Which hadronic decay modes are good for η_b searching: Double J/ψ or something else?

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It has been controversial whether η_b can be discovered in Tevatron Run 2 through the decay $\eta_b \to J/\psi J/\psi$ followed by $J/\psi \to \mu^+\mu^-$. I clear this controversy by an explicit calculation which predicts $\mathrm{Br}[\eta_b \to J/\psi J/\psi]$ to be of order 10^{-8}. It is concluded that observing η_b through this decay mode in Tevatron Run 2 may be rather unrealistic. The η_b may be observed in the forthcoming CERN LHC experiments through the 4-lepton channel, if the background events can be significantly reduced by imposing some kinematical cuts. By some rough but plausible considerations, I find that the analogous decay processes $\eta_b \to VV$, $D^*\bar{D}^*$ also have very suppressed branching ratios; nevertheless it may be worth looking for η_b at LHC and Super B factory through the decay modes $\eta_b \to K_S K^\pm \pi^\mp$, $D^*\bar{D}$.

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I. INTRODUCTION

Since the discovery of $\Upsilon(1S)$ nearly three decades ago, extensive search for its pseudoscalar partner, η_b, has been conducted in various experiments. Unfortunately, to date there is still no conclusive evidence that this elusive particle has been found [1].

The existence of η_b is a solid prediction of QCD. On the theoretical side, many works have attempted to unravel its various properties. In particular, its mass is believed to be among the simplest and most tractable observables. Numerous estimates for $\Upsilon - \eta_b$ mass splitting span the range 20–140 MeV [2–11]. Among different theoretical approaches, perturbative QCD is believed by many people to yield reliable predictions, because of decently heavy b mass and η_b being the lowest-lying $b\bar{b}$ state. By far the most sophisticated prediction along this direction, facilitated by the NRQCD renormalization group technique, gives $M_{\eta_b} = 9.421 \pm 0.013$ GeV [10]. An eventual unambiguous observation of η_b and precise measurement of its mass will decisively test the weakly coupled picture of the $\bar{b}b$ ground state.

Much effort is spent to search for η_b from $\gamma\gamma$ collisions in the full data samples of CERN LEP 2, where approximately two hundred η_b are expected to be produced. ALEPH has one candidate event $\eta_b \to K_S(\to \pi^+\pi^-)K^-\pi^+\pi^-\pi^+$ (possibly missing a π^0) with the reconstructed mass of 9.30 ± 0.03 GeV, but consistent with being a background event [12]. ALEPH, L3, DELPHI have also set upper limits on the branching fractions for the η_b decays into 4, 6, 8 charged particles [12–14]. Based on the 2.4 fb^{-1} data taken at the $\Upsilon(2S)$ and $\Upsilon(3S)$ resonances, CLEO has searched for distinctive single photons from hindered $M1$ transitions $\Upsilon(2S)$, $\Upsilon(3S) \to \eta_b\gamma$ and $\Upsilon(3S) \to h_b\pi^0$, $h_b\pi^+\pi^-$ followed by $E1$ transition $h_b \to \eta_b\gamma$, but no signals have been seen [15].

Hadron collider experiments provide an alternative environment to search for η_b. Unlike the e^+e^- machines which are limited by the low yield of η_b, hadron colliders generally possess much larger production cross sections for η_b, which in turn allows for searching for it through some relatively rare decay modes yet with clean signatures. However, one should bear in mind that a noteworthy disadvantage also accompanies hadron collision experiments, i.e., that the corresponding background events may also be copious, so the effectiveness of these decay modes might be seriously discounted (Such an example may be the decay mode $\eta_b \to \gamma\gamma$, with branching fraction $\sim 10^{-4}$, but the combinatorial background γ events can be enormous).

Encouraged by the large observed width of $\eta_c \to VV$ (V stands for light vector mesons), Braaten, Fleming, and Leibovich (hereafter BFL) have suggested that the analogous decay process $\eta_b \to J/\psi J/\psi$, followed by both J/ψ decays to muon pairs, may be used as a very clean trigger to search for η_b at Tevatron Run 2 [16]. Assuming $\mathrm{Br}[\eta_b \to J/\psi J/\psi] \sim 1/m_b^4$, they rescale the measured branching ratio of $\eta_c \to \phi\phi$ by a factor of $(m_c/m_b)^4$ to estimate[1]

$$\mathrm{Br}[\eta_b \to J/\psi J/\psi] = 7 \times 10^{-4\pm 1}. \tag{1}$$

Combining that with the knowledge about the production rate of η_b at the Tevatron, BFL conclude that the prospect of observing η_b through the 4μ decay mode in Run 2 is promising.

Following this suggestion, CDF has searched for the $\eta_b \to J/\psi J/\psi \to 4\mu$ events in the full Run 1 data sample [19]. A small cluster of seven events are seen in the search window, where 1.8 events are expected from background,

[1] Note that $\mathrm{Br}_{\mathrm{exp}}[\eta_c \to \phi\phi]$ has shifted from $(7.1 \pm 2.8) \times 10^{-3}$ given in the 2000 Particle Data Group (PDG) edition [17], which was quoted by BFL, to the latest PDG value $(2.7 \pm 0.9) \times 10^{-3}$ [18].

with the statistical significance of 2.2σ. A simple fit infers the cluster's mass to be $9.445 \pm 0.006(\mathrm{stat})$ GeV. If this cluster is truly due to η_b, then the product of its production cross section and decay branching ratio is close to the upper end of BFL expectation.

In a recent work, Maltoni and Polosa (henceforth MP) nevertheless argue that the BFL estimate, (1), may be overly optimistic [20]. MP suspect that the analogy between $\eta_c \to \phi\phi$ and $\eta_b \to J/\psi J/\psi$ is only superficial. It is known that the perturbative QCD (pQCD) framework has difficulty accounting for the large observed widths of the $\eta_c \to VV$ decay processes [21,22]. The consensus is that some nonperturbative mechanisms should be invoked to reconcile the discrepancy between the pQCD prediction and the actual measurement [23,24]. On the other hand, due to heavy b and c masses, it is rather reasonable to expect that $\eta_b \to J/\psi J/\psi$ can be safely tackled within the pQCD scheme. Therefore, the rescaling procedure used by BFL, whose validity should reside only in the domain of pQCD, may well be illegitimate when taking the measured ratio of $\eta_c \to \phi\phi$ as input, which is essentially dictated by nonperturbative dynamics.

The persuasive evidence in favor of MP's argument comes from their explicit calculation for the inclusive decay rate of η_b to 4-charm states,

$$\mathrm{Br}[\eta_b \to c\bar{c}c\bar{c}] = 1.8^{+2.3}_{-0.8} \times 10^{-5}. \tag{2}$$

This ratio is even smaller than the lower limit of $\mathrm{Br}[\eta_b \to J/\psi J/\psi]$ estimated by BFL. This is clearly at odds with the usual thought that the exclusive decay rate should be much smaller than the inclusive one.

One of the major concerns of this paper is to dispose of this controversy by performing an explicit calculation. Heavy b and c quark masses set hard scales so that one can confidently utilize pQCD to tackle this decay process, expecting those nonperturbative contributions plaguing the decay $\eta_c \to VV$ play an insignificant role here. Since each involved particle is a heavy quarkonium, I work with the color-singlet model, in line with the calculation done for double charmonium production at e^+e^- colliders [25–31]. It is found that at the lowest order in α_s, retaining the transverse momentum of c inside J/ψ is vital to obtain a nonvanishing result, and consequently, the correct asymptotic behavior of the hadronic decay branching ratio is $\alpha_s^2 v_c^{10}(m_c/m_b)^8$ (v_c stands for the typical velocity of c in J/ψ). Numerically, I predict

$$\mathrm{Br}[\eta_b \to J/\psi J/\psi] = (0.5\text{–}6.6) \times 10^{-8}, \tag{3}$$

which is much smaller than the BFL estimate, (1). Simple analysis indicates that the cluster reported by CDF [19] is extremely unlikely to be affiliated with η_b. I further argue that the potential of discovering η_b through this decay mode is gloomy even in Run 2.

To better guide the experimental search for η_b, an important issue is to know, besides $\eta_b \to J/\psi J/\psi$, which hadronic decay modes can serve as efficient triggers to detect η_b. Heavy η_b mass opens a huge phase space available for innumerable decay modes, but at a price that the branching ratio of each individual mode has been greatly diluted with respect to that from η_c decay.

