Primordial helium abundance from CMB: A constraint from recent observations and a forecast

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(Received 10 January 2008; revised manuscript received 19 May 2008; published 6 August 2008)

We studied a constraint on the primordial helium abundance Y_p from current and future observations of CMB. Using the currently available data from WMAP, ACBAR, CBI, and BOOMERANG, we obtained the constraint as $Y_p = 0.25^{+0.10}_{-0.07}$ at 68% confidence level. We also provide a forecast for the Planck experiment using the Markov chain Monte Carlo approach. In addition to forecasting the constraint on Y_p, we investigate how assumptions for Y_p affect constraints on the other cosmological parameters.

DOI: 10.1103/PhysRevD.78.043509 PACS numbers: 98.80.Ft, 26.35.+c, 98.70.Vc

I. INTRODUCTION

Current cosmological observations are very precise to give us a lot of information on the evolution and present state of the Universe. We usually extract the information by constraining cosmological parameters such as energy densities of baryon, dark matter and dark energy, the Hubble constant, reionization optical depth, spectral index of primordial fluctuation, and so on. Among them, in this paper, we focus on the primordial helium abundance Y_p, which has been of great interest in cosmology. One of the reasons why the primordial helium abundance has been considered to be interesting and important is that, in the context of the standard big bang nucleosynthesis (SBBN), we can know the baryon density once Y_p is determined from observations. However, it has been discussed that a significant systematic error dominates when one infers the value of Y_p from measurements in a low-metallicity extragalactic HII region [1–5]. Furthermore, there have been some discussions that there may be a large uncertainty in the neutron lifetime [6–8], which results in uncertainties in the predictions for the abundances of light elements. In this respect, the study of other independent measurements of the helium abundance would be interesting.

Recent precise measurements of cosmic microwave background (CMB) such as the Wilkinson microwave anisotropy probe (WMAP) can now enable us to constrain cosmological parameters with great accuracies. However, the helium abundance has not been discussed much in the literature when one study of cosmological constraints from CMB since Y_p has been considered to have little effect on CMB power spectrum. In most analyses, Y_p is fixed to be 0.24 which is probably motivated from a somewhat old value of the observed primordial helium abundance of $Y_p = 0.238$ [9].[1] But, in fact, there have been some works which discuss the effects of Y_p on CMB and some constraints were given [10–12]. Since the helium abundance affects the recombination history, the CMB power spectrum can be affected mainly through the diffusion damping. Although the constraint is not so severe, it is important to notice that they are obtained independently from BBN, which can be used to cross-check our understanding of the helium abundance. Furthermore, the value of Y_p at the time of BBN may be different from that at late times when CMB observations can probe.[2] After we studied the constraint on Y_p in [12], the data from WMAP has been updated [14–18]. In addition, the data at higher multipoles where the effects of Y_p become significant have been updated by the arcminute cosmology bolometer array receiver (ACBAR) [19] and cosmic background imager (CBI) Collaborations [20]. Thus it is a good time to investigate the constraint on the helium abundance using these CMB data, which is one of the aims of this paper.

Furthermore, we expect a more precise measurement of CMB from the future Planck satellite [21], which can give us a much better constraint on Y_p. In fact, a future constraint on Y_p has already been studied using the Fisher matrix formalism [10,12]. Although this method is fast and usually adopted to predict the precision of the future measurements of cosmological parameters, it can give some inaccurate predictions in some cases, for example, when the likelihood does not respect a Gaussian form. In addition, the Fisher matrix formalism predicts only the uncertainty for the parameter estimation since it is just concerned with the derivatives with respect to parameters around the fiducial values. However, since some parameters are correlated in general, fixing the values of some parameters can bias the estimation of other parameters. Namely, priors we assume on some parameters can cause the estimated central values to deviate from the input fiducial values, but such effects cannot be quantified by

[1]Recent observations give, e.g., $Y_p = 0.249 \pm 0.009$ [1,2].

[2]For example, in a scenario to solve the so-called "lithium problem" with Q balls, the decay of Q balls produces extra baryon after BBN has completed [13]. In a model of this kind, the value of Y_p can vary at different epochs.

the Fisher matrix approach. Thus, in this paper, we use the Markov chain Monte Carlo (MCMC) approach to extract reliable future constraints on Y_p and other cosmological parameters. In particular, when we forecast the sensitivity for other cosmological parameters, we assume some different priors on Y_p and investigate to what extent the information of Y_p is important to determine other cosmological parameters.

