $n = 0$, or they have a very small contribution for $n = 1$, when the theoretical side is computed by the integral over the circle of radius s_0 on the complex q^2 plain.

The mass is shown as a function of the threshold value s_0 in Fig. 14, where n is chosen to be 1. We find that there is a mass minimum. It is around 1.6 GeV for currents η_1^M, η_2^M, η_3^M, and η_4^M, whose quark contents are $qq\bar{q}\bar{q}$, while it is around 2.0 GeV for currents η_5^M, η_6^M, η_7^M, and η_8^M, whose quark contents are $qs\bar{q}\bar{s}$. Here we again find that the threshold values become smaller than the mass obtained in the mass minimum region. See Ref. [25] for details. In a short summary, we arrive at the same results as the previous SVZ QCD sum rule.

VI. DECAY PATTERNS OF THE 1^{-+} TETRAQUARK STATES

In this paper, we have verified that $(qq)(\bar{q}\bar{q})$ construction and $(\bar{q}q)(\bar{q}q)$ construction are equivalent (see Appendix A), and from the second one we can obtain some decay information. The four independent $(\bar{q}q)(\bar{q}q)$ currents $\xi_{i\mu}^M$ lead to the same mass, and therefore, we shall study the decay patterns from all these currents. We can obtain the S-wave decay patterns straightforwardly:

(1) The current $\xi_{1\mu}^M$ naively falls apart to one scalar meson and one vector meson:

$$\begin{aligned} \xi_{1\mu}^M\colon\ & \pi_1(1600) \to 0^+(\sigma(600), f_0(980)\cdots) \\ &\quad + 1^-(\rho(770), \omega(782)\cdots), \\ & \pi_1(2000) \to 0^+(\sigma(600), \kappa(800)\cdots) \\ &\quad + 1^-(\rho(770), K^*(892)\cdots). \end{aligned} \tag{19}$$

(2) The current $\xi_{2\mu}^M$ naively falls apart to one axial-vector meson and one pseudoscalar meson:

$$\begin{aligned} \xi_{2\mu}^M\colon\ & \pi_1(1600) \to 1^+(a_1(1260), b_1(1235)\cdots) \\ &\quad + 0^-(\pi(135)\cdots), \\ & \pi_1(2000) \to 1^+(a_1(1260), K_1(1270), \cdots) \\ &\quad + 0^-(\pi(135), K(498)\cdots). \end{aligned} \tag{20}$$

(3) The current $\xi_{3\mu}^M$ naively falls apart to one vector meson and one axial-vector meson:

$$\begin{aligned} \xi_{3\mu}^M\colon\ & \pi_1(1600) \to 1^-(\rho(770), \omega(782)\cdots) \\ &\quad + 1^+(a_1(1260), b_1(1235)\cdots), \\ & \pi_1(2000) \to 1^-(\rho(770), K^*(892)\cdots) \\ &\quad + 1^+(a_1(1260), K_1(1270)\cdots). \end{aligned} \tag{21}$$

(4) The current $\xi_{4\mu}^M$ naively falls apart to one axial-vector meson and one vector meson:

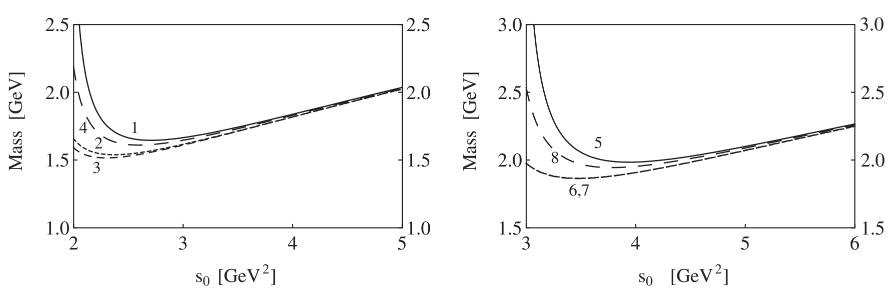

FIG. 14. The mass calculated using the finite energy sum rule. The mass for the currents $\eta_{1\mu}^M$, $\eta_{2\mu}^M$, $\eta_{3\mu}^M$, and $\eta_{4\mu}^M$ is shown on the left-hand side, and the mass for the currents $\eta_{5\mu}^M$, $\eta_{6\mu}^M$, $\eta_{7\mu}^M$, and $\eta_{8\mu}^M$ are shown on the right-hand side. The labels beside the lines indicate the suffix i of the current $\eta_{i\mu}^M$ ($i = 1, \cdots, 8$).

$$\begin{aligned} \xi^M_{4\mu}\colon\ \pi_1(1600) &\to 1^+(a_1(1260), b_1(1235)\cdots) \\ &\quad + 1^-(\rho(770), \omega(782)\cdots), \\ \pi_1(2000) &\to 1^+(a_1(1260), K_1(1270)\cdots) \\ &\quad + 1^-(\rho(770), K^*(892)\cdots). \end{aligned} \tag{22}$$

$\pi_1(2000)$ contains one $\bar{s}s$ pair, so its final states should also contain one $\bar{s}s$ pair, and its decay patterns are more complicated than $\pi_1(1600)$. We see that the decay modes (21) and (22) are kinematically forbidden (or strongly suppressed) due to energy conservation. The decay modes (19) are difficult to be observed in the experiments due to the large decay width of scalar mesons (σ and κ). Moreover, the scalar mesons below 1 GeV are sometimes interpreted as tetraquark states, and if so, these decay modes should be suppressed due to the extra $\bar{q}q$ pair [24]. Therefore, the decay modes (20) are preferred. The π_1 meson first decays to one axial-vector meson and one pseudoscalar meson. Then the axial-vector meson decays into two or more pseudoscalar mesons. However, the second step is a P-wave decay. Considering the conservation of G parity, the decay mode $a_1(1260)\pi$ is forbidden. One possible decay pattern is that $\pi_1(1600)$ first decays to $b_1(1235)\pi$, and then decays to $\omega\pi\pi$.

