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Towards the assignment for the 4^1S_0 meson nonet

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The strong decays of the $\pi(2070)$, $\eta(2010)$, $\eta(2100)$, $\eta(2190)$, and $\eta(2225)$ as the 4^1S_0 quark-antiquark states are investigated in the framework of the $^3P_0$ meson decay model. It is found that the $\pi(2070)$, $\eta(2100)$, and $\eta(2225)$ appear to be the convincing 4^1S_0 $q\bar{q}$ states, while the assignment of the $\eta(2010)$ and $\eta(2190)$ as the 4^1S_0 isoscalar states is not favored by their widths. In the presence of the $\pi(2070)$, $\eta(2100)$, and $\eta(2225)$ being the members of the 4^1S_0 meson nonet, the mass of the 4^1S_0 kaon is phenomenologically determined to be about 2153 MeV. The width of this unobserved kaon is expected to be about 197 MeV in the $^3P_0$ decay model.

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I. INTRODUCTION

From PDG2006 [1], the 1^1S_0 meson nonet (π, η, η', and K) as well as the 2^1S_0 members [$\pi(1300)$, $\eta(1295)$, and $\eta(1475)$] have been well established. In Ref. [2], we suggested that the $\pi(1800)$, $K(1830)$, together with the $X(1835)$ and $\eta(1760)$ observed by the BES Collaboration [3,4], constitute the 3^1S_0 meson nonet. More recently, we argued that the $\eta(2225)$ with a mass of (2240^{+30+30}_{-20-20}) MeV and a width of $(190 \pm 30^{+40}_{-60})$ MeV observed by the BES Collaboration [5] could be the 4^1S_0 $s\bar{s}$ in Ref. [6], where the other members of the 4^1S_0 meson nonet were not discussed. In the present work, we shall address the possible assignment for the 4^1S_0 meson nonet.

With the assignment of the $\eta(2225)$ as the $s\bar{s}$ member of the 4^1S_0 meson nonet, one can expect that both the isovector and other isoscalar members of the 4^1S_0 meson nonet should be lighter than the $\eta(2225)$. Experimentally, in the mass region 2000–2225 MeV, the pseudoscalar states $\pi(2070)$ (mass: 2070 ± 35 MeV; width: 310^{+100}_{-50} MeV), $\eta(2010)$ (mass: 2010^{+35}_{-60} MeV; width: 270 ± 60 MeV), $\eta(2100)$ (mass: 2103 ± 50 MeV; width: 187 ± 75 MeV), and $\eta(2190)$ (mass: 2190 ± 50 MeV; width: 850 ± 100 MeV) are reported [1]. Theoretically, some predicted values for the $\pi(4^1S_0)$ mass are 2.15 GeV by QCD sum rules [7,8], 2.009 GeV by the spectrum integral equation [9], 2.193 GeV by a covariant quark model [10], 2.039 GeV by a relativistic independent quark model [11], and 2.07 GeV by Regge phenomenology [12]. In addition, the mass of the third radial excitation of the η is predicted to be about 2.267 GeV by a covariant quark model [10] or 2.1 GeV by Regge phenomenology [12]. The $\pi(2070)$ mass is similar to the predicted $\pi(4^1S_0)$ mass, and all the masses of the $\eta(2010)$, $\eta(2100)$, and $\eta(2190)$ are close to the predicted mass range of the third radial excitation of the η. Only the mass information of these states is insufficient to classify them. The main purpose of this work is to discuss whether these reported pseudoscalar states can be assigned as the members of the 4^1S_0 meson nonet or not by investigating their decay properties in the $^3P_0$ meson decay model.

The organization of this paper is as follows. In Sec. II, we give a brief review of the $^3P_0$ decay model (for a detailed review, see e.g. Refs. [13–16].) In Secs. III and IV, the decay widths of the $\pi(2070)$, $\eta(2010)$, $\eta(2100)$, $\eta(2190)$, and $\eta(2225)$ as the 4^1S_0 $q\bar{q}$ state are presented. The decay widths of the 4^1S_0 kaon are predicted in Sec. V, and the summary and conclusion are given in Sec. VI.

II. THE $^3P_0$ MESON DECAY MODEL

The $^3P_0$ decay model, also known as the quark-pair creation model, was originally introduced by Micu [17] and further developed by Le Yaouanc *et al.* [13]. The $^3P_0$ decay model, which (in several variants) is the standard model for strong decays, at least for mesons in the initial state, has been widely used to evaluate the strong decays of hadrons [18–27], since it gives a good description of many of the observed decay amplitudes and partial widths of the hadrons. The main assumption of the $^3P_0$ decay model is that strong decays take place via the creation of a $^3P_0$ quark-antiquark pair from the vacuum. The newly produced quark-antiquark pair, together with the $q\bar{q}$ within the initial meson, regroups into two outgoing mesons in all possible quark rearrangements, which corresponds to the two decay diagrams shown in Fig. 1 for the meson decay process $A \to B + C$.

