Spin-string interaction in QCD strings

Vikram Vyas*

Physics Department St. Stephen's College, Delhi University, Delhi, India

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I consider the question of the interaction between a QCD string and the spin of a quark or an antiquark on whose worldline the string terminates. The problem is analyzed from the point of view of a string representation for the expectation value of a Wilson loop for a spin-half particle. A string representation of the super Wilson loop is obtained starting from an effective string representation of a Wilson Loop. The action obtained in this manner is invariant under a worldline supersymmetry and has a boundary term which contains the spin-string interaction. For rectangular loops the spin-string interaction vanishes and there is no spin-spin term in the resulting heavy quark potential. On the other hand if an allowance is made for the finite intrinsic thickness of the flux tube by assuming that the spin-string interaction takes place not just at the boundary of the string world sheet but extends to a distance of the order of the intrinsic thickness of the flux tube then we do obtain a spin-spin interaction which falls as the fifth power of the distance. Such a term was previously suggested by Kogut and Parisi in the context of a flux-tube model of confinement.

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I. INTRODUCTION

There is strong numerical evidence that a flux tube is formed between a static quark and an antiquark when the separation between them is of the order of a Fermi or even less, and that such flux tubes can be described by effective string models (for a review see, e.g, [1,2]). This evidence for the formation of a flux tube and its stringlike behavior matches well with the fact that the spectrum of highly excited mesons are well described by open-string models of mesons. Further, these facts are in concordance with the idea that in a suitable limit, namely, in the limit of a large number of colors, QCD is exactly equivalent to some unknown fundamental string theory (for a contemporary review of these idea see, e.g, [3]). It is therefore natural to ask a more detailed question about the dynamics of the QCD string, namely, do the spin of the quark and the antiquark interact with the string connecting them? Such an interaction could lead to a long range spin-spin term in the heavy quark potential [4]. Spin-string interaction could also perhaps be responsible for the pion-rho mass difference in effective string models of meson [5]. More generally the spin-string interaction could help answer the question of how is spontaneous breaking of chiral symmetry reflected in a fundamental string representation of QCD?

The nature of the interaction between the spin of the quark and the string has been investigated in the context of open-string models of mesons (see, e.g., [6–8]). In the present investigation, we will take a different approach. We will start with the assumption that the expectation value of the Wilson loop over the gauge fields can be written as a sum over surfaces whose boundary is the given loop [9–13]. These surfaces can be regarded as the world sheets of a string whose end points lie on the loop, while the loop represents the worldline of a scalar particle-antiparticle pair that is created at a point and is annihilated latter. If we replace the closed worldline of a scalar particle by a closed worldline of a spin-half particle then the amplitude for the corresponding process is given, apart from the kinematic factors, by the Wilson loop for a spin-half particle [14–18]. Such a Wilson loop is often referred to as a super Wilson loop as it is invariant under a one-dimensional supersymmetry [19]. If we can write the expectation value of a super Wilson loop as a sum over the surface whose boundary is the given loop, then the corresponding string action automatically includes the spin of the quark and the spin-string interaction [20]. The task of finding the string representation of a super Wilson loop is facilitated by the fact that the super Wilson loop is not an independent loop functional but is related to the Wilson loop via the area derivative of a loop [19].

The simplest string action used to model QCD strings is the Nambu-Goto action which is the area of the string world sheet. Though the Nambu-Goto string in four dimensions suffers from serious problems, it can be thought of as the leading term in an effective description [21–24]. The success of Nambu-Goto string in modelling the heavy quark potential as obtained from the lattice QCD simulations [25–27] indicates that the expectation value of the Wilson loop over the gauge fields can be well represented by a sum over surfaces with the surface being weighted by the exponential of the Nambu-Goto action, at least for rectangular loops. With this as our justification, we will obtain a string representation for the expectation value of the super Wilson loop via the area derivative of the Nambu-Goto action. The super Wilson loop, when written in terms of anticommuting variables, is invariant under a worldline

*vvyas@stephens.du.ac.in

supersymmetry. We will verify that the action of the string representing the super Wilson loop is also invariant under the worldline supersymmetry (SUSY).