We can also check the P-wave decay patterns besides S-wave decay patterns. We find that the current $\xi^M_{2\mu}$ leads to a decay mode of two P-wave pseudoscalar mesons by naively relating $\bar{q}\gamma_\mu\gamma_5 q$ and $\partial_\mu\pi$

$$\begin{aligned} \pi_1(1600) &\to 0^-(\pi, \eta, \eta'\cdots) + 0^-(\pi, \eta, \eta'\cdots), \\ \pi_1(2000) &\to 0^-(\pi, \eta, \eta'\cdots) + 0^-(\pi, \eta, \eta'\cdots). \end{aligned} \tag{23}$$

Considering the conservation of G parity, decay modes $\pi\pi$ and $\eta\eta$, etc. are forbidden, and possible decay modes are $\pi\eta$ and $\pi\eta'$, etc. Summarizing the decay patterns, there are two possible decay modes: P-wave many body decay, such as $\omega\pi\pi$, and P-wave two body decay, such as $\pi\eta$ and $\pi\eta'$. This is partly consistent with the experiments which observe $\pi_1(1600)$ and $\pi_1(2015)$ in the decay modes $\pi\eta'$, $\omega\pi\pi$, and $\eta\pi\pi\pi$. However, the experiment has not observed them in the final state $\pi\eta$. Certainly it is desired to study these decay patterns to obtain more information on the structure of the π_1s mesons.

VII. SUMMARY

In this paper we have performed the QCD sum rule analysis of the exotic tetraquark states with $I^G J^{PC} = 1^- 1^{-+}$. The tetraquark currents have rich internal structure. There are several independent currents for a given set of quantum numbers. We have classified the complete set of independent currents and constructed the currents in the form of either $(qq)(\bar{q}\bar{q})$ or $(\bar{q}q)(\bar{q}q)$. As expected, they are shown to be equivalent by having the complete set of independent currents. Physically, this seems to make it difficult to draw interpretation of the internal structure such as diquark (qq) dominated or meson $(\bar{q}\bar{q})$ dominated ones. Using the complete set of the currents, one can perform an optimal analysis of the QCD sum rule.

A somewhat complicated feature arises from the flavor structure. We have tested all possibilities for the isovector $I = 1$ states. In the $SU(3)$ limit, there are three cases of, in the diquark $(qq)(\bar{q}\bar{q})$ construction, $\mathbf{6} \otimes \bar{\mathbf{6}}$, $\bar{\mathbf{3}} \otimes \mathbf{3}$, and $(\bar{\mathbf{3}} \otimes \bar{\mathbf{6}}) \oplus (\mathbf{6} \otimes \mathbf{3})$. We find that the former two cases cannot result in a meaningful sum rule since the spectral functions become negative. On the other hand, the mixed case $(\bar{\mathbf{3}} \otimes \bar{\mathbf{6}}) \oplus (\mathbf{6} \otimes \mathbf{3})$ allows positive OPE with which we can perform the QCD sum rule analysis. Actual currents have been constructed in the limit of the ideal mixing where the currents are classified by the number of the strange quarks. Hence the quark contents are either $qq\bar{q}\bar{q}$ or $qs\bar{q}\bar{s}$.

We have then performed the SVZ and finite energy sum rules. The resulting masses are around 1.6 GeV for $qq\bar{q}\bar{q}$, and around 2.0 GeV for $qs\bar{q}\bar{s}$. The four independent currents lead to the same mass and couple to a single state as shown above. Hence one of our main conclusions is that the higher energy states $\pi_1(1600)$ and $\pi_1(2015)$ are well compatible with the tetraquark picture in the present QCD sum rule analysis. On the other hand, any combination of the independent currents does not seem to couple sufficiently to the lower mass state $\pi_1(1400)$, which was, however, described as a hybrid state by K. C. Yang in Ref. [12]. He obtained a low mass around 1.26 GeV by using the renormalization-improved QCD sum rules. The $\pi_1(1400)$ state seems somewhat special, as the experiments show the similarity between $\pi_1(1600)$ and $\pi_1(2015)$ as well as the difference between $\pi_1(1400)$ and the above two states, which we have discussed in the introduction.

We have also studied their decay patterns and found that these states can be searched for in the decay mode of the axial-vector and pseudoscalar meson pair such as $b_1(1235)\pi$, which is sometimes considered as the characteristic decay mode of the hybrid mesons. The P-wave modes $\pi\eta$, $\pi\eta'$ are also quite important.

It is also interesting to study the partners of π_1s. Especially, we can study the one with quark contents $ud\bar{s}\bar{s}$, which is at the top of the flavor representation $\overline{\mathbf{10}}$ (see Fig. 1). It has a mass around 2.0 GeV, and the decay modes are $K^+(\bar{s}u)K^0(\bar{s}d)$ (P-wave) and KKK (P-wave), etc. BESIII will start taking data very soon. The search/identification of exotic mesons is one of its important physical goals. Hopefully the dedicated experimental programs on the exotic mesons at BESIII and JLAB in the coming years will shed light on their existence, and then their internal structure. More work on the theoretical side is also needed. We will go on to study other tetraquark candidates.