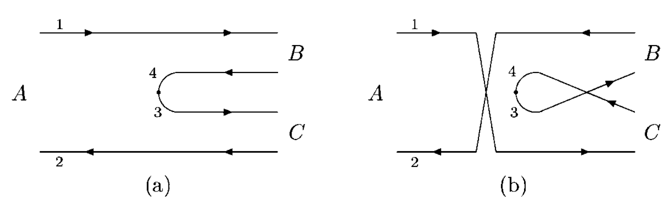

FIG. 1. The two possible diagrams contributing to $A \to B + C$ in the $^3P_0$ model.

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The transition operator T of the decay $A \to BC$ in the $^3P_0$ model is given by

$$T = -3\gamma\sum_m \langle 1m1-m|00\rangle \int d^3\vec{p}_3 d^3\vec{p}_4 \delta^3(\vec{p}_3+\vec{p}_4) \times \mathcal{Y}_1^m\Big(\frac{\vec{p}_3-\vec{p}_4}{2}\Big)\chi^{34}_{1-m}\phi_0^{34}\omega_0^{34} b_3^\dagger(\vec{p}_3)d_4^\dagger(\vec{p}_4), \quad (1)$$

where γ is a dimensionless parameter representing the probability of the quark-antiquark pair $q_3\bar{q}_4$ with $J^{PC} = 0^{++}$ creation from the vacuum, and $\vec{p}_3$ and $\vec{p}_4$ are the momenta of the created quark q_3 and antiquark $\bar{q}_4$, respectively. ϕ_0^{34}, ω_0^{34}, and $\chi^{34}_{1,-m}$ are the flavor, color, and spin wave functions of the $q_3\bar{q}_4$, respectively. The solid harmonic polynomial $\mathcal{Y}_1^m(\vec{p}) \equiv |p|^1 Y_1^m(\theta_p, \phi_p)$ reflects the momentum-space distribution of the $q_3\bar{q}_4$.

For the meson wave function, we adopt the mock meson $|A(n_A^{2S_A+1}L_{AJ_AM_{J_A}})(\vec{P}_A)\rangle$ defined by [28]

$$|A(n_A^{2S_A+1}L_{AJ_AM_{J_A}})(\vec{P}_A)\rangle \equiv \sqrt{2E_A}\sum_{M_{L_A},M_{S_A}}\langle L_AM_{L_A}S_AM_{S_A}|J_AM_{J_A}\rangle\int d^3\vec{p}_A\psi_{n_AL_AM_{L_A}}(\vec{p}_A)\chi^{12}_{S_AM_{S_A}}\phi_A^{12}\omega_A^{12} \times \Big|q_1\Big(\frac{m_1}{m_1+m_2}\vec{P}_A+\vec{p}_A\Big)\bar{q}_2\Big(\frac{m_2}{m_1+m_2}\vec{P}_A-\vec{p}_A\Big)\Big\rangle, \quad (2)$$

where m_1 and m_2 are the masses of the quark q_1 with a momentum of $\vec{p}_1$ and the antiquark $\bar{q}_2$ with a momentum of $\vec{p}_2$, respectively. n_A is the radial quantum number of the meson A composed of $q_1\bar{q}_2$. $\vec{S}_A = \vec{s}_{q_1}+\vec{s}_{\bar{q}_2}$, $\vec{J}_A = \vec{L}_A+\vec{S}_A$, $\vec{s}_{q_1}$ ($\vec{s}_{\bar{q}_2}$) is the spin of q_1 ($\bar{q}_2$), and $\vec{L}_A$ is the relative orbital angular momentum between q_1 and $\bar{q}_2$. $\vec{P}_A = \vec{p}_1+\vec{p}_2$, $\vec{p}_A = \frac{m_1\vec{p}_1-m_1\vec{p}_2}{m_1+m_2}$. $\langle L_AM_{L_A}S_AM_{S_A}|J_AM_{J_A}\rangle$ is a Clebsch-Gordan coefficient, and E_A is the total energy of the meson A. $\chi^{12}_{S_AM_{S_A}}$, ϕ_A^{12}, ω_A^{12}, and $\psi_{n_AL_AM_{L_A}}(\vec{p}_A)$ are the spin, flavor, color, and space wave functions of the meson A, respectively. The mock meson satisfies the normalization condition

$$\langle A(n_A^{2S_A+1}L_{AJ_AM_{J_A}})(\vec{P}_A)|A(n_A^{2S_A+1}L_{AJ_AM_{J_A}})(\vec{P}'_A)\rangle = 2E_A\delta^3(\vec{P}_A-\vec{P}'_A). \quad (3)$$

