

Non-Abelian strings in high-density QCD: Zero modes and interactionsEiji Nakano,¹ Muneto Nitta,² and Taeko Matsuura³¹*Department of Physics, National Taiwan University, Taipei 10617, Taiwan*²*Department of Physics, Keio University, Hiyoshi, Yokohama, Kanagawa 223-8521, Japan*³*ECT*, Villa Tambosi, strada delle Tabarelle, 286, I 38050 Villazzano (Tennessee), Italy, USA*

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The most fundamental strings in high-density color superconductivity are the non-Abelian semisuperfluid strings which have color-gauge flux tubes but behave as superfluid vortices in the energetic point of view. We show that in addition to the usual translational zero modes, these vortices have normalizable orientational zero modes in the internal space, associated with the color-flavor locking symmetry broken in the presence of the strings. The interaction among two parallel non-Abelian semisuperfluid strings is derived for general relative orientational zero modes to show the universal repulsion. This implies that the previously known superfluid vortices, formed by spontaneously broken $U(1)_B$, are unstable to decay. Moreover, our result proves the stability of color superconductors in the presence of external color-gauge fields.

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I. INTRODUCTION

Quark matter at high density is considered to exhibit color superconductivity [1]. There are lots of phases proposed in the color superconductivity and still there seems to be no agreement on which is the true ground state [2]. However, when the quark chemical potential becomes much larger than the strange quark mass $\mu \gg m_s$, the color-flavor locking (CFL) phase is expected to be realized [3]. In the CFL phase, the quark-quark pairing breaks the symmetry of QCD as $G = U(1)_B \times SU(N)_C \times SU(N)_F \rightarrow H = SU(N)_{C+F} \times \mathbf{Z}_N$ with $N = 3$, where $U(1)_B$ is a baryon number symmetry and $SU(N)_C$, $SU(N)_F$ and $SU(N)_{C+F}$ are color, flavor, and the color-flavor locked symmetries, respectively. The breaking of $U(1)_B$ produces the superfluid vortices [4,5] which may play a role in the neutron star physics. It has been shown, however, that they are not the fundamental strings in the color superconductivity [6]. The most fundamental strings are the semisuperfluid strings which are *non-Abelian* strings with color-gauge flux tube. A superfluid vortex in three flavor QCD can be topologically (and group-theoretically) decomposed into three non-Abelian strings. It remains as a significant open problem which is really energetically favored, three separated non-Abelian vortices or one superfluid vortex as a bound state of them.

In general, non-Abelian strings are strings which arise for symmetry breaking $G \rightarrow H$ in which the unbroken subgroup H is non-Abelian. This kind of string itself attracts lots of attention these several years. Recently the non-Abelian local strings have been found in superstring theory [7] and in supersymmetric QCD [8]. For instance, they have been used to show color confinement and non-Abelian duality in supersymmetric QCD [9]. These strings are Bogomol'nyi-Prasad-Sommerfield (BPS) monopoles, namely, at critical coupling, there thus exists no static force

between them and therefore the moduli space is admitted. The most generic solutions and their moduli space have been obtained [10] by introducing the method of the moduli matrix [11,12]. Dynamics of strings such as reconnection (intercommutation) of two strings has been studied using the moduli space approximation [13]. Non-Abelian semilocal strings have also been extensively studied [12,14]. As for non-Abelian global strings, there has not been as much work, but they may be generated during the chiral phase transition in high temperature QCD [15–17].

Different from Abelian strings, the distinct character of non-Abelian strings is their internal degrees of freedom which are called *orientation*. In the case of generalized QCD where the number of flavor equals the number of color, the presence of a string breaks the symmetry H further as $SU(N)_{C+F} \rightarrow SU(N-1)_{C+F} \times U(1)_{C+F}$. Consequently the zero modes corresponding to

$$\frac{SU(N)_{C+F}}{SU(N-1)_{C+F} \times U(1)_{C+F}} \simeq \mathbf{C}P^{N-1} \quad (1)$$

appear along the string. Then we have a continuously infinite number of strings with the same tension, which are parametrized by the *orientation*, namely, a point in $\mathbf{C}P^{N-1}$. In the case of local (global) non-Abelian strings, these orientational zero modes are (non-)normalizable because the transformation fixes [8] (changes [16,17]) the boundary condition of the strings at infinity. In the case of semisuperfluid strings, it is, however, a nontrivial problem whether orientations are normalizable or not, because $U(1)_B$ symmetry is a global symmetry, unlike the case of local strings [8] where $U(1)_B$ is gauged.

