Phenomenological analysis of quantum collapse as source of the seeds of cosmic structure

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The standard inflationary version of the origin of the cosmic structure as the result of the quantum fluctuations during the early universe is less than fully satisfactory as has been argued in [A. Perez, H. Sahlmann, and D. Sudarsky, Classical Quantum Gravity **23**, 2317 (2006).]. A proposal is made there of a way to address the shortcomings by invoking a process similar to the collapse of the quantum-mechanical wave function of the various modes of the inflaton field. This in turn was inspired by the ideas of R. Penrose about the role that quantum gravity might play in bringing about such a breakdown of the standard unitary evolution of quantum mechanics. In this paper we study in some detail the two schemes of collapse considered in the original work together with an alternative scheme, which can be considered as "more natural" than the former two. The new scheme assumes that the collapse follows the correlations indicated in the Wigner functional of the initial state. We end with considerations regarding the degree to which the various schemes can be expected to produce a spectrum that resembles the observed one.

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I. INTRODUCTION

In recent years, there have been spectacular advances in physical cosmology, resulting from a remarkable increase in the accuracy of the observational techniques and exemplified by the Supernova Surveys [1], the studies of large scale structure [2], and the highly accurate observations from various recent studies, in particular, those of the Wilkinson Microwave Anisotropy Probe (WMAP) [3]. These observations have strengthened the theoretical status of the inflationary scenarios among cosmologists.

We should note, however, that while much of the focus of the research in inflation has been directed towards the elucidation of the exact form of the inflationary model (i.e., the number of fields, the form of the potential, and the occurrence of nonminimal couplings to gravity to name a few), much less attention has been given to the questions of principle, how the initial conditions are determined, what accounts for the low entropy of the initial state, and how exactly does the Universe transit from a homogeneous and isotropic stage to one where the quantum uncertainties become actual inhomogeneous fluctuations. There are of course several works in which this issues are addressed [4,5], but as explained in [6–8] the fully satisfactory account of the last of them seems to require something beyond the current understanding of the laws of physics. The point is that the predictions of inflation in this regard cannot be fully justified in any known and satisfactory interpretational scheme for quantum physics. The Copenhagen interpretation, for instance, is inapplicable in that case, due to the fact that we, the observers, are part of the system, and to make things even worse we are in fact part of the outcome of the process we wish to understand, galaxies, stars, planets, and living creatures being impossible in a homogeneous and isotropic universe [9]. The arguments and counterarguments that have arisen in regard to this aspect of the article mentioned above have been discussed in various other places by now, and we point the reader who is interested in that debate to that literature [8,10]. In the present work we will focus on a more detailed study of the collapse schemes and on the traces they might leave on the observational data. Nevertheless, and in order to make the article self-contained, we will briefly review the motivation and line of approach described in detail in [6].

To clarify where the problem lies, and the way in which it is addressed in [6], we will review in a nutshell the standard explanation of the origin of the seeds of cosmic structure in the inflationary paradigm:

(i) One starts with an homogeneous and isotropic space-time [11]. The inflaton field is the dominant matter in this space-time, and it is in its vacuum quantum state, which is *homogeneous and isotropic too*. The field is in fact described in terms of its expectation value represented as a scalar field which depends only on cosmic time but not on the spatial coordinates, ϕ_0 and a quantum or "fluctuating part" $\delta\phi$ which is in the adiabatic vacuum state, which is an homogeneous and isotropic state (something that can be easily verified by applying the generators of rotations or translations to the state).

(ii) The quantum fluctuations of the inflaton act as perturbations [12] of the inflaton field and through the Einstein field equations (EFE) as perturbations of the metric.

(iii) As inflation continues the physical wavelength of the various modes of the inflaton field becomes

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larger than the Hubble radius (horizon-crossing as referred to commonly in the literature), and the quantum amplitudes of the modes freeze. At that moment one starts regarding such modes as actual waves in a classical field. Later on, after inflation ends, and as the Hubble radius grows, the fluctuations "reenter the horizon," transforming at that point into the seeds of the cosmic structure.

The last step is usually referred to as the quantum to classical transition. There are of course several schools of thought about the way one must consider such a transition: from those using the established physical paradigms [4,5], to views advocating a certain generalization of the standard formalisms [13]. The two works [4,13] focus concretely on a full-blown quantum cosmology, and its interpretational problems, which are even more severe than the ones we are dealing with here. In [6] it was argued that such schemes are insufficient, in particular, if one expects cosmology to provide a time evolution account starting from the totally symmetric state to an inhomogeneous and anisotropic universe in which creatures such as humans might eventually arise.

The view taken in [6] (and in this work) intends to be faithful to the notion that physics is always quantum mechanical, and that the only role for a classical description is that of an approximation where the uncertainties in the state of the system are negligibly small and one can take the expectation values as a fair description of the aspects of the state one is interested on. However one must keep in mind that behind any classical approximation there should always lie a full quantum description, and thus one should reject any scheme in which the classical description of the Universe is inhomogeneous and anisotropic but in which the quantum-mechanical description persists in associating to the Universe an homogeneous and isotropic state at all times. Thus in [6], one introduces a new ingredient to the inflationary account of the origin of the seeds of cosmic structure: *the self-induced collapse hypothesis*. That is, one considers a specific scheme by which a self-induced collapse of the wave function is taken as the mechanism by which inhomogeneities and anisotropies arise in each particular scale. This work was inspired by early ideas by Penrose [14], which regard the collapse of the wave function as an actual physical process (instead of just an artifact of our description of physics) and which is assumed to be caused somehow by quantum aspects of gravitation. We will not recapitulate the motivations and discussion of the original proposal and instead refer the reader to the abovementioned works.

The way we treat the transition of our system from a state that is homogeneous and isotropic to one that is not is to assume that at a certain cosmic time, something induces a jump in a state describing a particular mode of the quantum field, in a manner that would be similar to the standard quantum-mechanical collapse of the wave function associated with a measurement, but with the difference that in our scheme no external measuring device or observer is called upon as "triggering" that jump. (It is worth recalling that nothing of that sort exists, in the situation at hand, to play such role.)

The main aim of this article is to compare the results that emerge from the collapse schemes considered in [6] with an alternative scheme of collapse that can be said to be more natural than the previous two. In this new scheme [15] we take into account the correlations in the quantum state of the system before the collapse for the values of field and conjugate momentum variables as indicated by the Wigner functional analysis of the precollapse state.

This article is organized as follows: In Sec. II we review the formalism used in analyzing the collapse process. Section III reviews how to obtain the wave function for the field from its Fock space description, which is then used in evaluating the Wigner function for the state, and the state that results after the collapse. Section IV describes the details of the spectrum of cosmic fluctuations, resulting from such collapse, and finally, in Sec. V we discuss these results and those of other collapse schemes *vis-à-vis* the empirical data.

II. THE FORMALISM

The starting point is the action of a scalar field with minimal coupling to the gravity sector:

$$S[\phi, g_{ab}] = \int d^4x\sqrt{-g}\left(\frac{1}{16\pi G}R[g_{ab}] - \frac{1}{2}\nabla_a\phi\nabla_b\phi g^{ab} - V(\phi)\right). \tag{1}$$

One splits the corresponding fields into their homogeneous ("background") part and the perturbations ("fluctuation"), so the metric and the scalar field are written as $g = g_0 + \delta g$ and $\phi = \phi_0 + \delta\phi$. With the appropriate choice of gauge (conformal Newton gauge) and ignoring the vector and tensor part of the metric perturbations, the space-time metric is described by

$$ds^2 = a(\eta)^2[-(1 + 2\Psi)d\eta^2 + (1 - 2\Psi)\delta_{ij}dx^i dx^j], \tag{2}$$

where Ψ is called the *Newtonian potential*. One then considers the EFEs to zeroth and first order. The zeroth order gives rise to the standard solutions in the inflationary stage, where $a(\eta) = -\frac{1}{H_I\eta}$, with $H_I^2 \simeq (8\pi/3)GV$ with the scalar potential, ϕ_0 in slow-regime so $\phi_0' \simeq -\frac{1}{3H_I}\frac{dV}{d\phi}$; and the first-order EFEs reduce to an equation relating the gravitational perturbation and the perturbation of the field

$$\nabla^2\Psi = 4\pi G\phi_0'\delta\phi' \equiv s\delta\phi', \tag{3}$$

with $s \equiv 4\pi G\phi_0'$. The next step involves quantizing the fluctuating part of the inflaton field. In fact it is convenient to work with the rescaled field $y = a\delta\phi$. In order to avoid infrared problems we consider restriction of the system to a box of side L, where we impose, as usual, periodic boundary conditions. We thus write the fields as

$$\hat{y}(\eta, \vec{x}) = \frac{1}{L^3}\sum_k e^{i\vec{k}\cdot\vec{x}}\hat{y}_k(\eta),$$
$$\hat{\pi}(\eta, \vec{x}) = \frac{1}{L^3}\sum_k e^{i\vec{k}\cdot\vec{x}}\hat{\pi}_k(\eta), \tag{4}$$

where $\hat{\pi}_k$ is the canonical momentum of the scaled field. The wave vectors satisfy $k_i L = 2\pi n_i$ with $i = 1, 2, 3$, and $\hat{y}_k(\eta) \equiv y_k(\eta)\hat{a}_k + \bar{y}_k(\eta)\hat{a}_k^\dagger$, and $\hat{\pi}_k(\eta) \equiv g_k(\eta)\hat{a}_k + \bar{\pi}_k(\eta)\hat{a}_k^\dagger$. The functions $y_k(\eta)$, $g_k(\eta)$ reflect our election of the vacuum state, the so-called Bunch-Davies vacuum:

$$y_k(\eta) = \frac{1}{\sqrt{2k}}\left(1 - \frac{i}{\eta k}\right)e^{-ik\eta}, \qquad g_k(\eta) = -i\sqrt{\frac{k}{2}}e^{-ik\eta}. \tag{5}$$

The vacuum state is defined by the condition $\hat{a}_k|0\rangle = 0$ for all k, and is homogeneous and isotropic at all scales. As indicated before, according to the proposal, the self-induced collapse operates in close analogy with a "measurement" in the quantum-mechanical sense, and assumes that at a certain time η_k^c the part of the state that describes the mode $\vec{k}$ jumps to a new state, which is no longer homogeneous and isotropic. To proceed to the detailed description of this process, one decomposes the fields into their hermitian parts as follows: $\hat{y}_k = \hat{y}_k^R(\eta) + i\hat{y}_k^I(\eta)$ and $\hat{\pi}_k = \hat{\pi}_k^R(\eta) + i\hat{\pi}_k^I(\eta)$.