In the string representation of the super Wilson loop the spin-string interaction appears as a boundary term, representing interaction between the spin of the quark (or the antiquark) and the extrinsic curvature of the world sheet at the boundary. To obtain some intuition about the significance of the spin-string interaction we calculate the expectation value of a rectangular super Wilson loop, from which one can extract the spin-dependent heavy quark potential [28–35]. It turns out that for a rectangular super Wilson loop the spin-string term vanishes, and therefore there is no spin-spin dependent term in the heavy quark potential.

But if we think of a string as an effective description for a flux tube of finite *intrinsic* width that is formed between a static quark-antiquark pair, and evaluate the spin-string interaction not right at the boundary of the rectangular loop but average it over a distance of the order of the thickness of the flux tube, then we do obtain a spin-spin interaction term. The form of this term is precisely the one considered by Kogut and Parisi in the context of a fluctuating flux-tube model of confinement [4]. This term represents an attractive interaction between antialigned spins which falls as the fifth power of the inverse distance between the quark and the antiquark.

The outline of the paper is the following: in the next section the physical significance of the Wilson loop and the super Wilson loop are recalled and their relationship via area derivative of the loop is stated. A string representation of the super Wilson loop is obtained in Sec. II, assuming that the string representation of the Wilson loop is provided by the Nambu-Goto action. It is also shown that the string action for the super Wilson loop is invariant under the worldline SUSY, and a brief comment on the relationship between the string representation of a super Wilson loop and the vacuum expectation value of chiral condensate is made. In Sec. IV the string representation of the super Wilson loop is used to obtain the expectation value of a rectangular super Wilson loop from which the heavy quark potential is obtained. It is found that the spin-string interaction vanishes and therefore there is no spin-spin dependent correction to the heavy quark potential. Next, in Sec. V we evaluate the spin-string interaction in the spirit of the flux-tube model and obtain a nonvanishing spin-spin interaction. The conclusions are stated in the final section.

II. THE WILSON LOOP AND THE SUPER WILSON LOOP

The Wilson loop (WL) for a scalar particle in the fundamental representation of the gauge group is defined as,

$$W[x(\tau)] = \mathrm{Tr}\hat{P}\exp\left\{i\oint d\tau A.\frac{dx}{d\tau}\right\}, \tag{1}$$

where the trace is over the color indices of the matrix valued vector potential, $A = A^a.\tau_a$, with τ_a as the matrices providing the fundamental representation of the Lie algebra of the gauge group. $\hat{P}$ is the path ordering operator that instructs us to order the color matrices along the loop $x(\tau)$ in the order of the increasing value of the parameter τ. To recall the physical significance of the Wilson loop [9], consider the propagation of a meson which is created at x_i and annihilated at x_f in the approximation in which one neglects the virtual quark pairs. In this approximation, the amplitude for this process can be written as a sum over closed paths passing through points x_i and x_f, each path being weighted by the expectation value of the corresponding Wilson loop and some kinematic factors. The expectation value of the Wilson loop being defined as

$$\langle W[x(\tau)]\rangle_{\mathrm{YM}} = \frac{1}{Z_{\mathrm{YM}}}\int DA\exp(-S_{\mathrm{YM}}[A])W[x(\tau)], \tag{2}$$

$$Z_{\mathrm{YM}} = \int DA\exp(-S_{\mathrm{YM}}[A]), \tag{3}$$

where $S_{\mathrm{YM}}[A]$ is the Yang-Mills (YM) action for the gauge field in the Euclidean space. One way of formulating gauge-string duality is to assume that the expectation value of the Wilson loop can be written as a sum over surfaces,

$$\langle W[x(\tau)]\rangle_{\mathrm{YM}} = \int DX\exp(-S_{\mathrm{WL}}[X]), \tag{4}$$

where $X(\sigma)$ is the surface whose boundary is the loop $x(\tau)$ and $S_{\mathrm{WL}}[X]$ is some unknown string action [9–13].