The S matrix of the process $A \to BC$ is defined by

$$\langle BC|S|A\rangle = I - 2\pi i\delta(E_A-E_B-E_C)\langle BC|T|A\rangle, \quad (4)$$

with

$$\langle BC|T|A\rangle = \delta^3(\vec{P}_A-\vec{P}_B-\vec{P}_C)\mathcal{M}^{M_{J_A}M_{J_B}M_{J_C}}, \quad (5)$$

where $\mathcal{M}^{M_{J_A}M_{J_B}M_{J_C}}$ is the helicity amplitude of $A \to BC$. In the center-of-mass frame of meson A, $\mathcal{M}^{M_{J_A}M_{J_B}M_{J_C}}$ can be written as

$$\mathcal{M}^{M_{J_A}M_{J_B}M_{J_C}}(\vec{P}) = \gamma\sqrt{8E_AE_BE_C}\sum_{M_{L_A},M_{S_A},M_{L_B},M_{S_B},M_{L_C},M_{S_C},m}\langle L_AM_{L_A}S_AM_{S_A}|J_AM_{J_A}\rangle\langle L_BM_{L_B}S_BM_{S_B}|J_BM_{J_B}\rangle \times\langle L_CM_{L_C}S_CM_{S_C}|J_CM_{J_C}\rangle\langle 1m1-m|00\rangle\langle\chi^{14}_{S_BM_{S_B}}\chi^{32}_{S_CM_{S_C}}|\chi^{12}_{S_AM_{S_A}}\chi^{34}_{1-m}\rangle[f_1I(\vec{P},m_1,m_2,m_3) + (-1)^{1+S_A+S_B+S_C}f_2I(-\vec{P},m_2,m_1,m_3)], \quad (6)$$

with $f_1 = \langle\phi_B^{14}\phi_C^{32}|\phi_A^{12}\phi_0^{34}\rangle$ and $f_2 = \langle\phi_B^{32}\phi_C^{14}|\phi_A^{12}\phi_0^{34}\rangle$, corresponding to the contributions from Figs. 1(a) and 1(b), respectively, and

$$I(\vec{P},m_1,m_2,m_3) = \int d^3\vec{p}\,\psi^*_{n_BL_BM_{L_B}}\Big(\frac{m_3}{m_1+m_2}\vec{P}_B+\vec{p}\Big)\psi^*_{n_CL_CM_{L_C}}\Big(\frac{m_3}{m_2+m_3}\vec{P}_B+\vec{p}\Big)\psi_{n_AL_AM_{L_A}}(\vec{P}_B+\vec{p})\mathcal{Y}_1^m(\vec{p}), \quad (7)$$

where $\vec{P} = \vec{P}_B = -\vec{P}_C$, $\vec{p} = \vec{p}_3$, and m_3 is the mass of the created quark q_3.

The spin overlap in terms of Wigner's $9j$ symbol can be given by

$$\langle\chi^{14}_{S_BM_{S_B}}\chi^{32}_{S_CM_{S_C}}|\chi^{12}_{S_AM_{S_A}}\chi^{34}_{1-m}\rangle = \sum_{S,M_S}\langle S_BM_{S_B}S_CM_{S_C}|SM_S\rangle\langle S_AM_{S_A}1-m|SM_S\rangle(-1)^{S_C+1} \times\sqrt{3(2S_A+1)(2S_B+1)(2S_C+1)}\begin{Bmatrix}\frac{1}{2} & \frac{1}{2} & S_A\\ \frac{1}{2} & \frac{1}{2} & 1\\ S_B & S_C & S\end{Bmatrix}. \quad (8)$$

In order to compare with the experiment conventionally, $\mathcal{M}^{M_{J_A}M_{J_B}M_{J_C}}(\vec{P})$ can be converted into the partial amplitude by a recoupling calculation [29],

$$\mathcal{M}^{LS}(\vec{P}) = \sum_{M_{J_B},M_{J_C},M_S,M_L} \langle LM_L SM_S|J_A M_{J_A}\rangle \times \langle J_B M_{J_B} J_C M_{J_C}|SM_S\rangle \times \int d\Omega Y^*_{LM_L} \mathcal{M}^{M_{J_A}M_{J_B}M_{J_C}}(\vec{P}). \quad (9)$$

If we consider the relativistic phase space, the decay width $\Gamma(A \to BC)$ in terms of the partial wave amplitudes is

$$\Gamma(A \to BC) = \frac{\pi P}{4M_A^2}\sum_{LS}|\mathcal{M}^{LS}|^2. \quad (10)$$

Here $P = |\vec{P}| = \frac{\sqrt{[M_A^2-(M_B+M_C)^2][M_A^2-(M_B-M_C)^2]}}{2M_A}$, and M_A, M_B, and M_C are the masses of mesons A, B, and C, respectively.

