Dirac particle tunneling from black rings

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(Received 14 May 2008; published 7 August 2008)

Recent research shows that Hawking radiation can be treated as a quantum tunneling process, and Hawking temperatures of Dirac particles across the horizon of a black hole can be correctly recovered via the fermion tunneling method. In this paper, motivated by the fermion tunneling method, we attempt to apply the analysis to derive Hawking radiation of Dirac particles via tunneling from black ring solutions of 5-dimensional Einstein-Maxwell-dilaton gravity theory. Finally, it is interesting to find that, as in the black hole case, fermion tunneling can also result in correct Hawking temperatures for the rotating neutral, dipole, and charged black rings.

DOI: 10.1103/PhysRevD.78.044009 PACS numbers: 04.70.Dy, 04.62.+v

I. INTRODUCTION

Since Hawking proved that a black hole can radiate particles characterized by the thermal spectrum with the temperature $T = (1/2\pi)\kappa$, where κ is the surface gravity of the black hole, many papers have appeared that correctly derive Hawking temperature via different methods, such as the gravity collapsing method [1], the temperature Green function [2], the path integral [3], the Euclidean action integral [4], the second quantum method [5], the renormalization energy-momentum tensor [6], and more recently, a technique called generalized tortoise coordinate transformation (GTCT) which deals with Hawking radiation of evaporating black holes [7,8], etc. The study of Hawking radiation has long been attracting much attention from theoretical physicists. The reason is partly due to the fact that a deeper understanding of Hawking radiation may shed some light on seeking the underlying quantum gravity. And on the other hand, it is the key to making the second law of thermodynamics consistent in spacetimes involving black holes.

In recent years, a semiclassical quantum tunneling method, first put forward by Kraus and Wilczek [9] and then elaborated by Parikh and Wilczek [10], has already attracted much attention [11,12]. Here, the derivation of Hawking temperature mainly depends on the computation of the imaginary part of the action for the classically forbidden process of s-wave emission across the horizon. Normally, there are two approaches to obtain the imaginary part of the action. One, first used by Parikh and Wilczek [10] and later broadly discussed in many papers [11,12], is called the null geodesic method, where the contribution to the imaginary part of the action only comes from the integration of the radial momentum p_r for the emitted particles. The other method regarding the action of the emitted particles should satisfy the relativistic Hamilton-Jacobi equation, and solving it will yield the imaginary part of the action [13], which is an extension of the complex path analysis proposed by Padmanabhan *et al.* [14]. In the two tunneling modes, they use the fact that the tunneling rate for the classically forbidden trajectory from inside to outside the horizon is given by $\Gamma = \exp(-\frac{2}{\hbar}\,\mathrm{Im}I)$, where I is the classical action of the trajectory to leading order in $\hbar$. Where these two methods differ is in how the action is calculated. Reference [15] has given a detailed comparison between the Hamilton-Jacobi ansatz and the null geodesic methods.

Although the tunneling method is shown to be very robust in successfully deriving Hawking radiation of black holes and even black rings, most papers have only considered scalar particle tunneling radiation. In fact, a black hole can radiate all types of particles at the Hawking temperature, and the true emission spectrum should contain contributions of both scalar particles and fermions with all spins. Recently, applications of quantum tunneling methods to the fermion case have first been presented in Ref. [16] to correctly describe Hawking radiation of fermions with spin $1/2$ via tunneling from Rindler spacetime and that from the uncharged spherically symmetric black holes. Later, to further prove the robustness of the fermion tunneling method, some papers discuss Hawking radiation of fermions via tunneling from BTZ black holes [17], dynamical black holes [18], Kerr black holes [19], Kerr-Newman black holes [20], and more general and complicated black holes [21]. These involved black holes have in common taking 3- or 4-dimensional spacetimes. For spacetimes with different horizon topology and different dimensions, choosing a set of appropriate γ^μ matrices for general covariant Dirac equations is critical for the fermion tunneling method. In 3-dimensional cases, as the Pauli matrices σ^i $(i = 1, 2, 3)$ behave independently from each other, we can only introduce the matrices σ^i to act as γ^μ functions for the covariant Dirac equation [17]. However, for 4-dimensional spacetimes, we need four independent matri-

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ces to describe the matrices γ^μ well for the Dirac equation, and a detailed choice for the four matrices γ^μ; see Refs. [16,18–21]. Then, how does one choose the γ^μ matrices for 5-dimensional cases? To the best of our knowledge, five independent matrices should be involved in our discussion. On the other hand, the horizon topology also has an important impact on the choice for the matrices γ^μ [20]. In Secs. II and III, we will successfully introduce a set of appropriate matrices γ^μ for the 5-dimensional neutral, dipole, and charged black rings with the horizon topology $S^1 \times S^2$ to describe Dirac particle tunneling radiation well.