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APPENDIX A: $(\bar{q}q)(\bar{q}q)$ CURRENTS

In this appendix, we attempt to construct the tetraquark currents using quark-antiquark $(\bar{q}q)$ pairs. For each state containing a diquark and antidiquark having the symmetric flavor $\mathbf{6_f} \otimes \mathbf{6_f}$, there are four $(\bar{q}q)(\bar{q}q)$ currents:

$$
\begin{aligned}
\xi^S_{1\mu} &= (\bar{q}_{3a}\gamma_\mu\gamma_5 q_{1a})(\bar{q}_{4b}\gamma_5 q_{2b}) + (\bar{q}_{3a}\gamma_5 q_{1a}) \\
&\quad \times (\bar{q}_{4b}\gamma_\mu\gamma_5 q_{2b}) + (\bar{q}_{3a}\gamma_\mu\gamma_5 q_{2a})(\bar{q}_{4b}\gamma_5 q_{1b}) \\
&\quad + (\bar{q}_{3a}\gamma_5 q_{2a})(\bar{q}_{4b}\gamma_\mu\gamma_5 q_{1b}), \\
\xi^S_{2\mu} &= (\bar{q}_{3a}\gamma^\nu q_{1a})(\bar{q}_{4b}\sigma_{\mu\nu} q_{2b}) + (\bar{q}_{3a}\sigma_{\mu\nu} q_{1a})(\bar{q}_{4b}\gamma^\nu q_{2b}) \\
&\quad + (\bar{q}_{3a}\gamma^\nu q_{2a})(\bar{q}_{4b}\sigma_{\mu\nu} q_{1b}) + (\bar{q}_{3a}\sigma_{\mu\nu} q_{2a}) \\
&\quad \times (\bar{q}_{4b}\gamma^\nu q_{1b}), \\
\xi^S_{3\mu} &= \lambda_{ab}\lambda_{cd}\{(\bar{q}_{3a}\gamma_\mu\gamma_5 q_{1b})(\bar{q}_{4c}\gamma_5 q_{2d}) + (\bar{q}_{3a}\gamma_5 q_{1b}) \\
&\quad \times (\bar{q}_{4c}\gamma_\mu\gamma_5 q_{2d}) + (\bar{q}_{3a}\gamma_\mu\gamma_5 q_{2b})(\bar{q}_{4c}\gamma_5 q_{1d}) \\
&\quad + (\bar{q}_{3a}\gamma_5 q_{2b})(\bar{q}_{4c}\gamma_\mu\gamma_5 q_{1d})\}, \\
\xi^S_{4\mu} &= \lambda_{ab}\lambda_{cd}\{(\bar{q}_{3a}\gamma^\nu q_{1b})(\bar{q}_{4c}\sigma_{\mu\nu} q_{2d}) + (\bar{q}_{3a}\sigma_{\mu\nu} q_{1b}) \\
&\quad \times (\bar{q}_{4c}\gamma^\nu q_{2d}) + (\bar{q}_{3a}\gamma^\nu q_{2b})(\bar{q}_{4c}\sigma_{\mu\nu} q_{1d}) \\
&\quad + (\bar{q}_{3a}\sigma_{\mu\nu} q_{2b})(\bar{q}_{4c}\gamma^\nu q_{1d})\}.
\end{aligned}
$$

Among these currents, only two are independent. We can verify the following relations:

$$
\xi^S_{3\mu} = -\tfrac{5}{3}\xi^S_{1\mu} - i\xi^S_{2\mu}, \qquad \xi^S_{4\mu} = 3i\xi^S_{1\mu} + \tfrac{1}{3}\xi^S_{2\mu}.
$$

Moreover, they are equivalent to the $(qq)(\bar{q}\,\bar{q})$ currents

$$
\psi^S_{1\mu} = -\frac{1}{2}\xi^S_{1\mu} + \frac{i}{2}\xi^S_{2\mu}, \qquad \psi^S_{2\mu} = -\frac{3i}{2}\xi^S_{1\mu} + \frac{1}{2}\xi^S_{2\mu}.
$$

For each state containing a diquark and antidiquark having the antisymmetric flavor $\bar{\mathbf{3}}_\mathbf{f} \otimes \mathbf{3_f}$, there are also four $(\bar{q}q)(\bar{q}q)$ currents which are nonzero:

$$
\begin{aligned}
\xi^A_{1\mu} &= (\bar{q}_{3a}\gamma_\mu\gamma_5 q_{1a})(\bar{q}_{4b}\gamma_5 q_{2b}) + (\bar{q}_{3a}\gamma_5 q_{1a}) \\
&\quad \times (\bar{q}_{4b}\gamma_\mu\gamma_5 q_{2b}) - (\bar{q}_{3a}\gamma_\mu\gamma_5 q_{2a})(\bar{q}_{4b}\gamma_5 q_{1b}) \\
&\quad - (\bar{q}_{3a}\gamma_5 q_{2a})(\bar{q}_{4b}\gamma_\mu\gamma_5 q_{1b}), \\
\xi^A_{2\mu} &= (\bar{q}_{3a}\gamma^\nu q_{1a})(\bar{q}_{4b}\sigma_{\mu\nu} q_{2b}) + (\bar{q}_{3a}\sigma_{\mu\nu} q_{1a})(\bar{q}_{4b}\gamma^\nu q_{2b}) \\
&\quad - (\bar{q}_{3a}\gamma^\nu q_{2a})(\bar{q}_{4b}\sigma_{\mu\nu} q_{1b}) - (\bar{q}_{3a}\sigma_{\mu\nu} q_{2a}) \\
&\quad \times (\bar{q}_{4b}\gamma^\nu q_{1b}), \\
\xi^A_{3\mu} &= \lambda_{ab}\lambda_{cd}\{(\bar{q}_{3a}\gamma_\mu\gamma_5 q_{1b})(\bar{q}_{4c}\gamma_5 q_{2d}) + (\bar{q}_{3a}\gamma_5 q_{1b}) \\
&\quad \times (\bar{q}_{4c}\gamma_\mu\gamma_5 q_{2d}) - (\bar{q}_{3a}\gamma_\mu\gamma_5 q_{2b})(\bar{q}_{4c}\gamma_5 q_{1d}) \\
&\quad - (\bar{q}_{3a}\gamma_5 q_{2b})(\bar{q}_{4c}\gamma_\mu\gamma_5 q_{1d})\}, \\
\xi^A_{4\mu} &= \lambda_{ab}\lambda_{cd}\{(\bar{q}_{3a}\gamma^\nu q_{1b})(\bar{q}_{4c}\sigma_{\mu\nu} q_{2d}) + (\bar{q}_{3a}\sigma_{\mu\nu} q_{1b}) \\
&\quad \times (\bar{q}_{4c}\gamma^\nu q_{2d}) - (\bar{q}_{3a}\gamma^\nu q_{2b})(\bar{q}_{4c}\sigma_{\mu\nu} q_{1d}) \\
&\quad - (\bar{q}_{3a}\sigma_{\mu\nu} q_{2b})(\bar{q}_{4c}\gamma^\nu q_{1d})\},
\end{aligned}
$$