The decay width can be derived analytically if the simple harmonic oscillator (SHO) approximation for the meson space wave functions is used. In momentum space, the SHO wave function is

$$\psi_{nLM_L}(\vec{p}) = R^{\rm SHO}_{nL}(p)Y_{LM_L}(\Omega_p), \quad (11)$$

where the radial wave function is given by

$$R^{\rm SHO}_{nL} = \frac{(-1)^n(-i)^L}{\beta^{3/2}} \times \sqrt{\frac{2n!}{\Gamma(n+L+\frac{3}{2})}}\left(\frac{p}{\beta}\right)^L e^{-(p^2/2\beta^2)} L_n^{L+(1/2)}\left(\frac{p^2}{\beta^2}\right). \quad (12)$$

Here β is the SHO wave function scale parameter, and $L_n^{L+(1/2)}(\frac{p^2}{\beta^2})$ is an associated Laguerre polynomial.

The SHO wave functions cannot be regarded as realistic; however, they are a *de facto* standard for many nonrelativistic quark model calculations. Moreover, the more realistic space wave functions, such as those obtained from Coulomb, plus the linear potential model, do not always result in systematic improvements due to the inherent uncertainties of the $^3P_0$ decay model itself [19,20,22]. The SHO wave function approximation is commonly employed in the $^3P_0$ decay model in the literature. In the present work, the SHO wave function approximation for the meson space wave functions is taken.

Under the SHO wave function approximation, the parameters used in the $^3P_0$ decay model involve the $q\bar{q}$ pair production strength parameter γ, the SHO wave function scale parameter β, and the masses of the constituent quarks. In the present work, we take $\gamma = 8.77$ and $\beta_A = \beta_B = \beta_C = \beta = 0.4$ GeV, the values recently obtained by fitting 32 experimentally well-determined decay rates with the $^3P_0$ decay model[1], and $m_u = m_d = 0.33$ GeV, $m_s = 0.55$ GeV [26]. The meson masses used to determine the phase space and final state momenta are[2] $M_\pi = 138$ MeV, $M_K = 496$ MeV, $M_\eta = 548$ MeV, $M_{\eta'} = 958$ MeV, $M_\rho = 776$ MeV, $M_{K^*} = 894$ MeV, $M_\omega = 783$ MeV, $M_\phi = 1019$ MeV, $M_{a_2(1320)} = 1318$ MeV, $M_{K_2^*(1430)} = 1429$ MeV, $M_{f_2(1270)} = 1275$ MeV, $M_{f_2'(1525)} = 1525$ MeV, $M_{\pi(1300)} = 1240$ MeV, $M_{a_0(1450)} = 1474$ MeV, $M_{K_0^*(1430)} = 1414$ MeV, $M_{f_0(1370)} = 1370$ MeV, $M_{\rho(1450)} = 1459$ MeV, $M_{\omega(1420)} = 1420$ MeV, $M_{K^*(1580)} = 1580$ MeV, $M_{\rho(1700)} = 1720$ MeV, $M_{K^*(1680)} = 1717$ MeV, and $M_{K_3^*(1780)} = 1776$ MeV.

TABLE I. Decays of the $\pi(2070)$ as the 4^1S_0 isovector state in the $^3P_0$ model.

Mode	Γ_i (MeV)	Mode	Γ_i (MeV)
$\rho\omega$	3.1	$\rho\pi$	5.9
$\pi(1300)\rho$	52.0	$\rho(1700)\pi$	3.6
$\rho(1450)\pi$	112.5	$f_2(1270)\pi$	48.0
$f_0(1370)\pi$	0.3	$a_2(1320)\eta$	8.8
$a_0(1450)\eta$	5.1	KK^*	8.5
K^*K^*	14.9	$K_0^*(1430)K$	10.3
$K_2^*(1430)K$	4.6		
$\Gamma_{\rm thy} = 277.6$ MeV, $\Gamma_{\rm expt} = 310^{+100}_{-50}$ MeV			

III. DECAYS OF THE $\pi(2070)$

From (10), the numerical values of the partial decay widths of the $\pi(2070)$ as the 4^1S_0 isovector state are listed in Table I. The initial state mass is set to 2070 MeV.

Table I indicates that the total width of the $\pi(2070)$ as the 4^1S_0 isovector state predicted by the $^3P_0$ decay model is about 278 MeV, consistent with the observation (310^{+100}_{-50}) MeV within errors, and the dominant decay modes are expected to be $\pi(1300)\rho$, $\rho(1450)\pi$, and $f_2(1270)\pi$. Also, in order to check the dependence of the theoretical result on the initial state mass, the predicted total width of the $\pi(2070)$ is shown in Fig. 2 as a function of the initial state mass. Figure 2 shows that when the initial state mass varies from 2035 to 2105 MeV, the total width of the 4^1S_0 isovector state varies from about 230 to

[1]Our value of γ is higher than that used by Ref. [26] (0.505) by a factor of $\sqrt{96\pi}$, due to different field conventions, constant factors in T, etc. The calculated results of the widths are, of course, unaffected.