In the above discussion the particle was assumed to be a scalar particle, if we want to describe the propagation of a meson, including the spin of the quark and the antiquark, then the role of the Wilson loop is played by the Wilson loop for a spin-half particle [14] (see [29] for a review)

$$\mathcal{W}[x(\tau),\gamma_\mu(\tau)] = \mathrm{Tr}\hat{P}\exp\left\{i\oint d\tau\frac{dx}{d\tau}.A - \frac{i}{4}\times\oint d\tau\gamma_\mu\gamma_\nu F_{\mu\nu}\right\} \tag{5}$$

$$= \mathrm{Tr}\hat{P}\exp\left\{i\oint d\tau\dot{x}\cdot A + \frac{1}{4}\oint d\tau\Sigma_{\mu\nu}F_{\mu\nu}\right\} \tag{6}$$

where γ_μ are the Dirac gamma matrices and $\Sigma_{\mu\nu}$ are the corresponding spin matrices. Since these matrices do not commute therefore they too have to be path ordered and in that sense they are function of the loop parameter τ. In the context of the path integral for a spin-half particle the appropriate Wilson loop can also be written using Grassmann variables,

$$\mathcal{W}[x(\tau),\psi(\tau)] = \mathrm{Tr}\hat{P}\exp\left\{i\oint d\tau\left(\frac{dx}{d\tau}.A - \frac{1}{2}\psi_\mu\psi_\nu F_{\mu\nu}\right)\right\}, \tag{7}$$

where $\psi(\tau)$ are four independent anticommuting variables [15–18]. Their role is the same as that of gamma matrices, the integration over $\psi(\tau)$ with suitable action for a free spin-half particle is equivalent to taking trace over the gamma matrices. An immediate advantage of writing the Wilson loop for a spin-half particle using $\psi(\tau)$ is that it is invariant under the following one-dimensional supersymmetry

$$\delta x = \epsilon\psi; \qquad \delta\psi = -\epsilon\dot{x}. \tag{8}$$

For this reason, in what follows we will refer both to (5) and to (7) as the super Wilson Loop. The super Wilson loop is not an independent loop functional but is related via a linear operator to the Wilson loop

$$\exp\left\{-\frac{1}{2}\oint d\tau\psi_\mu\psi_\nu\frac{\delta}{\delta\sigma_{\mu\nu}}\right\}W[x(\tau)] = \mathcal{W}[x(\tau),\psi(\tau)], \tag{9}$$

where $\frac{\delta}{\delta\sigma_{\mu\nu}}$ is the area derivative of the loop [19,36].

III. STRING REPRESENTATION OF SUPER WILSON LOOP

The quark-antiquark potential is surprisingly well modeled by a Nambu-Goto string [25–27], suggesting that at least for rectangular loops the expectation value of the Wilson loop can be written as,

$$\langle W[x(\tau)]\rangle_{\rm YM} = \int DX(\sigma)\exp\{-S_{\rm NG}[X(\sigma)]\}. \tag{10}$$

The Nambu-Goto action (NG), $S_{\rm NG}$, is given by

$$S_{\rm NG}[X(\sigma)] = T_0\int d^2\sigma\sqrt{g}, \tag{11}$$

where T_0 is the string tension and g is the determinant of the induced metric. The induced metric can be written using the world sheet coordinates, (σ_1, σ_2), as

$$g_{ab}[\sigma] = \frac{\partial X}{\partial\sigma_a}\cdot\frac{\partial X}{\partial\sigma_b}. \tag{12}$$

Using Eq. (9) one can write the expectation value of the super Wilson loop in terms of the expectation value of the Wilson loop,

$$\begin{aligned}\langle\mathcal{W}[x(\tau),\psi(\tau)]\rangle_{\rm YM} &= \left\langle\exp\left\{-\frac{1}{2}\oint d\tau\psi_\mu\psi_\nu\frac{\delta}{\delta\sigma_{\mu\nu}}\right\}W\right\rangle_{\rm YM},\\ &= \exp\left\{i\oint d\tau\psi_\mu\psi_\nu\frac{\delta}{\delta\sigma_{\mu\nu}}\right\}\int DX\\ &\quad\times\exp\{-S_{\rm NG}[X]\},\\ &= \int DX\exp\{-S_{\rm SWL}[X,x(\tau),\psi(\tau)]\},\end{aligned} \tag{13}$$

where the string action for the super Wilson loop (SWL) is

$$S_{\rm SWL} = T_0\int d^2\sigma\sqrt{g} - \frac{T_0}{2}\oint d\tau\psi_\mu(\tau)\psi_\nu(\tau)t_{\mu\nu}(\tau). \tag{14}$$

In obtaining the above action we have used the fact [37] that the area derivative of the area functional is

$$\frac{\delta}{\delta\sigma_{\mu\nu}(\sigma(x(\tau))}\int d^2\sigma'\sqrt{g} = t_{\mu\nu}[\sigma(x(\tau))], \tag{15}$$

and $t_{\mu\nu}$ is given by

$$t_{\mu\nu}(\sigma) = \frac{\epsilon^{ab}\partial_a X_\mu\partial_b X_\nu}{\sqrt{g}} = \frac{X_{\mu\nu}(\sigma)}{\sqrt{g}}. \tag{16}$$

Thus, the action for the super Wilson loop differ from the Nambu-Goto action by the presence of an additional boundary term. The boundary term represents the interaction between the string variables and the spin of the quark whose worldline is the boundary of the given loop.