In this paper, we first show that orientational zero modes are in fact normalizable. We then calculate the force between two non-Abelian semisuperfluid strings with general orientations. We find that the static force is always repul-

sive and is $1/N$ of that between superfluid vortices. It does not depend on the orientations of the strings, which is somewhat surprising because the static force between two global non-Abelian strings *does* depend on the orientations [17]. Our result implies that a superfluid vortex of $U(1)_B$ breaking found in Refs. [4,5] is actually unstable to decay into N (three) non-Abelian strings by the repulsive force between them.

In the case of usual superconductors, the interaction between two strings is important; the repulsion (attraction) between strings in type II (I) superconductors implies their (in)stability in the presence of an external magnetic field. The universal repulsion found in this paper ensures the stability of color superconductors in the presence of external color-gauge fields, regardless of whether they are of type I or II.

II. NON-ABELIAN SEMISUPERFLUID STRINGS

Let us start from constructing the general form of the Ginzburg-Landau Lagrangian [18] on the basis of the generalized QCD symmetry: $G = SU(N)_C \times SU(N)_F \times U(1)_B$, where we set the number of flavor equals to the number of flavor and $SU(N)_C$ is the gauge symmetry. For this purpose, we first introduce an N by N matrix field $\Phi_{\alpha i}$ ($\alpha, i = 1, \dots, N$) where $\alpha(i)$ denote the color(flavor) indices. Φ belongs to $[N, N]$ representation of $SU(N)_C \times SU(N)_F$. The symmetry G transforms the matrix field as $\Phi \rightarrow e^{i\alpha} U_C \Phi U_F^\dagger$, where U_C and U_F are independent $SU(N)$ matrices and $e^{i\alpha}$ is a global $U(1)_B$ rotation associated with the baryon number conservation.

In the case of color superconductivity, Φ field corresponds to the pairing gap; the spin-zero pairing of the positive energy quarks in antisymmetric combinations of colors and flavors [19]. We may write Φ as $\Phi_{\alpha i} \sim \epsilon_{\alpha\beta\gamma} \epsilon_{ijk} \langle \psi_j^{T\beta} C \gamma_5 \psi_k^\gamma \rangle$, where ψ is the quark field and we assumed that the ground state is the positive parity state which would be determined by the instanton effect. Then the most general 3-d Ginzburg-Landau Lagrangian up to $\mathcal{O}(\Phi^4)$ is:

$$\begin{aligned} \mathcal{L} = & \text{tr}(D\Phi)^\dagger(D\Phi) - m^2 \text{tr}(\Phi^\dagger\Phi) - \lambda_1 (\text{tr}\Phi^\dagger\Phi)^2 \\ & - \lambda_2 \text{tr}[(\Phi^\dagger\Phi)^2] - \frac{1}{4} F_{ij}^a F^{aij}, \end{aligned} \quad (2)$$

where $D \equiv \partial - ig_s A^\alpha T_\alpha$ is the covariant derivative for the color symmetry, and T_a ($a = 1, 2, \dots, N^2 - 1$) are the generators of $SU(N)_C$ in the fundamental representation which we normalize as $\text{Tr}\{T_a T_b\} = \delta_{ab}$. Here g_s is the gauge coupling and $F_{ij}^a = \partial_i A_j^a - \partial_j A_i^a + g_s f_{abc} A_i^b A_j^c$ is the field strength [20].

The stability condition of vacua enforces $\lambda_1 + \lambda_2/N > 0$. When $m^2 < 0$ and $\lambda_2 > 0$, the vacuum expectation value takes $\langle \Phi \rangle = \mathbf{v} \mathbf{1} \equiv \Phi_0$ where $\mathbf{v} = \sqrt{-m^2/2(N\lambda_1 + \lambda_2)}$. The symmetry G is broken to $H = SU(N)_{C+F} \times \mathbf{Z}_N$. This condensation is called the color-flavor locking

(CFL) in the case of the color superconductivity. The action of H on $\langle \Phi \rangle$ is $\langle \Phi \rangle \rightarrow e^{i\alpha} U_C \langle \Phi \rangle U_F^\dagger$ with $(e^{i\alpha}, U_C, U_F^\dagger) = (\omega, \omega^{-1}U, U^\dagger)$: $\omega \in \mathbf{Z}_N, U \in SU(N)$. The coset space $G/H \simeq U(N)$ has the nontrivial first homotopy group $\pi_1[U(N)] \simeq \mathbf{Z}$, which develops non-Abelian as well as Abelian strings. In the vacua, only the $U(1)_B$ Nambu-Goldstone boson remains massless with the rest Higgsed. From now on, we will take $\nu = 1$ for simplicity.

