Generic estimates for magnetic fields generated during inflation including Dirac-Born-Infeld theories

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We estimate the strength of large-scale magnetic fields produced during inflation in the framework of Dirac-Born-Infeld (DBI) theories. This analysis is sufficiently general in the sense that it covers most of conformal symmetry breaking theories in which the electromagnetic field is coupled to a scalar field. In DBI theories there is an additional factor associated with the speed of sound, which allows a possibility to lead to an extra amplification of the magnetic field in a ultrarelativistic region. We clarify the conditions under which seed magnetic fields to feed the galactic dynamo mechanism at a decoupling epoch as well as present magnetic fields on galactic scales are sufficiently generated to satisfy observational bounds.

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I. INTRODUCTION

It is observationally known that there exist magnetic fields in clusters of galaxies with the field strength 10^{-7}–10^{-6} G on 10 kpc − 1 Mpc scales [1] as well as those with the field strength $\sim 10^{-6}$ G on 1–10 kpc scales in galaxies of all types [2] and in galaxies at cosmological distances [3]. In particular, it is very mysterious that magnetic fields in clusters of galaxies are as strong as galactic ones and that the coherence scale may be as large as ∼Mpc. Although galactic dynamo mechanisms [4] have been proposed to amplify very weak seed magnetic fields up to $\sim 10^{-6}$ G, seed magnetic fields to feed on is necessary at the initial stage, and the effectiveness of the dynamo amplification mechanism in galaxies at high redshifts and clusters of galaxies is not well established yet.

Proposed scenarios for the origin of cosmic magnetic fields fall into two broad categories. One is astrophysical processes [5] and the other is cosmological processes, e.g., cosmological phase transition [6] and primordial density perturbations before the epoch of recombination [7]. It is difficult, however, that these processes generate magnetic fields on megaparsec scales with sufficient strength consistent with observations of galaxies and clusters of galaxies without dynamo amplification mechanism.

The most natural origin of such a large-scale magnetic field is electromagnetic quantum fluctuations generated at the inflationary stage [8], because inflation has a causal mechanism to generate super-Hubble gauge fields from microphysical processes. When we assume the Friedmann-Robertson-Walker (FRW) spacetime usually considered, its metric is conformally flat. Moreover, the classical electrodynamics is conformally invariant. Hence, the conformal invariance of the Maxwell theory must have been broken at the inflationary stage in order that electromagnetic quantum fluctuations can be generated at that time [9]. We note that this does not apply when the FRW background has nonzero spatial curvature [10]. (In Refs. [11], the breaking of conformal flatness of the FRW metric induced by the evolution of scalar metric perturbations at the end of inflation was discussed. Moreover, the generation of magnetic fields from grand unified theories was studied in Ref. [12].)

So far various conformal symmetry breaking mechanisms have been proposed. An incomplete list includes nonminimal gravitational coupling [8,13], dilaton electromagnetism [14], coupling to a scalar field [15], that to a pseudoscalar field [16], that to a charged scalar field [17], scalar electrodynamics [18], general coupling to a time-dependent background field [19,20], the photon-graviphoton mixing [21], conformal anomaly induced by quantum effects [22], spontaneous breaking of the Lorentz invariance [23] (see also [24]), the generation of the mass of the gauge field due to a minimally supersymmetric standard model flat direction condensate [25], the photon mass generation due to the existence of the minimal fundamental scale [26], nonlinear electrodynamics [27], and cosmic defects [28].

In addition, as a breaking scenario based on the fundamental theory of particle physics, there exists a scenario in the framework of the Dirac-Born-Infeld (DBI) theory, which is a four-dimensional low-energy effective theory of string theories [29–31]. In this paper we shall derive the equation of electromagnetic fields for such theory and estimate the strength of magnetic fields generated during inflation. As we will see later, this analysis also covers theories that possess electromagnetic couplings of the form $I(\phi, R)F_{\mu\nu}F^{\mu\nu}$ [19], where I is an arbitrary function of a scalar field ϕ or a Ricci scalar R. Thus the strength of magnetic fields we will derive in this paper is applicable to many conformal symmetry violating models. In fact we shall apply our formula to several concrete models of inflation.