As mentioned earlier, super Wilson loop is invariant under a worldline SUSY (8), which we will refer to as SUSY1. One expects that the action (14) too should be invariant under SUSY1 [38]. We can check this using the methods of loop calculus [36,39]. To do so, let us write the action (14) as

$$\begin{aligned}S_{\rm SWL} &= T_0\int d^2\sigma\sqrt{g} - \frac{T_0}{2}\oint d\tau\psi_\mu(\tau)\psi_\nu(\tau)t_{\mu\nu}(\tau)\\ &= S_{\rm NG} + S_{\rm SS},\end{aligned} \tag{17}$$

and consider the variation of each of these terms under SUSY1. The general variation of a loop functional, $F[x(\tau)]$, can be written as

$$\delta F = \oint\delta x_\mu dx_\nu\frac{\delta F}{\delta\sigma_{\mu\nu}}. \tag{18}$$

Using this the variation of $S_{\rm NG}$ under (8) can be written as

$$\delta_{S1}S_{\rm NG} = T_0\oint\delta x_\mu dx_\nu\frac{\delta S_{\rm NG}}{\delta\sigma_{\mu\nu}} = T_0\oint d\tau\dot{x}_\nu\epsilon\psi_\mu t_{\mu\nu}. \tag{19}$$

The variation of $S_{\rm SS}$ under (8) is

$$\begin{aligned}\delta_{S1}S_{\rm SS} &= \delta_{S1}\left(-\frac{T_0}{2}\oint d\tau\psi_\mu\psi_\nu t_{\mu\nu}\right)\\ &= -T_0\oint d\tau\dot{x}_\nu\epsilon\psi_\mu t_{\mu\nu} - \frac{T_0}{2}\oint d\tau\psi_\mu\psi_\nu\delta_{S1}t_{\mu\nu},\end{aligned} \tag{20}$$

the first term in the above equation cancels with the variation of $S_{\rm NG}$ given by (19). Consider now the variation of $t_{\mu\nu}$ under SUSY1,

$$\delta_{S1}t_{\mu\nu}(x(\tau)) = t_{\mu\nu}(x(\tau) + \epsilon\psi(\tau)) - t_{\mu\nu}(x(\tau)), \tag{21}$$

in the context of loop calculus this quantity can be represented by a path derivative [36,39],

$$\delta_{S1}t_{\mu\nu}(x(\tau)) = \partial^x_\lambda t_{\mu\nu}\delta x_\lambda, \qquad = \epsilon\psi_\lambda\partial^x_\lambda t_{\mu\nu}, \tag{22}$$

where ∂^x_λ denotes the path derivative at point $x(\tau)$ and we have used (8). This allows us to write the variation in the second term of (20), using (15), as

$$\oint d\tau\psi_\mu\psi_\nu\delta_{S1}t_{\mu\nu} = \oint d\tau\epsilon\psi_\mu\psi_\nu\psi_\lambda\partial_\lambda(\tau)\frac{\delta}{\delta\sigma_{\mu\nu}(\tau)} \times\left(\int d^2\sigma\sqrt{g}\right). \tag{23}$$

The area derivative satisfies a Bianchi identity

$$\partial_\lambda(\tau)\frac{\delta}{\delta\sigma_{\mu\nu}(\tau)} + \partial_\mu(\tau)\frac{\delta}{\delta\sigma_{\nu\lambda}(\tau)} + \partial_\nu(\tau)\frac{\delta}{\delta\sigma_{\lambda\mu}(\tau)} = 0, \tag{24}$$

as a result the second term (23) in Eq. (20) vanishes and the action (17) is invariant under the worldline SUSY transformation (8).

