Critical magnetic field in a holographic superconductor

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We have studied a holographically dual description of superconductor in (2 + 1) dimensions in the presence of an applied magnetic field and observed that there exists a critical value of magnetic field, below which a charged condensate can form via a second-order phase transition.

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I. INTRODUCTION

The holographic correspondence between a gravitational theory and a quantum field theory, first emerged under the anti–de Sitter/conformal field theories (AdS/CFT) correspondence [1], has been proven useful to studying various aspects of nuclear physics such as relativistic heavy ion collisions (RHIC) and condensed matter phenomena, particularly in those recent studies [2–6].

In Refs. [7,8], the author proposed a gravity model in which Abelian symmetry of a Higgs field is spontaneously broken by the existence of a black hole. This mechanism was recently incorporated in the model of superconductivity and critical temperature was observed [9], and later on non-Abelian gauge condensate [10]. In this paper, we would like to extend the work to include the magnetic field, and we will show the existence of critical magnetic field as expected from physics of a superconductor.

To implement a magnetic field at finite temperature, we introduce a Reissner-Nordstrom charged black hole and a condensate through a charged scalar field. In the superconducting phase, the scalar field takes different values at the horizon for different condensate expectation values at the boundary, indicating the existence of a scalar hair; while in the normal phase, a vanishing scalar field tells the ordinary tale of a black hole with no hair.

II. THE MODEL WITH APPLIED MAGNETIC FIELD

Several important unconventional superconductors, such as the cuprates and organics, are layered in structure, and interesting physics can be captured by studying a (2 + 1)-dimensional system. We are now interested in building up a gravity model (coupled with other matter fields) in (3 + 1) dimensions which is holographically dual to the desired planar system which develops superconductivity below the critical temperature and critical magnetic field. We start with a model composed of the gravity sector and the matter sector. The gravity sector is given by the following Lagrangian density:

$$e^{-1}\mathcal{L}_g = R - \frac{6}{L^2} - \frac{1}{4}\mathcal{F}^{\mu\nu}\mathcal{F}_{\mu\nu}, \tag{1}$$

together with a solution of *magnetically* charged black holes in AdS_4, where [11]

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2(dx^2 + dy^2), \tag{2}$$

$$f(r) = \frac{r^2}{L^2} - \frac{M}{r} + \frac{H^2}{r^2}. \tag{3}$$

Throughout the paper we set the radius of curvature $L = 1$ for numerical computation. By assumption the only nonzero electromagnetic field is the magnetic component $\mathcal{F}_{xy} = \frac{H}{r^2}$, of which the energy density at any fixed radius coordinate r is always finite and constant, that is, $\mathcal{F}^{\mu\nu}\mathcal{F}_{\mu\nu} \propto H^2$. This serves the purpose of a constantly applied magnetic field at the boundary. The black hole is censored by a horizon provided the condition $27M^4 - 256H^6 \geq 0$ and the temperature, as a function of M and H, is determined via the relation

$$T = \frac{f'(r_+)}{4\pi}, \tag{4}$$

where r_+ is the most positive root of $f(r) = 0$ (outer horizon). We expect that the gravity sector, implied by its given name, can be easily obtained from a pure gravity theory of higher dimensions by appropriate reduction.

For the matter sector, we will use the Ginzburg-Landau (GL) action for a Maxwell field and a charged complex scalar, which does not backreact on the metric [8,9],

$$e^{-1}\mathcal{L}_m = -\frac{1}{4}F^{ab}F_{ab} + \frac{2}{L^2}|\Psi|^2 - |\partial\Psi - iA\Psi|^2. \tag{5}$$

This action differs from the usual GL theory by two places: the coefficient of the $|\Psi|^2$ term appears to be negative in both ordinary and superconducting phases, and a $|\Psi|^4$ term is not included. The AdS bulk geometry, however, plays the role of stabilization as long as the Breitenlohner-Freedman bound [12], say $m^2L^2 > -9/4$, is satisfied. We still expect some kind of Higgs mechanism triggered out-