We note that the vacuum state $|0\rangle$ is characterized in part by the following: its expectation values $\langle\hat{y}_k^{R,I}(\eta)\rangle = \langle\hat{\pi}_k^{R,I}(\eta)\rangle = 0$ and its uncertainties are $\Delta\hat{y}_k^{R,I} = 1/2|y_k|^2(\hbar L^3)$ and $\Delta\hat{\pi}_k^{R,I} = 1/2|g_k|^2(\hbar L^3)$.

For an arbitrarily given state of the field $|\Omega\rangle$, we introduce the quantity $d_k \equiv \langle\Omega|\hat{a}_k^{R,I}|\Omega\rangle \equiv |d_k^{R,I}|e^{i\alpha_k}$ so that,

$$\langle\hat{y}_k^{R,I}\rangle = \sqrt{2}\Re(y_k d_k^{R,I}), \qquad \langle\hat{\pi}_k^{R,I}\rangle = \sqrt{2}\Re(g_k d_k^{R,I}), \tag{6}$$

which shows that it specifies the main quantity of interest in characterizing the state of the field.

It is convenient for future use to define the following phases, $\beta_k = \arg(y_k)$ and $\gamma_k = \arg(g_k)$, keeping in mind that they depend on the conformal time η.

The analysis now calls for the specification of the scheme of collapse determining the state of the field after the collapse [16], which is the main purpose of the next section. With such a collapse scheme at hand one then proceeds to evaluate the perturbed metric using a semiclassical description of gravitation in interaction with quantum fields as reflected in the semiclassical EFEs: $G_{ab} = 8\pi G\langle T_{ab}\rangle$. To lowest order this set of equations reduces to

$$\nabla^2\Psi_k = s\langle\delta\hat{\phi}_k'\rangle_\Omega, \tag{7}$$

where $\langle\delta\hat{\phi}_k'\rangle_\Omega$ is the expectation value of the momentum field $\delta\hat{\phi}_k' = \hat{\pi}_k/a(\eta)$ on the state $|\Omega\rangle$ characterizing the quantum part of the inflaton filed. It is worthwhile emphasizing that *before* the collapse has occurred *there are not* metric perturbations [17], i.e., the right-hand side of the last equation is zero, so, it is only *after* the collapse that the gravitational perturbations appear, i.e., the collapse of each mode represents the onset of the inhomogeneity and anisotropy at the scale represented by the mode. Another point we must stress is that, after the collapse, and in fact at all times, our *Universe would be defined by a single state* $|\Omega\rangle$, *and not by an ensemble of states*. The statistical aspects arise once we note that we do not measure directly and separately each of the modes with specific values of $\vec{k}$, but rather the aggregate contribution of all such modes to the spherical harmonic decomposition of the temperature fluctuations of the celestial sphere (see below).

To make contact with the observations we note that the quantity that is experimentally measured (for instance by WMAP) is $\Delta T/T(\theta, \varphi)$, which is expressed in terms of its spherical harmonic decomposition $\sum_{lm}\alpha_{lm}Y_{lm}(\theta, \varphi)$. The contact with the theoretical calculations is made through the theoretical estimation most likely value of the α_{lm}'s, which are expressed in terms of the Newtonian potential on the 2-sphere corresponding to the intersection of our past light cone with the last scattering surface (LSS): $\Psi(\eta_D, \vec{x}_D)$, $\alpha_{lm} = \int\Psi(\eta_d, \vec{x}_D)Y_{lm}^* d^2\Omega$. We must then consider the expression for the Newtonian potential (7) at those points:

$$\Psi(\eta, \vec{x}) = \sum_k \frac{s\mathcal{T}(k)}{k^2L^3}\langle\delta\hat{\phi}_k'\rangle e^{i\vec{k}\cdot\vec{x}}, \tag{8}$$

where we have introduced the factor $\mathcal{T}(k)$ to represent the physics effects of the period between reheating and decoupling.

Writing the coordinates of the points of interest on the surface of last scattering as $\vec{x} = R_D(\sin\theta\sin\phi, \sin\theta\cos\phi, \cos\theta)$, where R_D is the comoving radius of that surface and θ, ϕ are the standard spherical coordinates of the sphere, and using standard results connecting Fourier and spherical expansions we obtain

$$\alpha_{lm} = \sum_k \frac{s\mathcal{T}(k)}{k^2L^3}\int\langle\delta\hat{\phi}_k'\rangle e^{i\vec{k}\cdot\vec{x}}Y_{lm}(\theta, \phi)d^2\Omega \tag{9}$$

$$= \frac{s}{L^3}\sum_k \frac{\mathcal{T}(k)}{k^2}\langle\delta\hat{\phi}_k'\rangle 4\pi i^l j_l(|\vec{k}|R_D)Y_{lm}(\hat{k}). \tag{10}$$

As indicated above statistical considerations arise when noting that Eq. (8) indicates that the quantity of interest is in fact the result of a large number (actually infinite) of harmonic oscillators, each one contributing with a complex number to the sum, leading to what is in effect a two-dimensional random walk whose total displacement corresponds to the observational quantity. Note that this part of the analysis is substantially different from the corresponding one in the standard approach. In order to obtain a prediction, we need to find the most likely value of the *magnitude* of such total displacement.

Thus we must concern ourselves with

$$|\alpha_{lm}|^2 = \frac{16s^2\pi^2}{L^6}\sum_{\vec{k}\vec{k}'}\frac{\mathcal{T}(k)}{k^2}\frac{\mathcal{T}(k')}{k'^2}\langle\delta\hat{\phi}_k'\rangle \times\langle\delta\hat{\phi}_{k'}'\rangle^* j_l(kR_D)j_l(k'R_D)Y_{lm}(\hat{k})Y_{lm}(\hat{k}'), \quad (11)$$

and to obtain the "most likely" value for this quantity. This we do with the help of the *imaginary* ensemble of universes [18] and the identification of the most likely value with the ensemble mean value.

As we will see, the ensemble mean value of the product $\langle\delta\hat{\phi}_k\rangle\langle\delta\hat{\phi}_{k'}\rangle^*$, evaluated in the post-collapse states [19], results in a form $\kappa C(k)\delta_{\vec{k}\vec{k}'}$, where $\kappa = \hbar L^3 k/(4a^2)$ and $C(k)$ is an adimensional function of k, which codifies the traces of detailed aspects of the collapse scheme. We are thus led to the following expression for the most likely (ML) value of the quantity of interest:

$$|\alpha_{lm}|^2_{ML} = s^2\frac{4\pi^2\hbar}{L^3a^2}\sum_{\vec{k}}\frac{C(k)\mathcal{T}(k)^2}{k^3}j_l^2(|\vec{k}|R_D)|Y_{lm}(\hat{k})|^2. \quad (12)$$

Writing the sum as an integral (using the fact that the allowed values of the components of $\vec{k}$ are separated by $\Delta k_i = 2\pi/L$):

$$|\alpha_{lm}|^2_{ML} = \frac{s^2\hbar}{2\pi a^2}\int\frac{C(k)\mathcal{T}(k)^2}{k^3}j_l^2(|\vec{k}|R_D)|Y_{lm}(\hat{k})|^2d^3k. \quad (13)$$

The last expression can be made more useful by changing the variables of integration to $x = kR_D$, leading to

$$|\alpha_{lm}|^2_{ML} = \frac{s^2\hbar}{2\pi a^2}\int\frac{C(x/R_D)}{x}\mathcal{T}(x/R_D)^2j_l^2(x)dx. \quad (14)$$

With this expression at hand we can compare the expectations from each of the schemes of collapse against the observations. We note, in considering the last equation, that the standard form of the spectrum corresponds to replacing the function C by a constant. In fact if one replaces C by 1 and one further takes the function $\mathcal{T}$ which encodes the late time physics including the plasma oscillations which are responsible for the famous acoustic peaks, and substitutes it by a constant, one obtains the characteristic signature of a scale-invariant spectrum: $|\alpha_{lm}|^2_{ML} \propto \frac{1}{l(l+1)}$.

In the remainder of the paper we will focus on the effects that a nontrivial form of the function C has on the predicted form of the observational spectrum.