At any rate, it is valuable to know which hadronic decay modes of η_b possess the largest branching ratios. I examine numerous hadronic decay channels, mostly two-body ones, e.g., η_b decays to two light mesons, and two charmed mesons. By some crude but plausible estimate, I find $\eta_b \to \phi\phi$, $D^*\bar{D}^*$ also have very tiny branching ratios, of the same order as that for $\eta_b \to J/\psi J/\psi$. In sharp contrast, the branching ratios for $\eta_b \to D^*\bar{D}$ might be as large as 10^{-5}; therefore they can be used as searching modes. Furthermore, stimulated by the experimental fact that η_c decays to three pseudoscalars have the largest branching ratios, I urge experimentalists to look into such 3-body channels as $\eta_b \to K\bar{K}\pi$, $\eta(\eta')\pi\pi$, $D\bar{D}\pi$. The corresponding ratios are estimated to be of order 10^{-4}.

The remainder of the paper will proceed as follows. In Sec. II, I discuss the asymptotic behavior of $\mathrm{Br}[\eta_b \to J/\psi J/\psi]$ in the limit $m_b \gg m_c \gg \Lambda_{\mathrm{QCD}}$, and elucidate the peculiarity of this decay process. I then present the actual calculation of the ratio in Sec. III, employing the color-singlet model which incorporates velocity expansion. In Sec. IV, I discuss the observation potential of η_b through this mode in Tevatron and LHC. In Sec. V, by some simple scaling analysis, I estimate the branching ratios for η_b decays to various final states, e.g., to two light mesons, to three light pseudoscalar mesons, and to two charmed mesons. For the decay process $\eta_b \to VV$, I also compare my estimates with those obtained from some nonperturbative mechanism. In Sec. VI I summarize and give an outlook.

II. ASYMPTOTIC BEHAVIOR AND UNNATURAL DECAY PROCESS

Before launching into the actual calculation, it is instructive to first envisage the general property of this exclusive decay process. A powerful tool in pQCD to count kinematical suppression factors for hard exclusive processes is the so-called *hadron helicity selection rule*, originally developed for the reactions involving light hadrons [32]. This rule has recently been applied to analyze the double charmonium production in e^+e^- annihilation in the limit $\sqrt{s} \gg m_c$ [25]. One can work out the asymptotic behavior for $\mathrm{Br}[\eta_b \to J/\psi J/\psi]$ in an analogous way. The decay $\eta_b \to J/\psi J/\psi$ can be initiated by either strong or electromagnetic interaction, with the corresponding lowest-order diagrams shown in Figs. 1 and 2, respectively. Generally speaking, the QCD contribution is dominating over the QED contribution. However, for the sake of completeness, the latter will be also included in my analysis.

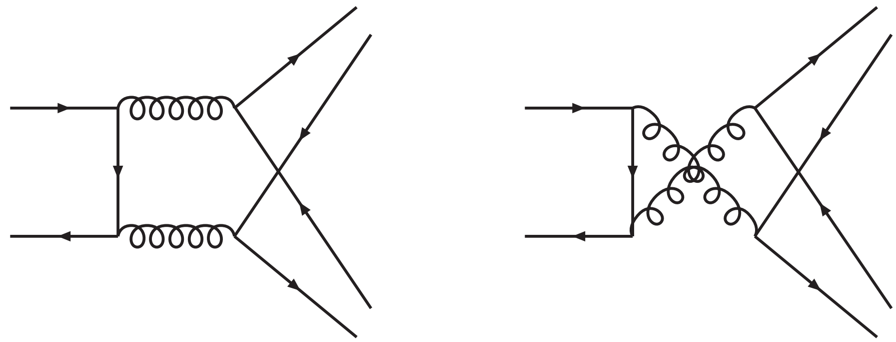

FIG. 1. Lowest-order QCD diagrams that contribute to $\eta_b \to J/\psi J/\psi$.

For the lowest-order strong decay process depicted in Fig. 1, a simple consideration may suggest that in the limit $m_b \to \infty$ with m_c fixed,

$$\mathrm{Br}_{\mathrm{str}}[\eta_b \to J/\psi(\lambda) + J/\psi(\tilde{\lambda})] \sim \alpha_s^2 v_c^6 \left(\frac{m_c^2}{m_b^2}\right)^{2+|\lambda+\tilde{\lambda}|}, \quad (4)$$

where λ and $\tilde{\lambda}$ represent the helicities of two J/ψ viewed in the η_b rest frame, and v_c denotes the characteristic velocity of c in J/ψ. Obviously the scaling behavior depends on the helicity configurations of both J/ψ. The factor v_c^6 is expected because there is a factor of wave function at the origin, $\psi_{J/\psi}(0)$, for each $c\bar{c}$ pair to emerge with small relative momentum to form an S-wave bound state, and $\psi_{J/\psi}(0) \sim (m_c v_c)^{3/2}$.

The expectation (4) is compatible with the helicity selection rule that the decay configuration which conserves the hadron helicity, i.e. $\lambda + \tilde{\lambda} = 0$, exhibits the slowest asymptotic decrease, $\mathrm{Br}_{\mathrm{str}} \sim 1/m_b^4$. The only helicity state bearing this least suppressed ratio which is also compatible with the angular momentum conservation $\lambda = \tilde{\lambda}$ is thus $(\lambda, \tilde{\lambda}) = (0, 0)$. The helicity conservation can be violated either by the nonzero charm mass m_c or by the transverse momentum of c inside J/ψ, $q_\perp$. For every unit of violation of the selection rule, there is a further suppression factor of m_c^2/m_b^2 or $q_\perp^2/m_b^2$. For the other physically allowed configurations $(\lambda, \tilde{\lambda}) = (\pm 1, \pm 1)$, the helicity conservation is violated by two units, so one expects $\mathrm{Br}_{\mathrm{str}} \sim 1/m_b^8$. By writing (4) the way it is, I have temporarily assumed that the cause of violation is entirely due to the quark mass m_c.

The essential assumption of BFL is the "leading twist" scaling behavior $\mathrm{Br}_{\mathrm{str}} \sim 1/m_b^4$, which is tacitly associated with the η_b decay to two longitudinally polarized J/ψ.[2] However, one may recall that $\eta_b \to J/\psi J/\psi$, like $\eta_c \to VV$, belongs to a class of so-called *unnatural* decay processes [33], for which the helicity state $(0, 0)$ is strictly forbidden due to the conflict between parity and angular momentum conservation.[3] In the operational basis, it arises because the decay amplitude for such a process involves the Levi-Civita tensor and there are not enough numbers of independent Lorentz vectors to contract with it, for vector mesons are longitudinally polarized. Further examples of the *unnatural* processes for a bottomonium decay to two S-wave charmonia include $\Upsilon(\chi_{b2}) \to J/\psi \eta_c$ and $\chi_{b1} \to J/\psi J/\psi$, in contrast with the *natural* decay processes such as $\chi_{b0,2} \to J/\psi J/\psi$ [34], $\eta_c \eta_c$, and $h_b \to J/\psi \eta_c$.

One may wish to examine this assertion closely for my case. Parity and Lorentz invariance constrain the decay amplitude to have the following tensor structure:

$$\mathcal{M}(\lambda, \tilde{\lambda}) \propto \epsilon^{\mu\nu\alpha\beta} P_\mu \tilde{P}_\nu \varepsilon_\alpha^*(\lambda) \tilde{\varepsilon}_\beta^*(\tilde{\lambda}), \quad (5)$$

where P, $\tilde{P}$, ε, and $\tilde{\varepsilon}$ are momenta and polarization vectors for both J/ψ. If both J/ψ are longitudinally polarized, ε and $\tilde{\varepsilon}$ then can be expressed as linear combinations of P and $\tilde{P}$; $M(0, 0)$ thus vanishes.[4] It is worth emphasizing that this result is based solely on the basic principle of parity and angular momentum conservation, so it will not depend on dynamical details. For example, it will be true irrespective of whether this process is initiated by QCD or QED, whether higher order perturbative corrections are included or not, and whether nonperturbative QCD effects are incorporated or not.

In passing, one may notice an equivalent but more intuitive explanation for $M(0, 0) = 0$ for the unnatural decay processes [22]. It can be attributed to a peculiar property of Clebsch-Gordon coefficients that $\langle 10|10; 10\rangle = 0$. I take $\eta_b \to J/\psi J/\psi$ as an example to illustrate this. Because of parity and angular momentum conservation, two J/ψ must have the relative orbital angular momentum $L = 1$ and the total spin $S = 1$. However, it is impossible for the two longitudinally polarized J/ψ to

[2]The decay $\eta_b \to J/\psi \eta_c$ would exhibit the "leading twist" scaling behavior, $\mathrm{Br} \sim \alpha_s^2 v_c^6 (m_c/m_b)^4$, were it not inhibited by C invariance.

[3]An unnatural process, according to Ref. [33], is defined as a heavy quarkonium two-meson decay process that does not conserve a multiplicative quantum number called *naturalness*, which is defined by $\sigma = (-1)^J P$ (J, P stand for the spin and parity of a meson).

[4]Exactly with the same argument, the decay $e^+e^- \to \gamma^* \to J/\psi(\lambda = 0) + \eta_c$ turns out to be strictly forbidden.