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side the horizon [8]. Enough for our purpose, we will also assume the planar symmetry ansatz for the scalar potential $A_t = \Phi(r)$ and the complex scalar $\Psi(r)$, where we have already fixed the phase to be constant. Then we need to solve a pair of coupled second-order differential equations

$$\Psi'' + \left(\frac{f'}{f} + \frac{2}{r}\right)\Psi' + \frac{\Phi^2}{f^2}\Psi + \frac{2}{L^2 f}\Psi = 0, \quad \Phi'' + \frac{2}{r}\Phi' - \frac{2\Psi^2}{f}\Phi = 0 \tag{6}$$

with appropriate boundary conditions at the horizon and at asymptotic infinity. They can be solved numerically regardless of difficulty which appears in finding nontrivial analytic solutions. In particular, for a normalizable scalar potential, we require at the horizon [8,9]

$$\frac{\Psi'}{\Psi}\bigg|_{r=r_+} = \frac{-2r_+}{3r_+^2 - \frac{H^2}{r_+^2}}, \qquad \Phi(r_+) = 0. \tag{7}$$

Nevertheless, we still have freedom for a two-parameter family of solutions by assigning Φ' and Ψ at the horizon; therefore, we have a scalar hair from a black hole for nonvanishing Ψ. At the boundary, the solutions behave like

$$\Psi = \frac{\Psi_{(1)}}{r} + \frac{\Psi_{(2)}}{r^2} + \cdots, \qquad \Phi = \mu - \frac{\rho}{r} + \cdots, \tag{8}$$

where μ and ρ are interpreted as chemical potential and charge density in the dual field theory. We are interested in the case where either $\Psi_{(1)}$ or $\Psi_{(2)}$ vanishes for stability concern at the asymptotic AdS region, then read off the pairing operator $\mathcal{O}$ dual to Ψ from the bulk-boundary coupling [9],

$$\langle \mathcal{O}_i \rangle = \sqrt{2}\Psi_{(i)}. \tag{9}$$

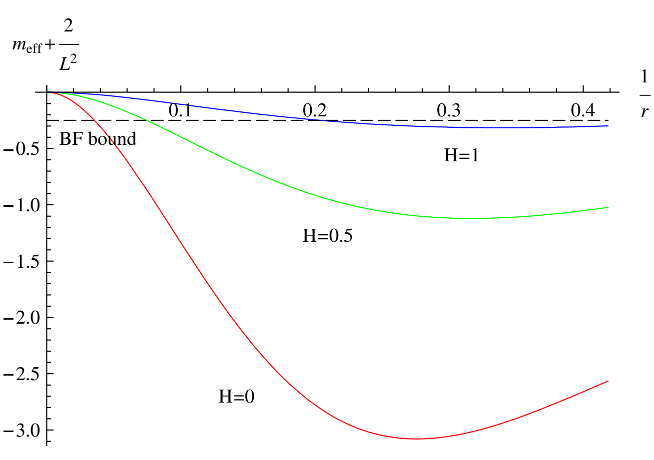

FIG. 1 (color online). The effective mass $m_{\rm eff}^2$ evaluated at fixed temperature and boundary conditions at the horizon. From bottom up, the curves are with $H = 0$, 0.5 and 1, respectively. We also plot the Bretenlohner-Freedman bound, dashed line, below which the AdS vacuum is unstable under perturbation of Ψ.

To gain a better intuition of how a condensate is realized in this gravity setup, we may investigate the effective mass of the Ψ field along the radius direction, that is

$$m_{\rm eff}^2(r) = -\frac{2}{L^2} - \frac{\Phi^2}{f}. \tag{10}$$

We observe in Fig. 1 that provided fixed temperature and boundary condition at the horizon, the effective mass gets more negative along the r direction for a smaller magnetic field. In other words, the condensate happens more easily while the magnetic field is smaller. This implies the existence of a critical magnetic field below which the condensation can take place.

III. CRITICAL MAGNETIC FIELD

In the normal phase, we always have solutions to Eqs. (6), that is $\Psi = 0$ and $\Phi = \mu - \frac{\rho}{r}$; while in the superconducting phase, we may have nontrivial $\Psi(r)$ and its boundary value serves as an order parameter for condensate. In the absence of an applied magnetic field, for any fixed ρ, there exists a critical temperature T_c, above which there is no longer a nontrivial solution [9]. In the presence of an applied magnetic field, however, the Meissner effect is expected and there exists both T_c and a critical magnetic field H_c, above which the nontrivial solution is again not admissible. As argued in the previous section, we expect that the stronger applied magnetic field H is, the lower the critical temperature T_c is. This statement is supported by our numerical results for $\langle \mathcal{O}_2 \rangle$ as shown in Fig. 2. The operator $\mathcal{O}_2$ corresponds to a pair of fermions, while $\mathcal{O}_1$ to a pair of bosons [9]. We have also found similar results for $\langle \mathcal{O}_1 \rangle$ only at a small H region.