Having obtained the spin-string interaction, one would like to know whether one can relate it to spontaneous breaking of chiral symmetry. This can be done, at least formally, in the large N limit using Banks and Casher's relation [40] that expresses the vacuum expectation value of chiral condensate, V_χ, in terms of the expectation value of a super Wilson loop

$$V_\chi = m\int_0^\infty dT\exp\left\{-\frac{m^2}{2}T\right\}\int_{y,\psi}\exp\{-S_0\}\langle\mathcal{W}\rangle_{\rm YM}, \tag{25}$$

where the subscript y, ψ under the integral represents a sum over all closed paths of spin-half particle whose length is T, and S_0 is the action for a free spin-half particle,

$$S_0 = \int_0^T d\tau\left\{\frac{\dot{x}^2}{2} + \frac{1}{2}\psi_\mu\dot{\psi}_\mu\right\}. \tag{26}$$

To check for spontaneous breaking of chiral symmetry one has to consider the above expression in the limit $m \to 0$, where m is the current quark mass. Using the string representation for the expectation value of super Wilson loop (13), we can write chiral condensate as

$$V_\chi = \lim_{m\to 0} m\int_0^\infty dT\exp\left\{-\frac{m^2}{2}T\right\}\int_{y,\psi}\exp\{-S_0\} \times\int DX\exp\{-S_{\rm SWL}[X, x(\tau), \psi(\tau)]\}. \tag{27}$$

Unfortunately, this is cumbersome and intractable as it involves sum over an infinite number of boundaries, and for each boundary one has to sum over surfaces. But it does indicate the role of spin-string interaction for describing the spontaneous breaking of chiral symmetry.

IV. SUPER WILSON LOOP AND THE HEAVY QUARK POTENTIAL

The spin-dependent corrections to the heavy quark potential can be obtained from the expectation value of a rectangular super Wilson loop [28–31,33]. For this purpose, it will be more convenient to consider super Wilson loop written in terms of the Dirac gamma matrices, Eq. (5), and then consider the nonrelativistic limit of the following amplitude

$$Z_{q\bar{q}} = \int_0^\infty dT\int Dx\exp\left\{-\int_0^T d\tau\frac{1}{2}\left(\dot{x}^2 + m^2\right)\right\}\langle\mathcal{W}\rangle_{\rm YM}. \tag{28}$$

In the nonrelativistic (NR) limit the parameter τ is related to the Euclidean time by

$$\tau = \frac{x_0}{m} = \frac{t}{m}, \tag{29}$$

where m is the quark mass [29], and in the same limit the super Wilson loop associated with a rectangular loop, Fig. 1, is

$$\mathcal{W}_{\rm NR}[T, R] = {\rm Tr}\hat{P}\left\{i\oint dt(\dot{x}\cdot A) + \frac{1}{4m}\oint dt(\Sigma_{\mu\nu}F_{\mu\nu})\right\}, \tag{30}$$

where we have taken the limit $T \to \infty$ and ignored the contribution from the short sides of the rectangular loop. The rectangular loop can be thought of as being made of the worldline of a quark at origin and a worldline of an antiquark located at a distance R from it. According to our assumptions the expectation value of such a super Wilson loop is given by

$$\langle\mathcal{W}_{\rm NR}\rangle_{\rm YM} = \int DX\exp\{-S_{\rm SWL}\}, \tag{31}$$

with the string action

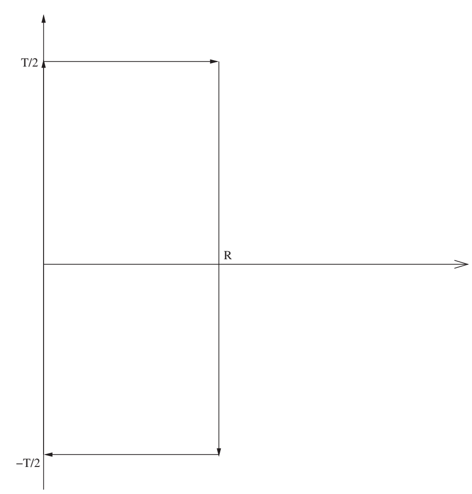

FIG. 1. Loop for calculating spin-dependent heavy quark potential.

$$S_{\mathrm{SWL}}[T,R] = T_0 \int d^2\sigma \sqrt{g} + i\frac{T_0}{4m}\oint \Sigma_{\mu\nu}t_{\mu\nu}(x_0)dt. \tag{32}$$

The expectation value of a rectangular super Wilson loop, in the limit $T \to \infty$, can be expressed as

$$\langle \mathcal{W}[T,R]\rangle_{\mathrm{YM}} = \exp\{i\phi(T,R)\}\exp\{-V(R)T\}, \tag{33}$$

where $\phi(T, R)$ is a phase factor which is a peculiarity of Euclidean path integrals for fermions, while $V(R)$ is the spin-dependent potential between the quark and the antiquark separated by a distance R (the use of euclidean path integral to obtain spin-dependent potentials is reviewed in [29]).