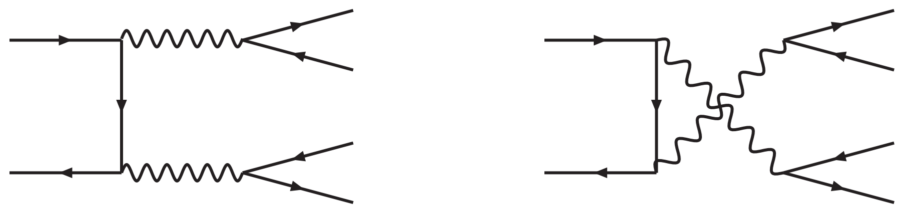

FIG. 2. Lowest-order QED diagrams that contribute to $\eta_b \to J/\psi J/\psi$. Only the fragmentation type diagrams are retained, whereas the other two, which can be obtained by replacing the gluons in Fig. 1 by photons, have been suppressed.

couple to a $(S, S_z) = (1, 0)$ state because of this vanishing Clebsch-Gordon coefficient.

Since both J/ψ in the final state must be transversely polarized, the helicity conservation is violated by two units, and Eq. (4) then indicates $\mathrm{Br}_{\mathrm{str}}[\eta_b \to J/\psi_\perp J/\psi_\perp] \sim \alpha_s^2 v_c^6 (m_c/m_b)^8$. Nevertheless, as the explicit calculation will reveal, this behavior is not quite correct in counting powers of v_c. It turns out that the true asymptotic behavior is even more suppressed,

$$\mathrm{Br}_{\mathrm{str}}[\eta_b \to J/\psi(\pm 1) + J/\psi(\pm 1)] \sim \alpha_s^2 v_c^6 \left(\frac{m_c^2}{m_b^2}\right)^2 \left(\frac{q_\perp^2}{m_b^2}\right)^2 \sim \alpha_s^2 v_c^{10} \left(\frac{m_c}{m_b}\right)^8, \qquad (6)$$

where $q_\perp \sim m_c v_c$ is assumed in the last term. This implies that, at the lowest order in α_s, the violation of the rule should be ascribable to the nonzero transverse momentum of c in J/ψ, instead of the nonzero m_c. This is compatible with the earlier finding that the amplitude for $\eta_c \to V_\perp V_\perp$ vanishes in the collinear quark configuration, even though the quark masses are kept nonzero [21]. After $q_\perp$ is included for the light vector mesons, the decay rate then becomes nonzero, though still too tiny to account for the measured rates [22].

One can also infer the asymptotic behavior of the electromagnetic contribution in Fig. 2. The photon fragmentation produces transversely polarized J/ψ. Since the hard scale is set by the virtuality of fragmenting photon $\sim m_c^2$, no suppression factor $\propto 1/m_b^2$ can arise in the branching ratio. A simple counting rule suggests that the QED fragmentation contribution to the ratio exhibits the following behavior:

$$\mathrm{Br}_{\mathrm{em}}[\eta_b \to J/\psi(\pm 1) + J/\psi(\pm 1)] \sim \left(\frac{\alpha^4}{\alpha_s^2}\right) v_c^6. \qquad (7)$$

Although the electromagnetic contribution is free of suppression by inverse powers of m_b, in general it still cannot counteract the adversity caused by $\alpha \ll \alpha_s$.

There is also the interference term between QCD and QED contributions, with a scaling behavior intermediate between (6) and (7).

III. COLOR-SINGLET MODEL CALCULATION

In this section, I present a calculation for $\eta_b \to J/\psi J/\psi$ in the perturbative QCD scheme. As stressed before, this scheme is expected to generate reliable answer for this process, since the annihilation of $b\bar{b}$ and creations of $c\bar{c}$ pairs take place in rather short distances. This decay process should be contrasted with the analogous process $\eta_c \to VV$. To account for the unnaturally large measured width of the latter, it is proposed that some nonperturbative mechanisms, e.g., the mixing among pseudoscalar states $\eta - \eta' - \eta_c$, either due to QCD axial anomaly [23], or due to perturbative box diagram [24], together with quark pair creation from vacuum, should play a prominent role. The impact of final state interaction on the doubly Okubo-Zweig-Iizuka–suppressed process $\eta_c \to \omega\phi$ has also been addressed [35]. Fortunately, neither of these nagging complications will bother us, because the η_b is too heavy to bear a significant mixing with η_c, and c is too heavy to easily pop out of the vacuum. The weak interquarkonium van der Waals interaction [36] implies that the final state effects are also unimportant.

I first consider the QCD contribution in Fig. 1 for $\eta_b(K) \to J/\psi(P) + J/\psi(\tilde{P})$, where K, P, and $\tilde{P}$ signify momenta of each quarkonium. In this work, I will only consider the color-singlet Fock state of the quarkonium, completely ignoring the possible color-octet effect, which is difficult to analyze for exclusive processes in a clear-cut way.[5]

In the color-singlet model calculation, it is customary to begin with the parton level matrix element $b(k)\bar{b}(\bar{k}) \to c(p)\bar{c}(\bar{p}) + c(\tilde{p})\bar{c}(\tilde{\bar{p}})$, then project this matrix element onto the corresponding color-singlet quarkonium Fock states. It is worth stressing that because of the impossibility to find a frame such that two back-to-back fast moving J/ψ become simultaneously at rest, it is imperative to admit a manifestly Lorentz-covariant projector.

[5]Note the color-singlet model can be viewed as a truncated version of the nonrelativistic QCD (NRQCD) factorization approach [37], which still admits a factorization of the calculable short-distance part and universal long-distance factors in the color-singlet channel. For the exclusive processes involving heavy quarkonium, the color-single model and NRQCD are often used interchangeably in the literature.

For the $c\bar{c}$ pair with total momentum P, I assign the momentum carried by each constituent as

$$p = \frac{P}{2} + q, \qquad \bar{p} = \frac{P}{2} - q, \tag{8}$$

where q is the relative momentum satisfying $P \cdot q = 0$. In the rest frame of the $c\bar{c}$ pair, P^μ is thus purely timelike and q^μ purely spacelike. Since the $c\bar{c}$ pair forms J/ψ, it is necessarily in a spin-triplet color-singlet state, and one can replace the product of the Dirac and color spinors for c and $\bar{c}$ in the final state with the following Lorentz-covariant projector [25,38]:

$$v(\bar{p})\bar{u}(p) \to \int [dq] \frac{1}{4\sqrt{2}E(E+m_c)} (\bar{\not{p}} - m_c)\not{\varepsilon}^*(\lambda)(\not{P} + 2E) \times (\not{p} + m_c)\left(\frac{1}{\sqrt{m_c}}\phi_{J/\psi}(-q^2)\right) \otimes \left(\frac{\mathbf{1}_c}{\sqrt{N_c}}\right), \tag{9}$$

where $p^2 = \bar{p}^2 = m_c^2$, $E = \sqrt{P^2}/2 = \sqrt{m_c^2 - q^2}$ is a Lorentz scalar, and ε^* is the polarization vector of J/ψ satisfying $\varepsilon(\lambda) \cdot \varepsilon^*(\lambda') = -\delta^{\lambda\lambda'}$ and $P \cdot \varepsilon^* = 0$. $N_c = 3$ and $\mathbf{1}_c$ stands for the unit color matrix. The momentum space wave function $\phi_{J/\psi}(-q^2)$ is explicitly included and the Lorentz-invariant measure is defined as

$$\int [dq] \equiv \int \frac{d^4 q}{(2\pi)^3} 2E\delta(P \cdot q). \tag{10}$$

The introduction of the covariant projector (9) unites the nonrelativistic internal motion and highly relativistic external motion of J/ψ in a coherent fashion, which is indispensable if one wishes to systematically implement relativistic corrections for highly energetic processes like this one. At the final stage of the calculation, one can always choose to perform the momentum integrals in the rest frame of each J/ψ successively, thanks to the Lorentz-invariant measure (10). I take the following integral, which arises in the zeroth order of relativistic expansion, as an example:

$$\int [dq]\phi_{J/\psi}(-q^2) = \int \frac{d^3 q}{(2\pi)^3} \phi_{J/\psi}(\mathbf{q}^2)|_{\text{rest frame}} = \psi_{J/\psi}(0), \tag{11}$$

where $\psi_{J/\psi}(0)$ is the spatial Schrödinger wave function at the origin for J/ψ.

The above formulas obviously also apply to the second J/ψ, once the replacements $P \to \tilde{P}$, $q \to \tilde{q}$, $E \to \tilde{E}$, and $\varepsilon(\lambda) \to \tilde{\varepsilon}(\tilde{\lambda})$ are made.