where two are independent

$$
\xi^A_{3\mu} = \tfrac{1}{3}\xi^A_{1\mu} + i\xi^A_{2\mu}, \qquad \xi^A_{4\mu} = -3i\xi^A_{1\mu} - \tfrac{5}{3}\xi^A_{2\mu}.
$$

They are equivalent to the $(qq)(\bar{q}\,\bar{q})$ currents

$$
\psi^A_{1\mu} = -\frac{1}{2}\xi^A_{1\mu} + \frac{i}{2}\xi^A_{2\mu}, \qquad \psi^A_{2\mu} = -\frac{3i}{2}\xi^A_{1\mu} + \frac{1}{2}\xi^A_{2\mu}.
$$

For the currents which have a mixed flavor symmetry, we just show the $(\bar{q}q)(\bar{q}q)$ currents which belong to the flavor representation $\bar{\mathbf{3}}_\mathbf{f} \otimes \bar{\mathbf{6}}_\mathbf{f}$.

$$
\begin{aligned}
\xi^{ML}_{1\mu} &= (\bar{q}_{3a} q_{1a})(\bar{q}_{4b}\gamma_\mu q_{2b}) - (\bar{q}_{3a}\gamma_\mu q_{1a})(\bar{q}_{4b} q_{2b}) \\
&\quad - (\bar{q}_{3a} q_{2a})(\bar{q}_{4b}\gamma_\mu q_{1b}) + (\bar{q}_{3a}\gamma_\mu q_{2a})(\bar{q}_{4b} q_{1b}), \\
\xi^{ML}_{2\mu} &= (\bar{q}_{3a}\gamma^\mu\gamma_5 q_{1a})(\bar{q}_{4b}\gamma_5 q_{2b}) - (\bar{q}_{3a}\gamma_5 q_{1a}) \\
&\quad \times (\bar{q}_{4b}\gamma^\mu\gamma_5 q_{2b}) - (\bar{q}_{3a}\gamma^\mu\gamma_5 q_{2a})(\bar{q}_{4b}\gamma_5 q_{1b}) \\
&\quad + (\bar{q}_{3a}\gamma_5 q_{2a})(\bar{q}_{4b}\gamma^\mu\gamma_5 q_{1b}), \\
\xi^{ML}_{3\mu} &= (\bar{q}_{3a}\gamma^\nu q_{1a})(\bar{q}_{4b}\sigma_{\mu\nu} q_{2b}) - (\bar{q}_{3a}\sigma_{\mu\nu} q_{1a})(\bar{q}_{4b}\gamma^\nu q_{2b}) \\
&\quad - (\bar{q}_{3a}\gamma^\nu q_{2a})(\bar{q}_{4b}\sigma_{\mu\nu} q_{1b}) + (\bar{q}_{3a}\sigma_{\mu\nu} q_{2a}) \\
&\quad \times (\bar{q}_{4b}\gamma^\nu q_{1b}), \\
\xi^{ML}_{4\mu} &= (\bar{q}_{3a}\gamma^\nu\gamma_5 q_{1a})(\bar{q}_{4b}\sigma_{\mu\nu}\gamma_5 q_{2b}) - (\bar{q}_{3a}\sigma_{\mu\nu}\gamma_5 q_{1a}) \\
&\quad \times (\bar{q}_{4b}\gamma^\nu\gamma_5 q_{2b}) - (\bar{q}_{3a}\gamma^\nu\gamma_5 q_{2a})(\bar{q}_{4b}\sigma_{\mu\nu}\gamma_5 q_{1b}) \\
&\quad + (\bar{q}_{3a}\sigma_{\mu\nu}\gamma_5 q_{2a})(\bar{q}_{4b}\gamma^\nu\gamma_5 q_{1b}).
\end{aligned}
$$

There are also four currents which have a color $\mathbf{8_c} \otimes \mathbf{8_c}$ structure, and they can be written as a combination of these color $\mathbf{1_c} \otimes \mathbf{1_c}$ currents. The relations between $\phi^{ML}_{i\mu}$ and $\xi^{ML}_{i\mu}$ are

$$\psi_{1\mu}^{ML} = -\frac{1}{4}\xi_{1\mu}^{ML} + \frac{1}{4}\xi_{2\mu}^{ML} + \frac{i}{4}\xi_{3\mu}^{ML} - \frac{i}{4}\xi_{4\mu}^{ML},$$
$$\psi_{2\mu}^{ML} = \frac{3i}{4}\xi_{1\mu}^{ML} + \frac{3i}{4}\xi_{2\mu}^{ML} + \frac{1}{4}\xi_{3\mu}^{ML} + \frac{1}{4}\xi_{4\mu}^{ML},$$
$$\psi_{3\mu}^{ML} = \frac{1}{4}\xi_{1\mu}^{ML} + \frac{1}{4}\xi_{2\mu}^{ML} + \frac{i}{4}\xi_{3\mu}^{ML} + \frac{i}{4}\xi_{4\mu}^{ML},$$
$$\psi_{4\mu}^{ML} = -\frac{3i}{4}\xi_{1\mu}^{ML} + \frac{3i}{4}\xi_{2\mu}^{ML} + \frac{1}{4}\xi_{3\mu}^{ML} - \frac{1}{4}\xi_{4\mu}^{ML}.$$