III. PROPOSAL OF COLLAPSE *à LA* WIGNER

As indicated in the Introduction, the schemes of collapse considered in the first work following the present approach, [6], essentially ignored the correlations between the canonical variables that are present in the precollapse vacuum state. In the present analysis, we will focus on this feature, characterizing such correlations via the Wigner distribution function [20], and requiring the collapse state to reflect those aspects. The choice of the Wigner distribution function to describe these correlations in this setting is justified by some of its standard properties regarding the "classical limit" (see for instance [21]), and, by the fact that there is a precise sense in which it is known to encode the correlations in question [22]. The Wigner distribution function for pure quantum states characterized by a position space wave function $\Psi(q)$ is defined as

$$\mathcal{W}(q,p) = \frac{1}{2\pi\hbar}\int_{-\infty}^{\infty}dy\Psi^*(q+y)\Psi(q-y)\exp\left(\frac{ipy}{\hbar}\right), \quad (15)$$

with (q, p) corresponding to the canonical conjugate variables.

In our case the wave function for each mode of the field (characterized by its wave vector number $\vec{k}$) corresponds, initially, to the ground state of an harmonic oscillator. It is a well-known result that the Wigner distribution function gives for a quantum harmonic oscillator in its vacuum state a bidimensional Gaussian function. This fact will be used to model the result of collapse of the quantum field state. The assumption will be that at a certain (conformal) time η_k^c, the part of the state characterizing the mode k, will collapse (in a way that is similar to what in the Copenhagen interpretation is associated with a measurement), leading to a new state $|\Omega\rangle$ in which the fields (expressed by its hermitian parts) will have expectation values given by

$$\langle\hat{y}_k^{R,I}\rangle_\Omega = x_k^{(R,I)}\Lambda_k\cos\Theta_k, \qquad \langle\hat{\pi}_k^{R,I}\rangle_\Omega = x_k^{(R,I)}\Lambda_k k\sin\Theta_k, \quad (16)$$

where $x^{(R,I)}$ is a random variable, characterized by a Gaussian distribution centered at zero with a spread one; Λ_k is given by the major semiaxis of the ellipse characterizing the bidimensional Gaussian function (the ellipse corresponds to the boundary of the region in "phase space" where the Wigner function has a magnitude larger than 1/2 its maximum value), and Θ_k is the angle between that axis and the $y_k^{R,I}$ axis.

Comparing (6) with (16) we obtain,

$$|d_k^{R,I}|\cos(\alpha_k+\beta_k) = \frac{1}{\sqrt{2}|y_k|}x_k^{R,I}\Lambda_k\cos\Theta_k, \quad (17)$$

$$|d_k^{R,I}|\cos(\alpha_k+\gamma_k) = \frac{1}{\sqrt{2}|g_k|}x_k^{R,I}\Lambda_k k\sin\Theta_k. \quad (18)$$

From these expressions we can solve for the constants $d_k^{R,I} = |d_k^{R,I}|e^{i\alpha_k}$. In fact using the polar representation of the y_k and g_k we find

$$\tan(\alpha_k - k\eta) = \frac{k^2\eta^c|y_k|\sin\Theta_k}{|g_k|\cos\Theta_k\sqrt{1+k^2(\eta^c)^2} - k|y_k|\sin\Theta_k} \tag{19}$$

obtaining

$$|d_k^{R,I}| = \frac{x_k^{R,I}\Lambda_k}{\sqrt{2}|y_k||g_k|}\cdot\frac{\sqrt{1+k^2\eta_c^2}}{k\eta_c}\times\sqrt{|y_k|^2k^2\sin^2\Theta_k + |g_k|^2\cos^2\Theta_k - \frac{2|y_k||g_k|k\cos\Theta_k\sin\Theta_k}{(1+k^2\eta_c^2)^{1/2}}}, \tag{20}$$

where in all of the expressions above the conformal time η is set to the time of collapse η_c^k of the corresponding mode.

In order to obtain the expression for Λ_k it is necessary to find the wave-function representation of the vacuum state for the variable $y_k^{R,I}$. Following a standard procedure, we apply the annihilation operator, $\hat{a}^{R,I}$, to the vacuum state $|0\rangle$, obtaining the well-known equation for the harmonic oscillator in the vacuum state, and from the result we extract the wave function of the k-mode of the inflaton field:

$$\Psi^{R,I}(y_k^{R,I},\eta) = \left(\frac{2k}{(1+\frac{i}{k\eta})\pi\hbar L^3}\right)^{1/4} \times \exp\left(-\frac{k}{\hbar L^3(1+\frac{i}{k\eta})}(y_k^{R,I})^2\right). \tag{21}$$

We next substitute this in the expression for the Wigner function, $\mathcal{W}(y_k^{R,I}, \pi_k^{R,I}, \eta)$, obtaining,

$$\mathcal{W}(y_k^{R,I}, \pi_k^{R,I}, \eta) = 2\left(1+\frac{1}{k^2\eta^2}\right)^{1/4}\exp\left(-\frac{2k}{\hbar L^3}(y_k^{R,I})^2\right) \times \exp\left(\frac{2}{k\eta\hbar L^3}y_k^{R,I}\pi_k^{R,I}\right) \times \exp\left(-\frac{(1+k^2\eta^2)}{2\hbar L^3k^3\eta^2}(\pi_k^{R,I})^2\right). \tag{22}$$

This has the form of a bidimensional Gaussian distribution as expected from the form of the vacuum state. The cross term is telling us that the support of the Wigner function is rotated with respect to the original axes. Rescaling the π_k-axe to $\Pi_k = \pi_k/k$ and doing a simple 2D rotation (i.e., $y_k'^{R,I} = y_k^{R,I}\cos\Theta_k + \Pi_k^{R,I}\sin\Theta_k$, $\Pi_k'^{R,I} = \Pi_k^{R,I}\cos\Theta_k - y_k^{R,I}\sin\Theta_k$) we find the principal axes of the Wigner function:

$$\mathcal{W}(y'^{R,I}_k\Pi'^{R,I}_k,\eta) = 2\left(1+\frac{1}{k^2\eta^2}\right)^{1/4} \times \exp\left(-\left(\frac{y'^{R,I}_k}{\sigma'_{y_k}}\right)^2\right) \times \exp\left(-\left(\frac{\Pi'^{R,I}_k}{\sigma_{\Pi'_k}}\right)^2\right), \tag{23}$$

with the corresponding widths given by

$$\sigma_{y'_k} = \frac{4\hbar L^3k\eta^2}{1+5k^2\eta^2+\sqrt{1+10k^2\eta^2+9k^4\eta^4}}, \tag{24}$$

$$\sigma_{\Pi'_k} = \frac{4\hbar L^3k\eta^2}{1+5k^2\eta^2-\sqrt{1+10k^2\eta^2+9k^4\eta^4}}. \tag{25}$$

Note that $\sigma_{\Pi'_k} > \sigma_{y'_k}$. The rotation angle, θ_k is given by

$$2\Theta_k = \arctan\left(\frac{4k\eta}{1-3k^2\eta^2}\right). \tag{26}$$

It is clear then that $\Lambda_k \equiv 2\sigma_{\Pi'_k}$.

Substituting $\hat{\pi}_k$ in $\delta\hat{\phi}'_k$ (defined by Eq. (8)) and calculating the expectation value of it in the post-collapse state, $|\Omega\rangle$, we obtain

$$\langle\delta\hat{\phi}'_k\rangle_\Omega = \sqrt{\frac{k}{2}}\cdot\frac{1}{a}[|d_k^R|\cos(\alpha_k^R+\gamma_k+\Delta_k) + i|d_k^I|\cos(\alpha_k^I+\gamma_k+\Delta_k)], \tag{27}$$

where we have defined the "collapse to observation delay" from the collapse time of the mode k, η_k^c as $\Delta_k = k(\eta - \eta_k^c)$ where η represents the time of interest which in our case will be the "observation time."

Inserting Eq. (20) in the last expression, we can rewrite $\langle\delta\hat{\phi}'_k\rangle_\Omega$ as

$$\langle\delta\hat{\phi}'_k\rangle_\Omega = \frac{2}{a(\eta_c)}\cdot\frac{k\eta_c\sqrt{\hbar L^3 k}}{(1+10k^2\eta_c^2+9k^4\eta_c^4)^{1/4}}\cdot\frac{x_k^R+ix_k^I}{\sqrt{1+5k^2\eta_c^2-\sqrt{1+10k^2\eta_c^2+9k^4\eta_c^4}}}\times\left\{\cos\Delta_k\sqrt{\sqrt{1+10k^2\eta_c^2+9k^4\eta_c^4}-1+3k^2\eta_c^2}+\sin\Delta_k\left[\sqrt{\sqrt{1+10k^2\eta_c^2+9k^4\eta_c^4}+1-3k^2\eta_c^2}-\frac{1}{k\eta_c}\sqrt{\sqrt{1+10k^2\eta_c^2+9k^4\eta_c^4}-1+3k^2\eta_c^2}\right]\right\}. \quad (28)$$

Now we take the ensemble mean value of the square of $\langle\delta\hat{\phi}'_k\rangle_\Omega$, taking out a factor of κ (remember that $\kappa = \hbar L^3 k/4a^2$, see last section) and call it $C_{\rm wigner}(k)$

$$C_{\rm wigner}(k) = \frac{32z_k^2}{\sqrt{1+10z_k^2+9z_k^4}}\times\frac{1}{1+5z_k^2-\sqrt{1+10z_k^2+9z_k^4}}\left\{\left[\sqrt{1+10z_k^2+9z_k^4}-1+3z_k^2\right]\left(\cos\Delta_k-\frac{\sin\Delta_k}{z_k}\right)^2+\sin^2\Delta_k\left[\sqrt{1+10z_k^2+9z_k^4}-3z_k^2-7\right]+8z_k\cos\Delta_k\sin\Delta_k\right\}, \quad (29)$$

where we replaced $k\eta_k^c(k)$ by z_k. Henceforth (14) is

$$|\alpha_{lm}|^2_{ML} = \frac{s^2\hbar}{2\pi a^2}\int\frac{C_{\rm wigner}(x/R_D)}{x}\mathcal{T}(x/R_D)^2 j_l^2(x)dx. \quad (30)$$

Now we are prepared to compare the predictions of the various schemes of collapse with observations.