In extracting the spin-dependent potential, it is both suggestive and convenient to write the spin-string interaction term as

$$\Sigma_{\mu\nu}t_{\mu\nu} = \sigma\cdot\mathcal{B} - \sigma\cdot\mathcal{E}, \tag{34}$$

where the worldline quantities $\mathcal{B}$ and $\mathcal{E}$ are defined as

$$\mathcal{B}_i = \tfrac{1}{2}\epsilon_{ijk}t_{jk}, \qquad \mathcal{E}_i = t_{oi}, \tag{35}$$

and σ are the Pauli-spin matrices. In the nonrelativistic limit we can restrict to the upper two components of the Dirac spinors. For a rectangular super Wilson loop the "electric term," $\sigma\cdot\mathcal{E}$, only contributes to a phase factor in Eq. (33) and the spin-spin term arises from the "magnetic term," $\sigma\cdot\mathcal{B}$. The string action for a rectangular super Wilson loop that contributes to the heavy quark potential takes the form

$$S_{\mathrm{SWL}}[T,R] = T_0 \int d^2\sigma \sqrt{g} + i\frac{T_0}{4m}\int dt^+ \sigma^+\cdot\mathcal{B}^+ - i\frac{T_0}{4m}\int dt^- \sigma^-\cdot\mathcal{B}^-, \tag{36}$$

where the superscripts $\pm$ denote the quark and the antiquark S.

It will be convenient to introduce dimensionless coordinates,

$$M = \sqrt{T_0}, \qquad Y(\sigma_0,\sigma_1) = MX(\sigma_0,\sigma_1), \tag{37}$$

and the small transverse fluctuations of the minimal surface

$$\boldsymbol{\phi} = (Y_2, Y_3) = (\phi_y, \phi_z), \tag{38}$$

can be parametrized using

$$\sigma_0 = Y_0 = \bar{t}; \qquad \sigma_1 = Y_1 = \bar{r}. \tag{39}$$

In terms of these dimensionless variables the action for the rectangular super Wilson loop is

$$S_{\mathrm{SWL}}[T,R] = \int d\bar{t}\int d\bar{r}\sqrt{g} + i\frac{M}{4m}\int d\bar{t}^+\sigma^+\cdot\mathcal{B}^+ - i\frac{M}{4m}\int d\bar{t}^-\sigma^-\cdot\mathcal{B}^-. \tag{40}$$

The appropriate boundary conditions for a rectangular super Wilson loop are

$$\partial_{\bar{t}}\boldsymbol{\phi}(\bar{t},0) = \partial_{\bar{t}}\boldsymbol{\phi}(\bar{t},\bar{R}) = 0, \tag{41}$$

with these boundary conditions, and using

$$t_{ij} = \frac{1}{\sqrt{g}}(\dot{\phi}_i\phi'_j - \phi'_i\dot{\phi}_j), \tag{42}$$

we immediately see that

$$\mathcal{B}^+ = \mathcal{B}^- = 0 \tag{43}$$

and therefore there is no contribution from the spin-string interaction to the heavy quark-antiquark potential, and, in particular, there is no spin-spin dependent term in the heavy quark potential.

Absence of a spin-spin term in the heavy quark potential seems to be consistent with the experimental results and lattice simulations. These results suggest that quarks see purely chromoelectric fields in their rest frame [41], and is the starting point for introducing spin degrees of freedom in open-string models of mesons in [8].