Black rings in five dimensions have many unusual properties not shared by Myers-Perry black holes with spherical topology; for instance, their event horizon topology is $S^1 \times S^2$, which is not spherical for the neutral, dipole, and charged black rings. (Actually, some topological black holes also have nontrivial topology; see, for example, [22].) Therefore, it is very interesting to study Hawking radiation from these black ring solutions. In Ref. [23], scalar particles via tunneling from black rings have already been discussed by using the so-called Hamilton-Jacobi method. And in [24], following a recent hot discussion on the anomalous derivation of Hawking radiation, the authors attempt to recover Hawking temperature of black rings via gauge and gravitational anomalies at the horizon. However, when reducing the higher dimensional theory to the effective 2-dimensional theory, they also only consider the scalar field near the horizon. As far as I know, till now, there have been no references to report Hawking radiation of Dirac particles across black rings. So it is interesting to see if the fermion tunneling method is still applicable in such exotic spacetimes, and to see how to choose the matrices γ^μ for the covariant Dirac equation of 5-dimensional black rings. In this paper, we shall concentrate on Dirac particle tunneling radiation from 5-dimensional black rings via the fermion tunneling method. We finally find, as in the black hole case, fermion tunneling results in correct Hawking temperatures for the rotating neutral, dipole, and charged black rings.

The remainder of this paper is organized as follows. In Sec. II, Hawking radiation of Dirac particles via tunneling from the 5-dimensional rotating neutral black ring is studied by improving the fermion tunneling method. To make an analysis of the rotating dipole and charged black rings in a more unified form, in Sec. III we deduce a general 5-dimensional metric from the rotating neutral black ring, and discuss its Hawking radiation of Dirac particles. In fact, the involved 5-dimensional metric is not arbitrarily taken, and after some substitutions, has a unified form for the rotating neutral, dipole, and charged black rings (see Ref. [23]). Section III is devoted, once again, to checking the validity of the fermion tunneling method for the rotating dipole and charged black rings. Section IV contains some conclusions and discussions.

II. DIRAC PARTICLE TUNNELING FROM NEUTRAL BLACK RINGS

In this section, we focus on studying Hawking radiation of Dirac particles via tunneling from 5-dimensional neutral black rings. In this paper, black rings involved are only special solutions of the Einstein-Maxwell-dilaton gravity model (EMD) in 5 dimensions, and the corresponding action takes the forms as

$$S = \frac{1}{16\pi}\int d^5x\sqrt{-g}\left(\mathcal{R} - \frac{1}{2}(\partial\phi)^2 - \frac{1}{4}e^{-\alpha\Phi}F^2\right), \quad (1)$$

where F is a three-form field strength and Φ is a dilaton. Black ring solutions of the action (1) have special characteristics: (1) they all have horizon of topology $S^1 \times S^2$; (2) there exist three Killing coordinates to determine their local symmetries; (3) there exist infinitely many different black ring solutions carrying the same mass, angular momentum, and electric charge, etc. In this paper, the rotating neutral, dipole, and charged black rings accompanied by the action (1) are involved in our discussion. First, we consider the case of the 5-dimensional neutral black ring. The neutral black ring in 5-dimensional EMD theory has been given by [25]

$$\begin{aligned} ds^2 = &-\frac{F(y)}{F(x)}\left(dt - C(\nu,\lambda)R\frac{1+y}{F(y)}d\psi\right)^2 + \frac{R^2}{(x-y)^2}F(x) \\ &\times\left[-\frac{G(y)}{F(y)}d\psi^2 - \frac{dy^2}{G(y)} + \frac{dx^2}{G(x)} + \frac{G(x)}{F(x)}d\varphi^2\right], \end{aligned} \quad (2)$$

where

$$F(\xi) = 1 + \lambda\xi, \qquad G(\xi) = (1-\xi^2)(1+\nu\xi),$$
$$C(\nu,\lambda) = \sqrt{\lambda(\lambda-\nu)\frac{1+\lambda}{1-\lambda}}.$$

The parameters λ and ν are dimensionless and take values in the range $(0 < \nu \leq \lambda < 1)$, and to avoid the conical singularity also at $x = 1$, λ and ν must be related to each other via $\lambda = 2\nu/(1+\nu^2)$. The coordinates ϕ and ψ are two cycles of the black ring, and x and y take the range as $-1 \leq x \leq 1$ and $-\infty \leq y \leq -1$. The constant R has the dimension of length and, for large thin rings, corresponds roughly to the radius of the ring circle [26]. The horizon is located at $y = y_h = -1/\nu$. The mass of the black ring is $M = 3\pi R^2\lambda/[4(1-\nu)]$, and its angular momentum takes $J = \pi R^3\sqrt{\lambda(\lambda-\nu)(1+\lambda)}/[2(1-\nu)^2]$. In addition, the spacetime contains three Killing coordinates t, φ, and ψ. Next, we shall study Dirac particle tunneling from the above neutral black ring. For simplicity, we take