Before doing so it is worth recalling that the standard results are obtained if the function C is a constant, and to mention that it turns out that in order to obtain a constant C (in this and any collapse scheme) there seems to be a single simple option: that the z_k be essentially independent of k indicating that the time of collapse for the mode k, η_k^c should depend on the mode frequency according to $\eta_k^c = z/k$. For a more detailed treatment we refer to the article [6].

IV. COMPARING WITH OBSERVATIONS

This is going to be a rather preliminary analysis concentrating on the main features of the resulting spectrum and ignoring the late time physics corresponding to the effects of reheating and acoustic oscillations (represented by $\mathcal{T}(k)$). Actual comparison with empirical data requires a more involved analysis which is well outside the scope of the present paper.

We remind the reader that $C(k)$ encapsulates all of the imprint of the details of the collapse scheme on the observational power spectrum.

The functional form of this quantity for the scheme considered in this article, $C_{\rm wigner}$ (29), has a more complicated form than the corresponding quantities that resulted from the schemes of collapse considered in [6]. Here we reproduce those expressions for comparison with the scheme considered here and with observations. In the first collapse scheme (31), the expectation values for the field $\hat{y}_k$ and its canonical conjugate momentum $\hat{\pi}_k$ after the collapse are randomly distributed within the respective ranges of uncertainties in the precollapsed state, and are uncorrelated. The resulting power spectrum has

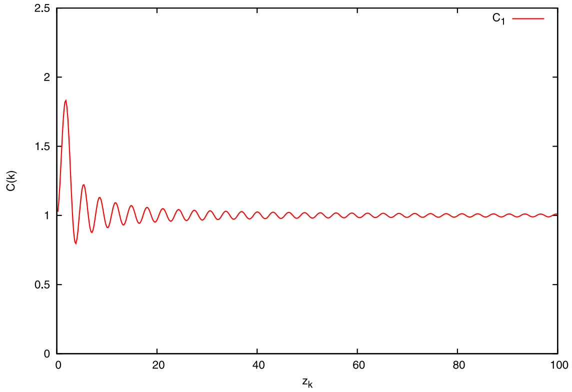
$$C_1(k) = 1 + \frac{2}{z_k^2}\sin^2\Delta_k + \frac{1}{z_k}\sin(2\Delta_k). \quad (31)$$

In the second scheme considered in [6] only the conjugate momentum changes its expectation value from zero to a value in such range; this second scheme is proposed since in the first-order Eq. (7) only this variable appears as a source. This leads to a spectrum with

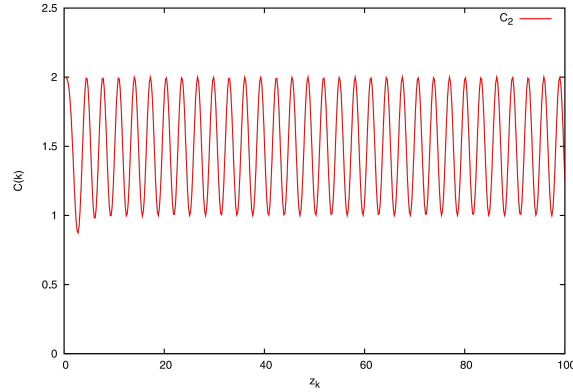
$$C_2(k) = 1 + \sin^2\Delta_k\left(1-\frac{1}{z_k^2}\right) - \frac{1}{z_k}\sin(2\Delta_k). \quad (32)$$

Despite the fact that the expression for $C_{\rm wigner}$ looks by far more complicated than C_2, their dependence in z_k is very similar, except for the amplitude of the oscillations [see Figs. 1(b) and 1(c)]. Another interesting fact that can be easily detected in the behavior of the different schemes of collapse is that if we consider the limit $z_k \to \pm\infty$, then $C_1(k)\to 1$ and we recover the standard scale-invariant spectrum. This does not happen with $C_2(k)$ or $C_{\rm wigner}(k)$ (see Fig. 1).

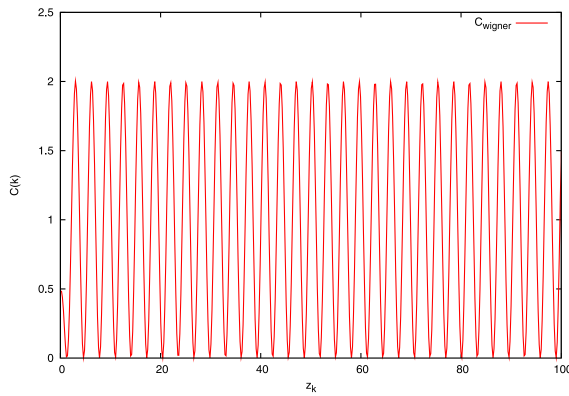
We recall that the standard form of the predicted spectrum is recovered by taking $C(k) = 1$. Therefore, we can consider the issue of how the various collapse schemes approach the standard answer (given the fact that the standard answer seems to fit the observations rather well). In particular, we want to investigate how sensitive the predictions are for the various schemes, to small departures from the case where z_k is independent of k, which as we argued above would lead to a precise agreement with the standard spectral form. In order to carry out this analysis, we must obtain the integrals (14) for the various collapse schemes characterized by the various functions $C_1(k)$, $C_2(k)$ and $C_{\rm wigner}(k)$. It is convenient to define the



(a) C_1, the two field variables $\langle \hat{y}_k \rangle$ and $\langle \hat{\pi}_k \rangle$, collapses to a random value of the dispersion of the vaccum state independently



(b) C_2, this scheme is proposed taking in account the fact that only $\langle \hat{\pi}_k \rangle$ appears in the EFE at first order.



(c) C_{wigner}, this scheme proposes a kind of correlation between the post-collapse values taking the Wigner functional of the vaccum state as an indicator of this correlation.

FIG. 1 (color online). Plots of the three collapse schemes. We could appreciate that C_2 (middle) and C_{wigner} have a similar behavior despite their dissimilar functional form.

adimensional quantity $\tilde{z}_x \equiv xN(x)$, where $x = kR_D$ and $N(x) \equiv \eta^c_{k(x)}/R_D$. We will be working under the following assumptions: (1) The changes in scale during the time elapsed from the collapse to the end of inflation are much more significant than those associated with the time elapsed from the end of inflation to our days, thus we will use the approximation $\Delta_k = -\tilde{z}_x$; (2) We will explore the sensitivity for small deviations of the "z_k independent of k recipe" by considering a linear departure from the k independent z_k characterized by $\tilde{z}_x$ as $\tilde{z}_x = A + Bx$ in order to examine the robustness of the collapse scheme in predicting the standard spectrum. We note that A and B are adimensional.

Figs. 2–4 reflect the way the spectrum behaves as a function of l, where we must recall that standard prediction (ignoring the late physics input of plasma oscillations, etc.) is a horizontal line. Those graphs represent various values of A and B chosen to sample a relatively ample domain. The graphs (5–7) show the form of the spectrum for various choices for the value of B keeping the value of A fixed.

It is important at this point to remind the reader—in the order to avoid possible misinterpretations—that these graphs are ignoring the effect of late physics phenomena (plasma oscillations, etc.). Our aim at this stage is to compare these graphs with the scale-invariant spectrum predicted by standard inflationary scenarios (i.e., a constant value for $2l(l+1)|\alpha_{lm}|^2$) and not—directly—with the observed spectrum.

As we observed before, the behavior of C_2 and C_{wigner} is qualitatively similar, the main difference comes from the amplitude of the oscillations of the functional.

From these results we can obtain some reasonable constraints on the values of the A and B for the different schemes of collapse. We start by defining for a given

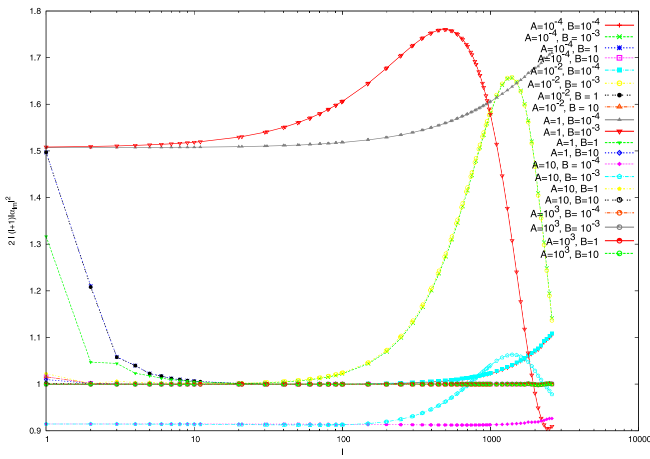


FIG. 2 (color online). Semilog plot of $|\alpha_{lm}|^2(C_1(k))$ for different values of (A, B), representing how robust is the scheme of collapse when it departs from z_k constant. The abscissa is l until $l = 2600$.