[2]The assignment of the $K^*(1410)$ as the 2^3S_1 kaon is problematic [25,30]. The quark model [31] and other phenomenological approaches [32] consistently suggest that the 2^3S_1 kaon has a mass of about 1580 MeV; here we take 1580 MeV as the mass of the 2^3S_1 kaon [$K^*(1580)$]. Also, we assume that the $a_0(1450)$, $K_0^*(1430)$, and $f_0(1370)$ are the ground scalar mesons as in Refs. [23–25].

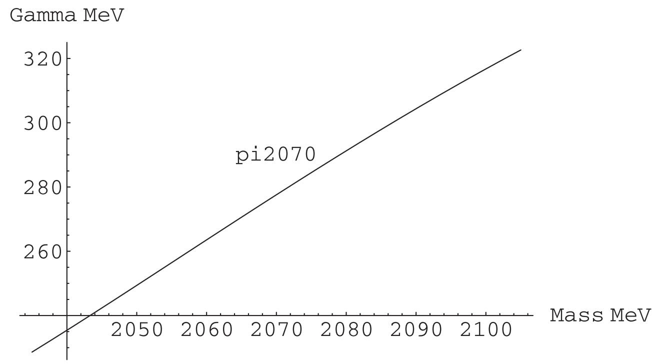

FIG. 2. The predicted total width of the $\pi(2070)$ as the 4^1S_0 isovector versus the initial state mass.

320 MeV, generally in accord with the width range of the $\pi(2070)$. Both the mass and width of the $\pi(2070)$ are consistent with the predicted 4^1S_0 isovector state, which therefore suggests that the assignment of the $\pi(2070)$ as the 4^1S_0 isovector state seems plausible.

IV. DECAYS OF THE $\eta(2010)$, $\eta(2100)$, $\eta(2190)$, AND $\eta(2225)$

In the presence of the $\eta(2225)$, we shall discuss the possibility of the $\eta(2010)$, $\eta(2100)$, or $\eta(2190)$ being another isoscalar member. It is well known that in a meson nonet, the two physical isoscalar states can mix. The mixing of the two isoscalar states can be parametrized as

$$\eta(x) = \cos\phi n\bar{n} - \sin\phi s\bar{s}, \tag{13}$$

$$\eta(2225) = \sin\phi n\bar{n} + \cos\phi s\bar{s}, \tag{14}$$

where $n\bar{n} = (u\bar{u} + d\bar{d})/\sqrt{2}$ and $s\bar{s}$ are the pure 4^1S_0 nonstrange and strange states, respectively, and $\eta(x)$ denotes the $\eta(2010)$, $\eta(2100)$, or $\eta(2190)$.

According to (10), the partial widths of $\eta(x)$ and $\eta(2225)$ become, with mixing,

$$\Gamma(\eta(x) \to BC) = \frac{\pi P}{4M^2_{\eta(x)}} \sum_{LS} |\cos\phi \mathcal{M}^{LS}_{n\bar{n}\to BC} - \sin\phi \mathcal{M}^{LS}_{s\bar{s}\to BC}|^2, \tag{15}$$

$$\Gamma(\eta(2225) \to BC) = \frac{\pi P}{4M^2_{\eta(2225)}} \sum_{LS} |\sin\phi \mathcal{M}^{LS}_{n\bar{n}\to BC} + \cos\phi \mathcal{M}^{LS}_{s\bar{s}\to BC}|^2. \tag{16}$$

Based on (15) and (16), the predicted total widths of the $\eta(2010)$, $\eta(2100)$, $\eta(2190)$, and $\eta(2225)$ are shown in Fig. 3 as functions of the initial state mass and the mixing angle ϕ. From Fig. 3, one can see that, with the variations of the initial state mass and ϕ, only the measured widths of the $\eta(2100)$ and $\eta(2225)$ can possibly be reasonably reproduced in the $^3P_0$ model. We therefore suggest that the assignment of the $\eta(2010)$ and $\eta(2190)$) as the 4^1S_0 isoscalar states seems unfavorable. We shall focus on the possibility of the $\eta(2100)$ being the partner of the $\eta(2225)$. Taking $M_{\eta(2100)} = 2103$ MeV and $M_{\eta(2225)} = 2240$ MeV, we list the numerical values of the partial decay widths of the $\eta(2100)$ and $\eta(2225)$ in Table II. The variation of the theoretical total widths of the $\eta(2100)$ and $\eta(2225)$ with the mixing angle ϕ is shown in Fig. 4.