For the η_b in the initial state, it turns out that the $\mathcal{O}(v_b^2)$ correction is less relevant, so I will neglect the relative momentum and simply take $k = \bar{k} = K/2$. Consequently, the following simplified projector will be used:

$$u(k)\bar{v}(\bar{k}) \to \frac{1}{2\sqrt{2}}(\not{K} + 2m_b)i\gamma_5 \times \left(\frac{1}{\sqrt{m_b}}\psi_{\eta_b}(0)\right) \otimes \left(\frac{\mathbf{1}_c}{\sqrt{N_c}}\right). \tag{12}$$

For processes involving heavy quarkonium, one usually organizes the amplitude in powers of the relative momenta, to accommodate the NRQCD ansatz. Some simple algebra shows that the η_b part of the matrix element contains the factor

$$\mathrm{Tr}[\gamma_5 \not{K}\gamma^\mu(\not{q} - \tilde{\not{q}})\gamma^\nu] = 4i\epsilon^{\mu\nu\rho\sigma}K_\rho(q - \tilde{q})_\sigma. \tag{13}$$

In the leading order in v_c expansion, $q = \tilde{q} = 0$, the amplitude hence vanishes. As a matter of fact, Anselmino *et al.* have drawn the same conclusion for $\eta_c \to VV$ while using the light-cone scheme in the collinear quark configuration but retaining nonzero light quark masses [21]. This color-singlet-model result can be viewed as a specific example of theirs by invoking a narrow-peak approximation to the light-cone wave function for vector mesons.

In order to obtain a nonvanishing amplitude, one must go to the next-to-leading order in v_c. In particular, one should expand the remaining part of the amplitude to linear order in q and $\tilde{q}$, to pair up with the ones in Eq. (13).[6]

First I expand the product of three propagators in Fig. 1:

$$\frac{1}{(q-\tilde{q})^2 - m_b^2} \frac{1}{(K/2 - q + \tilde{q})^2} \frac{1}{(K/2 + q - \tilde{q})^2} \approx -\frac{1}{m_b^6}\left[1 + \mathcal{O}\left(\frac{q^2}{m_b^2}, \frac{\tilde{q}^2}{m_b^2}, \frac{q \cdot \tilde{q}}{m_b^2}\right)\right]. \tag{14}$$

Because the subleading terms are at least quadratic in q or $\tilde{q}$, they can be dropped.

The missing q or $\tilde{q}$ thus must arise from the J/ψ part of the matrix element, or more precisely, from the projectors for J/ψ, Eq. (9). After some straightforward Dirac trace algebra, one can express the full QCD amplitude in the Lorentz-invariant form:

$$\mathcal{M}_{\text{str}} = -4\sqrt{2}C_{\text{str}}g_s^4 \frac{\psi_{\eta_b}(0)}{m_b^{13/2}m_c} A, \tag{15}$$

where the color factor $C_{\text{str}} = N_c^{-3/2}\,\mathrm{Tr}(T^aT^b)\,\mathrm{Tr}(T^aT^b) = 4^{-1}N_c^{-3/2}(N_c^2 - 1)$, and the double integral A reads

$$A = \iint [dq][d\tilde{q}]\phi_{J/\psi}(-q^2)\phi_{J/\psi}(-\tilde{q}^2) \times [\epsilon_{\mu\nu\alpha\beta}P^\mu\tilde{P}^\nu q^\alpha(\tilde{\varepsilon}^{*\beta} q \cdot \varepsilon^* - \varepsilon^{*\beta} q \cdot \tilde{\varepsilon}^*) + (q \leftrightarrow \tilde{q})], \tag{16}$$

where only the terms with the intended accuracy of $\mathcal{O}(v_c^2)$

[6]Retaining the relative momentum in η_b but not q and $\tilde{q}$ still leads to vanishing amplitude.

are kept. I have substituted E, $\tilde{E}$ everywhere by m_c, which is legitimate in the current level of accuracy.[7]

One immediately observes that, to survive when contracted with the antisymmetric tensor, q and $\tilde{q}$, as well as ε^* and $\tilde{\varepsilon}^*$ in (16), must be *transverse*. I thus confirm the earlier assertion that transverse momentum is the agent to violate the helicity conservation.

Because the transverse components of a 4-vector are invariant under the boost along the moving direction of J/ψ, one can perform the integrals in the rest frame of J/ψ without concerning the boost effect. Using the spherical symmetry of S-wave wave function, I have

$$\int [dq] q_\perp^i q_\perp^j \phi_{J/\psi}(-q^2) = \frac{\delta^{ij}}{3} \times \int \frac{d^3q}{(2\pi)^3} \mathbf{q}^2 \phi_{J/\psi}(\mathbf{q}^2)|_{\text{rest frame}} \equiv \frac{\delta^{ij}}{3} m_c^2 \langle v^2\rangle_{J/\psi} \psi_{J/\psi}(0). \tag{17}$$

Here $\langle v^2\rangle_{J/\psi}$ is a quantity governing the size of relativistic corrections. Loosely speaking, it is related to the second derivative of the wave function at the origin for J/ψ, and characterizes the average v^2 of c inside J/ψ. Inspecting Eq. (17), however, one immediately realizes at large $\mathbf{q}$, the Coulomb wave function should dominate and this quantity turns out to be linearly ultraviolet divergent, hence its meaning becomes obscure. In fact, this factor admits a rigorous definition as a ratio of NRQCD matrix elements [25,37]:

$$\langle v^2\rangle_{J/\psi} = \frac{\boldsymbol{\epsilon}\cdot\langle J/\psi(\boldsymbol{\epsilon})|\psi^\dagger \boldsymbol{\sigma}(-\mathbf{D}^2)\chi|0\rangle}{m_c^2 \boldsymbol{\epsilon}\cdot\langle J/\psi(\boldsymbol{\epsilon})|\psi^\dagger\boldsymbol{\sigma}\chi|0\rangle}, \tag{18}$$

where ψ and χ represent Pauli spinor fields in NRQCD, and $\mathbf{D}$ is the spatial covariant derivative. The matrix elements appearing in the above ratio should be understood to be the renormalized ones. Lattice QCD extraction of this quantity has been available long ago, but the precision is quite poor [39]. I will specify my choice for the numerical value of $\langle v^2\rangle_{J/\psi}$ in Sec. IV.

Substituting (17) into (16), I obtain

$$A = -\tfrac{4}{3} m_c^2 \langle v^2\rangle_{J/\psi} \psi_{J/\psi}^2(0) \epsilon_{\mu\nu\alpha\beta} P^\mu \tilde{P}^\nu \varepsilon^{*\alpha}\tilde{\varepsilon}^{*\beta}, \tag{19}$$

which has the desired tensor structure as in (5). The strong decay amplitude then reads

$$\mathcal{M}_{\text{str}} = \frac{512\sqrt{6}\pi^2\alpha_s^2 m_c}{27 m_b^{13/2}} \psi_{\eta_b}(0)\psi_{J/\psi}^2(0) \times \langle v^2\rangle_{J/\psi} \epsilon^{\mu\nu\alpha\beta} P_\mu \tilde{P}_\nu \varepsilon_\alpha^* \tilde{\varepsilon}_\beta^*. \tag{20}$$

[7] For the same reason, I have not bothered to include explicitly the relativistic effects due to 2-body phase space and normalization of $c\bar{c}$ states as considered in Refs. [25,38], which are powers of E/m_c or $\tilde{E}/m_c$.

Next I turn to the electromagnetic contribution to $\eta_b \to J/\psi J/\psi$. Two QED diagrams which have the same topology as Fig. 1, but with gluons replaced by photons, lead to the amplitude of the same form as (20) except α_s^2 is replaced by $e_b^2 e_c^2 \alpha^2$. Obviously, their contributions are much more suppressed than those from the fragmentation diagrams in Fig. 2, hence will not be considered. The QED fragmentation contribution to the amplitude can be easily worked out,

$$\mathcal{M}_{\text{em}} = \frac{24\sqrt{6}\pi^2 e_b^2 e_c^2\alpha^2}{m_b^{5/2} m_c^3} \psi_{\eta_b}(0)\psi_{J/\psi}^2(0) \times \left(1 - \frac{2m_c^2}{m_b^2}\right)^{-1} \epsilon^{\mu\nu\alpha\beta} P_\mu \tilde{P}_\nu \varepsilon_\alpha^* \tilde{\varepsilon}_\beta^*. \tag{21}$$

Adding (20) and (21) together, squaring, summing over transverse polarizations of both J/ψ, and integrating over half of the phase space, I then obtain the partial width $\Gamma[\eta_b \to J/\psi J/\psi]$. Nevertheless, it is more convenient to have a direct expression for the branching ratio, where $\psi_{\eta_b}(0)$ drops out,

$$\mathrm{Br}[\eta_b \to J/\psi J/\psi] = \frac{2^{13}\pi^2\alpha_s^2}{3^4} \frac{m_c^2}{m_b^8} \psi_{J/\psi}^4(0) \left(1 - \frac{4m_c^2}{m_b^2}\right)^{3/2} \times \left[\langle v^2\rangle_{J/\psi} + \left(\frac{9 e_b e_c \alpha}{8\alpha_s} \frac{m_b^2}{m_c^2}\right)^2 \times \left(1 - \frac{2m_c^2}{m_b^2}\right)^{-1}\right]^2. \tag{22}$$

In deriving this, I have approximated the total width of η_b by its gluonic width:

$$\Gamma_{\text{tot}}[\eta_b] \approx \Gamma[\eta_b \to gg] = \frac{8\pi\alpha_s^2}{3m_b^2}\psi_{\eta_b}^2(0), \tag{23}$$

where the LO expression in α_s and v_b is used for simplicity.