The organization of this paper is as follows. In the next section, the effects of Y_p on the CMB power spectrum are briefly discussed. In Sec. III, we study the constraint on Y_p using CMB data currently available including ACBAR, BOOMERANG (balloon observations of millimetric extragalactic radiation and geophysics), and CBI as well as WMAP5. In Sec. IV, we investigate a constraint on Y_p and other cosmological parameters from the future Planck experiment. In addition, we also study how the prior on Y_p affects the determination of other cosmological parameters. A brief discussion on the significance of the uncertainties of the recombination theory in deriving cosmological constraint is given too. The final section is devoted to the summary of this paper.

II. EFFECTS OF Y_p ON CMB

Here we briefly discuss the effects of the primordial helium abundance Y_p on the CMB power spectrum where $Y_p = 4n_{\rm He}/(n_{\rm H} + 4n_{\rm He})$ with $n_{\rm H}$ and $n_{\rm He}$ being the number density of hydrogen and helium-4, respectively. As has been discussed in [10,12], the value of Y_p can affect the recombination history, which changes the structure of acoustic peaks. The effects of Y_p on acoustic peaks mainly come from the diffusion damping which suppresses the power on small scales and the shift of the position of acoustic peaks due to the change of the recombination epoch. Before recombination, the number density of electron n_e can be given by $n_e = n_b(1 - Y_p)$, where n_b is the baryon number density. Thus the increase of Y_p indicates the decrease of the number of electrons. When the number of electrons is reduced, the mean free path of the Compton scattering becomes larger, which means that fluctuations on larger scales can be more affected by the diffusive mixing and rescattering. Thus the damping scale below which fluctuation of the photon is exponentially suppressed becomes larger. Furthermore, due to the change of the number density of electrons, the epoch of recombination is also affected even though its effect is not so significant. This effect shifts the position of acoustic peaks slightly. In Fig. 1, we show the CMB TT spectrum with several values of Y_p. For reference, we also plot the current data (left panel) and the expected data from the future Planck experiment (right panel). As mentioned above, by increasing the value of Y_p, the power on small scales is damped more. In addition, it is noticeable that the position of acoustic peaks also shifts slightly.

To characterize the effects of the change in Y_p and other cosmological parameters on the CMB TT power spectrum $C_l^{\rm TT}$, we consider some useful quantities [22]. First of all, to see how cosmological parameters affect the position of acoustic peaks, we investigated the response of the position of the first peak by the change of the parameters, which we denote Δl_1. In addition, to see the effects of the diffusion damping and some other effects by cosmological parameters, we study the height of the first peak relative to that at $l = 10$ and the height of the second peak (and higher peaks up to the 5th peak) relative to the first peak, which are denoted as H_1, H_2, H_3, H_4, and H_5, respectively. For clarity, we give the definitions of these quantities. The definition of H_1 is

$$H_1 \equiv \left(\frac{\Delta T(l = l_1)}{\Delta T(l = 10)}\right)^2, \tag{1}$$

and the height of the ith peak relative to the first peak is defined as

$$H_i \equiv \left(\frac{\Delta T(l = l_i)}{\Delta T(l = l_1)}\right)^2 \quad (\text{for } i \geq 2), \tag{2}$$