$$\mathcal{M}(x,y)=\frac{F(y)}{F(x)}\bigg(1-\frac{C^2(\nu,\lambda)(1+y)^2(x-y)^2}{F^2(x)G(y)+C^2(\nu,\lambda)(1+y)^2(x-y)^2}\bigg),$$
$$\mathcal{N}(x,y)=-\bigg(\frac{R^2}{(x-y)^2}\frac{F(x)}{G(y)}\bigg)^{-1},$$
$$N^{\psi}(x,y)=-\frac{C(\nu,\lambda)R(1+y)F(y)(x-y)^2}{C^2(\nu,\lambda)(x-y)^2R^2(1+y)^2+R^2F^2(x)G(y)},$$
$$g_{\psi\psi}(x,y)=-\frac{C^2(\nu,\lambda)(x-y)^2R^2(1+y)^2+R^2F^2(x)G(y)}{F(x)F(y)(x-y)^2},$$
$$g_{xx}(x,y)=\frac{R^2F(x)}{(x-y)^2G(x)},\quad g_{\varphi\varphi}(x,y)=\frac{R^2G(x)}{(x-y)^2}. \tag{3}$$

Now the new form of the neutral black ring (2) changes as

$$ds^2=-\mathcal{M}(x,y)dt^2+\frac{1}{\mathcal{N}(x,y)}dy^2+g_{\psi\psi}(x,y)(d\psi+N^{\psi}(x,y)dt)^2+g_{xx}(x,y)dx^2+g_{\varphi\varphi}(x,y)d\varphi^2. \tag{4}$$

At the event horizon of the neutral black ring, the coefficients in Eq. (3) obviously obey

$$\mathcal{M}(x,y_h)=\mathcal{N}(x,y_h)=0,\qquad N^{\psi}(x,y_h)=-\Omega_h, \tag{5}$$

where $y=y_h$ is the event horizon of the neutral black ring and Ω_h is the angular velocity of the black ring at the event horizon. Throughout this paper, the 5-dimensional spacetime coordinates are always chosen as $x^{\mu}=(t,y,\varphi,x,\psi)$.

Now we focus on studying Dirac particle tunneling from the rotating neutral black ring. In curved spacetime, the Dirac particle motion equation satisfies the following covariant Dirac equation:

$$i\gamma^a e_a^{\mu}D_{\mu}\Psi-\frac{m}{\hbar}\Psi=0, \tag{6}$$

where D_{μ} is the spinor covariant derivative defined by $D_{\mu}=\partial_{\mu}+\frac{1}{4}\omega_{\mu}^{ab}\gamma_{[a}\gamma_{b]}$, and ω_{μ}^{ab} is the spin connection corresponding to the tetrad e_a^{μ}. In this paper, we choose the matrices $\gamma^a=(\gamma^0,\gamma^3,\gamma^4,\gamma^1,\gamma^2)$ for the 5-dimensional rotating neutral black ring, where

$$\gamma^0=\begin{pmatrix}0 & I\\ -I & 0\end{pmatrix},\qquad \gamma^1=\begin{pmatrix}0 & \sigma^1\\ \sigma^1 & 0\end{pmatrix},$$
$$\gamma^2=\begin{pmatrix}0 & \sigma^2\\ \sigma^2 & 0\end{pmatrix},\qquad \gamma^3=\begin{pmatrix}0 & \sigma^3\\ \sigma^3 & 0\end{pmatrix}, \tag{7}$$
$$\gamma^4=\begin{pmatrix}-I & 0\\ 0 & I\end{pmatrix},$$

and the σ^i ($i=1,2,3$) are the Pauli matrices, which are given by

$$\sigma^1=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix},\qquad \sigma^2=\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix},\qquad \sigma^3=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}. \tag{8}$$

According to the new form of the rotating neutral black ring (4), the tetrad field e_a^{μ} can be constructed as

$$e_0^{\mu}=\bigg(\frac{1}{\sqrt{\mathcal{M}}},0,0,0,-\frac{N^{\psi}}{\sqrt{\mathcal{M}}}\bigg),\qquad e_1^{\mu}=(0,\sqrt{\mathcal{N}},0,0,0),$$
$$e_2^{\mu}=\bigg(0,0,\frac{1}{\sqrt{g_{\varphi\varphi}}},0,0\bigg),\qquad e_3^{\mu}=\bigg(0,0,0,\frac{1}{\sqrt{g_{xx}}},0\bigg),$$
$$e_4^{\mu}=\bigg(0,0,0,0,\frac{1}{\sqrt{g_{\psi\psi}}}\bigg). \tag{9}$$

As Dirac particles take spin 1/2, when measuring spin along the y direction, there would be two cases. One is the spin-up case, which shares the same direction as y, and the other (spin-down) case takes the opposite direction. In the Pauli matrix σ^3 representation, they can explicitly be expressed by the eigenvectors $\xi_{\uparrow/\downarrow}$, and the corresponding eigenvalues are $1/-1$. In this paper, we only refer to the spin field for the upper case ($\xi_{\uparrow}$). In fact, after the same step for the spin-down ($\xi_{\downarrow}$) case, we can also get the same result. We employ the following ansatz for the Dirac field in the spin-up case:

$$\Psi_{\uparrow}(t,y,\varphi,x,\psi)=\begin{pmatrix}A(t,y,\varphi,x,\psi)\xi_{\uparrow}\\ B(t,y,\varphi,x,\psi)\xi_{\uparrow}\end{pmatrix}\times\exp\bigg[\frac{i}{\hbar}I_{\uparrow}(t,y,\varphi,x,\psi)\bigg]=\begin{pmatrix}A(t,y,\varphi,x,\psi)\\ 0\\ B(t,y,\varphi,x,\psi)\\ 0\end{pmatrix}\times\exp\bigg[\frac{i}{\hbar}I_{\uparrow}(t,y,\varphi,x,\psi)\bigg]. \tag{10}$$