This paper is organized as follows. In Sec. II we consider the evolution of the $U(1)$ gauge field and derive the general formula for the field strength of the large-scale magnetic fields. We apply the derived formula to several inflation models in Sec. III. Finally, Sec. IV is devoted to conclusions.

We use units in which $k_B = c = \hbar = 1$, and adopt Heaviside-Lorentz units in terms of electromagnetism.

II. GENERATION OF MAGNETIC FIELDS

Let us start with the following four-dimensional action

$$S = -\int d^4x f_1(\phi) \times \sqrt{-\det(g_{\mu\nu} + f_2(\phi)\partial_\mu\phi\partial_\nu\phi + f_3(\phi)F_{\mu\nu})} + \tilde{S}(\phi, R, g_{\mu\nu}), \quad (1)$$

where $f_1(\phi)$, $f_2(\phi)$, $f_3(\phi)$ are the functions of ϕ, $g_{\mu\nu}$ is the metric tensor, and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the electromagnetic field-strength tensor. The action $\tilde{S}$ depends on ϕ, R and $g_{\mu\nu}$ but not on $F_{\mu\nu}$. The DBI scenario proposed in Ref. [31] corresponds to $f_1(\phi) = 1/f(\phi) = \phi^4/\lambda$, $f_2(\phi) = f(\phi)$, and $f_3(\phi) = \sqrt{f(\phi)}$ for the anti–de Sitter (AdS) throat. The rolling tachyon scenario [30] corresponds to $f_1(\phi) = V(\phi)$, $f_2(\phi) = 1$, and $f_3(\phi) = 2\pi/M_s^2$, where M_s is the string mass scale.

When the action (1) is varied with respect to the $U(1)$ gauge field A_μ, we neglect those terms whose orders are higher than $F_{\mu\nu}F^{\mu\nu}$. We then obtain

$$\partial_\mu\left(\frac{f_1(\phi)f_3^2(\phi)}{\sqrt{-G}} G F^{\mu\nu}\right) = 0, \quad (2)$$

where $G = \det(G_{\mu\nu})$, $G_{\mu\nu} = g_{\mu\nu} + f_2(\phi)\partial_\mu\phi\partial_\nu\phi$, and $F^{\mu\nu} = G^{\mu\alpha}G^{\nu\beta}F_{\alpha\beta}$. Let us consider the flat FRW spacetime with scale factor $a(t)$, where t is a cosmic time. For the Coulomb gauge, $\partial^j A_j(t, \boldsymbol{x}) = 0$ and $A_0(t, \boldsymbol{x}) = 0$, the equation of motion for A_i is given by

$$\ddot{A}_i(t, \boldsymbol{x}) + \frac{\dot{\mathcal{F}}}{\mathcal{F}}\dot{A}_i(t, \boldsymbol{x}) - \frac{1}{\gamma^2}\frac{1}{a^2}\overset{(3)}{\Delta}A_i(t, \boldsymbol{x}) = 0, \quad (3)$$

where a dot represents a derivative with respect to t and

$$\mathcal{F} \equiv f_1 f_3^2 a\gamma, \qquad \gamma \equiv [1 - f_2(\phi)\dot{\phi}^2]^{-1/2}. \quad (4)$$

One can expand the gauge field A_i by using annihilation and creation operators together with two orthonormal transverse polarization vectors [19]. Then the Fourier mode $A(\eta, k)$, with a conformal time $\eta = \int a^{-1}dt$ and a comoving wave number k, satisfies the following equation of motion:

$$\frac{d^2}{d\eta^2}A(\eta, k) + \frac{1}{J}\frac{dJ}{d\eta}\frac{d}{d\eta}A(\eta, k) + \frac{k^2}{\gamma^2}A(\eta, k) = 0, \quad (5)$$

where $J = f_1 f_3^2 \gamma$. Introducing another time $\tau = \int \gamma^{-1}d\eta$, Eq. (5) reduces to