From Fig. 4, we find that if the $\eta(2100)$–$\eta(2225)$ mixing angle ϕ lies in the range from about -0.6 to $+0.7$ radians, both the measured widths of the $\eta(2100)$ and $\eta(2225)$ can be reasonably reproduced. In order to check whether the

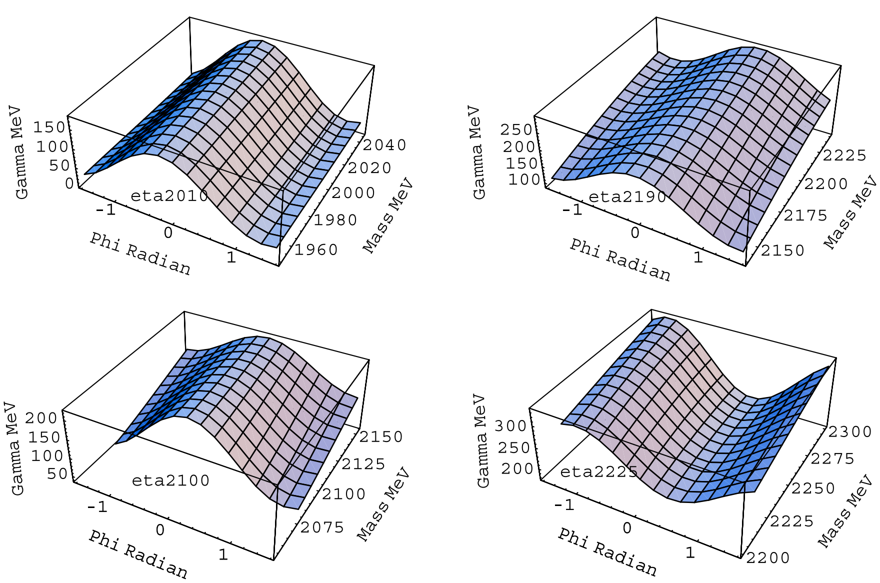

FIG. 3 (color online). The predicted total widths of the $\eta(2010)$, $\eta(2190)$, $\eta(2100)$, and $\eta(2225)$ as the 4^1S_0 isoscalar states versus the initial state mass and the mixing angle ϕ.

TABLE II. Decays of the $\eta(2100)$ and $\eta(2225)$ as the 4^1S_0 isoscalar states in the $^3P_0$ model. $c \equiv \cos\phi$, $s \equiv \sin\phi$.

Mode	$\eta(2100)$ Γ_i (MeV)	$\eta(2225)$ Γ_i (MeV)
$\rho\rho$	$2.1c^2$	$1.4s^2$
$\omega\omega$	$0.9c^2$	$0.3s^2$
$\phi\phi$	$9.7s^2$	$20.1c^2$
$a_2(1320)\pi$	$143.7c^2$	$158.8s^2$
$a_0(1450)\pi$	$0.1c^2$	$4.5s^2$
KK^*	$7.4c^2 - 15.0cs + 7.6s^2$	$14.4c^2 + 12.7cs + 2.8s^2$
K^*K^*	$15.4c^2 - 13.4cs + 2.9s^2$	$0.8c^2 - 6.6cs + 13.6s^2$
$KK_0^*(1430)$	$8.8c^2 - 7.9cs + 1.8s^2$	$2.5c^2 - 4.7cs + 2.2s^2$
$KK_2^*(1430)$	$7.7c^2 - 31.1cs + 31.3s^2$	$69.4c^2 + 91.9cs + 30.4s^2$
$KK^*(1580)$	$5.4c^2 + 17.2cs + 13.6s^2$	$90.7c^2 - 158.1cs + 68.9s^2$
$KK^*(1680)$		$1.6c^2 - 1.4cs + 0.3s^2$
	$\Gamma_{\rm thy} = 191.5c^2 - 50.2cs + 66.9s^2$	$\Gamma_{\rm thy} = 199.4c^2 - 66.2cs + 283.2s^2$
	$\Gamma_{\rm expt} = 187 \pm 75$	$\Gamma_{\rm expt} = 190 \pm 30^{+40}_{-60}$

possibility of $-0.6 \le \phi \le +0.7$ radians exists or not, below we shall estimate the $\eta(2100)$–$\eta(2225)$ mixing angle phenomenologically.