Substituting the above ansatz (10) for the upper-spinning state into the covariant Dirac equation (6) and then applying the WKB approximation and keeping the prominent terms, we can get the following equations:

$$B\bigg(\frac{1}{\sqrt{\mathcal{M}}}\partial_t I_{\uparrow}+\sqrt{\mathcal{N}}\partial_y I_{\uparrow}-\frac{N^{\psi}}{\sqrt{\mathcal{M}}}\partial_{\psi}I_{\uparrow}\bigg)+A\bigg(m-\frac{1}{\sqrt{g_{\varphi\varphi}}}\partial_{\varphi}I_{\uparrow}\bigg)=0, \tag{11}$$

$$B\bigg(\frac{1}{\sqrt{g_{xx}}}\partial_x I_{\uparrow}+\frac{i}{\sqrt{g_{\psi\psi}}}\partial_{\psi}I_{\uparrow}\bigg)=0, \tag{12}$$

$$A\bigg(\frac{1}{\sqrt{\mathcal{M}}}\partial_t I_{\uparrow}-\sqrt{\mathcal{N}}\partial_y I_{\uparrow}-\frac{N^{\psi}}{\sqrt{\mathcal{M}}}\partial_{\psi}I_{\uparrow}\bigg)-B\bigg(m+\frac{1}{\sqrt{g_{\varphi\varphi}}}\partial_{\varphi}I_{\uparrow}\bigg)=0, \tag{13}$$

$$A\bigg(\frac{1}{\sqrt{g_{xx}}}\partial_x I_{\uparrow}+\frac{i}{\sqrt{g_{\psi\psi}}}\partial_{\psi}I_{\uparrow}\bigg)=0. \tag{14}$$

In fact, the derivatives of A and B, and the components

$\frac{1}{4}\omega_\mu^{ab}\gamma_{[a}\gamma_{b]}$ are all of order $\mathcal{O}(\hbar)$ and according to the WKB approximation, have already been neglected for the above equations. Considering the symmetries of the rotating neutral black ring, we employ the following ansatz:

$$I_\uparrow = -\mathcal{E}t + \mathcal{J}\psi + \mathcal{L}\varphi + \mathcal{W}(x, y) + \mathcal{K}, \tag{15}$$

where $\mathcal{E}$, $\mathcal{J}$, and $\mathcal{L}$ are all real constants which, respectively, represent the emitted particle's energy and angular momentum corresponding to the angles ψ and φ, and $\mathcal{K}$ is a complex constant (where we consider only the positive frequency contributions without loss of generality). Inserting the ansatz (15) into Eqs. (11)–(14), and expanding the resulting equations near the event horizon of the black ring, we have

$$B\bigg(\frac{-\mathcal{E} + \Omega_h\mathcal{J}}{\sqrt{\mathcal{M}_{,y}(x, y_h)(y - y_h)}} + \sqrt{\mathcal{N}_{,y}(x, y_h)(y - y_h)}\partial_y\mathcal{W}(x, y)\bigg) + A\bigg(m - \frac{\mathcal{L}}{\sqrt{g_{\varphi\varphi}(x, y_h)}}\bigg) = 0, \tag{16}$$

$$B\bigg(\frac{1}{\sqrt{g_{xx}(x, y_h)}}\partial_x\mathcal{W}(x, y) + \frac{i}{\sqrt{g_{\psi\psi}(x, y_h)}}\mathcal{J}\bigg) = 0, \tag{17}$$

$$A\bigg(\frac{-\mathcal{E} + \Omega_h\mathcal{J}}{\sqrt{\mathcal{M}_{,y}(x, y_h)(y - y_h)}} - \sqrt{\mathcal{N}_{,y}(x, y_h)(y - y_h)}\partial_y\mathcal{W}(x, y)\bigg) - B\bigg(m + \frac{\mathcal{L}}{\sqrt{g_{\varphi\varphi}(x, y_h)}}\bigg) = 0, \tag{18}$$

$$A\bigg(\frac{1}{\sqrt{g_{xx}(x, y_h)}}\partial_x\mathcal{W}(x, y) + \frac{i}{\sqrt{g_{\psi\psi}(x, y_h)}}\mathcal{J}\bigg) = 0. \tag{19}$$