$$A''(\tau, k) + \frac{I'}{I}A'(\tau, k) + k^2 A(\tau, k) = 0, \quad (6)$$

where a prime represents a derivative with respect to τ and

$$I = f_1 f_3^2. \quad (7)$$

If we consider conformal symmetry violating Maxwell theories with the action

$$S = -\int d^4x\sqrt{-g}\left[\frac{1}{4}I(\phi, R)F_{\mu\nu}F^{\mu\nu} + \mathcal{L}(\phi, R, g_{\mu\nu})\right], \quad (8)$$

we get the same form of equation as (6) apart from the fact that τ is replaced by the conformal time η [19].

The Hubble parameter, $H = \dot{a}/a$, needs to satisfy the condition $|\dot{H}/H^2| \ll 1$ during inflation. Then we have $\tau \simeq -(\gamma a H)^{-1}$ under the condition $|\dot{\gamma}/H\gamma| \ll 1$. The modes starting from the "sub-Hubble" regime ($k \gg \gamma a H$) enter the "super-Hubble" regime ($k \ll \gamma a H$) at a time τ_k characterized by the condition $\tau_k \simeq -1/k$.

The WKB sub-Hubble solution to Eq. (6) is $A_{in} = e^{-ik\tau}/\sqrt{2kI}$, which approaches the Minkowski vacuum state in the limit $\tau \to -\infty$. Meanwhile the super-Hubble solution neglecting correction terms of the order k^2 is given by $A_{out} = C(k) + D(k)\int_\tau^{\tau_R} d\tilde{\tau}/I(\tilde{\tau})$, where $C(k)$ and $D(k)$ are constants and τ_R corresponds to the time at reheating. Matching these two solutions at time $\tau = \tau_k$ using the junction conditions $A_{out}(\tau_k) = A_{in}(\tau_k)$ and $A'_{out}(\tau_k) = A'_{in}(\tau_k)$, the coefficients $C(k)$ and $D(k)$ are determined accordingly. Neglecting the decaying mode for A_{out}, we get the late-time solution $|A(\tau, k)|^2 = |C(k)|^2$ at the end of inflation:

$$|A(\tau, k)|^2 = \frac{1}{2kI(\tau_k)} \times \left|1 - \left(\frac{I'(\tau_k)}{2kI(\tau_k)} + i\right)k\int_{\tau_k}^{\tau_R}\frac{I(\tau_k)}{I(\tilde{\tau})}d\tilde{\tau}\right|^2. \quad (9)$$

In the following we assume that the energy density of the field ϕ is converted to radiation almost instantly right after the end of inflation and that the conductivity σ_c of the Universe jumps to a value much larger than the Hubble rate at reheating. Then the proper magnetic field, $B_i^{proper}(t, \boldsymbol{x}) = a^{-2}\epsilon_{ij\ell}\partial_j A_\ell(t, \boldsymbol{x})$, evolves as $B_i^{proper}(t, \boldsymbol{x}) \propto a^{-2}$ in the reheating and subsequent radiation/matter/dark energy dominated stages. Taking into account two polarization degrees of freedom, the spectrum of the magnetic field is given by

$$|B^{proper}(\tau, k)|^2 = 2\frac{k^2}{a^4}|A(\tau, k)|^2. \quad (10)$$

The energy density of the magnetic field per unit logarithmic interval of k is defined by

$$\rho_B(\tau, k) \equiv \frac{1}{2}\frac{4\pi k^3}{(2\pi)^3}|B^{\rm proper}(\tau, k)|^2 I(\tau). \quad (11)$$

Since the radiation density evolves as $\rho_\gamma(\tau) = \rho_\gamma(\tau_R) \times (a_R/a)^4$, it is convenient to introduce the density parameter $\Omega_B(\tau, k) = \rho_B(\tau, k)/\rho_\gamma(\tau)$. From Eqs. (9)–(11) we obtain

$$\Omega_B(\tau, k) = \frac{15}{2\pi^4 N_{\rm eff}}\left(\frac{k}{a_R T_R}\right)^4 \frac{I(\tau)}{I(\tau_k)} \times \left|1 - \left(\frac{I'(\tau_k)}{2kI(\tau_k)} + i\right)k\int_{\tau_k}^{\tau_R}\frac{I(\tau_k)}{I(\tilde\tau)}d\tilde\tau\right|^2. \quad (12)$$

Here we used $\rho_\gamma(\tau_R) = \pi^2 N_{\rm eff} T_R^4/30$, where $N_{\rm eff}$ is the effective massless degree of freedom and T_R is the reheating temperature.