where $(\Delta T(l))^2 = l(l + 1)C_l^{\rm TT}/2\pi$. We varied cosmologi-

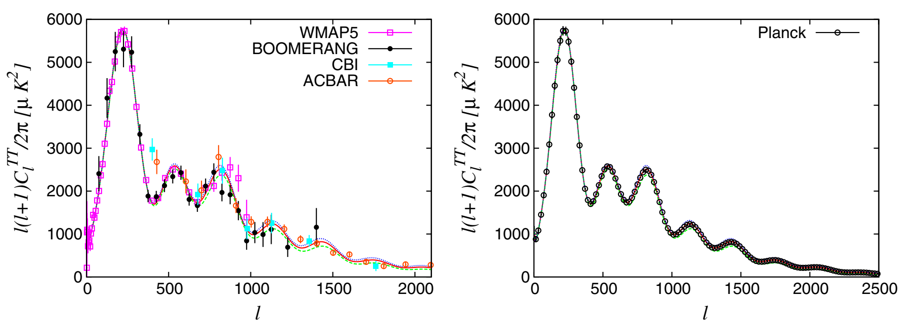

FIG. 1 (color online). CMB TT power spectra for several values of Y_p. In this figure, we take $Y_p = 0.1$ (blue dotted line), 0.24 (red solid line), and 0.4 (green dashed line). Other cosmological parameters are assumed to be the mean value of WMAP5 for a power-law ΛCDM model. For reference, in the left panel, data from WMAP5, ACBAR, BOOMERANG, and CBI are also depicted. In the right panel, the expected data from the Planck experiment are also shown.

cal parameters including Y_p around a fiducial model and obtained partial derivatives by fitting linearly around the fiducial value. For the fiducial cosmological values, we assumed the mean values of the WMAP 5-yr result (WMAP5) for a power-law ΛCDM model. Regarding Y_p, we take $Y_p = 0.248$ which corresponds to the value obtained in the SBBN for the WMAP5 baryon density. The resulting derivatives are

$$\Delta l_1 = 15.6\frac{\Delta\omega_b}{\omega_b} - 27.0\frac{\Delta\omega_m}{\omega_m} + 36.0\frac{\Delta n_s}{n_s} + 0.94\frac{\Delta Y_p}{Y_p} - 44.5\frac{\Delta h}{h}, \quad (3)$$

$$\Delta H_1 = 2.87\frac{\Delta\omega_b}{\omega_b} - 3.13\frac{\Delta\omega_m}{\omega_m} + 16.7\frac{\Delta n_s}{n_s} - 2.30\frac{\Delta h}{h}, \quad (4)$$

$$\Delta H_2 = -0.290\frac{\Delta\omega_b}{\omega_b} + 0.023\frac{\Delta\omega_m}{\omega_m} + 0.396\frac{\Delta n_s}{n_s} - 0.013\frac{\Delta Y_p}{Y_p}, \quad (5)$$

$$\Delta H_3 = -0.177\frac{\Delta\omega_b}{\omega_b} + 0.206\frac{\Delta\omega_m}{\omega_m} + 0.514\frac{\Delta n_s}{n_s} - 0.028\frac{\Delta Y_p}{Y_p}, \quad (6)$$

$$\Delta H_4 = -0.102\frac{\Delta\omega_b}{\omega_b} + 0.082\frac{\Delta\omega_m}{\omega_m} + 0.317\frac{\Delta n_s}{n_s} - 0.025\frac{\Delta Y_p}{Y_p}, \quad (7)$$

$$\Delta H_5 = -0.040\frac{\Delta\omega_b}{\omega_b} + 0.084\frac{\Delta\omega_m}{\omega_m} + 0.236\frac{\Delta n_s}{n_s} - 0.023\frac{\Delta Y_p}{Y_p}, \quad (8)$$

where ω_b and ω_m are energy densities of baryon and matter, n_s is the scalar spectral index of primordial fluctuation, h is the Hubble constant in units of $100\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$. In the formula for H_1, we do not show the dependence on $\Delta Y_p/Y_p$ since its effect on H_1 is very small compared to that of the other parameters. As seen from the negative signs of $\Delta H_i/\Delta Y_p$ for $i = 2$–5, the diffusion damping becomes more efficient as Y_p increases. We can also see the correlation of Y_p with other cosmological parameters, which can be useful when we interpret the results, in particular, for a Planck forecast.

TABLE I. The prior ranges for the parameters used in the analysis. Priors shown in the 1st and 2nd columns are adopted for current and expected Planck data, respectively. Note that we adopt the prior range for Y_p shown above only in the cases with Y_p being treated as a free parameter whereas Y_p is a derived parameter in the case where we assume the SBBN relation. For the analysis with the current data, we also vary the amplitude of the Sunyaev-Zel'dovich effect A_{SZ}, which is omitted in the Planck data analysis. We also include two additional parameters F_H and b_{He} which represent uncertainties in the theory of recombination (see Sec. IV for more details).