Here $\mathcal{M}_{,y}(x, y_h) = \partial_y\mathcal{M}(x, y)|_{y=y_h}$ and $\mathcal{N}_{,y}(x, y_h) = \partial_y\mathcal{N}(x, y)|_{y=y_h}$. Now we carry out an explicit analysis of the above equations. From Eqs. (17) and (19) we can obtain

$$\partial_x\mathcal{W}(x, y) = -i\sqrt{\frac{g_{xx}(x, y_h)}{g_{\psi\psi}(x, y_h)}}\mathcal{J}. \tag{20}$$

And from Eqs. (16) and (18), one can easily see that the two equations have a nontrivial solution for A and B if and only if the determinant of the coefficient matrix vanishes, so we have

$$\partial_y\mathcal{W}(x, y) = \pm\frac{\sqrt{(\mathcal{E} - \Omega_h\mathcal{J})^2 + \mathcal{M}_{,y}(x, y_h)(y - y_h)(m^2 - \frac{\mathcal{L}^2}{g_{\varphi\varphi}})}}{\sqrt{\mathcal{M}_{,y}(x, y_h)\mathcal{N}_{,y}(x, y_h)}(y - y_h)}. \tag{21}$$

It should be noted that Eq. (20) implies that near the horizon of the black ring $\partial_x\mathcal{W}(x, y)$ has no explicit y dependence. On the other hand, in Eq. (21) $\mathcal{M}_{,y}(x, y_h)$ and $\mathcal{N}_{,y}(x, y_h)$ are both related to the coordinate x, but their product $\mathcal{M}_{,y}(x, y_h)\cdot\mathcal{N}_{,y}(x, y_h)$ is independent of x. So, near the horizon ($y \simeq y_h$), $\partial_y\mathcal{W}(x, y)$ is independent of x. Now the function $\mathcal{W}(x, y)$ can be separated as $\mathcal{W}(x, y) = \mathcal{W}(x) + \mathcal{W}(y)$, which means that near the horizon of the black ring $\partial_x\mathcal{W}(x, y) = \partial_x\mathcal{W}(x)$ and $\partial_y\mathcal{W}(x, y) = \partial_y\mathcal{W}(y)$.

The WKB approximation tells us that the tunneling rate for the classically forbidden trajectory from inside to outside the horizon is related to the imaginary part of the emitted particle's action across the event horizon. Now our first job is to find the imaginary part of the action. From Eq. (15), we find that only $\mathcal{W}(x, y)$ and $\mathcal{K}$ yield contributions to the imaginary part of the action. As $\mathcal{K}$ is a complex constant, the focus is on computing $\mathcal{W}(x)$ and $\mathcal{W}(y)$. In fact, after an integration on Eq. (20), $\mathcal{W}(x)$ must be given by a complex constant, so it will yield a contribution to the imaginary part of the action. From Eq. (21) we get

$$\mathcal{W}_\pm(y) = \pm i\pi\frac{\mathcal{E} - \Omega_h\mathcal{J}}{\sqrt{\mathcal{M}_{,y}(x, y_h)\mathcal{N}_{,y}(x, y_h)}}, \tag{22}$$

where the $+/-$ sign corresponds to outgoing/incoming solutions. As we all know, the tunneling probability is proportional to the imaginary part of the action. So when particles tunnel across the horizon each way, the outgoing and ingoing rates are, respectively, given by

$$\begin{aligned} P_{\text{out}} &= \exp\bigg[-\frac{2}{\hbar}\,\mathrm{Im}I_\uparrow\bigg] \\ &= \exp\bigg[-\frac{2}{\hbar}(\mathrm{Im}\mathcal{W}_+(y) + \mathrm{Im}\mathcal{W}(x) + \mathrm{Im}\mathcal{K})\bigg], \\ P_{\text{in}} &= \exp\bigg[-\frac{2}{\hbar}\,\mathrm{Im}I_\uparrow\bigg] \\ &= \exp\bigg[-\frac{2}{\hbar}(\mathrm{Im}\mathcal{W}_-(y) + \mathrm{Im}\mathcal{W}(x) + \mathrm{Im}\mathcal{K})\bigg]. \end{aligned} \tag{23}$$

Note that any particles can classically enter the horizon with no barrier, which means the tunneling rate should be unity for incoming particles crossing the horizon. In our case, this implies $\mathrm{Im}\mathcal{W}_-(y) = -\mathrm{Im}\mathcal{W}(x) - \mathrm{Im}\mathcal{K}$. If we set $\hbar$ to unity, the tunneling probability of Dirac particles crossing from inside to outside the horizon is naturally written as

$$\Gamma = \exp[-4\,\mathrm{Im}\,\mathcal{W}_+(y)] = \exp\left[-\frac{4\pi}{\sqrt{\mathcal{M}_{,y}(x, y_h)\mathcal{N}_{,y}(x, y_h)}}(\mathcal{E} - \Omega_h \mathcal{J})\right], \quad (24)$$

which results in the expected temperature of the rotating neutral black ring,

$$T = \frac{\sqrt{\mathcal{M}_{,y}(x, y_h)\mathcal{N}_{,y}(x, y_h)}}{4\pi} = \frac{1}{4\pi R}\frac{1+\nu}{\sqrt{\nu}}\sqrt{\frac{1-\lambda}{\lambda(1+\lambda)}}. \quad (25)$$