In order to estimate the strength of magnetic fields, let us consider the case in which the evolution of the quantity I during inflation is given by

$$I = I_*(\tau/\tau_*)^{-\alpha}, \quad (13)$$

where I_*, τ_*, and α are constants. This choice is made to get a quantitative estimate of the generated magnetic field and is general enough to cover many models including those discussed in the following section. On using the relations $\tau_R \simeq -(\gamma_R a_R H_R)^{-1}$ and $3H_R^2 \simeq \rho_\gamma(\tau_R)/M_{\rm pl}^2$ (where $M_{\rm pl}$ is a reduced Planck mass), we get

$$\Omega_B(\tau, k) = \mathcal{C}\frac{N_{\rm eff}}{1080}\left(\frac{T_R}{M_{\rm pl}}\right)^4\left(\frac{k}{a_R H_R}\right)^{4-\alpha}\frac{I(\tau)}{I(\tau_R)}\gamma_R^\alpha, \quad (14)$$

where $\mathcal{C} = |1 - \frac{\alpha+2i}{2(\alpha+1)}|^2$. Hence the spectral index of the magnetic field is given by

$$n_B = 4 - \alpha. \quad (15)$$

For larger positive α it is possible to generate large-scale magnetic fields. Note that the reheating temperature generally has an upper bound from the cosmic microwave background (CMB) observations ($T_R \lesssim 10^{15}$ GeV). Because of the presence of the γ factor there is an extra amplification of the magnetic field for $\gamma_R \gg 1$ and $\alpha > 0$.

Let us first estimate the quantity $k/a_R H_R$ for the scale $L = 2\pi/k$ [Mpc]. Using the present value $H_0^{-1} = 3.0 \times 10^3 h^{-1}$ Mpc and the relation $a_0/a_R = T_R/T_0$ we have $k/a_R H_R \simeq (1.88/h)(10^4\ {\rm Mpc}/L)(T_R/T_0)(H_0/H_R)$. Since $H_R^2 \simeq \pi^2 N_{\rm eff} T_R^4/90M_{\rm pl}^2$, $T_0 = 2.73$ K, and $H_0 = 2.47h \times 10^{-29}$ K, we find

$$\frac{k}{a_R H_R} = 5.1 \times 10^{-25}\frac{1}{\sqrt{N_{\rm eff}}}\frac{M_{\rm pl}}{T_R}\frac{1}{L/{\rm Mpc}}. \quad (16)$$

The energy density $\rho_B(\tau_0)$ at the present epoch is given by $\rho_B(\tau_0) = (1/2)|B(\tau_0)|^2 = \Omega_B(\tau_0, k)\rho_\gamma(\tau_0)$, where $B(\tau_0)$ is an observed magnetic field. Since $\rho_\gamma(\tau_0) \simeq 2 \times 10^{-51}$ GeV^4 and 1 G $= 1.95 \times 10^{-20}$ GeV^2, we obtain

$$|B(\tau_0)| = 2.7 \times 10^{-56+25\alpha/2}\cdot\left[\mathcal{C}\frac{I(\tau_0)}{I(\tau_R)}\right]^{1/2} N_{\rm eff}^{\alpha/4-1/2} \times \left(\frac{1}{5.1}\frac{T_R}{M_{\rm pl}}\gamma_R\right)^{\alpha/2}\left(\frac{L}{\rm Mpc}\right)^{\alpha/2-2}\ {\rm G}. \quad (17)$$