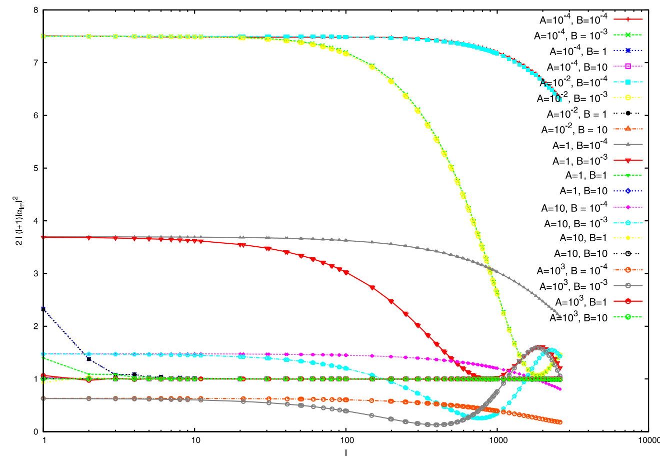


FIG. 4 (color online). Semilog plot of $|\alpha_{lm}|^2(C_{\rm wigner}(k))$ for different values of (A, B), representing how robust the scheme of collapse is when it departs from z_k constant. The abscissa is l until $l = 2600$.

predicted spectrum the degree of deviation from the flat spectrum to be simply $\Delta_{l\,\max} \equiv (\frac{1}{l\,\max}\sum_{l=1}^{l=l\,\max}[(l(l+1)\times \frac{1}{2l+1}\sum_m|\alpha_{l,m}|^2 - S]^2)^{1/2}/S$ where S represents the flat spectrum that would best approximate the corresponding imaginary data and is given by $S \equiv \frac{1}{l\,\max}\sum_{l=1}^{l=l\,\max}(l(l+1)\frac{1}{2l+1}\sum_m|\alpha_{l,m}|^2)$. If we set a bound on the departure from scale invariance up to $l = 1500$ of 10% measured by $\Delta_{l\,\max}$ (i.e., requiring $\Delta_{l\,\max} < 0.1$) we obtain for the various collapse schemes the corresponding allowed range of values for the parameters A and B. The results from these analyses are presented in Tables I, II, and III. We see that the restriction of range in B becomes weaker for larger values of A, something that can be described by stating that the earlier the collapse occurs the larger the possible departures from the behavior $\eta_k^c k =$ constant.

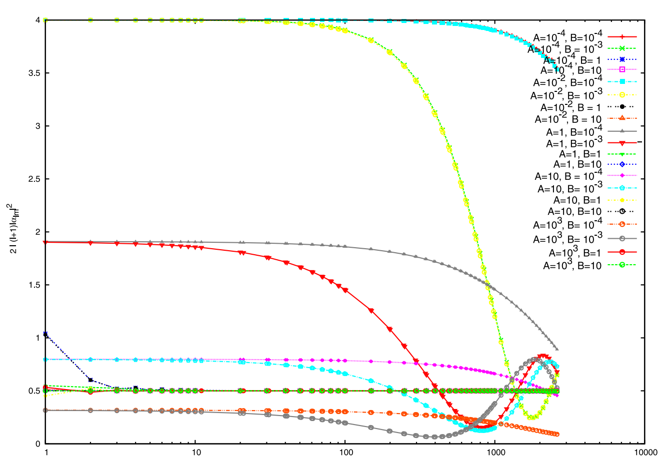


FIG. 3 (color online). Semilog plot of $|\alpha_{lm}|^2(C_2(k))$ for different values of (A, B), representing how robust the scheme of collapse is when it departs from z_k constant. The abscissa is l until $l = 2600$.

We note that we can recover the range of times of collapse for the different values of A and B. We can solve $N(x) = A/x + B$, therefore $|\eta_k^c(k)| = A/k + R_D B$. Note that R_D is the *comoving* radii of the last scattering surface. Considering the radial null geodesics we find $R_D = \eta_0 - \eta_d$, where η_d is the time of the decoupling. The decoupling of photons occurs in the matter domination epoch, so we can use the expression for R_D in terms of the scale factor, using the corresponding solution to the Friedman equation

TABLE I. Robustness of C_1 when the parameters (A, B) were varied from $10^{-4} \leq A \leq 10^3$ and $10^{-4} \leq B \leq 10$.

	$C_1(k)$	
A	B	$\Delta_{l\,\max} \times 100$
0.0001	0.0001	6.63019
0.0001	0.001	28.3844
0.0001	1	0.288273
0.0001	10	0.301883
0.01	0.0001	6.84475
0.01	0.001	28.3706
0.01	1	0.282546
0.01	10	0.301614
1	0.0001	10.1258
1	0.001	21.3117
1	1	0.247444
1	10	0.341509
10	0.0001	1.67782
10	0.001	15.8869
10	1	0.195523
10	10	0.384265
1000	0.0001	0.44236
1000	0.001	1.58567
1000	1	0.394892
1000	10	0.402706

TABLE II. Robustness of C_2 when the parameters (A, B) were varied from $10^{-4} \le A \le 10^3$ and $10^{-4} \le B \le 10$.

A	$C_2(k)$ B	$\Delta_{l\max} \times 100$
0.0001	0.0001	7.92849
0.0001	0.001	53.9872
0.0001	1	0.423473
0.0001	10	0.249129
0.01	0.0001	8.12093
0.01	0.001	54.2265
0.01	1	0.277929
0.01	10	0.251313
1	0.0001	21.8266
1	0.001	50.6328
1	1	0.312876
1	10	0.443572
10	0.0001	18.4953
10	0.001	46.1397
10	1	0.917963
10	10	0.445398
1000	0.0001	28.9085
1000	0.001	56.2369
1000	1	0.208227
1000	10	0.434914

$$R_D = \frac{2}{H_0}(1 - \sqrt{a_d}), \qquad (33)$$

where we have normalized the scale factor so today is $a_0 = 1$, so, $a_d \equiv a(\eta_d) \simeq 10^{-3}$ and H_0 is the Hubble variable today. The numerical value is $R_D = 5807.31h^{-1}$ Mpc. Henceforth

$$|\eta_k^c(k)| = \frac{A}{k} + \frac{2B}{H_0}(1 - \sqrt{a_d}). \qquad (34)$$

Thus, we can use this formula and calculate the collapse time of the interesting values of k we observe in the cosmic microwave background (CMB), namely, the range between 10^{-3} Mpc$^{-1} \le k \le 1$ Mpc^{-1}. These modes cover the range of the multipoles l of interest: $1 \le l \le 2600$, where we made use of the relation [23] $l = kR_D$. The collapse times for these modes can be regarded as the times in which inhomogeneities and anisotropies first emerged at the corresponding scales. These collapse times are shown in Fig. 8 for the best values of (A, B) given in the tables [24] (I, II, and III).

We can compare the value of the scale factor at the collapse time $a(\eta_k^c)$ with the traditional scale factor at "horizon crossing" that marks the "quantum to classical transition" in the standard explanation of inflation: a_k^H. The "horizon crossing" occurs when the length corresponding to the mode k has the same size as the "Hubble Radius," H_I^{-1}, (in comoving modes $k = aH_I$) therefore, $a_k^H \equiv a(\eta_k^H) = \frac{k}{H_I} = \frac{3k}{8\pi GV}$. Thus the ratio of the value of the scale factor at horizon crossing for mode k and its value at collapse time for the same mode is

$$\frac{a_k^H}{a_k^c} = k\eta_k^c(k) = A + BR_Dk = A + Bl. \qquad (35)$$

Using the best-fit values for the different collapse schemes, we can plot the e-folds elapsed between the mode's collapse and its horizon crossing. As we can see in Fig. 9 this quantity changes—at most—of 1 order of magnitude in the range for k for the values of A and B that were considered more reasonable, i.e., $a_k^H > a_k^c$, the time of collapse $\eta_k^c \simeq 10^{-3}\eta_k^H$ in this range. The door is clearly open for a more detailed analysis and comparison to the actual empirical data, whereby one could hope to extract robust information of the type discussed above.

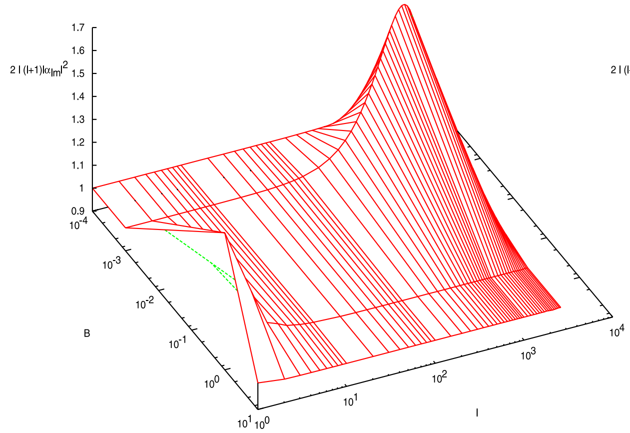
TABLE III. Robustness of $C_{\rm wigner}$ when the parameters (A, B) were varied from $10^{-4} \le A \le 10^3$ and $10^{-4} \le B \le 10$.