Here we concentrate on the most fundamental string out of which all the other string configurations are made, and it actually has the lowest energy configuration with a nontrivial loop. Since the common element \mathbf{Z}_N of $SU(N)_{C,F}$ and $U(1)_B$ provides the warp points to make the nontrivial loops therein, the fundamental string (non-Abelian string) is generated by *both* $SU(N)_{C,F}$ and $U(1)_B$ generators. To make this statement clear, we consider the cylindrically symmetric string configuration along the z -axis. In the polar coordinates (ρ, θ) in the x - y plane, an isolated fundamental string has the form

$$\begin{aligned} \Phi(\theta, \rho) = & e^{i(1/N)\theta} e^{-iT_{N^2-1}\sqrt{(N-1)/N}\theta} \text{diag}(f, g, \dots, g) \\ = & \text{diag}(e^{i\theta} f, g, \dots, g). \end{aligned} \quad (3)$$

We take the basis so that the $(N^2 - 1)$ th generator is $T_{N^2-1} = \frac{1}{\sqrt{N(N-1)}} \text{diag}(1 - N, 1, \dots, 1)$. f and g are functions of ρ , and $f(0) = 0$ and $f(\infty) = g(\infty) = 1$ at boundaries. At sufficiently large distance from the core, where $\rho \gg \lambda$ ($\lambda \equiv m^{-1}$ the coherence length), the profile of the string is well described by

$$\Phi(\theta, \rho) \simeq \text{diag}(e^{i\theta}, 1, \dots, 1). \quad (4)$$

One can see that the nontrivial loop must be made from both generators of $SU(N)$ and $U(1)_B$. Although from a topological reason we could use either $SU(N)_C$ or $SU(N)_F$ or both of them for $SU(N)$, the energy consideration forces us to use the gauge symmetry $SU(N)_C$ to minimize the energy of the covariant derivative term. Therefore, we use both global $U(1)_B$ and local $SU(N)_C$ symmetries to construct the string. The name *semisuperfluid* originates this fact.

The gauge fields associated with the string are determined by minimizing the kinetic energy as much as they can. Note that, different from local strings, the covariant derivative term does not vanish at infinity but it is finite, which makes the energy of the string logarithmically divergent as the system size is infinite, just like superfluid vortices. Given Eq. (4), the form of the gauge field can be deduced as $A_\theta^{N^2-1} = \frac{\xi h(\rho)}{g_s \rho}$, and the other components of the gauge field vanish. Here the gauge field only has a nonzero component for θ direction, which we have denoted as A_θ . The function $h(\rho)$ satisfies $h(\infty) = 1$ at boundary. A constant ξ should be determined so as to minimize the kinetic

energy density at infinity: $\text{tr}|D\Phi|^2 = [(\xi + \sqrt{\frac{N-1}{N}})^2 + \frac{1}{N}]/\rho^2$. The minimization is achieved at $\xi = -\sqrt{\frac{N-1}{N}}$. The kinetic energy then becomes $1/(N\rho^2)$ which is $1/N$ of that of the global $U(1)_B$ string [4]. Thus, at large distance over the penetration depth $\lambda_v \equiv m_g^{-1} \sim g_s^{-1}v^{-1}$, the gauge field configuration becomes

$$ig_s A_\theta^{N^2-1} T_{N^2-1} \simeq -\frac{i}{N\rho} \begin{pmatrix} 1-N & 0 \\ 0 & \mathbf{1}_{N-1} \end{pmatrix}. \quad (5)$$

The numerical solution of f , g , and h for the semisuperfluid string in the color-flavor locked phase is found in Ref. [6], where the color and electromagnetic fields are mixed and only one of their linear combinations is relevant for flux.

III. INTERNAL SPACE AND COLOR-GAUGE TRANSFORMATION

Before going ahead, it is instructive to clarify the internal space of the string. The presence of the string (3) breaks the symmetry H further to $SU(N)_{C+F} \rightarrow SU(N-1)_{C+F} \times U(1)_{C+F}$. The internal space corresponds to the coset space (1). Zero modes parametrizing the space (1) appear along the string, i.e., Eq. (3) denotes just one particular string of a continuously infinite number of strings with the same string tension (flux energy) which are parametrized by the *orientation* in the CP^{N-1} . Unlike the case of global strings [16,17] these zero modes are normalizable as seen below.