V. SPIN-STRING INTERACTION IN THE FLUX-TUBE MODEL

The absence of a spin-spin dependent correction to the heavy quark potential is perhaps surprising, for there is an argument due to Kogut and Parisi [4], in the context of the flux-tube model of confinement, for the existence of a long range spin-spin dependent term in the heavy quark potential. They argue, using the language of $U(1)$ gauge theory, that the zero-point fluctuations of the flux tube creates time-dependent electric flux lines which in turn produces a magnetic field. This magnetic field interacts with the spin of the quark and the antiquark, leading to a spin-spin interaction term in the heavy quark potential. The argument in the previous section implies that the "magnetic" field vanishes on the quark worldline for a static quark or antiquark, but the argument is for the magnetic field produced by a string with no intrinsic thickness and could get modified for a flux tube which has finite intrinsic thickness. One possible way of taking into account the intrinsic thickness of the flux tube, while still retaining the effective string description, is to evaluate the magnetic field t_{ij}, not at the boundary, but to average it over a longitudinal distance of the order of the intrinsic thickness of the string, r_T,

$$\bar{t}_{ij}(t, x^+) = \frac{1}{r_T}\int_0^{r_T} dr t_{ij}(t, r) = \frac{1}{r_T}\int_0^{r_T} dr(t_{ij}(t, x^+) + r\partial_r t_{ij}(t, x^+)) = \frac{r_T}{2}(\partial_r t_{ij})_{x^+}. \quad (44)$$

Evaluating $\bar{t}_{ij}$ for small transverse fluctuations, $\phi_i \ll 1$, for which

$$\sqrt{g} = 1 + \tfrac{1}{2}(\partial_{\bar{t}}\boldsymbol{\phi}^2 + \partial_{\bar{r}}\boldsymbol{\phi}^2), \quad (45)$$

and keeping only the leading terms in $\boldsymbol{\phi}$, we find a nonvanishing spin-string interaction

$$\sigma \cdot \bar{\mathcal{B}}(x^\pm) = \frac{1}{M_g}\sigma \cdot (0, \partial_r\dot{\phi}_z(x^\pm), -\partial_r\dot{\phi}_y(x^\pm)), \quad (46)$$

where $M_g^{-1} = r_T/2$ is some measure of the intrinsic thickness of the flux tube. Apart from the factor of M_g^{-1}, this is precisely the interaction assumed by Kogut and Parisi in Ref. [4]. Using this spin-string interaction, the action for small transverse fluctuations about the minimal surface binding the rectangular loop is

$$S_{sd} = \bar{R}\bar{T} + \frac{1}{2}\int d\bar{t}d\bar{r}(\partial_{\bar{t}}\dot{\boldsymbol{\phi}}^2 + \partial_{\bar{r}}\dot{\boldsymbol{\phi}}^2) + i\alpha_{ss}\int d\bar{t}^+\sigma \cdot \mathbf{b} - i\alpha_{ss}\int d\bar{t}^-\sigma \cdot \mathbf{b}, \quad (47)$$

where the spin-string coupling constant is

$$\alpha_{\rm ss} = \frac{T_0}{4mM_g} \quad (48)$$

and the dimensionless magnetic field is

$$\mathbf{b} = (0, \partial_r\dot{\phi}_z(x^\pm), \qquad -\partial_r\dot{\phi}_y(x^\pm)). \quad (49)$$

The expectation value of the rectangular super Wilson loop then is

$$\langle \mathcal{W}_{\rm NR}\rangle_{\rm YM} = \exp\{-\bar{R}\bar{T}\}Z_{\rm RT}\Big\langle \exp\Big\{-i\alpha_{\rm ss}\Big(\int d\bar{t}^+\sigma \cdot \mathbf{b} - \int d\bar{t}^-\sigma \cdot \mathbf{b}\Big)\Big\}\Big\rangle_\phi, \quad (50)$$

where the average over the string fluctuations $\boldsymbol{\phi}$ is given by

$$Z_{\rm RT} = \int_{\boldsymbol{\phi}} \exp\Big\{-\frac{1}{2}\int d\bar{t}d\bar{r}(\partial_{\bar{t}}\dot{\boldsymbol{\phi}}^2 + \partial_{\bar{r}}\dot{\boldsymbol{\phi}}^2)\Big\}. \quad (51)$$

If we set $\alpha_{\rm ss}$ to zero then the super Wilson loop reduces to the Wilson loop and we recover the linear potential along with the Lüscher term. The effect of the spin-string interaction $\mathbf{b}\cdot\sigma$ can be evaluated in perturbation theory in a manner identical to that of Ref. [4] and the first nonvanishing term appears in the fourth order in $\alpha_{\rm ss}$ and gives rises to the

$$V_{\rm ss} = \frac{T_0^2}{(mM_g)^4}\frac{\sigma^+\cdot\hat{R}\sigma^-\cdot\hat{R}}{R^5}, \quad (52)$$

where $\hat{R}$ is a unit vector pointing from the quark to the antiquark and dimensionless numerical factors have been absorbed in M_g whose inverse we have taken as a measure of the thickness of the flux tube. In the limit $M_g^{-1}\to 0$, which corresponds to a flux tube with no intrinsic thickness, $V_{\rm ss}$ vanishes and there is no spin-spin correction to the heavy quark potential due to spin-string interaction.