Equation (22) constitutes the main formula of this paper. One can readily confirm the asymptotic behavior of QCD and QED contributions first given in Sec. II, Eq. (6) and (7).

IV. OBSERVATION POTENTIAL OF $\eta_b \to J/\psi J/\psi$ AT TEVATRON AND LHC

I now explore the phenomenological implication of (22). The input parameters are m_b, m_c, α, α_s, $\psi_{J/\psi}(0)$, and $\langle v^2\rangle_{J/\psi}$, all of which can be inferred from other independent sources. The wave function at the origin for J/ψ can be extracted from its dielectron width:

$$\Gamma[J/\psi \to e^+e^-] = \frac{4\pi e_c^2\alpha^2}{m_c^2}\psi_{J/\psi}^2(0). \tag{24}$$

(The LO formula in α_s and v_c^2 is used for simplicity.) Using the measured dielectron width 5.55 keV [18], I obtain $\psi_{J/\psi}(0) = 0.205\ \mathrm{GeV}^{3/2}$ for $m_c = 1.5$ GeV.

Among various input parameters, the least precisely known is $\langle v^2\rangle_{J/\psi}$. Since this is a subtracted quantity, it can be either positive or negative. There is a useful relation, first derived by Gremm and Kapustin using the equation of motion of NRQCD [40], relating this quantity with the pole mass of the charm quark and the J/ψ mass.[8] To my purpose this relation reads [25]

$$\langle v^2\rangle_{J/\psi} \approx \frac{M_{J/\psi}^2 - 4m_{c\,\mathrm{pole}}^2}{4m_{c\,\mathrm{pole}}^2}. \tag{25}$$

In the analysis of the process $e^+e^- \to J/\psi\eta_c$ at B factory [25], the charm quark pole mass is taken as the commonly quoted value, 1.4 GeV; consequently $\langle v^2\rangle_{J/\psi} = 0.22$ and $\langle v^2\rangle_{\eta_c} = 0.13$. However, since $m_{c\,\mathrm{pole}}$ contains renormalon ambiguity, it cannot be determined better within an accuracy of order Λ_{QCD}. Therefore one should not be surprised that a somewhat larger value of $m_{c\,\mathrm{pole}}$ may be occasionally reported in literature. For example, a recent QCD moment sum rule analysis claims $m_{c\,\mathrm{pole}} = 1.75 \pm 0.15$ GeV [42]. For this specific value of charm quark pole mass, one would obtain instead $\langle v^2\rangle_{J/\psi} = -0.22 \pm 0.15$.

The situation seems to be rather obscure since even the sign of $\langle v^2\rangle_{J/\psi}$ cannot be unambiguously determined. To proceed, I take a practical attitude and adopt the value $\langle v^2\rangle_{J/\psi} = 0.25 \pm 0.09$ from a recent Cornell potential model based analysis [43]. This *positive* value seems to be most welcomed to alleviate the alarming discrepancy between the predicted and the measured cross section for $e^+e^- \to J/\psi + \eta_c$ [25], and also seems compatible with a recent QCD sum rule determination of $\langle v^2\rangle_{\eta_c}$ [44]. However, I leave the possibility open that this quantity may turn out to be *negative* after future scrutiny. Taking $m_b = M_{\eta_b}/2 \approx 4.7$ GeV, $m_c = 1.5$ GeV, $\alpha = 1/137$, and $\alpha_s(m_b) = 0.22$, I then find[9]

$$\mathrm{Br}[\eta_b \to J/\psi J/\psi] = 2.4^{+4.2}_{-1.9} \times 10^{-8}. \tag{26}$$

The uncertainty is estimated by varying m_b and m_c in the ± 100 MeV range, varying $\alpha_s(\mu)$ in the -0.04 range (which corresponds to slide the scale from $\mu = m_b$ to $2m_b$), as well as taking into account the errors in measured $\Gamma_{e^+e^-}$ (of ± 0.14 keV) and in $\langle v^2\rangle_{J/\psi}$. The constructive interference between electromagnetic and strong amplitudes has modest effect, i.e., neglecting the QED contribution decreases the branching ratio by a few to ten percent.

My prediction for the branching ratio is at least 3 orders of magnitude smaller than the BFL estimate, (1). Despite the large uncertainties inherent in various input parameters, I believe my prediction captures the correct order of magnitude, 10^{-8}. It is also worth noting that my prediction for this exclusive decay ratio is about 1000th of the inclusive 4-charm ratio, (2), which seems fairly reasonable.

Experimentally J/ψ can be tagged cleanly by its decay to a muon pair. Multiplying (26) by the branching ratios of 6% for each of the decays $J/\psi \to \mu^+\mu^-$, I obtain $\mathrm{Br}[\eta_b \to J/\psi J/\psi \to 4\mu] \approx (0.2\text{–}2.4) \times 10^{-10}$. The total cross section for η_b production at Tevatron energy is about 2.5 μb [20]. Therefore, the production cross section for this 4 μ decay mode is about 0.05–0.6 fb. For the full Tevatron Run 1 data of 100 pb^{-1}, I then obtain between 0.005 and 0.06 produced events. I now can safely assert that the seven 4 μ events reported by Ref. [19] must come from sources other than η_b decay.

Tevatron Run 2 plans to achieve an integrated luminosity of 8.5 fb^{-1} by 2009. Assuming equal $\sigma(p\bar{p} \to \eta_b + X)$ at $\sqrt{s} = 1.96$ and 1.8 TeV, I then estimate there are about 0.4–5 produced events. Since the kinematical cuts, as well as taking into account the acceptance and efficiency for detecting muon, will further cut down this number, I conclude it is not realistic to search η_b through this decay mode in Tevatron Run 2.

To fathom the observability of η_b through this mode at LHC, I need first to know the inclusive η_b production rate. There are rough estimates for the $\chi_{b0,2}$ cross sections at LHC, which are about 6 times larger than the corresponding cross sections at Tevatron [34]. Assuming the same scaling also holds for η_b, I then expect the cross section for η_b at LHC to be about 15 μb, and subsequently the production cross section for the 4μ events to be about 0.3–3.6 fb. For a 300 fb^{-1} data, which is expected to be accumulated in 1 yr running at LHC design luminosity, the number of produced events may reach between 100 to 1000. The product of acceptance and efficiency for detecting J/ψ decay to $\mu^+\mu^-$ is estimated to be $\epsilon \approx 0.1$ [16], which is perhaps a conservative estimate for LHC. Multiplying the number of the produced events by ϵ^2, I expect between 1 and 10 observed events per year. If I lose the constraint that J/ψ must be tagged by a $\mu^+\mu^-$ pair and also allow its reconstruction through e^+e^- mode, I can have 4–40 observed 4-lepton events per year.

The above analysis seems to indicate that the chance of observing η_b at LHC through the 4-lepton mode subsists, but critically hinges on whether the signal events can be singled out from the abundant background events. The most important background events may come from the direct double J/ψ production from gg fusion [45,46]. From previous analysis, I know that all of the seven 4μ candidate events selected by CDF based on Tevatron Run 1 data [19] should be regarded as this kind of background event, which seems to outnumber the expected signal

[8] For a reformulation of the Gremm-Kapustin relation from a even lower energy effective theory of NRQCD, dubbed *potential* NRQCD, we refer the interested readers to Ref. [41].

[9] Note the actual prediction, $\mathrm{Br} \sim 10^{-8}$, is much larger than the expectation based on the asymptotic scaling behavior $\mathrm{Br} \sim \alpha_s^2 v_c^{10} (m_c/m_b)^8 \sim 10^{-11}$. This can be attributed to the large prefactors in the right-hand side of (22).

events by several orders of magnitude. The same situation may also apply to LHC. It might be possible for experimentalists to judiciously choose kinematical cuts to significantly suppress the background events while retaining as many signal events as possible.