If we take the maximum reheating temperature $T_R \simeq 10^{15}$ GeV $= 4 \times 10^{-4} M_{\rm pl}$ with $N_{\rm eff} = 100$, one can estimate the order of the present magnetic field to be

$$|B(\tau_0)| \simeq 10^{11\alpha-57}\left[\mathcal{C}\frac{I(\tau_0)}{I(\tau_R)}\right]^{1/2}\gamma_R^{\alpha/2}\left(\frac{L}{\rm Mpc}\right)^{\alpha/2-2}{\rm G}. \quad (18)$$

We must have $|B(\tau_0)| \gtrsim 10^{-9}$ GeV to explain observed magnetic fields on the scales 1 kpc − 1 Mpc without the mechanism of galactic dynamo.

At the decoupling epoch with $z = 1000$, the radiation energy density is given by $\rho_\gamma(\tau_{\rm dec}) \simeq 10^{12}\rho_\gamma(\tau_0)$. Then the magnetic field strength at this epoch is given by

$$|B(\tau_{\rm dec})| = 2.7 \times 10^{-50+25\alpha/2}\cdot\left[\mathcal{C}\frac{I(\tau_{\rm dec})}{I(\tau_R)}\right]^{1/2} N_{\rm eff}^{\alpha/4-1/2} \times \left(\frac{1}{5.1}\frac{T_R}{M_{\rm pl}}\gamma_R\right)^{\alpha/2}\left(\frac{L}{\rm Mpc}\right)^{\alpha/2-2}\ {\rm G}. \quad (19)$$

When $T_R \simeq 10^{15}$ GeV and $N_{\rm eff} = 100$, the order of $|B(\tau_{\rm dec})|$ is

$$|B(\tau_{\rm dec})| \simeq 10^{11\alpha-51}\left[\mathcal{C}\frac{I(\tau_{\rm dec})}{I(\tau_R)}\right]^{1/2}\gamma_R^{\alpha/2}\left(\frac{L}{\rm Mpc}\right)^{\alpha/2-2}\ {\rm G}. \quad (20)$$

The seed field with an amplitude $|B(\tau_{\rm dec})| \gtrsim 10^{-23}$ G is required to explain the present size of the magnetic field through the galactic dynamo mechanism for a flat universe without cosmological constant. However, this limit is relaxed up to $|B(\tau_{\rm dec})| \gtrsim 10^{-30}$ G on ~kpc scale in the presence of cosmological constant at late times [32].

We would like to stress here that the above results are valid even for the theories with the action (8) by setting $\gamma = 1$.

III. APPLICATION TO CONCRETE MODELS

We shall apply the formula derived in the previous section to several conformal symmetry breaking models. We adopt the reheating temperature $T_R = 10^{15}$ GeV to estimate the maximum allowed size of magnetic fields. Note that the factor $\mathcal{C}$ in Eqs. (17)–(20) is of the order of unity.

A. Power-law inflation with $\gamma = 1$

Let us consider the dilatonic coupling $I(\phi) = e^{\lambda\phi}$ and the Lagrangian $\mathcal{L} = (1/2)(\nabla\phi)^2 + V(\phi)$ in Eq. (8). This corresponds to the case $\gamma = 1$, i.e., $\tau = \eta$. If the potential is given by $V(\phi) = V_0\exp(-\sqrt{2/p}\phi)$, where ϕ is normalized by $M_{\rm pl}$, power-law inflation with $a \propto t^p$ ($p > 1$) is realized. Since the field evolves as $\phi = \phi_0 + \sqrt{2p}\ln(t)$, the coupling I has a time dependence $I \propto t^{\lambda\sqrt{2p}} \propto (-\eta)^{-\alpha}$, where

$$\alpha = \lambda \frac{\sqrt{2p}}{p-1}. \tag{21}$$

We shall study the case in which the field ϕ is frozen right after the end of inflation due to the appearance of a potential minimum. We then have $I(\tau_R) = I(\tau_0) = I(\tau_{\rm dec})$ in Eqs. (18) and (20). In order to get the present size of magnetic fields $[|B(\tau_0)| \gtrsim 10^{-9}$ G] on the scale $L = 1$ Mpc without the mechanism of galactic dynamo, we must have $\alpha > 4.4$. To explain the origin of seed magnetic fields $|B(\tau_{\rm dec})| > 10^{-30}$ G on the scale $L = 1$ Mpc at the decoupling epoch, we need $\alpha > 1.9$. This condition is relaxed to $\alpha > 1.6$ for the magnetic fields on the scale $L = 1$ kpc.