We can obtain similar results for $\xi_{i\mu}^{MR}$, which belong to the flavor representation $\mathbf{6_f} \otimes \mathbf{3_f}$ can be obtained similarly, and the currents with $J^{PC} = 1^{-+}$ are

$$\xi_{i\mu}^{M} = \xi_{i\mu}^{ML} + \xi_{i\mu}^{MR}. \tag{A1}$$

APPENDIX B: TWO-POINT CORRELATION FUNCTIONS

In this appendix we show the results for the Borel transformed correlation functions as defined in Eq. (8). Results for the currents η_1^A, η_2^M, η_3^M, η_4^M, η_6^M, η_7^M, and η_8^M are indicated by the same upper and lower indices.

$$\begin{aligned}\Pi_1^A(M_B^2) = {}& \int_{s_<}^{s_0}\Big[\frac{1}{36\,848\pi^6}s^4 - \frac{17m_s^2}{15\,360\pi^6}s^3 + \Big(\frac{\langle g_s^2GG\rangle}{18\,432\pi^6} + \frac{m_s\langle\bar qq\rangle}{192\pi^4} + \frac{m_s\langle\bar ss\rangle}{96\pi^4}\Big)s^2 \\ & + \Big(-\frac{\langle\bar qq\rangle^2}{72\pi^2} - \frac{\langle\bar ss\rangle^2}{72\pi^2} - \frac{\langle\bar qq\rangle\langle\bar ss\rangle}{18\pi^2} + \frac{m_s\langle g_s\bar q\sigma Gq\rangle}{96\pi^4} + \frac{m_s\langle g_s\bar s\sigma Gs\rangle}{192\pi^4} - \frac{m_s^2\langle g_s^2GG\rangle}{4608\pi^6}\Big)s \\ & - \frac{\langle\bar qq\rangle\langle g_s\bar q\sigma Gq\rangle}{48\pi^2} - \frac{\langle\bar ss\rangle\langle g_s\bar s\sigma Gs\rangle}{48\pi^2} - \frac{\langle\bar qq\rangle\langle g_s\bar s\sigma Gs\rangle}{24\pi^2} - \frac{\langle\bar ss\rangle\langle g_s\bar q\sigma Gq\rangle}{24\pi^2} + \frac{m_s\langle g_s^2GG\rangle\langle\bar qq\rangle}{256\pi^4} - \frac{m_s^2\langle\bar qq\rangle^2}{12\pi^2} \\ & + \frac{m_s^2\langle\bar ss\rangle^2}{48\pi^2} + \frac{m_s^2\langle\bar qq\rangle\langle\bar ss\rangle}{4\pi^2}\Big]e^{-s/M_B^2}ds + \Big(-\frac{\langle g_s\bar q\sigma Gq\rangle^2}{192\pi^2} - \frac{\langle g_s\bar s\sigma Gs\rangle^2}{192\pi^2} - \frac{\langle g_s\bar q\sigma Gq\rangle\langle g_s\bar s\sigma Gs\rangle}{48\pi^2} \\ & - \frac{5\langle g_s^2GG\rangle\langle\bar qq\rangle\langle\bar ss\rangle}{864\pi^2} + \frac{m_s\langle\bar qq\rangle^2\langle\bar ss\rangle}{3} - \frac{2m_s\langle\bar qq\rangle\langle\bar ss\rangle^2}{9} + \frac{5m_s\langle g_s^2GG\rangle\langle g_s\bar q\sigma Gq\rangle}{4608\pi^4} + \frac{m_s^2\langle\bar qq\rangle\langle g_s\bar s\sigma Gs\rangle}{12\pi^2} \\ & + \frac{m_s^2\langle\bar ss\rangle\langle g_s\bar q\sigma Gq\rangle}{8\pi^2}\Big) + \frac{1}{M_B^2}\Big(-\frac{16g_s^2\langle\bar qq\rangle^2\langle\bar ss\rangle^2}{81} + \frac{\langle g_s^2GG\rangle\langle\bar qq\rangle\langle g_s\bar s\sigma Gs\rangle}{1152\pi^2} + \frac{\langle g_s^2GG\rangle\langle\bar ss\rangle\langle g_s\bar q\sigma Gq\rangle}{1152\pi^2} \\ & - \frac{m_s\langle\bar qq\rangle^2\langle g_s\bar s\sigma Gs\rangle}{9} - \frac{m_s\langle\bar ss\rangle^2\langle g_s\bar q\sigma Gq\rangle}{18} - \frac{5m_s\langle\bar qq\rangle\langle\bar ss\rangle\langle g_s\bar q\sigma Gq\rangle}{18} - \frac{m_s\langle\bar qq\rangle\langle\bar ss\rangle\langle g_s\bar s\sigma Gs\rangle}{18} + \frac{m_s^2\langle g_s\bar q\sigma Gq\rangle^2}{48\pi^2} \\ & + \frac{m_s^2\langle g_s\bar q\sigma Gq\rangle\langle g_s\bar s\sigma Gs\rangle}{48\pi^2}\Big),\end{aligned}$$