V. OTHER DECAY CHANNELS OF η_b

The decay channel I have considered so far, $\eta_b \to J/\psi J/\psi$, which has a very clean signature, is unfortunately very much suppressed because of its maximal violation of the helicity selection rule.[10] There are other two-body decay channels, e.g., η_b decays to two light mesons and to two charmed mesons, some of which do conserve the hadron helicity, and thus may have much larger branching ratios. In this section, I attempt to estimate the order of magnitude of the branching ratios for these processes. In addition, I also present some crude estimation for the η_b decays to three pseudoscalar states.

A. η_b (η_c) decays to two and three light mesons

When contending with light mesons in hard exclusive processes, the most appropriate description of them is in terms of the light-cone expansion approach. On the other hand, the constituent quark model, which treats the light mesons as nonrelativistic bound states, is also frequently invoked as an alternative method for a quick order-of-magnitude estimate. In this sense, the preceding formulas derived for $\eta_b \to J/\psi J/\psi$ can be applied to describe the unnatural decay processes $\eta_b \to VV$, once one understands one is working with the constituent quark model.

I take $\eta_b \to \phi\phi$ as a representative. By regarding ϕ as a strangeonium, I can directly use (22), only with some trivial changes of input parameters. I take the constituent quark mass $m_s \approx M_\phi/2 = 0.5$ GeV. The wave function at the origin of ϕ, $\psi_\phi(0)$, can be extracted analogously from its measured dielectron width of 1.27 ± 0.04 keV [18] through (24). I take $\langle v^2\rangle_\phi \approx 1$ to reflect the fact that s is inherently not a heavy quark and ϕ is not truly a heavy quarkonium. Taking $m_b = 4.7 \pm 0.1$ GeV, varying the strong coupling constant between $\alpha_s(m_b) = 0.22$ and $\alpha_s(2m_b) = 0.18$, and including the experimental uncertainty in $\Gamma_{e^+e^-}$, I obtain

$$\mathrm{Br}[\eta_b \to \phi\phi] = (0.9\text{–}1.4) \times 10^{-9}. \tag{27}$$

The interference effect is more pronounced in this case due to the larger ratio of m_b to m_s. Neglecting the QED contribution will decrease the branching fraction by about 20%. If this estimate is reliable, such a rare decay mode perhaps will never be observed experimentally.

It is interesting also to consider the similar decay process $\eta_c \to \phi\phi$, which has been measured long ago. Parallel to the preceding procedure, varying the strong coupling constant from $\alpha_s(m_c) = 0.36$ to $\alpha_s(2m_c) = 0.26$, taking $m_c = 1.5 \pm 0.1$ GeV, I obtain

$$\mathrm{Br}[\eta_c \to \phi\phi] = (0.3\text{–}1.5) \times 10^{-5}. \tag{28}$$

The interference effect is rather modest, i.e., neglecting the QED contribution only decreases the branching ratio by less than one percent. This prediction is consistent with an early constituent quark model based analysis using Bethe-Salpeter (BS) bound state formalism [22].[11] Note this estimate is indeed much smaller than the measured value $\mathrm{Br}_{\exp}[\eta_c \to \phi\phi] = (2.7 \pm 0.9) \times 10^{-3}$, which reflects a generic symptom in pQCD calculation, also present in the η_c decays to $\rho\rho$ and $K^*\bar{K}^*$.

As stressed several times before, some nonperturbative mechanisms must be called for to rescue this discrepancy. Among different proposals, a particularly attractive and predictive scheme has been put forward by Feldmann and Kroll (henceforth FK) [23], by generalizing their influential work together with Stech on η–η' mixing in quark flavor basis [48], to include η_c. FK models the amplitude of $\eta_c \to VV$ as the product of a factor governing the small admixture amplitude between η_c and $\eta^{(\prime)}$, which presumably can be inferred from QCD $U_A(1)$ anomaly, times a soft vertex function parametrizing the amplitude for virtual η, η' transiting into VV. To be specific, the $\eta_c \to \phi\phi$ in their ansatz can be described as

$$\Gamma_{\mathrm{FK}}[\eta_c \to \phi\phi] = \frac{1}{32\pi M_{\eta_c}}\left(1 - \frac{4M_\phi^2}{M_{\eta_c}^2}\right)^{1/2} \times |c_{\phi\phi}^{\mathrm{mix}} g^{\mathrm{mix}}(\eta_c)|^2, \tag{29}$$

where the subscript FK is used to distinguish from my earlier prediction based on the pQCD plus constitute quark model ansatz, Eq. (28). The parameter c^{mix} depends on the specific flavor content of VV states but not on the initial charmonium state, whereas the charm mass dependence is encoded in

$$g^{\mathrm{mix}}(\eta_c) = \frac{1}{f_{\eta_c}} F_{\eta^{(\prime)}VV}(s = M_{\eta_c}^2) = \frac{1}{f_{\eta_c}} F_{\eta^{(\prime)}VV}(s = 0)\left(\frac{\Lambda^2}{M_{\eta_c}^2 - \Lambda^2}\right)^n, \tag{30}$$

where f_{η_c} is the decay constant of η_c, the second factor indicates the on-shell coupling $\eta^{(\prime)}VV$, and the last factor parametrizes the s-dependence of this vertex function. Λ is the cutoff of typical size around 1 GeV, and the variable $n = 1$ corresponds to the familiar monopole ansatz for form factor, and $n = 2$ characterizes the dipole ansatz. One noteworthy fact is that this simple model seems able

[10] This process is the only possible one for η_b decays to two ground charmonium states allowed by C and P invariance.

[11] The predictions of Ref. [22], which employ three different forms of BS wave functions for V, are duplicated in Table 1 of Ref. [47].

to account for both the absolute and the relative strengths of partial widths for each decay channel ($VV = \rho\rho$, K^*K^*, and $\phi\phi$), to a reasonably satisfactory degree.

One curious question is to ask whether the FK mixing mechanism can be extrapolated to $\eta_b \to VV$, and if so, whether the corresponding prediction to the branching ratios differs drastically from my pQCD-based estimate. Let us now examine this question. After some straightforward derivation, it is easy to find

$$\frac{\mathrm{Br_{FK}}[\eta_b \to \phi\phi]}{\mathrm{Br_{FK}}[\eta_c \to \phi\phi]} \approx \left(1 - \frac{4M_\phi^2}{M_{\eta_c}^2}\right)^{-1/2}\left(\frac{f_{\eta_c}M_{\eta_c}^n}{f_{\eta_b}M_{\eta_b}^n}\right)^4 = 0.16 \times 0.01^n, \tag{31}$$

where I have resorted to heavy quark spin symmetry for decay constants $f_{\eta_c} \approx f_{J/\psi}$ and $f_{\eta_b} \approx f_\Upsilon$, and consequently taken $f_{\eta_c} = 417$ MeV and $f_{\eta_b} = 715$ MeV through the relation $f_{J/\psi} = \sqrt{12/M_{J/\psi}}\psi_{J/\psi}(0)$. For simplicity, I have utilized the hierarchy $m_b, m_c \gg \Lambda$ in deriving (31). If $\mathrm{Br_{FK}}[\eta_c \to \phi\phi]$ is identified with the measured value 2.7×10^{-3}, I then obtain $\mathrm{Br_{FK}}[\eta_b \to \phi\phi]$ to be about 4×10^{-6} for $n = 1$ and 4×10^{-8} for $n = 2$. It is easy to see that the $\mathrm{Br_{FK}}[\eta_b \to VV]$ scales as $m_b^{-4(1+n)}v_b^{-6}$, since $f_{\eta_b} \sim m_b v_b^{3/2}$. This indicates this nonperturbative mechanism also resembles the pQCD analysis in that the branching ratio is highly suppressed with respect to "leading twist" scaling $\propto 1/m_b^4$. Despite large uncertainty, the predicted branching ratios of $\eta_b \to VV$ in this mixing ansatz are (much) larger than my earlier prediction based on "NRQCD" plus constitute quark model, (27). It is reasonable to question the validity of the above nonperturbative mechanism at η_b energy; in my opinion, perhaps the pQCD result in (27) is more trustworthy. At any rate, it will be of interest for future experiments to distinguish these different mechanisms, though the task of recording this rare decay mode in hadron machine could be extremely challenging.