In the $n\bar{n}$ and $s\bar{s}$ bases, the mass-squared matrix describing the $\eta(2100)$ and $\eta(2225)$ mixing can be written as [33,34]

$$M^2 = \begin{pmatrix} M_{n\bar{n}}^2 + 2A_m & \sqrt{2}A_mX \\ \sqrt{2}A_mX & M_{s\bar{s}}^2 + A_mX^2 \end{pmatrix}, \tag{17}$$

where $M_{n\bar{n}}$ and $M_{s\bar{s}}$ are the masses of the states $n\bar{n}$ and $s\bar{s}$, respectively, A_m denotes the total annihilation strength of the $q\bar{q}$ pair for the light flavors u and d, and X describes the $SU(3)$-breaking ratio of the nonstrange and strange quark masses via the constituent quark mass ratio m_u/m_s. The masses of the two physical states $\eta(2100)$ and $\eta(2225)$ can be related to the matrix M^2 by the unitary matrix

$$U = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix},$$

$$UM^2U^\dagger = \begin{pmatrix} M_{\eta(2100)}^2 & 0 \\ 0 & M_{\eta(2225)}^2 \end{pmatrix}. \tag{18}$$

$n\bar{n}$ is the orthogonal partner of the $\pi(4^1S_0)$, the isovector state of the 4^1S_0 meson nonet, and one can expect that $n\bar{n}$ degenerates with $\pi(4^1S_0)$ in effective quark masses; here we take $M_{n\bar{n}} = M_{\pi(4^1S_0)} = M_{\pi(2070)}$. With the help of the Gell-Mann-Okubo mass formula $M_{s\bar{s}}^2 = 2M_{K(4^1S_0)}^2 - M_{n\bar{n}}^2$ [35], the following relations can be derived from (18),

$$8X^2(M_{K(4^1S_0)}^2 - M_{\pi(2070)}^2)^2 = [4M_{K(4^1S_0)}^2 - (2 - X^2)M_{\pi(2070)}^2 - (2 + X^2)M_{\eta(2100)}^2] \times [(2 - X^2)M_{\pi(2070)}^2 + (2 + X^2)M_{\eta(2225)}^2 - 4M_{K(4^1S_0)}^2], \tag{19}$$

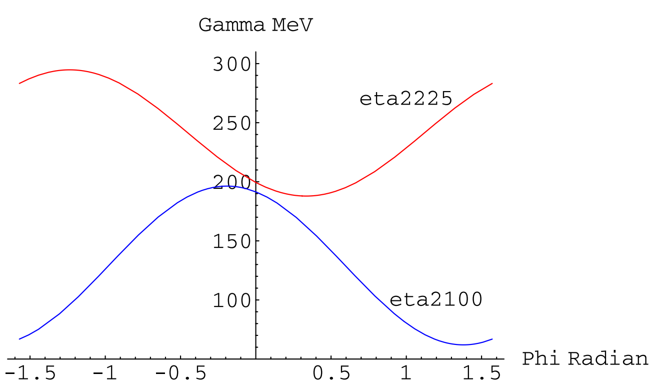

FIG. 4 (color online). The predicted total widths of the $\eta(2100)$ and $\eta(2225)$ as the 4^1S_0 isoscalar states versus the mixing angle ϕ.

$$A_m = \frac{(M^2_{\eta(2225)} - 2M^2_{K(4^1S_0)} + M^2_{\pi(2070)})(M^2_{\eta(2100)} - 2M^2_{K(4^1S_0)} + M^2_{\pi(2070)})}{2(M^2_{\pi(2070)} - M^2_{K(4^1S_0)})X^2}. \tag{20}$$

If the $SU(3)$-breaking effect is not considered, i.e., $X = 1$, relation (19) can be reduced to Schwinger's original nonet mass formula [36]. Taking $X = m_u/m_s = 0.33/0.55 = 0.6$, from (19) and (20) we have

$$M_{K(4^1S_0)} = 2.153\ \mathrm{GeV}, \qquad A_m = 0.07\ \mathrm{GeV}^2. \tag{21}$$

Based on the values of the above parameters involved in (17), the unitary matrix U can be given by

$$U = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} = \begin{pmatrix} +0.995 & -0.104 \\ +0.104 & +0.995 \end{pmatrix}, \tag{22}$$

which gives $\phi = +0.1$ radians, just lying in the range from about -0.6 to $+0.7$ radians. From Table II, this estimated mixing angle leads to $\Gamma_{\mathrm{thy}}(\eta(2100)) = 185.2$ MeV and $\Gamma_{\mathrm{thy}}(\eta(2225)) = 193.7$ MeV, both in good agreement with the experimental results. The $\eta(2100)$ and $\eta(2225)$, together with the $\pi(2070)$, therefore appear to be the convincing 4^1S_0 states.

V. DECAYS OF THE 4^1S_0 KAON

The above two sections show that in the presence of the $\pi(2070)$, $\eta(2100)$, and $\eta(2225)$ belonging to the 4^1S_0 meson nonet, the total widths of these three states can be naturally accounted for in the $^3P_0$ decay model, and the 4^1S_0 kaon is expected to have a mass of about 2153 MeV by the mass formula (19). Below, the $K(2150)$ denotes the 4^1S_0 kaon. We note that the K, $K(1460)$,[3] $K(1830)$, and $K(2150)$ approximately populate a common trajectory, as shown in Fig. 5. The quasilinear trajectories at the (n, mass-squared) plots are able to describe the light mesons with a good accuracy [12]. Figure 5 therefore indicates that the $K(1460)$, $K(1830)$, and $K(2150)$ could be good candidates for the 2^1S_0, 3^1S_0, and 4^1S_0 kaons, respectively.