$$\begin{aligned}\Pi_2^M(M_B^2) = {}& \int_0^{s_0}\Big[\frac{1}{6144\pi^6}s^4 + \frac{11\langle g_s^2GG\rangle}{18\,432\pi^6}s^2 + \frac{\langle\bar qq\rangle^2}{6\pi^2}s + \frac{\langle\bar qq\rangle\langle g_s\bar q\sigma Gq\rangle}{4\pi^2}\Big]e^{-s/M_B^2}ds + \Big(\frac{\langle g_s\bar q\sigma Gq\rangle^2}{16\pi^2} + \frac{5\langle g_s^2GG\rangle\langle\bar qq\rangle^2}{864\pi^2}\Big) \\ & + \frac{1}{M_B^2}\Big(-\frac{32g_s^2\langle\bar qq\rangle^4}{27} - \frac{\langle g_s^2GG\rangle\langle\bar qq\rangle\langle g_s\bar q\sigma Gq\rangle}{576\pi^2}\Big),\end{aligned}$$

$$\begin{aligned}\Pi_3^M(M_B^2) = {}& \int_0^{s_0}\Big[\frac{1}{36\,864\pi^6}s^4 + \frac{\langle g_s^2GG\rangle}{18\,432\pi^6}s^2 + \frac{\langle\bar qq\rangle^2}{36\pi^2}s + \frac{\langle\bar qq\rangle\langle g_s\bar q\sigma Gq\rangle}{24\pi^2}\Big]e^{-s/M_B^2}ds + \Big(\frac{\langle g_s\bar q\sigma Gq\rangle^2}{96\pi^2} + \frac{5\langle g_s^2GG\rangle\langle\bar qq\rangle^2}{864\pi^2}\Big) \\ & + \frac{1}{M_B^2}\Big(-\frac{16g_s^2\langle\bar qq\rangle^4}{81} - \frac{\langle g_s^2GG\rangle\langle\bar qq\rangle\langle g_s\bar q\sigma Gq\rangle}{576\pi^2}\Big),\end{aligned}$$

$$\begin{aligned}\Pi_4^M(M_B^2) = {}& \int_0^{s_0}\Big[\frac{1}{12\,288\pi^6}s^4 + \frac{\langle g_s^2GG\rangle}{18\,432\pi^6}s^2 + \frac{\langle\bar qq\rangle^2}{12\pi^2}s + \frac{\langle\bar qq\rangle\langle g_s\bar q\sigma Gq\rangle}{8\pi^2}\Big]e^{-s/M_B^2}ds + \Big(\frac{\langle g_s\bar q\sigma Gq\rangle^2}{32\pi^2} - \frac{5\langle g_s^2GG\rangle\langle\bar qq\rangle^2}{864\pi^2}\Big) \\ & + \frac{1}{M_B^2}\Big(-\frac{16g_s^2\langle\bar qq\rangle^4}{27} + \frac{\langle g_s^2GG\rangle\langle\bar qq\rangle\langle g_s\bar q\sigma Gq\rangle}{576\pi^2}\Big),\end{aligned}$$

$$\Pi_6^M(M_B^2) = \int_{4m_s^2}^{s_0} \bigg[\frac{1}{6144\pi^6} s^4 - \frac{17m_s^2}{2560\pi^6} s^3 + \bigg(\frac{11\langle g_s^2 GG\rangle}{18\,432\pi^6} - \frac{m_s\langle \bar{q}q\rangle}{32\pi^4} + \frac{m_s\langle \bar{s}s\rangle}{16\pi^4} \bigg) s^2 + \bigg(-\frac{\langle \bar{q}q\rangle^2}{12\pi^2} + \frac{\langle \bar{q}q\rangle\langle \bar{s}s\rangle}{3\pi^2} - \frac{\langle \bar{s}s\rangle^2}{12\pi^2} - \frac{m_s\langle g_s\bar{q}\sigma Gq\rangle}{16\pi^4} + \frac{m_s\langle g_s\bar{s}\sigma Gs\rangle}{32\pi^4} - \frac{109 m_s^2\langle g_s^2 GG\rangle}{18432\pi^6} \bigg) s - \frac{\langle \bar{q}q\rangle\langle g_s\bar{q}\sigma Gq\rangle}{8\pi^2} + \frac{\langle \bar{q}q\rangle\langle g_s\bar{s}\sigma Gs\rangle}{4\pi^2} + \frac{\langle \bar{s}s\rangle\langle g_s\bar{q}\sigma Gq\rangle}{4\pi^2} - \frac{\langle \bar{s}s\rangle\langle g_s\bar{s}\sigma Gs\rangle}{8\pi^2} - \frac{3m_s\langle g_s^2 GG\rangle\langle \bar{q}q\rangle}{128\pi^4} + \frac{5m_s\langle g_s^2 GG\rangle\langle \bar{s}s\rangle}{256\pi^4} - \frac{m_s^2\langle \bar{q}q\rangle^2}{2\pi^2} - \frac{3m_s^2\langle \bar{q}q\rangle\langle \bar{s}s\rangle}{2\pi^2} + \frac{m_s^2\langle \bar{s}s\rangle^2}{8\pi^2} \bigg] e^{-s/M_B^2} ds + \bigg(-\frac{\langle g_s\bar{q}\sigma Gq\rangle^2}{32\pi^2} + \frac{\langle g_s\bar{q}\sigma Gq\rangle\langle g_s\bar{s}\sigma Gs\rangle}{8\pi^2} - \frac{\langle g_s\bar{s}\sigma Gs\rangle^2}{32\pi^2} - \frac{25\langle g_s^2 GG\rangle\langle \bar{q}q\rangle^2}{1728\pi^2} + \frac{5\langle g_s^2 GG\rangle\langle \bar{q}q\rangle\langle \bar{s}s\rangle}{144\pi^2} - \frac{25\langle g_s^2 GG\rangle\langle \bar{s}s\rangle^2}{1728\pi^2} - \frac{5m_s\langle g_s^2 GG\rangle\langle g_s\bar{q}\sigma Gq\rangle}{768\pi^4} + \frac{25m_s\langle g_s^2 GG\rangle\langle g_s\bar{s}\sigma Gs\rangle}{4608\pi^4} + 2m_s\langle \bar{q}q\rangle^2\langle \bar{s}s\rangle + \frac{4m_s\langle \bar{q}q\rangle\langle \bar{s}s\rangle^2}{3} - \frac{m_s^2\langle \bar{q}q\rangle\langle g_s\bar{s}\sigma Gs\rangle}{2\pi^2} - \frac{3m_s^2\langle \bar{s}s\rangle\langle g_s\bar{q}\sigma Gq\rangle}{4\pi^2} \bigg) + \frac{1}{M_B^2} \bigg(-\frac{32 g_s^2\langle \bar{q}q\rangle^2\langle \bar{s}s\rangle^2}{27} + \frac{5\langle g_s^2 GG\rangle\langle \bar{q}q\rangle\langle g_s\bar{q}\sigma Gq\rangle}{1152\pi^2} - \frac{\langle g_s^2 GG\rangle\langle \bar{q}q\rangle\langle g_s\bar{s}\sigma Gs\rangle}{192\pi^2} - \frac{\langle g_s^2 GG\rangle\langle \bar{s}s\rangle\langle g_s\bar{q}\sigma Gq\rangle}{192\pi^2} + \frac{5\langle g_s^2 GG\rangle\langle \bar{s}s\rangle\langle g_s\bar{s}\sigma Gs\rangle}{1152\pi^2} - \frac{2m_s\langle \bar{q}q\rangle^2\langle g_s\bar{s}\sigma Gs\rangle}{3} - \frac{5m_s\langle \bar{q}q\rangle\langle \bar{s}s\rangle\langle g_s\bar{q}\sigma Gq\rangle}{3} + \frac{m_s\langle \bar{q}q\rangle\langle \bar{s}s\rangle\langle g_s\bar{s}\sigma Gs\rangle}{3} + \frac{m_s\langle \bar{s}s\rangle^2\langle g_s\bar{q}\sigma Gq\rangle}{3} - \frac{5m_s^2\langle g_s^2 GG\rangle\langle \bar{s}s\rangle^2}{1152\pi^2} + \frac{m_s^2\langle g_s\bar{q}\sigma Gq\rangle^2}{8\pi^2} - \frac{m_s^2\langle g_s\bar{q}\sigma Gq\rangle\langle g_s\bar{s}\sigma Gs\rangle}{8\pi^2} \bigg),$$