One related class of decay channels may deserve some attention. The helicity-conserving decay, such as $\eta_c \to K_\parallel^*\bar{K}$ ("$\parallel$" implies longitudinally polarized), has still not been observed yet. The experimental bound for this channel is [18][12]

$$\mathrm{Br_{exp}}[\eta_c \to K^*\bar{K} + \text{c.c.}] < 1.28\%. \tag{32}$$

Since the coupling $\eta^{(\prime)}VP$ is negligible, it is reasonable to assume that the aforementioned nonperturbative mechanism responsible for $\eta_c \to VV$, may not play a significant role for $\eta_c \to VP$. One may then make use of the scaling behaviors derived earlier to interconnect each of them. First I expect that flavor SU(3) is respected in pQCD calculation of $\eta_c \to VV$; accordingly my prediction is $\mathrm{Br}[\eta_c \to K^*\bar{K}^*] \sim \mathrm{Br}[\eta_c \to \phi\phi] \sim 10^{-5}$. As we have learned from Sec. II, at tree level, the maximal helicity violation in $\eta_b \to J/\psi J/\psi$ is entirely due to the charm quark transverse momentum, as is evident in (6). I assume the similar pattern occurring here, i.e., the branching ratios of $\eta_c \to VV$ are also suppressed by $(q_\perp/m_c)^4$ relative to those of the corresponding helicity-conserving channels. I then have

$$\mathrm{Br}[\eta_c \to K^*\bar{K}] \sim \mathrm{Br}[\eta_c \to \phi\phi]\left(\frac{m_c}{\Lambda_{\mathrm{QCD}}}\right)^4 \epsilon_{\mathrm{SU(3)}}^2 \sim 10^{-3} \times \epsilon_{\mathrm{SU(3)}}^2, \tag{33}$$

where I have taken $q_\perp \sim \Lambda_{\mathrm{QCD}} = 500$ MeV. I have intentionally included a parameter $\epsilon_{\mathrm{SU(3)}}$, to embody the extent of SU(3) flavor violation at the amplitude level. Obviously, this parameter should be proportional to current quark mass difference $m_s - m_{d,u}$, divided by some typical hadronic scale (presumably independent of m_c). In contrast to the isospin violation effect, the U-spin violating process should not receive too severe suppression. If assuming a somewhat optimistic value $\epsilon_{\mathrm{SU(3)}} \approx 0.3$, I then obtain $\mathrm{Br}[\eta_c \to K^*\bar{K}] \sim 10^{-4}$, which is safely below the experimental bound, (32). Needless to say, it will be very beneficial for the ongoing high-luminosity charmonium facility such as BES III to impose more tight constraint on this branching ratio.

Carrying over the same line of argument to η_b decay to VP, I obtain

$$\mathrm{Br}[\eta_b \to K^*\bar{K}] \sim \mathrm{Br}[\eta_b \to \phi\phi]\left(\frac{m_b}{\Lambda_{\mathrm{QCD}}}\right)^4 \epsilon_{\mathrm{SU(3)}}^2 \sim 10^{-5} \times \epsilon_{\mathrm{SU(3)}}^2. \tag{34}$$

If I again take $\epsilon_{\mathrm{SU(3)}}^2 \approx 0.1$, the branching ratio is estimated to be around 10^{-6}, about 100 times larger than $\mathrm{Br}[\eta_b \to J/\psi J/\psi]$. Of course, one should not take this crude estimate too seriously, and a concrete pQCD calculation based on the light-cone expansion approach, which incorporates m_s and m_d difference, might be illuminating. If this estimate is trustworthy, one then expects there are already about $O(10^2)$ produced events in the Tevatron Run 1. Since K^* almost exclusively decays to $K\pi$, one needs to select those resonant $K\bar{K}\pi$ events. However, there are practical problems about the usefulness of this kind of decay mode in hadron collision experiments. Because of copiously produced kaons and pions in a typical hadron collision, huge combinatorial backgrounds might make it very difficult to identify the true signal. On the other hand, the prospective Super B factory, which runs at several $\Upsilon(nS)$ resonances with an unprecedented luminosity [49], may produce an enormous amount of η_b through $M1$ transition from $\Upsilon(nS)$ states. With much suppressed backgrounds, the Super B factory may offer a viable environment to detect this decay mode.

[12]Although this decay channel is favored by helicity selection rule, it violates the U-spin conservation [1,23].

Finally I exploit one experimental fact that some 3-body channels, i.e., η_c decays to three pseudoscalar states, have exceedingly large branching ratios. To be concrete, three η_c decay modes with largest branching ratios are [18]

$$\mathrm{Br}_{\mathrm{exp}}[\eta_c \to K\bar{K}\pi] = (7.0 \pm 1.2)\%, \tag{35}$$

$$\mathrm{Br}_{\mathrm{exp}}[\eta_c \to \eta\pi\pi] = (4.9 \pm 1.7)\%, \tag{36}$$

$$\mathrm{Br}_{\mathrm{exp}}[\eta_c \to \eta'\pi\pi] = (4.1 \pm 1.8)\%. \tag{37}$$

Although we do not know how to calculate these processes in practice, some general pattern may still be identified. Subtracting off the phase space effects, one finds these three amplitudes roughly respect the SU(3) flavor symmetry. This may be taken as a sign that these processes could in principle be described by the pQCD scheme. These processes proceed as two steps. First η_c annihilates to two highly virtual gluons, which then transit into four energetic light quarks. This is a short-distance process. Subsequently these light quarks materialize into three pseudoscalars via soft nonpertubative dynamics, which is a long-distance process. Factorization is expected to hold between these two stages due to the hard scale set by large c mass. Because of their large decay ratios, these processes are naturally expected to possess the "leading twist" scaling behaviors, $\mathrm{Br} \sim 1/m_c^4$. This scaling assumption can then be used to infer the ratios for η_b decays to the same pseudoscalar states. For example, one may expect

$$\mathrm{Br}[\eta_b \to K\bar{K}\pi] \sim \mathrm{Br}_{\mathrm{exp}}[\eta_c \to K\bar{K}\pi]\left(\frac{m_c}{m_b}\right)^4 \sim 10^{-4}. \tag{38}$$

This is by far the largest branching ratio I have found in all exclusive hadronic decay channels of η_b. If this is the case, these channels will be worth pursuing. Practically, the $\eta\pi\pi$ mode may not be very useful in hadron hadron collider experiments, since the ordinary way of reconstructing η, which goes through the 2γ decay, may suffer enormous contamination from combinatorial background. For similar reason, the $K^+K^-\pi^0$ channel may also be difficult to detect due to the presence of π^0. In contrast, the decay mode $K_S K^\pm \pi^\mp$ is much more advantageous since K_S can be reconstructed cleanly via its decay to $\pi^+\pi^-$. In any event, all these decay channels have promising observation potential at future Super B factory.

B. η_b decays to two charmed mesons

Heavy η_b mass opens the gate for it to decay into charmed particles as well as light ones. In fact, Maltoni and Polosa have suggested that η_b decays to two charmed mesons could be the most promising channels for observing η_b in Tevatron Run 2 [20]. They first perform a perturbative calculation for η_b decay to the inclusive $c\bar{c}g$ state,

$$\mathrm{Br}[\eta_b \to c\bar{c}g] = 1.5^{+0.8}_{-0.4}\%, \tag{39}$$

then interpret this value as an upper limit for the ratios of the exclusive decays to double charmed mesons. MP continue to argue that the exclusive decays to $D^*\bar{D}^{(*)}$ (the charge conjugate is implicitly implied) may saturate the inclusive charmful decay ratio, and subsequently estimate $10^{-3} < \mathrm{Br}[\eta_b \to D^*\bar{D}^{(*)}] < 10^{-2}$.

In my opinion, this saturation assumption seems to be physically unwarranted; consequently MP's predictions for the ratios may well be an overestimate. First, there seems no reason to expect that the parton process $\eta_b \to c\bar{c}g$ will be dominated by the two-body exclusive decays, since the g jet may readily hadronize in an independent direction, which will result in a multibody decay configuration. This can be exemplified by the fact I just mentioned, that η_c decays to 3 pseudoscalars have larger branching ratios than any 2-body decay channels. One may notice at the lowest order in α_s, η_b decay to double charm mesons is also depicted by Fig. 1, with one $c\bar{c}$ pair replaced by a $q\bar{q}$ pair. The gluon which is on-shell in the inclusive $\eta_b \to c\bar{c}g$ process now has to convert to a light quark pair with large invariant mass, so is highly virtual; the corresponding ratios of double charm mesons thus must be at least suppressed by one factor of α_s and one factor of $1/m_b^2$ relative to (39). Taking into account the nonperturbative binding probability, which is much less than 1, will further suppress the exclusive 2-body decay rates. Moreover, there seems also no strong reason to believe that the binding of c with $\bar{q}$ will necessarily be saturated by the ground state charm meson only.

Despite the lack of an explicit pQCD calculation, one may still proceed with some physical consideration. The branching ratios of η_b decays to two charm mesons can depend on three dimensional parameters: m_b, m_c, and Λ_{QCD}. Since the decay $\eta_b \to D^*_\parallel\bar{D}$ conserves the helicity, one thus expects the corresponding branching fraction scales as $1/m_b^4$. For each pair of c and $\bar{q}$ to form a $D^{(*)}$ meson, there is a factor proportional to $\Lambda_{\mathrm{QCD}}/m_c$ which characterizes the binding probability [50]. This is the only place where Λ_{QCD} dependence can enter. Therefore, I expect the following asymptotic behavior:

$$\mathrm{Br}[\eta_b \to D^*\bar{D}] \sim \alpha_s^2\left(\frac{\Lambda_{\mathrm{QCD}}^2 m_c^2}{m_b^4}\right) \sim 10^{-5}, \tag{40}$$

where m_c^2 is inserted to make the ratio dimensionless. Notice this value is reasonably compatible with the inclusive upper bound, (39).[13]

[13]However, as will be reported in Ref. [51], the decay rate of this process vanishes if D and D^* are exactly degenerate. Therefore, the nonvanishing decay amplitude must be induced by heavy-quark-spin-symmetry violating effects, $\propto \Lambda_{\mathrm{QCD}}/m_c$. This indicates the estimate (40) may well be subject to further suppression.