A	$C_{\rm wigner}(k)$ B	$\Delta_{l\max} \times 100$
0.0001	0.0001	10.0763
0.0001	0.001	47.3616
0.0001	1	0.506768
0.0001	10	0.162458
0.01	0.0001	10.2874
0.01	0.001	47.494
0.01	1	0.359852
0.01	10	0.165756
1	0.0001	18.445
1	0.001	34.1731
1	1	0.358535
1	10	0.394309
10	0.0001	19.3128
10	0.001	45.1946
10	1	0.51842
10	10	0.430548
1000	0.0001	28.9273
1000	0.001	56.2646
1000	1	0.197662
1000	10	0.445794

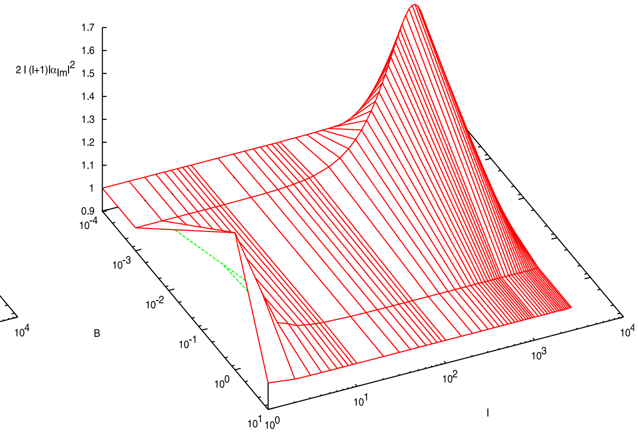
V. DISCUSSION

We have considered various relatively *ad hoc* recipes for the form of the state of the quantum inflationary field that results, presumably from a gravitationally induced collapse of the wave function. The breakdown of unitarity that this entails is thought to be associated with drastic departures from standard quantum mechanics once the fundamental quantum gravity phenomena come into play. We have not discussed at any length this issue here and have focused in the present treatment on purely phenomenological aspects of the problem.

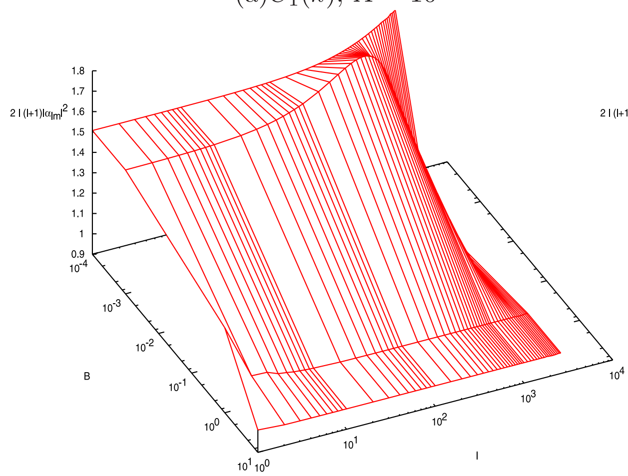
The analysis of the signatures of the different schemes of collapse illustrate various generic points worth mention-



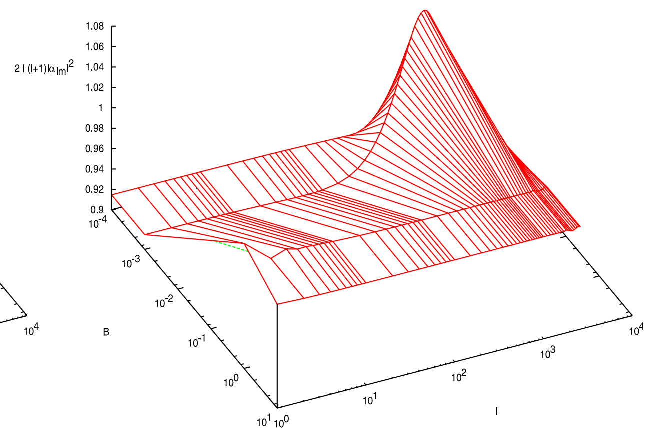
(a)$C_1(k),\ A = 10^{-4}$



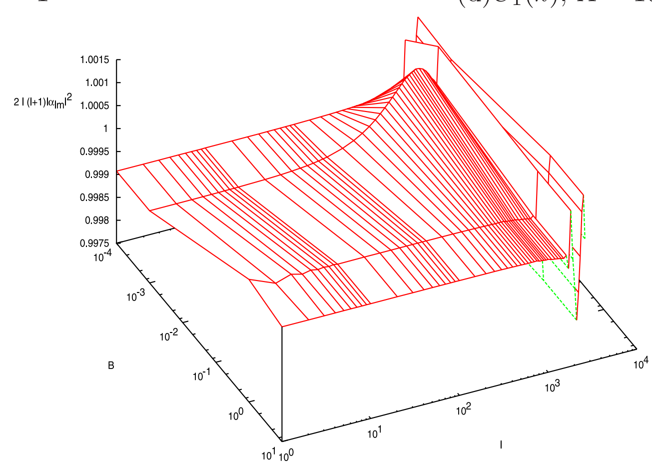
(b)$C_1(k),\ A = 10^{-3}$



(c)$C_1(k),\ A = 1$



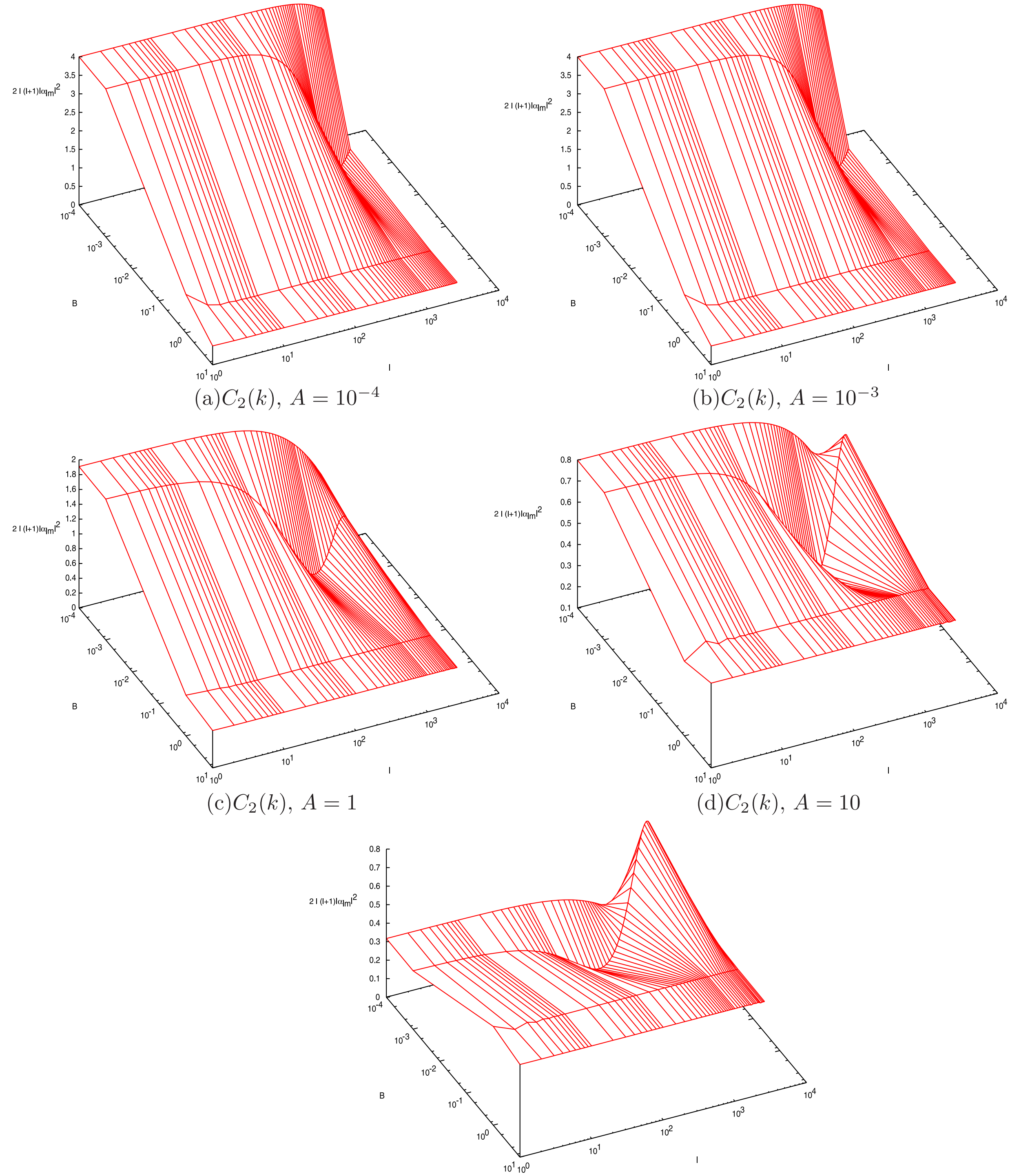
(d)$C_1(k),\ A = 10$



(e)$C_1(k),\ A = 1000$

FIG. 5 (color online). Plot showing how the integral of $|\alpha_{lm}|^2(C_1)$ varies with respect to changes in B $(10^{-4} - 10)$, keeping A fixed. Both axes B and l are in logscale. See the main text for a more extensive explanation.

ing: First, that, depending on the details of the collapse scheme and its parameters, there can be substantial departures in the resulting power spectrum, from the standard scale-invariant spectrum usually expected to be a generic prediction from inflation. Of course it is known that there exist other ways to generate modifications in the predicted spectrum, such as considering departures from slow roll and modifications of the inflaton potential and so forth. In the approach we have been following the modifications arise from the details of a quantum collapse mechanism, a