This result is exactly consistent with that in Refs. [23–25], which, respectively, present the correct Hawking temperature of the rotating neutral black ring by using the so-called Hamilton-Jacobi method, the anomalous cancellation method, and the original definition of the surface gravity. Note that the resulting temperature (25) is only for Dirac particles with spin-up. For the spin-down case, taking a manner fully analogous to the spin-up case will produce the same result, which means both spin-up and spin-down particles are emitted at the same rate. So, such a treatment does not lose the generality of the fermion tunneling method. In addition, the tunneling rate (24) is derived by neglecting the higher terms about $\mathcal{E}$ and $\mathcal{J}$, and the resulting spectrum is purely thermal. If we consider energy and angular momentum conservation when particles are tunneling out of the horizon, the higher terms will be present in the tunneling rate, and the radiation spectrum is not thermal, and related to the change of Bekenstein-Hawking entropy, which was discussed a lot in Refs. [10–12,23]. In the next section, to further verify the validity of the application of the fermion tunneling method to black rings, we additionally take dipole and charged black rings as examples to discuss the Hawking radiation of Dirac particles.

III. DIRAC PARTICLE TUNNELING FROM DIPOLE AND CHARGED BLACK RINGS

In the section, we will discuss Hawking radiation of Dirac particles via tunneling from dipole and charged black rings, and we expect to get the correct Hawking temperatures.

A. Dipole black rings

Dipole black rings share the same action (1) as neutral black rings, so they physically take many similar characteristics. The 5-dimensional dipole black ring was first constructed in [25]; its metric takes the form

$$\begin{aligned} ds^2 = &-\frac{F(y)}{F(x)}\left(\frac{H(x)}{H(y)}\right)^{N/3}\left(dt - C(\nu,\lambda)R\frac{1+y}{F(y)}d\psi\right)^2 \\ &+ \frac{R^2}{(x-y)^2}F(x)(H(x)H^2(y))^{N/3}\left[-\frac{G(y)}{F(y)H^N(y)}d\psi^2\right. \\ &\left. - \frac{dy^2}{G(y)} + \frac{dx^2}{G(x)} + \frac{G(x)}{F(x)H^N(x)}d\varphi^2\right], \end{aligned} \quad (26)$$

where $F(\xi)$, $G(\xi)$, and $C(\nu, \lambda)$ are of the same form as neutral black rings, and $H(\xi) = 1 - \mu\xi$ $(0 \le \mu < 1)$. The dilaton coupling constant is related to the dimensionless constant N as $\alpha^2 = (\frac{4}{N} - \frac{4}{3})(0 < N \le 3)$. The horizon is also located at $y = y_H = -1/\nu$. Taking the limit of $\mu = 0$ in Eq. (26), this solution degenerates into neutral black rings [25]. In suitable limits, dipole black rings also contain Myers-Perry black holes [27]. The metric (26) takes the same form as (2), so we can apply the same procedure as in Sec. II to correctly recover the Hawking temperature of the dipole black ring. Before that, we take

$$\begin{aligned} \mathcal{M}(x,y) &= \frac{F(y)}{F(x)}\left(\frac{H(x)}{H(y)}\right)^{N/3}\left(1 - \frac{C^2(\nu,\lambda)(1+y)^2(x-y)^2}{F^2(x)G(y) + C^2(\nu,\lambda)(1+y)^2(x-y)^2}\right), \\ \mathcal{N}(x,y) &= -\left(\frac{R^2}{(x-y)^2}\frac{F(x)}{G(y)}(H(x)H^2(y))^{N/3}\right)^{-1}, \qquad N^{\psi}(x,y) = -\frac{C(\nu,\lambda)R(1+y)F(y)(x-y)^2}{C^2(\nu,\lambda)(x-y)^2R^2(1+y)^2 + R^2F^2(x)G(y)}, \\ g_{\psi\psi}(x,y) &= -\frac{C^2(\nu,\lambda)(x-y)^2R^2(1+y)^2 + R^2F^2(x)G(y)}{F(x)F(y)(x-y)^2}\left(\frac{H(x)}{H(y)}\right)^{N/3}, \qquad g_{xx}(x,y) = \frac{R^2F(x)}{(x-y)^2G(x)}(H(x)H^2(y))^{N/3}, \\ g_{\varphi\varphi}(x,y) &= \frac{R^2G(x)}{(x-y)^2}\frac{(H(x)H^2(y))^{N/3}}{H^N(x)}, \end{aligned} \quad (27)$$

which results in the metric (26) taking the same form as (4). At the horizon, the functions $\mathcal{M}(x, y)$, $\mathcal{N}(x, y)$, and $N^{\psi}(x, y)$ still satisfy Eq. (5). Now substituting the matrices γ^a (7) and the tetrad e^{μ}_a (9) into the covariant Dirac equation (6) and then adopting the same procedure presented in Sec. II, one can read out the Hawking temperature of Dirac particles via tunneling from the dipole black ring,

$$T = \frac{\sqrt{\mathcal{M}_{,y}(x, y_h)\mathcal{N}_{,y}(x, y_h)}}{4\pi} = \frac{1}{4\pi R}\frac{\nu^{(N-1)/2}(1+\nu)}{(\mu+\nu)^{N/2}}\sqrt{\frac{1-\lambda}{\lambda(1+\lambda)}}. \quad (28)$$