The $K(2150)$ is not related to any current experimental candidate. The predicted decay widths of the $K(2150)$ are listed in Table III. The initial state mass is set to 2153 MeV. The total width of the $K(2150)$ is predicted to be about 197 MeV, and the dominant decay modes of the $K(2150)$ are expected to be $K_2^*(1430)\pi$, $K^*(1580)\pi$, $\rho(1450)K$, and $a_2(1320)K$. These results could be of use in looking for the candidate for the 4^1S_0 kaon experimentally.

VI. SUMMARY AND CONCLUSION

With the assignment of the $\eta(2225)$ recently observed by the BES Collaboration as the $s\bar{s}$ member of the 4^1S_0 meson nonet, the possibility of the $\pi(2070)$, $\eta(2010)$, $\eta(2100)$, and $\eta(2190)$ being the 4^1S_0 $q\bar{q}$ states is discussed. With respect to the $\pi(2070)$, its assignment to the 4^1S_0 isovector state is favored not only by its mass, but also by its width. The assignment of the $\eta(2010)$ and $\eta(2190)$ as the 4^1S_0 isoscalar states is not favored by their widths. Both the widths of the $\eta(2100)$ and $\eta(2225)$ can be reasonably reproduced with the mixing angle lying in the range from about -0.6 to $+0.7$ radians. The assignment of the $\pi(2070)$, $\eta(2100)$, and $\eta(2225)$ as the members of the 4^1S_0 meson nonet not only leads to the $\eta(2100)$–$\eta(2225)$ mixing angle being about $+0.1$ radians, which naturally accounts for the widths of the $\eta(2100)$ and $\eta(2225)$, but also shows that the 4^1S_0 kaon has a mass of about 2153 MeV. The K, $K(1460)$, $K(1830)$, and $K(2150)$ approximately populate a common (n, mass-squared) trajectory. We tend to conclude that the observed pseudoscalar states $\pi(2070)$, $\eta(2100)$, $\eta(2225)$, together with the unobserved $K(2150)$, appear to be good candidates for the

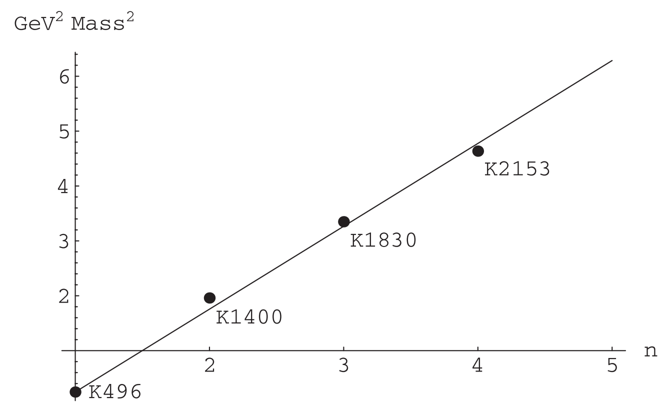

FIG. 5. The (n, mass-squared) trajectory for the K.

TABLE III. Decays of the $K(2150)$ as the 4^1S_0 isodoublet in the $^3P_0$ model.

Mode	Γ_i (MeV)	Mode	Γ_i (MeV)
ρK	3.9	ωK	1.3
ϕK	1.3	ρK^*	0.08
ωK^*	0.04	ϕK^*	12.0
πK^*	4.5	ηK^*	1.5
$\eta' K^*$	0.09	$K_0^*(1430)\pi$	1.6
$K_2^*(1430)\pi$	29.7	$K^*(1580)\pi$	34.5
$K^*(1680)\pi$	2.5	$K_3^*(1780)\pi$	1.0
$\pi(1300)K^*$	6.3	$\rho(1450)K$	42.8
$\omega(1420)K$	14.6	$a_2(1320)K$	26.7
$f_2(1270)K$	9.9	$f_2'(1525)K$	2.9
$\Gamma_{\mathrm{thy}} = 197.2$ MeV			

[3]The $K(1460)$ mass is taken to be 1400 MeV, as reported by [37].

members of the 4^1S_0 meson nonet. The $K(2150)$ width is predicted to be about 197 MeV, and the dominant decay modes of the $K(2150)$ are expected to be $K_2^*(1430)\pi$, $K^*(1580)\pi$, $\rho(1450)K$, and $a_2(1320)K$. These results could be of use in looking for the candidate for the 4^1S_0 kaon experimentally.

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