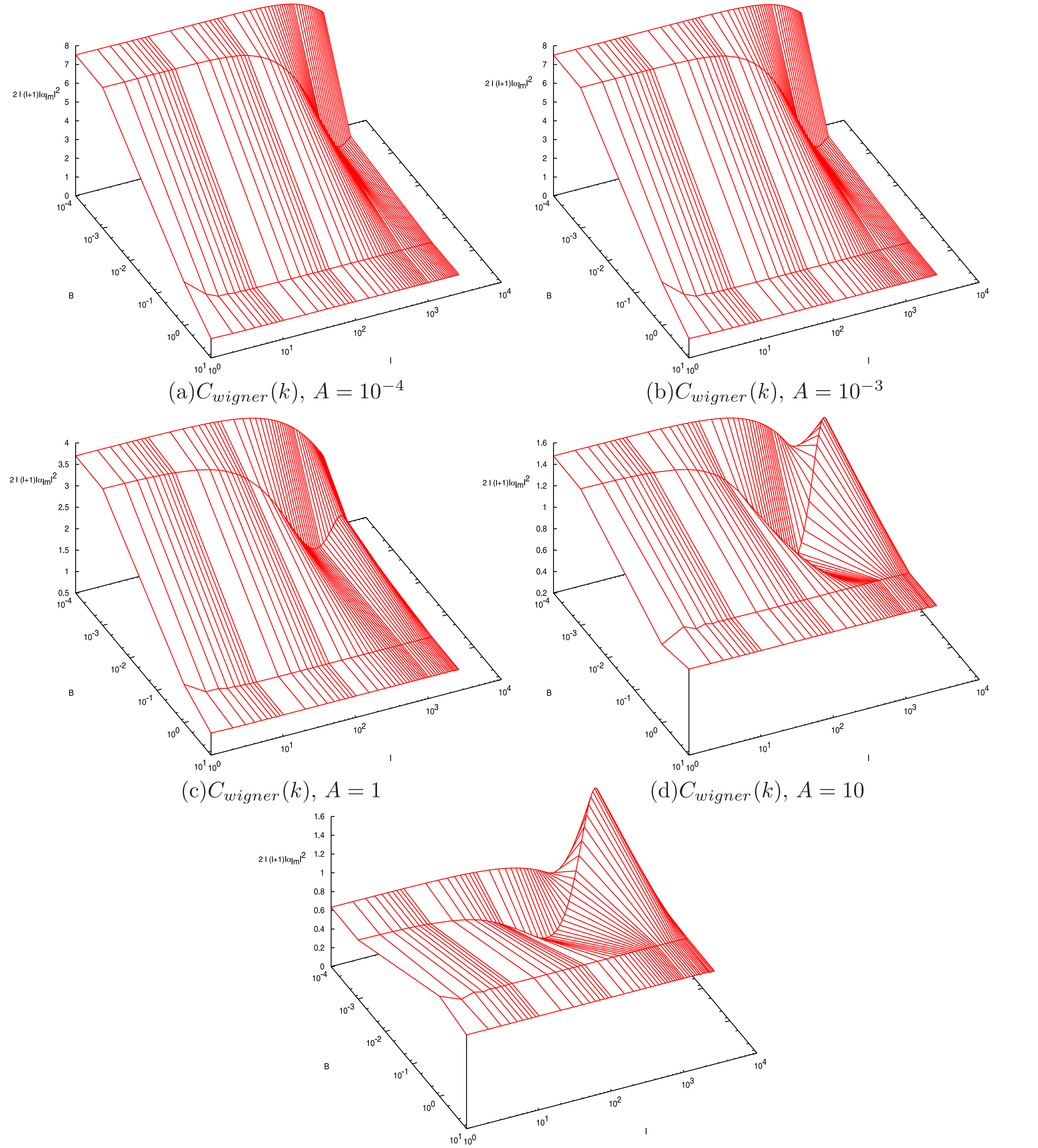
(a) $C_2(k),\ A = 10^{-4}$ (b) $C_2(k),\ A = 10^{-3}$

(c) $C_2(k),\ A = 1$ (d) $C_2(k),\ A = 10$

(e) $C_2(k),\ A = 1000$

FIG. 6 (color online). Plot showing how the integral of $|\alpha_{lm}|^2(C_2)$ varies with respect to changes in B $(10^{-4} - 10)$, keeping A fixed. Both axes B and l are in logscale. See the main text for a more extensive explanation.

feature tied to a dramatic departure from the standard unitary evolution of quantum of physics that we have argued must be invoked if we are to have a satisfactory understanding of the emergence of structure from quantum fluctuations. In fact, by fitting the predicted and observational spectra, these sorts of modifications are possible sources of clues about what exactly is the physics behind the quantum-mechanical collapse or whatever replaces it. We saw that generically one recovers the standard scale-invariant Harrison-Z'eldovich spectrum if the collapse



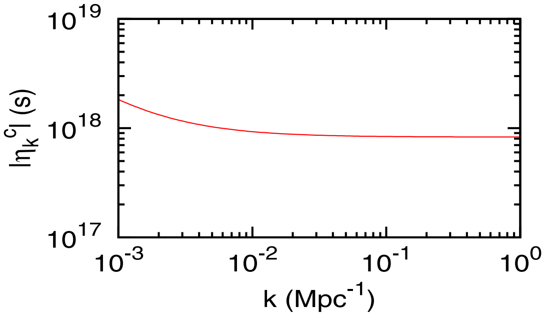
(a) $C_{wigner}(k),\ A = 10^{-4}$ (b) $C_{wigner}(k),\ A = 10^{-3}$

(c) $C_{wigner}(k),\ A = 1$ (d) $C_{wigner}(k),\ A = 10$

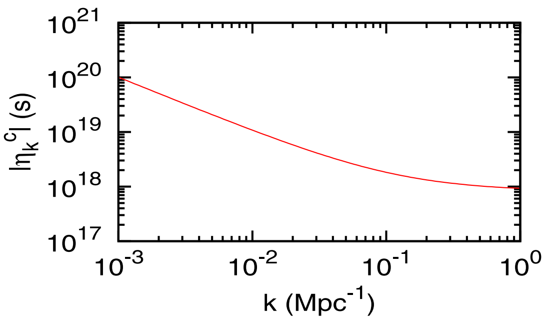
(e) $C_{wigner}(k),\ a = 1000$

FIG. 7 (color online). Plot showing how the integral of $|\alpha_{lm}|^2(C_{\rm wigner})$ varies with respect to changes in B $(10^{-4} - 10)$, keeping A fixed. Both axes B and l are in logscale. See the main text for a more extensive explanation.

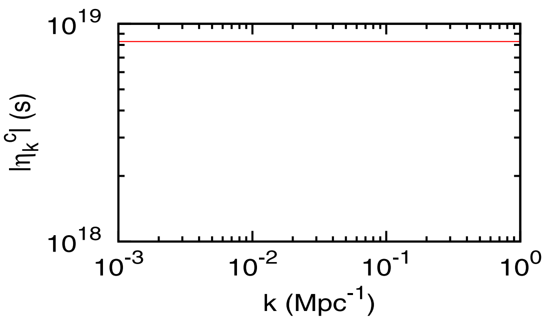
time (conformal time) of the modes is such that $\eta_k^c k =$ constant [25]. On the other hand and as shown in detail in [6] the simple generalization of the ideas of Penrose about the conditions that would trigger the quantum gravity induced collapse leads precisely to such a prediction for η_k^c. We should however keep in mind that, even if something of that sort is operating, the stochastic nature of any sort of quantum-mechanical collapse leads us to expect that such a pattern would not be followed with arbitrarily high precision. In this regard we have studied the robustness of the various schemes in leading to an almost scale-invariant spectrum. To this end we have considered in this



(a) C_1, with $A = 10, B = 1$.

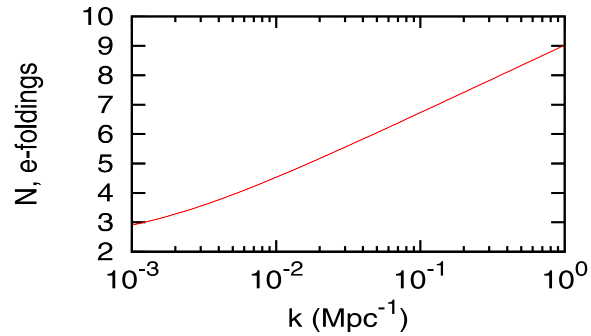


(b) C_2, with $A = 1000, B = 1$.

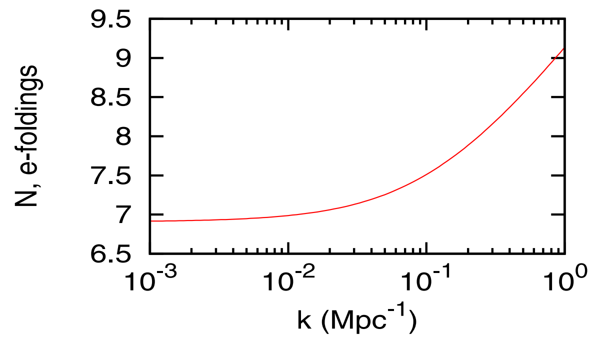


(c) C_{wigner}, with $A = 0.01, B = 10$. Note how in this scheme almost all the modes must collapse at the same time.