This result has been identically derived by using the Hamilton-Jacobi method [23] and the anomalous cancellation method [24], where particles across the horizon are only for scalar cases. Note that the dipole black ring actually contains a gauge field. Here we do not consider its effect because it is magnetic, and its electric dual consists of two-form fields that do not couple to point particles (see Chen and He's paper in [24]). In the next subsection, we will further study Hawking radiation of a rotating black ring with a single electric charge by using the fermion tunneling method.

B. Charged black rings

In this subsection, we consider Hawking radiation from black rings with only one electric charge [28]. For black rings with two or three charges [29], we can take a similar procedure to get the correct results. The metric of black rings with a single electric charge can be written in a form consistent with the neutral and dipole cases as

$$ds^2 = -\frac{F(y)}{F(x)K^2(x,y)}\left(dt - C(\nu,\lambda)R\frac{1+y}{F(y)}\cosh^2\alpha d\psi\right)^2 + \frac{R^2}{(x-y)^2}F(x)\left[-\frac{G(y)}{F(y)}d\psi^2 - \frac{dy^2}{G(y)} + \frac{dx^2}{G(x)} + \frac{G(x)}{F(x)}d\varphi^2\right], \tag{29}$$

where some tricks are needed to reduce the original metric of the black ring with a single electric charge to the form of (29) (refer to Chen and He's paper in [24]). Here $F(\xi)$ and $G(\xi)$ are defined as before, and $K(x,y) = 1 + \lambda(x-y)\sinh^2\alpha/F(x)$, where α is the parameter representing the electric charge. The metric also has a Killing horizon at $y = y_h = -1/\nu$. The dilaton field is $e^{-\Phi} = K(x,y)$, and the gauge fields accompanied by the metric are

$$\mathcal{A}_t = \frac{\lambda(x-y)\sinh\alpha\cosh\alpha}{F(x)K(x,y)}, \qquad \mathcal{A}_\psi = \frac{C(\nu,\lambda)R(1+y)\sinh\alpha\cosh\alpha}{F(x)K(x,y)}, \tag{30}$$

with the electric charge $Q = 2M\sinh 2\alpha/(3(1+\frac{4}{3}\sinh^2\alpha))$. To do an explicit computation of Hawking radiation of the black ring, we first introduce the following substitution:

$$\begin{aligned}
\mathcal{M}(x,y) &= \frac{F(y)}{F(x)K^2(x,y)}\left(1 - \frac{C^2(\nu,\lambda)(1+y)^2(x-y)^2\cosh^4\alpha}{F^2(x)G(y)K^2(x,y) + C^2(\nu,\lambda)(1+y)^2(x-y)^2\cosh^4\alpha}\right),\\
\mathcal{N}(x,y) &= -\left(\frac{R^2}{(x-y)^2}\frac{F(x)}{G(y)}\right)^{-1}, \qquad N^\psi(x,y) = -\frac{C(\nu,\lambda)R(1+y)F(y)(x-y)^2\cosh^2\alpha}{C^2(\nu,\lambda)(x-y)^2R^2(1+y)^2\cosh^4\alpha + R^2F^2(x)G(y)K^2(x,y)},\\
g_{\psi\psi}(x,y) &= -\frac{C^2(\nu,\lambda)(x-y)^2R^2(1+y)^2\cosh^4\alpha + R^2F^2(x)G(y)K^2(x,y)}{F(x)F(y)(x-y)^2K^2(x,y)},\\
g_{xx}(x,y) &= \frac{R^2F(x)}{(x-y)^2G(x)}, \qquad g_{\varphi\varphi}(x,y) = \frac{R^2G(x)}{(x-y)^2},
\end{aligned} \tag{31}$$

where at the event horizon $\mathcal{M}(x,y)$, $\mathcal{N}(x,y)$, and $N^\psi(x,y)$ take the values in Eq. (5). Now the metric (29) has the same form as (4). In the spacetime, gauge fields (30) couple to Dirac particles, so we should introduce the following covariant Dirac equation:

$$i\gamma^a e_a^\mu\left(D_\mu + \frac{ie}{\hbar}\mathcal{A}_\mu\right)\Psi - \frac{m}{\hbar}\Psi = 0. \tag{32}$$