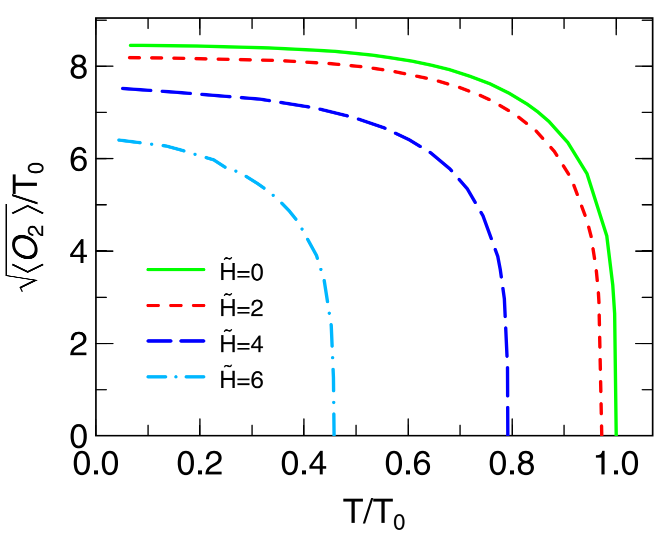

FIG. 2 (color online). We plot order parameter $\langle \mathcal{O}_2 \rangle$ as a function of temperature. The critical temperature T_c decreases as the applied magnetic field increases. Here $\tilde{H}$ is the normalized H given by $H^{2/3}/T_0$, where $T_0 = T_c$ at $H = 0$.

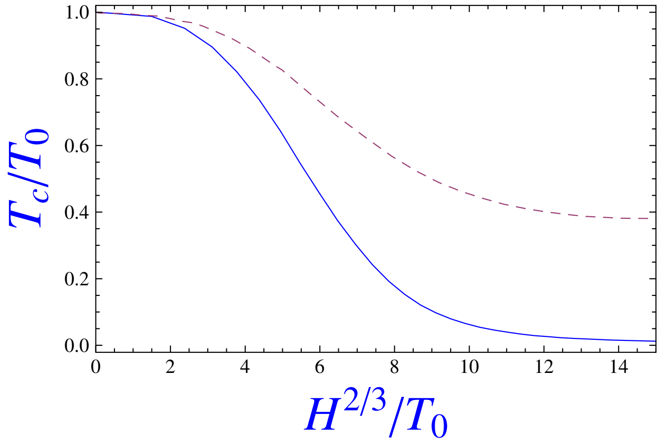

FIG. 3 (color online). The phase diagram of T_c against H_c. The superconducting phase where $\langle \mathcal{O}_2 \rangle \neq 0$ ($\langle \mathcal{O}_1 \rangle \neq 0$) exists in the lower left part below the solid (dashed) curve, while the normal phase is in the upper right part above the curve.

In Fig. 3 we also plot the phase diagram of the critical magnetic field against critical temperature.

IV. DISCUSSION

In this paper, we have considered a hybrid model for AdS/CFT superconductors in the presence of a magnetic field. Several comments are in order: First, a magnetic field is provided in the gravity sector as a background, independent of the probed sector. We argue that this is perfectly fine as long as we only consider a constant magnetic field at the boundary. Second, the matter sector has no backreaction to the gravity sector; therefore, the equation of motion for total Lagrangian is not satisfied. Although this may not be crucial to the occurrence of a superconducting phase, it is still interesting to investigate a fully backreacted action which can be derived from some higher-dimensional theory such as string theory or M theory. Third, in order to discuss possible formation of vortex lattice and distinguish between type I and II superconductors, one is tempted to relax the ansatz of planar symmetry. This will complicate the construction and analysis and we hope to report it in the near future. At last, this construction is a tractable model of a strongly coupled system which may capture some physics of unconventional superconductors, in contrast to the conventional superconductors well described by GL theory macroscopically and BCS theory microscopically. Though we do not see fermionic degrees of freedom from this macroscopic construction, the complex scalar, serving as an order parameter, seems sufficient to explain such a critical phenomenon as good as the usual GL theory. In order to pursue a microscopic model along this line of reasoning, one may still need to understand better how to realize underlying fermionic degrees of freedom in the context of AdS/CFT correspondence.

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