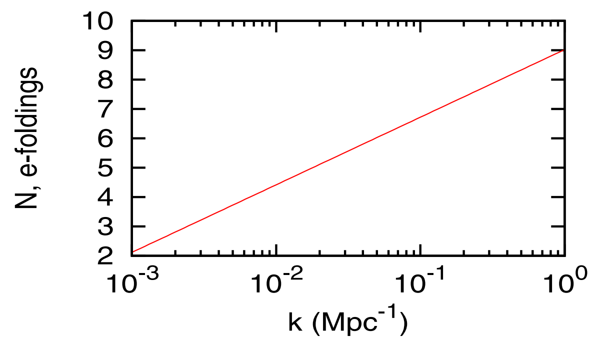
FIG. 8 (color online). Logarithmic plot in both axes of the collapse times $|\eta_k^c|$ (in seconds), for the three schemes, taking in account only the best values of (A, B) in the range of $10^{-3}\ \mathrm{Mpc}^{-1} < k < 1\ \mathrm{Mpc}^{-1}$. For these plots $h = 0.7$.



(a) C_1, with $A = 10, B = 1$.



(b) C_2, with $A = 1000, B = 1$.



(c) C_{wigner}, with $A = 0.01, B = 10$.

FIG. 9 (color online). Semilogarithmic plot of the number of e-foldings between a_k^H and a_k^c for the three schemes, taking in account only the best values of (A, B) in the range of $10^{-3}\ \mathrm{Mpc}^{-1} < k < 1\ \mathrm{Mpc}^{-1}$. For these plots $h = 0.7$.

work the simplest (linear) deviations from the behavior of η_k^c as a function of k, i.e., we have explored in the three existing collapse schemes the effects of having a time of collapse given by $\eta_k^c = A/k + BR_D$. The results of these studies are summarized in Figs. 2–4 and Tables I, II, and III, so here we will only point out one of the most salient features: We note that the different collapse schemes lead to different types of departures of the spectrum from the scale-invariant one, for instance the schemes $C_2(k)$ and $C_{\mathrm{wigner}}(k)$ lead naturally to a turning down of the spectrum as we increase l.

It is worth noting that a turning down in the spectrum is observed in the CMB data [3], which is attributed as a whole in literature to *the damping effect* [26], i.e., to the fact that inhomogeneities are dampened to the nonzero mean-free-path of photons at that time of decoupling [27]. As observed in Figs. 3 and 4 for some values of (A, B) we obtain an additional source of "damping" due to fluctuations in the time of collapse about the pattern characterized by $\eta_k^c k =$ constant. It is expected that the PLANCK probe will provide more information on the spectrum for large values of l, so hopefully this characteristic of our analysis could be analyzed and distinguished from the standard damping in order to obtain interesting constraints on the parameters (A, B). In fact we believe that one should be able to disentangle the two effects, because in the cases in which our model leads to additional damping in the spectrum, it also predicts that there should be a rebound at even higher values of l (see Figs. 2–4).

However, the most remarkable conclusion, illustrated by the present analysis, is that by focusing on issues that could be thought to be only philosophical and of principle, we have been led to the possibility of addressing issues pertaining to some novel aspects of physics which could be confronted with empirical observations. Further and more detailed analyses based on direct comparisons with observations are indeed possible, and should be carried out. This together with the foreseeable improvements in the empirical data on the spectrum, particularly in the large l region, and the large scale matter distribution studies, should permit even more detailed analysis of the novel aspects of physics that we believe are behind the origin of structure in our Universe.

ACKNOWLEDGMENTS

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[8] D. Sudarsky, J. Phys. Conf. Ser. **67**, 012054 (2007).

[9] Further issues about the identification of ensemble averages with time averages, which are valid under the ergodic hypothesis, can be raised in the cosmological context where assumptions of ergodicity and equilibrium seem to be much less justified.

[10] C Kiefer, I Lohmar, D Polarski, and A. S. Starobinsky, in *3rd International Workshop DICE2006: Quantum Mechanics Between Decoherence and Determinism: New Aspects from Particle Physics to Cosmology* (Castello di Piombino, Tuscany, Italy, 2006); J. Phys. Conf. Ser. **67**, 012023 (2007); J. Martin, arXiv:hep-th/07043540; D. Sudarsky, *Proceedings of From Quantum to Emergent Gravity: Theory and Phenomenology, Trieste, 2007* (to be published).

[11] Inflation could work if we do not start strictly with this condition, but after some e-foldings the Universe reaches this stage.

[12] We find this wording unfortunate because it leads people to think that something is fluctuating in the sense of Brownian motion, while a wording such as "quantum uncertainties" would evoke something like the wave packet associated with a ground state of an harmonic oscillator which is a closer analogy with what we have at hand.

[13] J.B. Hartle, arXiv:gr-qc/9701022; in *Directions in Relativity. Vol. 2: Proceedings*, edited by B.L. Hu and T.A. Jacobson (Cambridge University Press, Cambridge, 1993).

[14] R. Penrose, *The Emperor's New Mind* (Oxford University Press, New York, 1989); R. Penrose, in *Physics Meets Philosophy at the Planck Scale*, edited by C. Callender and N. Huggett (Cambridge University Press, Cambridge, 2001) pp 290–304; R. Penrose, *The Road to Reality: A Complete Guide to the Laws of the Universe* (Jonathan Cape, London, 2005).

[15] As we will show, the relevant quantities that one is interested in computing are determined once one characterizes the time of collapse and describes the state after the collapse in terms of the expectation values of the field and momentum conjugate variables.

[16] At this point, in fact, all we require is the specification of the expectation values of certain operators in this new quantum state.

[17] This might seem awkward to some readers. It is then worth emphasizing that our view is that, in contrast with what happens with other fields, the fundamental degrees of freedom of gravitation are not related to the metric degrees of freedom in any simple way, but instead the latter appear as effective degrees of freedom of a nonquantum effective

theory. Therefore, the quantum uncertainties (we feel "uncertainties" is a more appropriate word than "fluctuations," as the latter suggests that something is actually changing constantly in a random way) associated with the gravitational degrees of freedom are most naturally thought of as not having a metric description (as occurs for instance in the Loop Quantum Gravity program where the fundamental degrees of freedom are holonomies and fluxes), and thus that the metric can appear only at the classical level of description, where it satisfies something close to the semiclassical Einstein equation. In other words, from our point of view, it would be incorrect to think of the quantum uncertainties of the metric as an appropriate description of the quantum aspects of gravitation, and much less, as satisfying Einstein's equations. From our point of view, this would be analogous to imagining the quantum indeterminacies associated with the ground state of the hydrogen atom, as described in terms of a perturbation of the orbit of an electron in the hydrogen atom, and satisfying Keppler's equations for the classical Coulomb potential. For more details about this point of view see [6–8]. The reader should be aware that this is not a view shared by most cosmologists.

[18] This is just a mathematical evaluation device and no assumption regarding the existence of such an ensemble of universes is made or needed. These aspects of our discussion can be regarded as related to the so-called cosmic variance problem.

[19] Note here again the difference with the standard treatment of this part of the calculation which calls for the evaluation of the expectation value $\langle \delta\hat{\phi}_k \delta\hat{\phi}_{k'} \rangle^*$ on the vacuum state which as already emphasized is completely homogeneous and isotropic.

[20] E. Wigner, Phys. Rev. **40**, 749 (1932).

[21] L. E. Ballentine, *Quantum Mechanics: A Modern Development* (World Scientific Publishing Co. Pte. Ltd., Singapore, 2000).

[22] S. Brandt and H. D. Dahmen, *The Picture Book of Quantum Mechanics* (Springer-Verlag, Berlin, 2001); E. Wigner, *Perspectives in Quantum Theory* edited by W. Yourgrad and A. van der Merwe (Dover, New York, 1971), pp 25–36; M. Hillery, R. O'Connell, M. Scully, and E. Wigner, Phys. Rep. **106**, No. 3, 121 (1984).

[23] The relation between the angular scale θ and the multipole l is $\theta \sim \pi/l$. The comoving angular distance, d_A, from us to an object of physical linear size L, is $d_A = L/(a\theta)$. $L/a \sim 1/k$, $d_A = R_D$ if the object is in the LSS, and using the first expression in this footnote, we get $l = kR_D$.

[24] The reader should keep in mind that our parametrization of the inflationary regime has the conformal time running from large negative values to small negative values.

[25] This resembles the condition that is sometimes considered in the context of the so-called trans-Plankian problem. There is, however, an important difference of what is supposed to occur at the (conformal) time that appears in this condition. In addressing, the trans-Plankian problem the time indicates when the mode actually comes into existence. In contrast, in our approach, the mode has always existed—modes are not created or destroyed—but the state of the field in the corresponding mode *changes (or jumps)* from the adiabatic vacuum before the condition to the so-called post-collapsed state after this, or a similar condition, is reached.

[26] This effect basically is a damping for the photon density and velocity at scale k at the time of decoupling by a factor of e^{-k^2/k_D^2}, where k_D is the diffusion scale and depends on the physics of the collisions between electrons and photons. Accordingly, the C_l spectrum is also damped as e^{-l^2/l_D^2} where $l_D \sim k_D d_A(\eta_d) \sim 1500$, for typical cosmological parameters.

[27] W. Hu and M. White, Astrophys. J. **479**, 568 (1997); S. Dodelson, *Modern Cosmology* (Academic Press, New York, 2003), Chap. 8; J. A. Peacock, *Cosmological Physics* (Cambridge University Press, Cambridge, England, 2000), Chaps. 15, 18; P. Anninos, *Computational Cosmology: From the Early Universe to the Large Scale Structure*, Living Rev. Relativity (2001), http://www.livingreviews.org/lrr-2001-2; A. Jones and A. N. Lasenby, *The Cosmic Microwave Background*, Living Rev. Relativity (1998), Vol. 1, p. 11, http://www.livingreviews.org/lrr-1998-11.