The recent Wilkinson Microwave Anisotropy Probe (WMAP) data of density perturbations constrains the power p to be $p > 70$ at the 95% confidence level [33]. We then find that the parameter λ must satisfy at least the relation, $\lambda > 9.4$, from the condition $\alpha > 1.6$.

B. Tachyon inflation

The rolling tachyon scenario [30,34–36] corresponds to the choice $f_1(\phi) = V(\phi)$, $f_2(\phi) = 1$, and $f_3(\phi) = 2\pi/M_s^2$, where M_s is the string mass scale. We then have $I(\phi) = 4\pi^2 V(\phi)/M_s^4$, which decreases during inflation.

Consider the inverse power-law potential $V(\phi) = V_0\phi^{-2}$ with $V_0 = 4p(1 - 2/3p)^{1/2}M_{\rm pl}^2$. This leads to the power-law expansion $a \propto t^p$ $(p \gg 1)$ with $\phi = \sqrt{2/3p}\,t$ [35–37], in which case γ is a constant $[\gamma = (1 - 2/3p)^{-1/2} \simeq 1]$. Hence one has $I \propto t^{-2} \propto (-\tau)^{2/(p-1)}$, thereby giving

$$\alpha = \frac{2}{1-p} < 0. \tag{22}$$

This shows that the spectral index n_B given in Eq. (15) is highly blue-tilted. Hence it is difficult to generate sufficient amounts of large-scale magnetic fields. Moreover the quantity $I(\phi)$ ($\propto V(\phi)$) decreases toward zero after inflation for the standard tachyon models in which the field rolls down toward infinity. In tachyon inflation the magnetic field at the present epoch is vanishingly small.

C. DBI inflation

The DBI inflation for the AdS throat corresponds to the choice $f_1(\phi) = 1/f(\phi) = \phi^4/\lambda$, $f_2(\phi) = f(\phi)$, and $f_3(\phi) = \sqrt{f(\phi)}$ [31]. In this case one has $I(\phi) = 1$, which means that the generation of the magnetic field does not occur unlike the results found in Ref. [38]. Since the coupling $f_3(\phi)$ given above is chosen to reproduce the standard Maxwell Lagrangian in the low-energy regime $(f\dot{\phi}^2 \ll 1)$, the field ϕ is not directly coupled to the electromagnetic field.

One may consider a scenario in which the coupling $f_3(\phi)$ takes a different form in the ultrarelativistic regime $(\gamma = [1 - f\dot{\phi}^2]^{-1/2} \gg 1)$. For example, let us study the case

$$f_3(\phi) \propto \phi^{-n}, \quad \text{i.e.,} \quad I \propto \phi^{4-2n}. \tag{23}$$

For the potential $V(\phi) = (1/2)m^2\phi^2$, inflationary solutions in the regime $\gamma \gg 1$ are given by $\phi = \sqrt{\lambda}/t$, $\gamma \simeq mM_{\rm pl}\sqrt{2\lambda/3}/\phi^2 \propto t^2$, and $a \propto t^p$, where $p = \sqrt{\lambda/6}(m/M_{\rm pl})$ [31,39]. Since t has a dependence $t \propto (-\tau)^{-1/(p+1)}$ in this case we get $I \propto (-\tau)^{-\alpha}$ with