	Prior ranges	
Parameters	Current data	Planck
ω_b	$0.005 \to 0.1$	$0.005 \to 0.1$
ω_c	$0.01 \to 0.99$	$0.01 \to 0.99$
θ_s	$0.5 \to 10$	$0.5 \to 10$
τ	$0.01 \to 0.8$	$0.01 \to 0.8$
n_s	$0.5 \to 1.5$	$0.5 \to 1.5$
$\ln(10^{10}A_s)$	$2.7 \to 4$	$2.7 \to 4$
Y_p	$(0 \to 1)$	$(0 \to 1)$
A_{SZ}	$0 \to 2$	...
F_H	...	$(0 \to 2)$
b_{He}	...	$(0 \to 1.5)$

III. CURRENT CONSTRAINT ON Y_p

Now we discuss the constraint on Y_p from current cosmological observations. For this purpose, we make use of the CMB data from WMAP5 [14–18], ACBAR [19], BOOMERANG [23–25], and CBI [20]. To investigate the constraint, we performed a MCMC analysis by using a modified version of COSMOMC code [26]. We sampled in an eight-dimensional parameter space with $(\omega_b, \omega_c, \tau, \theta_s, n_s, A_s, Y_p, A_{SZ})$ where ω_c is the energy density of dark matter, τ is the optical depth of reionization, θ_s is the acoustic peak scale [27], A_s is the amplitude of primordial curvature fluctuation at the pivot scale $k_0 = 0.05\ \mathrm{Mpc^{-1}}$, and A_{SZ} is the amplitude of the thermal Sunyaev-Zel'dovich (SZ) effect which is normalized to the C_l^{SZ} template from [28].[3] In this paper, we consider a flat universe and assume a cosmological constant as dark energy. We also assume no running for primordial scalar fluctuation and no tensor mode. When we report our results in the following, we also use other customarily used cosmological parameters such as the Hubble constant $H_0 = 100h\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$ and energy density of matter $\Omega_m = (\omega_b + \omega_c)/h^2$. In performing a MCMC analysis, we impose top-hat priors on the primary parameters given above, which are summarized in Table I.

[3] However, the SZ effect may be so large at very high multipoles that this template may not be appropriate to adopt. Hence, we conservatively do not use the ACBAR and CBI data with $l \geq 2100$.

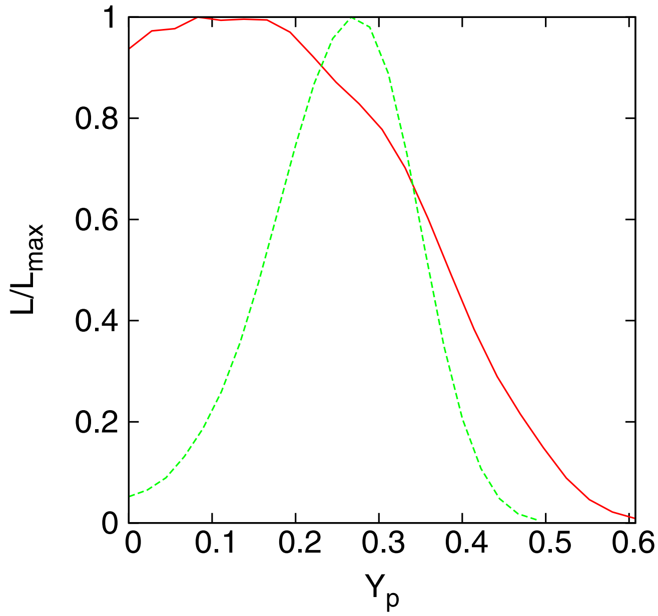

FIG. 2 (color online). One-dimensional marginalized distribution of Y_p for the cases with WMAP5 alone (red solid line) and all CMB data combined (green dashed line).