$$\Pi_7^M(M_B^2) = \int_{4m_s^2}^{s_0} \bigg[\frac{1}{36\,864\pi^6} s^4 - \frac{17m_s^2}{15\,360\pi^6} s^3 + \bigg(\frac{\langle g_s^2 GG\rangle}{18\,432\pi^6} - \frac{m_s\langle \bar{q}q\rangle}{192\pi^4} + \frac{m_s\langle \bar{s}s\rangle}{96\pi^4} \bigg) s^2 + \bigg(-\frac{\langle \bar{q}q\rangle^2}{72\pi^2} + \frac{\langle \bar{q}q\rangle\langle \bar{s}s\rangle}{18\pi^2} - \frac{\langle \bar{s}s\rangle^2}{72\pi^2} - \frac{m_s\langle g_s\bar{s}\sigma Gs\rangle}{96\pi^4} + \frac{m_s\langle g_s\bar{s}\sigma Gs\rangle}{192\pi^4} - \frac{m_s^2\langle g_s^2 GG\rangle}{4608\pi^6} \bigg) s - \frac{\langle \bar{q}q\rangle\langle g_s\bar{q}\sigma Gq\rangle}{48\pi^2} + \frac{\langle \bar{q}q\rangle\langle g_s\bar{s}\sigma Gs\rangle}{24\pi^2} + \frac{\langle \bar{s}s\rangle\langle g_s\bar{q}\sigma Gq\rangle}{24\pi^2} - \frac{\langle \bar{s}s\rangle\langle g_s\bar{s}\sigma Gs\rangle}{48\pi^2} - \frac{m_s\langle g_s^2 GG\rangle\langle \bar{q}q\rangle}{256\pi^4} - \frac{m_s^2\langle \bar{q}q\rangle^2}{12\pi^2} - \frac{m_s^2\langle \bar{q}q\rangle\langle \bar{s}s\rangle}{4\pi^2} + \frac{m_s^2\langle \bar{s}s\rangle^2}{48\pi^2} \bigg] e^{-s/M_B^2} ds + \bigg(-\frac{\langle g_s\bar{q}\sigma Gq\rangle^2}{192\pi^2} + \frac{\langle g_s\bar{q}\sigma Gq\rangle\langle g_s\bar{s}\sigma Gs\rangle}{48\pi^2} - \frac{\langle g_s\bar{s}\sigma Gs\rangle^2}{192\pi^2} + \frac{5\langle g_s^2 GG\rangle\langle \bar{q}q\rangle\langle \bar{s}s\rangle}{864\pi^2} + \frac{m_s\langle \bar{q}q\rangle^2\langle \bar{s}s\rangle}{3} + \frac{2m_s\langle \bar{q}q\rangle\langle \bar{s}s\rangle^2}{9} - \frac{5m_s\langle g_s^2 GG\rangle\langle g_s\bar{q}\sigma Gq\rangle}{4608\pi^4} - \frac{m_s^2\langle \bar{s}s\rangle\langle g_s\bar{q}\sigma Gq\rangle}{8\pi^2} - \frac{m_s^2\langle \bar{q}q\rangle\langle g_s\bar{s}\sigma Gs\rangle}{12\pi^2} \bigg) + \frac{1}{M_B^2} \bigg(-\frac{16 g_s^2\langle \bar{q}q\rangle^2\langle \bar{s}s\rangle^2}{81} - \frac{\langle g_s^2 GG\rangle\langle \bar{q}q\rangle\langle g_s\bar{s}\sigma Gs\rangle}{1152\pi^2} - \frac{\langle g_s^2 GG\rangle\langle \bar{s}s\rangle\langle g_s\bar{q}\sigma Gq\rangle}{1152\pi^2} - \frac{m_s\langle \bar{q}q\rangle^2\langle g_s\bar{s}\sigma Gs\rangle}{9} - \frac{5m_s\langle \bar{q}q\rangle\langle \bar{s}s\rangle\langle g_s\bar{q}\sigma Gq\rangle}{18} + \frac{m_s\langle \bar{q}q\rangle\langle \bar{s}s\rangle\langle g_s\bar{s}\sigma Gs\rangle}{18} + \frac{m_s\langle \bar{s}s\rangle^2\langle g_s\bar{q}\sigma Gq\rangle}{18} + \frac{m_s^2\langle g_s\bar{q}\sigma Gq\rangle^2}{48\pi^2} - \frac{m_s^2\langle g_s\bar{q}\sigma Gq\rangle\langle g_s\bar{s}\sigma Gs\rangle}{48\pi^2} \bigg),$$