I next turn to the other charmful decay channel, $\eta_b \to D^*_\perp \bar{D}^*_\perp$ ($\eta_b \to D\bar{D}$ is forbidden by P invariance), which violates the helicity selection rule maximally. Assuming again the helicity violation is caused by the transverse momentum of the light quark in D^* meson, the same as in $\eta_b \to J/\psi J/\psi$, I then obtain

$$\mathrm{Br}[\eta_b \to D^* \bar{D}^*] \sim \mathrm{Br}[\eta_b \to D^* \bar{D}] \left(\frac{\Lambda_{\mathrm{QCD}}}{m_b}\right)^4 \sim \alpha_s^2 \left(\frac{\Lambda_{\mathrm{QCD}}^6 m_c^2}{m_b^8}\right) \sim 10^{-8}, \tag{41}$$

where $q_\perp \sim \Lambda_{\mathrm{QCD}} = 500$ MeV is used. The rather small branching ratio renders this decay mode not so useful for detecting η_b. Nevertheless, it might be possible that a large prefactor may arise in the actual calculation, like what occurs in $\eta_b \to J/\psi J/\psi$ (see footnote 9), so that the actual ratio may be somewhat larger than this naive estimate. In any event, a reliable pQCD calculation of the decay rates for $\eta_b \to D^* \bar{D}^{(*)}$ will be helpful.

Based on the previous discussions about 3-body decay of η_b, one may wish that the branching ratio for $\eta_b \to D\bar{D}\pi$ might be as large as 10^{-4}, similar to that for $\eta_b \to K\bar{K}\pi$. However, this decay mode may not be as competitive as the $K_S K^\pm \pi^\mp$ mode, since the D meson does not possess a clean signature comparable with K_S.

To summarize, the $\eta_b \to D^* \bar{D}$ channel, with an expected branching ratio $\sim 10^{-5}$, may be regarded as a valuable searching mode. The $D^{*0}\bar{D}^0$ channel (charge conjugate state implicitly included) may not be so useful, since D^{*0} predominantly decays to $D^0\pi^0$ and $D^0\gamma$, where neither π^0 nor γ can be cleanly tagged in hadron collision environment. In contrast, it is more advantageous to use the $D^{*+}D^-$ mode as a trigger. D^{*+} can be tagged through its decay to $D^0\pi^+$; subsequently D^0 may be reconstructed from $K^-\pi^+$, while D^- can be tagged through its decay to $K^+\pi^-\pi^-$. It is worth emphasizing that due to the proximity of D^{*+} mass to the sum of masses of D^0 and π^+, D^{*+} can be cleanly identified with a rather narrow peak in the $D\pi$ invariant mass spectrum. Using the measured branching ratios $\mathrm{Br}[D^{*+} \to D^0\pi^+] \approx 70\%$, $\mathrm{Br}[D^0 \to K^-\pi^+] \approx 4\%$, and $\mathrm{Br}[D^- \to K^+\pi^-\pi^-] \approx 10\%$ [18], I estimate $\mathrm{Br}[\eta_b \to D^{*+}D^- \to K^+K^-\pi^+\pi^-\pi^+\pi^-] \sim 10^{-8}$. There are roughly $O(1)$ produced events in Tevatron Run 1, about $O(10^2)$ produced events in Run 2, and about $O(10^4)$ produced events in 1 yr run at LHC. The statistics seems to be enough in the forthcoming hadron collider program for observing η_b through this decay mode, provided that the signal events are not swallowed by the possibly large combinatorial backgrounds.

VI. SUMMARY AND OUTLOOK

The major motif of this work is to clarify a controversy about whether double J/ψ can be a useful decay mode to detect η_b in Tevatron Run 2. I have shown this process is subject to large kinematical suppression due to the maximal violation of the helicity selection rule. By an explicit pQCD calculation based on NRQCD, I predict the corresponding branching ratio to be only of order 10^{-8}, thus making the search for η_b through this mode rather unrealistic in Tevatron Run 2. Nevertheless, I anticipate that at LHC, the observation potential of this decay mode may not be so pessimistic, if experimentalists can find a good way to suppress the rather copious QCD background events.

The large mass of η_b renders any of its exclusive decay channels in general very small. To provide some useful guidance for an experimental search for this elusive particle, it is valuable to identify those decay modes with relatively large branching fractions and also with clean signature. To make progress along this direction, in the following I outline some issues which I think may deserve further studies:

(1) Stimulated by rather large QCD radiative correction to exclusive double charmonium production at B factory [30,31], one may inquire how large the effect of the next-to-leading order QCD correction to $\eta_b \to J/\psi J/\psi$ is. It turns out that, at NLO in α_s for this process, the helicity selection rule can be violated by finite charm mass; consequently one obtains the nonvanishing result even at the leading order in velocity expansion [52]. At the amplitude level, the ratio of the radiative correction piece to the relativistic correction piece as considered in this work is about $\frac{\alpha_s}{\pi} : \langle v^2 \rangle_{J/\psi} \sim \mathcal{O}(1)$, which implies both effects are equally important, and the more accurate prediction will crucially depend on their relative phase. To this end, a precise determination of the quantity $\langle v^2 \rangle_{J/\psi}$ would be helpful.

(2) The process $\eta_b \to K^* \bar{K}$ + c.c., favored by the helicity selection rule but at the same time violating U-spin symmetry, is estimated to have a branching fraction $\sim 10^{-6}$–10^{-7}. It might be worthwhile if an actual pQCD calculation which implements the $m_s - m_d$ difference can be carried out in the light-cone expansion scheme, to compare with this rough estimate. Moreover, the eventual experimental sighting of $\eta_c \to K^* \bar{K}$ in a charmonium factory like BES III will definitely enrich our understanding toward this class of helicity-conserving yet flavor SU(3) violating quarkonium decay processes.

(3) The individual decay modes of η_c with the largest branching ratios are $\eta_c \to K\bar{K}\pi$, $\eta\pi\pi$, and $\eta'\pi\pi$. Stimulated by this experimental fact, one may hope that the 3-body decay channels of η_b, such as $K_S K^\pm \pi^\mp$, with an expected branching ratio of order 10^{-4}, might be a potentially useful searching mode for η_b in current and forthcoming hadron collider programs, if it can survive from the copious combinatorial background events. This mode will defi-

nitely have promising potential to be observed in the prospective Super B factory.

(4) Another helicity-conserving decay process, $\eta_b \rightarrow D^* \bar{D}$ + c.c., with an expected branching ratio of order 10^{-5}, may also be well worth searching for experimentally. It is also profitable to carry out a concrete calculation of this exclusive double charm decay process from pQCD scheme, but one may be obliged to incorporate the heavy-quark-spin-symmetry violating effect [51].

(5) In this work I have not discussed the possibility of observing η_b through its decay to a baryon pair, such as $\eta_b \rightarrow p\bar{p}$. If the corresponding ratio is not too small, this is potentially a good searching mode thanks to relatively fewer baryonic background events in hadron collision experiments. Experimentally $\eta_c \rightarrow p\bar{p}$ is observed to have a branching ratio of order 10^{-3} [18]. However, one should be aware that this process also violates the helicity selection rule, and the available pQCD prediction, when taking into account the nonzero light quark mass but still in the collinear quark configuration, is still far smaller than the measured value [53]; therefore some nonperturbative mechanism needs to be called for to explain this discrepancy [23]. Because of the rather heavy mass of η_b, one may hope the pQCD framework should provide a reliable prediction for $\eta_b \rightarrow p\bar{p}$. It will be valuable if a more systematic calculation can be carried out.

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Note added.—While the revised version of the manuscript was to be submitted, there appeared an eprint in the arXiv by the *BABAR* collaboration [54], which claims the first unambiguous discovery of η_b through the hindered $M1$ transition process $\Upsilon(3S) \rightarrow \eta_b \gamma$. It is interesting to note that the rather precisely measured η_b mass, $9388.9^{+3.1}_{-2.3}(\text{stat}) \pm 2.7(\text{syst})$ MeV, seems not compatible with the predictions from many potential models as well as the weakly coupled potential NRQCD; instead it is consistent with the lattice QCD prediction within error [11].

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