Here we consider a fundamental string with general orientation in the internal space. We first take the fundamental string Eq. (3) as a reference string ϕ_0 . Then the string ϕ with general orientation in CP^{N-1} relative to the reference string should be obtained by $SU(N)_{C+F}$ transformation to ϕ_0 . However, there are some redundancies in this transformation, i.e., only an $SU(2)_{C+F} \subset SU(N)_{C+F}$ rotation is enough to be considered for orientation relative to ϕ_0 without loss of generality. This corresponds to a CP^1 submanifold in the whole CP^{N-1} . Furthermore, since any regular color-gauge transformation does not change the physical situation, we omit $SU(2)_C$ rotation to ϕ_0 for the moment. Thus we transform ϕ_0 by an element U_F of flavor $SU(2)_F$:

$$\phi = \phi_0 U_F^t = \begin{pmatrix} \begin{pmatrix} e^{i\theta} f & 0 \\ 0 & g \end{pmatrix} u_F^{-1} & 0 \\ 0 & g \mathbf{1}_{N-2} \end{pmatrix}, \quad (6)$$

where

$$u_F \equiv \begin{pmatrix} a^* & -b \\ b^* & a \end{pmatrix}$$

(with $|a|^2 + |b|^2 = 1$) is an element of $SU(2)_F$. This is a general expression for the fundamental string. However, the flavor-rotated string configuration (6) can be transformed back to the original form $\phi \simeq \text{diag}(e^{i\theta}, 1, \dots, 1)$

at a long distance by a *twisted* color-gauge transformation. The explicit form of this (upper left 2×2 matrix $\in SU(2)_C$) is given by

$$u_C(\theta, \rho) = \begin{pmatrix} a^* & -be^{i\theta F(\rho)} \\ b^* e^{-i\theta F(\rho)} & a \end{pmatrix}, \quad (7)$$

where $F(\rho)$ is an arbitrary regular function with boundary conditions $F(0) = 0$ and $F(\infty) = 1$. The former condition has been imposed to make the transformation regular at the center of string. This is possible because of $\pi_1[SU(2)_C] = 0$. Then the upper left 2×2 minor matrix of ϕ in (6) is transformed to

$$u_C \begin{pmatrix} f e^{i\theta} & 0 \\ 0 & g \end{pmatrix} u_F^{-1} \simeq \begin{pmatrix} e^{i\theta} & 0 \\ 0 & 1 \end{pmatrix} \quad \text{for } \rho \gg \lambda. \quad (8)$$

This result means $\phi \simeq \phi_0$ for $\rho \gg \lambda$. Also, the fully opposite orientation can be obtained for $\rho \gg \lambda$ by another color-gauge transformation to (8): $\text{diag}(e^{-i\theta F(\rho)}, e^{i\theta F(\rho)}) \text{diag}(e^{i\theta}, 1) \simeq \text{diag}(1, e^{i\theta})$. Note that one cannot change the topological number by use of this kind of regular gauge transformation. We thus have seen that spatial infinity of the string configurations is the same and does not depend on the orientational zero modes. This implies that the orientational zero modes are *normalizable*, unlike the case of global strings [16,17].

IV. INTERACTION BETWEEN TWO STRINGS

Here we consider the interaction between arbitrary two strings named ϕ_1 and ϕ_2 . The situation we will consider is sorted out as follows: (1) Strings ϕ_1 and ϕ_2 are placed at $(\rho, \theta) = (a, \pi)$ and $(a, 0)$ in parallel with the z -axis, respectively. Their interval is much larger than both the coherence length and the penetration depth: $a \gg \lambda, \lambda_v$. (2) The configuration of the ϕ_1 is given by the asymptotic profile (4): $\phi_1 = \text{diag}(e^{i\theta_1}, 1, \dots, 1)$, while the ϕ_2 has the profile (6) with general orientation relative to ϕ_1 . At large distance of our interest, however, it is equivalent to the reference string configuration (4), i.e., $\phi_2 = \text{diag}(e^{i\theta_2}, 1, \dots, 1)$. (3) The total profile of the two string system is given by the Abrikosov ansatz: $\Phi_{\text{tot}} = \phi_1 \phi_2$ [21] and $A_{\text{tot}}^\theta = A_1^\theta + A_2^\theta$. $A_{1,2}^\theta$ is the gauge field configuration (5) accompanied with the single string system of $\phi_{1,2}$. For the antistring, $\theta_{1,2}$ and $A_{1,2}^\theta$ change their signs.