$$\alpha = \frac{2n-4}{p+1}. \tag{24}$$

For $\gamma \gg 1$ and $\alpha > 0$, the magnetic field can be more significantly amplified relative to the case $\gamma = 1$ because a mode with the wave number k crosses the point $k = \gamma aH$ earlier for larger γ. In the ultrarelativistic regime of the DBI inflation the non-Gaussian parameter $f_{\rm nl}$ in CMB observations is given by $f_{\rm nl} = \frac{35}{108}(\gamma^2 - 1)$ [40]. Using the latest WMAP bound $|f_{\rm nl}| < 253$ based on the equilateral models [33], we obtain the constraint $\gamma_{\rm CMB} < 28$ on the scale relevant to 7CMB anisotropies. Since γ grows as $\gamma \propto a^{2/p}$ during inflation, one can estimate the value γ_R to be $\gamma_R = \gamma_{\rm CMB}e^{2N/p}$, where N is the number of e-foldings from the epoch at which CMB fluctuations are generated to the end of inflation ($N = 50 \sim 60$). In the following we adopt the value $N = 55$ for concreteness.

Let us consider the case in which the field ϕ is frozen right after the end of inflation so that $I(\tau_R)$ is the same order as $I(\tau_{\rm dec})$ and $I(\tau_0)$. On using Eq. (18), we find that the present magnetic field greater than the order of 10^{-9} G can be obtained for

$$n > 2 + \frac{2p(p+1)[24 + \log_{10}(L/\text{Mpc})]}{48 + p[22 + \log_{10}(\gamma_{\rm CMB}\cdot L/\text{Mpc})]}. \tag{25}$$

From Eq. (20) the condition to get the seed magnetic field larger than the order of 10^{-30} G is given by

$$n > 2 + \frac{p(p+1)[21 + 2\log_{10}(L/\text{Mpc})]}{48 + p[22 + \log_{10}(\gamma_{\rm CMB}\cdot L/\text{Mpc})]}. \tag{26}$$

In the relativistic regime of DBI inflation the tensor-to-scalar ratio in CMB anisotropies is given by $r \simeq 16\epsilon/\gamma = (48/\lambda)(M_{\rm pl}/m)^2(\phi/M_{\rm pl})^2$ (where $\epsilon = -\dot{H}/H^2$ is the slow-roll parameter). Using the latest WMAP bound $r < 0.2$ [33] together with the non-Gaussianity bound $\gamma = mM_{\rm pl}\sqrt{2\lambda/3}/\phi^2 < 28$, we find that $\phi_{\rm CMB}$ is bounded from both above and below. For the consistency of this inequality, we must require that $\lambda(m/M_{\rm pl})^2 > 49$, i.e., $p > 2.9$.

If we adopt the values $L = 1$ Mpc, $\gamma_{\rm CMB} = 28$, and $p = 3$ in Eqs. (25) and (26), then we get the bounds $n > 6.9$ and $n > 4.1$, respectively. The constraint on n is weakened for smaller scales. For example, when $L = 1$ kpc, $\gamma_{\rm CMB} = 28$, and $p = 3$, Eq. (26) gives the bound $n > 3.6$. Meanwhile the constraint on n tends to be tighter for larger p. Since $\gamma_{\rm CMB}$ is bounded from above ($\gamma_{\rm CMB} < 28$), one cannot choose arbitrary large values of $\gamma_{\rm CMB}$ to make the right-hand side of Eqs. (25) and (26) smaller.

IV. CONCLUSIONS

In the present paper, we have studied the generation of large-scale magnetic fields due to the breaking of the conformal invariance of the electromagnetic field through its coupling to a scalar field in the framework of DBI theory. Introducing a time $\tau = \int \gamma^{-1} d\eta$, the Fourier component of the gauge field satisfies the equation of motion (6). This is the same form of equation derived for the electromagnetic coupling given in Eq. (8) apart from the difference that τ is replaced by conformal time η for the action (8). Hence our analysis is applicable to many conformal symmetry breaking Maxwell theories.

By matching two solutions in "sub-Hubble" ($k \gg \gamma aH$) and "super-Hubble" ($k \ll \gamma aH$) regimes during the inflationary epoch, the strength of the magnetic field at the end of inflation can be estimated as Eq. (9). Under the assumptions that the energy density of inflaton is almost instantly converted to radiation after inflation and that the conductivity during reheating is much higher than the Hubble rate at that epoch, we derived the size of the magnetic field both at the present and at the decoupling epoch. Note that we have not assumed any other mechanisms for the amplification of the magnetic field. The results (17) and (19) are sufficiently general to cover the theories described by the action (8).