$$\Pi_8^M(M_B^2) = \int_{4m_s^2}^{s_0}\Big[\frac{1}{12\,288\pi^6}s^4 - \frac{17m_s^2}{5120\pi^6}s^3 + \Big(\frac{\langle g_s^2GG\rangle}{18\,432\pi^6} - \frac{m_s\langle\bar{q}q\rangle}{64\pi^4} + \frac{m_s\langle\bar{s}s\rangle}{32\pi^4}\Big)s^2 + \Big(-\frac{\langle\bar{q}q\rangle^2}{24\pi^2} + \frac{\langle\bar{q}q\rangle\langle\bar{s}s\rangle}{6\pi^2} - \frac{\langle\bar{s}s\rangle^2}{24\pi^2} - \frac{m_s\langle g_s\bar{q}\sigma Gq\rangle}{32\pi^4} + \frac{m_s\langle g_s\bar{s}\sigma Gs\rangle}{64\pi^4} - \frac{17m_s^2\langle g_s^2GG\rangle}{18\,432\pi^6}\Big)s - \frac{\langle\bar{q}q\rangle\langle g_s\bar{q}\sigma Gq\rangle}{16\pi^2} + \frac{\langle\bar{q}q\rangle\langle g_s\bar{s}\sigma Gs\rangle}{8\pi^2} + \frac{\langle\bar{s}s\rangle\langle g_s\bar{q}\sigma Gq\rangle}{8\pi^2} - \frac{\langle\bar{s}s\rangle\langle g_s\bar{s}\sigma Gs\rangle}{16\pi^2} + \frac{m_s\langle g_s^2GG\rangle\langle\bar{s}s\rangle}{256\pi^4} - \frac{m_s^2\langle\bar{q}q\rangle^2}{4\pi^2} - \frac{3m_s^2\langle\bar{q}q\rangle\langle\bar{s}s\rangle}{4\pi^2} + \frac{m_s^2\langle\bar{s}s\rangle^2}{16\pi^2}\Big]e^{-s/M_B^2}ds + \Big(-\frac{\langle g_s\bar{q}\sigma Gq\rangle^2}{64\pi^2} + \frac{\langle g_s\bar{q}\sigma Gq\rangle\langle g_s\bar{s}\sigma Gs\rangle}{16\pi^2} - \frac{\langle g_s\bar{s}\sigma Gs\rangle^2}{64\pi^2} - \frac{5\langle g_s^2GG\rangle\langle\bar{q}q\rangle^2}{1728\pi^2} - \frac{5\langle g_s^2GG\rangle\langle\bar{s}s\rangle^2}{1728\pi^2} + \frac{5m_s\langle g_s^2GG\rangle\langle g_s\bar{s}\sigma Gs\rangle}{4608\pi^4} + m_s\langle\bar{q}q\rangle^2 \times\langle\bar{s}s\rangle + \frac{2m_s\langle\bar{q}q\rangle\langle\bar{s}s^2\rangle}{3} - \frac{3m_s^2\langle\bar{s}s\rangle\langle g_s\bar{q}\sigma Gq\rangle}{8\pi^2} - \frac{m_s^2\langle\bar{q}q\rangle\langle g_s\bar{s}\sigma Gs\rangle}{4\pi^2}\Big) + \frac{1}{M_B^2}\Big(-\frac{16g_s^2\langle\bar{q}q\rangle^2\langle\bar{s}s\rangle^2}{27} + \frac{\langle g_s^2GG\rangle\langle\bar{q}q\rangle\langle g_s\bar{q}\sigma Gq\rangle}{1152\pi^2} + \frac{\langle g_s^2GG\rangle\langle\bar{s}s\rangle\langle g_s\bar{s}\sigma Gs\rangle}{1152\pi^2} - \frac{m_s\langle\bar{q}q\rangle^2\langle g_s\bar{s}\sigma Gs\rangle}{3} - \frac{5m_s\langle\bar{q}q\rangle\langle\bar{s}s\rangle\langle g_s\bar{q}\sigma Gq\rangle}{6} + \frac{m_s\langle\bar{q}q\rangle\langle\bar{s}s\rangle\langle g_s\bar{s}\sigma Gs\rangle}{6} + \frac{m_s\langle\bar{s}s\rangle^2\langle g_s\bar{q}\sigma Gq\rangle}{6} - \frac{m_s^2\langle g_s^2GG\rangle\langle\bar{s}s\rangle^2}{1152\pi^2} + \frac{m_s^2\langle g_s\bar{q}\sigma Gq\rangle^2}{16\pi^2} - \frac{m_s^2\langle g_s\bar{q}\sigma Gq\rangle\langle g_s\bar{s}\sigma Gs\rangle}{16\pi^2}\Big).$$

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