In order to evaluate the static force between two non-Abelian strings with a general orientation, we first calculate the interaction energy density F , which is obtained by subtracting two individual string energies from the total configuration energy. According to the above situation,

$$\begin{aligned} F(\rho, \theta, a) &\simeq \text{tr}(|D\Phi_{\text{tot}}|^2 - |D\phi_1|^2 - |D\phi_2|^2) \\ &= \pm \frac{2}{N} \left[\frac{-a^2 + \rho^2}{a^4 + \rho^4 - 2a^2 \rho^2 \cos(2\theta)} \right], \quad (9) \end{aligned}$$

where we have used the fact that $V(\Phi_{\text{tot}}) = V(\phi_1) = V(\phi_2) = 0$ and $F_{ij}^a F^{aij} = 0$ at large distance [22]. Here and below, the upper (lower) sign indicates the quantity for string-string (string-anti-string) configuration.

The tension, the energy of the string per unit length, is obtained by integrating the energy density,

$$E(a, L) = \pm \int_0^L d\rho \int_0^{2\pi} d\theta \rho F = \pm \frac{2\pi}{N} \ln \frac{a^2 + L^2}{(2a)^2} \quad (10)$$

with L the IR cutoff for making the integral finite. The force between the two strings is then obtained by differentiating E by the interval

$$f(a, L) = \mp \frac{\partial E}{\partial a} = \pm \frac{2\pi}{N} \left(\frac{1}{a} - \frac{a}{a^2 + L^2} \right) \simeq \pm \frac{2\pi}{Na}, \quad (11)$$

where the last expression is for $a \ll L \rightarrow \infty$. We can see that the force is *repulsive (attractive)* for string-string (anti-string-string) configuration. The overall factors $1/N$ in Eqs. (9)–(11) are attributed to the fact that the tension of the fundamental non-Abelian string is reduced by $1/N$ compared to the usual Abelian string, then leading to $1/N$ erosion in magnitude of the force.

Note that our result does not depend on whether the superconductivity is of type I or II. This has an important meaning in the case of color superconductivity since although the perturbation theory indicates the color superconductivity is of type I for whole density regime [23], the most fundamental strings, semisuperfluid strings, can be stable at any density regime where the CFL is realized. This result also implies that the global $U(1)_B$ superfluid strings $\Phi \simeq \text{diag}(e^{i\theta}, \dots, e^{i\theta})$ found in Refs. [4,5] as well as the M_2 strings $\Phi \simeq \text{diag}(1, e^{-i\theta}, e^{-i\theta}) \simeq (e^{2i\theta}, 1, 1)$ suggested in Ref. [6] are unstable to decay into N or 2 semisuperfluid strings, respectively. It contrasts to the case of global non-Abelian strings [17], where the $U(1)$ Abelian

string is marginally unstable, i.e., no force exists between two strings with opposite orientations.

From all the above arguments we conclude that there exists the long range force between two semisuperfluid strings, which is independent of the orientations in the internal space, and the sign of the force is determined by the difference in their topological charges.

V. DISCUSSION AND OUTLOOK

The Abrikosov approximation employed here for a long range force might not be applicable to describe a fine structure of the force with a short interval, where the string cores overlap. However, our conclusion of the instability of $U(1)_B$ strings (or the stability of color superconductors) is unchanged, i.e., even if the short range force is attractive the $U(1)_B$ string will decay by long range repulsion through large classical fluctuations, thermal fluctuations, or the quantum tunneling effect.

As applications of our results, the universal repulsion implies that a lattice structure appears in a many-string system, e.g., during the rapid cooling of the protoneutron stars or in response to an external magnetic field (or rotation). This might have impacts on observables such as the pulsar glitch phenomenon.

Another interesting issue is how the non-Abelian string releases or interacts with the Nambu-Goldstone bosons corresponding to the global $U(1)_B$ breaking. This may be described using the two-index antisymmetric tensor representation in which the Kalb-Ramond action appropriately describes the string [24]. It is also interesting to explore interactions between strings and other topological solitons, e.g., $U(1)_A$ domain walls [25], and Skyrmions as fundamental quarks in the CFL phase [26].

Note added—: Non-Abelian strings discussed in Ref. [27] are completely different from ours, but are local strings with gauged $U(1)_B$.

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