It is worth emphasizing that our calculation is entirely within an effective string description. We have only modified the spin-string interaction, in Eq. (44), by averaging it along the string rather than restricting it to the boundary. Thus, the dynamics are that of a string with no intrinsic thickness but with a modified spin-string interaction. It is because of this and particularly because of the ground state fluctuations of the string, that we obtain a long range spin-spin interaction (52) and this is also the reason for the vanishing of the second order term in spin-spin interaction which is proportional to $1/m^2$ (see [4] for details). Our effective string model, by definition, does not include the short-range correlation which is responsible for the formation of the flux tube and which gives rise to exponentially decaying spin-spin interaction of the order $1/m^2$ with a decay length proportional to the intrinsic thickness of the flux tube [42,43]. We comment on a possible way of exploring the relationship between a fundamental string and a flux tube in the next section.

VI. CONCLUSIONS

In a string description of QCD it is important to find out the nature of the spin-string interaction, as it can illuminate both the spin-dependent corrections to the heavy quark potential and within the context of a fundamental string description it may also help us in understanding the existence of a massless pion in chiral limit and more generally understand the pion-rho mass difference. The approach we have taken to analyze this question is to write the expectation value of a super Wilson loop as a sum over surfaces whose boundary is the given loop. Each surface appearing in the sum can be interpreted as a world sheet of an open string that terminates on a worldline of a spin-half particle, the quark in our case. The action appearing in the string representation then naturally includes the spin-string interaction.

In order to obtain a string representation for the expectation value of the super Wilson loop, we used the fact that the super Wilson loop is related to the Wilson loop via the area derivative of a loop. Then we assumed that the expectation value of the Wilson loop has a string representation with the string action for large loops being the Nambu-Goto action. The resulting string action for the super Wilson loop is the Nambu-Goto action with an additional boundary term that incorporates the interaction between the spin degrees of freedom and the string degrees of freedom. The super Wilson loop is invariant under a world-line SUSY, the string action that we have obtained has the desired property that it too is invariant under this symmetry. An important question that we have not discussed in the present investigation is the relationship between the string representation of the super Wilson loop and the string model of mesons which include spin quantum number. A formal string representation for the meson propagator can of course be written in terms of the expectation value of super Wilson loop, in a manner very similar to the expression for chiral condensate (27), but it does not provide a direct string representation for the mesons.

One can extract the spin-dependent potential from the expectation value of a rectangular super Wilson loop. We found that the spin-string interaction does not contribute to heavy quark potential. But if we try and incorporate the effect of the finite intrinsic thickness of the flux tube by averaging the spin-string interaction over a longitudinal distance of the order of thickness of the flux tube, then we do obtain a spin-spin term in the heavy quark potential. The form of the resulting term is precisely the one suggested by Kogut and Paris based on the fluctuation of the electric field lines forming a flux tube [4]. The spin-spin interaction that we obtained depends, in addition to the mass of the quark, on the square of the string tension and on the intrinsic thickness of the flux tube.

In the context of an effective string description of QCD the idea of an intrinsic thickness of a flux tube remains heuristic. It is quite plausible that the flux tube in QCD has an intrinsic thickness, but in the absence of our understanding of the physics behind confinement we cannot identify an operator whose expectation value would give the thickness of the flux tube. AdS/CFT correspondence could perhaps illuminate this issue. In Ref. [44] the authors have argued, using AdS/CFT correspondence, that while the hadrons are represented by an ideal fundamental string with no intrinsic thickness in the bulk of the five-dimensional anti-de Sitter (AdS) space, but their holographic projection on to the four-dimensional boundary theory does have a finite intrinsic thickness. Therefore it would be very interesting and useful to try and obtain a string representation for the expectation value of super Wilson loop using AdS/CFT correspondence and to see if there are any spin-spin terms in the heavy quark potential so obtained.

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