Taking the same matrices γ^a and tetrad fields e_a^μ as those in Eqs. (7) and (9) for the black ring, employing the ansatz (10) for the spin-up Dirac particles, and then expanding the resulting equation near the horizon yields

$$B\left(\frac{-\mathcal{E} + \Omega_h\mathcal{J} + e\Phi_h}{\sqrt{\mathcal{M}_{,y}(x,y_h)(y-y_h)}} + \sqrt{\mathcal{N}_{,y}(x,y_h)(y-y_h)}\partial_y\mathcal{W}(x,y)\right) + A\left(m - \frac{\mathcal{L}}{\sqrt{g_{\varphi\varphi}(x,y_h)}}\right) = 0, \tag{33}$$

$$B\left(\frac{1}{\sqrt{g_{xx}(x,y_h)}}\partial_x\mathcal{W}(x,y) + \frac{i}{\sqrt{g_{\psi\psi}(x,y_h)}}(\mathcal{J} + \mathcal{A}_\psi(x,y_h))\right) = 0, \tag{34}$$

$$A\left(\frac{-\mathcal{E}+\Omega_h\mathcal{J}+e\Phi_h}{\sqrt{\mathcal{M}_{,y}(x,y_h)(y-y_h)}}-\sqrt{\mathcal{N}_{,y}(x,y_h)(y-y_h)}\partial_y\mathcal{W}(x,y)\right)-B\left(m+\frac{\mathcal{L}}{\sqrt{g_{\varphi\varphi}(x,y_h)}}\right)=0, \quad (35)$$

$$A\left(\frac{1}{\sqrt{g_{xx}(x,y_h)}}\partial_x\mathcal{W}(x,y)+\frac{i}{\sqrt{g_{\psi\psi}(x,y_h)}}(\mathcal{J}+\mathcal{A}_\psi(x,y_h))\right)=0, \quad (36)$$

where $\Phi_h=\mathcal{A}_t(x,y_h)+\Omega_h\mathcal{A}_\psi(x,y_h)$ is the electric chemical potential at the horizon and Ω_h is the angular velocity at the horizon. Carrying out a similar analysis of the neutral black ring, we easily find the tunneling rate of charged Dirac particles across the horizon of the charged black ring taking the form as

$$\Gamma=\exp\left[-\frac{4\pi}{\sqrt{\mathcal{M}_{,y}(x,y_h)\mathcal{N}_{,y}(x,y_h)}}(\mathcal{E}-\Omega_h\mathcal{J}-e\Phi_h)\right]. \quad (37)$$

The Hawking temperature of the charged black ring is then given by

$$T=\frac{\sqrt{\mathcal{M}_{,y}(x,y_h)\mathcal{N}_{,y}(x,y_h)}}{4\pi}=\frac{1}{4\pi R\cosh^2\alpha}\frac{1+\nu}{\sqrt{\nu}}\sqrt{\frac{1-\lambda}{\lambda(1+\lambda)}}. \quad (38)$$

This result is exactly consistent with the Hawking temperature derived by canceling gauge and gravitational anomalies at the horizon of the charged black ring (see Chen and He's paper in [24]). Here to reduce the higher dimensional theory to the effective 2-dimensional theory, a dimensional reduction technique is carried out by using the scalar field near the horizon of the charged black ring. So, the resulting Hawking temperature is only for scalar particles across the horizon. Now we can also conclude that scalar and Dirac particles can tunnel across the horizon of black rings at the same Hawking temperature.

IV. CONCLUSIONS AND DISCUSSIONS

Hawking radiation of scalar particles across black holes or black rings has been discussed a lot via different methods, such as the recently discussed tunneling method and the anomalous cancellation method, etc. Hawking radiation of Dirac particles across 3- or 4-dimensional black holes has also been presented in recent papers via the fermion tunneling method. In this paper, choosing a set of appropriate matrices γ^μ for the 5-dimensional neutral, dipole, and charged black rings, we successfully recover Hawking temperatures of these black rings via the fermion tunneling method.

The fermion tunneling method has already been successfully applied to derive Hawking radiation of Dirac particles across stationary back holes [16,17,19–21] and black rings (as shown in this paper). For a nonstationary black hole, although [18] has discussed fermion tunneling from Bardeen-Vaidya and cosmological black holes, there is no coupling effect between the spin of Dirac particles and the angular momentum of the black hole in the tunneling rate. This is because the involved nonstationary black holes in [18] are of spherical symmetry and have no angular momentum. So we expect that when Dirac particles are tunneling from nonstationary black holes with one or more angular momentum, the spin coupling effect should be present. This is our next task. In addition, note that choosing a set of appropriate matrices γ^μ is an important technique for the fermion tunneling method; otherwise, we cannot correctly recover the Hawking temperature that we expected. Finally, it is necessary to say that, in Secs. II and III, we only considered the case of Dirac particles with spin-up. In fact, adopting a similar procedure, we will find the same result for Dirac particles with spin-down. This means that both spin-up and spin-down Dirac particles tunnel across the horizon at the same Hawking temperature [20], and such handling does not result in loss of generality.

ACKNOWLEDGMENTS

This work is supported by the National Natural Science Foundation of China through Grant No. 10675051, No. 70571027, No. 10635020; the Ministry of Education of China under Grant No 306022; and by the Graduate Innovation Foundation of Central China Normal University.

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