We applied our formula for three cases: (i) power-law inflation with $\gamma = 1$, (ii) tachyon inflation, and (iii) DBI inflation. The power α defined in Eq. (13) that characterizes the evolution of the quantity I during inflation is important to determine the spectral index of the magnetic field. For the theories with $\gamma = 1$, it should be generally required that the spectrum is red-tilted ($\alpha > 4$) to realize the present field strength $|B(\tau_0)|$ larger than 10^{-9} G on the scales 1 kpc $-$ 1 Mpc. The constraint on α is not so severe to obtain seed magnetic fields to feed the galactic dynamo mechanism [$|B(\tau_{\rm dec})| > 10^{-30}$ G]. In power-law inflation, for example, we found that the constant λ for the electromagnetic coupling $I(\phi) = e^{\lambda\phi}$ is constrained to be $\lambda > 9.4$ to satisfy the condition required for the seed field on the scale $L = 1$ kpc ($\alpha > 1.6$).

In the theories with $\gamma \neq 1$ there exists an extra factor $\gamma_R^{\alpha/2}$ that can lead to additional amplification of the magnetic field. In tachyon inflation, in addition to the fact that γ_R is very close to 1, the quantity $I(\phi)$ is proportional to the field potential $V(\phi)$, which decreases during inflation (i.e., $\alpha < 0$). Hence we cannot expect the generation of large-scale magnetic fields consistent with observations.

In DBI inflation, if we wish to reproduce the standard Maxwell theory in low-energy regimes, we have $f_1(\phi) = 1/f(\phi) = \phi^4/\lambda$ and $f_3(\phi) = \sqrt{f(\phi)}$ in the action (1). This corresponds to the effective coupling with $I(\phi) = 1$, which means that the generation of magnetic fields cannot be expected. This situation changes if we allow the possibility that the coupling $f_3(\phi)$ takes a different form in the ultrarelativistic regime ($\gamma \gg 1$). We adopted the coupling of the form $f_3(\phi) \propto \phi^{-n}$ and derived the bounds (25) and (26) to get observationally required magnetic fields at the present and at the decoupling epoch. It is worth mentioning that the presence of the $\gamma_R^{\alpha/2}$ factor leads to the larger magnetic field relative to the theories with $\gamma = 1$.

It will be certainly of interest to apply our formula to many other conformal symmetry breaking models. While we have assumed instant reheating with large conductivity, the details of the reheating process actually depends upon models of inflation. It is generally difficult to construct string/brane inflation models with successful reheating, so we need to wait for the construction of such viable models to carry out detailed analysis for the dynamics of magnetic fields in the reheating phase.

Finally, we remark interesting cosmological effects of large-scale magnetic fields generated during inflation on the CMB radiation. In Ref. [41], the effect of gravity waves induced by a possible helicity-component of a primordial magnetic field on CMB temperature anisotropies and polarization has been considered. According to it, the effect could be sufficiently large to be observable if the spectrum of the primordial magnetic field is close to scale-invariant and if its helical component is stronger than $\sim 10^{-10}$ G. In Ref. [41], only the tensor mode, whose contribution is significant for low multipoles ($l < 100$), has been considered, while the vector mode has an imprint for higher multipoles too [42]. Thus, the tensor mode alone cannot significantly limit the magnetic field amplitude. According to Ref. [41], the amplitude of the helical magnetic field (and not the helical component) must be larger than a few $\times 10^{-9}$ G to be detectable by current CMB measurements. Similar bounds have been derived in Ref. [43]. However, the future missions, for example, PLANCK, will be able to test the cosmological magnetic field with an amplitude 10^{-10} G or even lower [44]. The current (best) limit on the amplitude of the magnetic field from the CMB polarization Faraday rotation effect using WMAP 5-year data is around 5×10^{-10} G [45] for the magnetic field generated from inflation.

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