Now we discuss the constraints on Y_p when WMAP5 alone is used and when the data of ACBAR, BOOMERANG, and CBI are used in addition. In Fig. 2, we show one-dimensional marginalized distributions of Y_p for these two cases and, in the 1st and 2nd columns of Table II, we list the parameter estimations for Y_p and other cosmological parameters. When we use WMAP5 data alone, the constraint is given as $Y_p \leq 0.44$ at 95% C.L., where we only give the upper bound since the likelihood has a sizable value at $Y_p = 0$. On the other hand, when all data are combined, the constraint is $Y_p = 0.25^{+0.10(+0.15)}_{-0.07(-0.17)}$ at 68% (95%) C.L. For the analysis using WMAP5 alone, the limit we obtained is consistent with the result given in [15]. We see that by including the data from ACBAR, BOOMERANG, and CBI the likelihood distribution has a well-defined peak which is close to Gaussian. It may also be interesting to notice that when the data from ACBAR, BOOMERANG, and CBI are combined, the mean value becomes as $Y_p = 0.25$, which is very close to the value obtained from the HII region observations although the uncertainty is still large. With the help of this data set, we may begin to see the concordance with regards to the Y_p measurement from CMB and that from the HII regions.

Next we discuss the effects of the prior of Y_p on the determination of other cosmological parameters. For this purpose, we repeated a MCMC analysis fixing the value of the helium abundance to $Y_p = 0.24$ as in usual analyses. We use all the CMB data (i.e., WMAP5, ACBAR, BOOMERANG, and CBI) here. In Table II, in the last column, the constraints on cosmological parameters for the case with fixing $Y_p = 0.24$ are shown. When we compare the constraints for the cases with and without fixing Y_p, the central values as well as the errors at 68% C.L. are almost unchanged. Thus we can conclude that the usual practice of fixing of $Y_p = 0.24$ scarcely affects the constraints on other cosmological parameters with the current precision of CMB data. However, since we can expect more precise measurements of CMB in the near future, the prior on Y_p may become important and can affect the constraints on other cosmological parameters. We study this issue in the next section.

IV. FUTURE CONSTRAINT FROM THE PLANCK EXPERIMENT

In this section, we forecast a constraint for the Planck experiment [21] focusing on the constraint on Y_p itself and how the prior on Y_p affects the constraints on other cosmological parameters. In fact, constraints from the future Planck experiment from this viewpoint have already been

TABLE II. Mean values and 68% errors from current observations of CMB for the cases with WMAP5 alone and all data combined. (Regarding Y_p, an upper bound at 95% C.L. is given for the case with WMAP5 alone.) In the last column, the value of Y_p is fixed as $Y_p = 0.24$.

Parameters	WMAP5 alone (Y_p free)	CMB all (Y_p free)	CMB all ($Y_p = 0.24$)
ω_b	0.0228 ± 0.0006	0.0229 ± 0.0005	$0.0229^{+0.0006}_{-0.0005}$
ω_c	$0.109^{+0.006}_{-0.009}$	$0.113^{+0.006}_{-0.007}$	$0.112^{+0.005}_{-0.006}$
θ_s	$1.040^{+0.004}_{-0.006}$	1.043 ± 0.004	1.043 ± 0.003
τ	$0.088^{+0.016}_{-0.018}$	$0.087^{+0.016}_{-0.018}$	$0.087^{+0.016}_{-0.018}$
n_s	$0.964^{+0.016}_{-0.018}$	$0.967^{+0.016}_{-0.015}$	$0.966^{+0.013}_{-0.014}$
$\ln(10^{10}A_s)$	3.06 ± 0.06	$3.08^{+0.04}_{-0.05}$	3.07 ± 0.04
Y_p	$<0.44(95\%)$	$0.25^{+0.10}_{-0.07}$	...
A_{SZ}	$1.1^{+0.9}_{-0.3}$	$1.1^{+0.9}_{-0.3}$	$1.0^{+1.0}_{-0.4}$
Ω_m	0.25 ± 0.03	0.27 ± 0.03	$0.26^{+0.02}_{-0.03}$
H_0	$72.3^{+2.6}_{-2.8}$	$71.7^{+2.2}_{-2.6}$	$71.7^{+2.3}_{-2.5}$

discussed in Refs. [10,12] by using the Fisher matrix analysis. As mentioned in the Introduction, when the likelihood of cosmological parameters can be approximated by a multivariate Gaussian function, the Fisher matrix analysis can give a reliable prediction. However, in practice, the likelihood function deviates from the Gaussian form. Furthermore, since the Fisher matrix analysis can only predict the uncertainty for a fiducial value, it cannot extract a bias effect (i.e., the estimated central value deviates from the fiducial value) which is caused by assuming priors on parameters and possible correlations among parameters. Thus it may be better to make a more reliable prediction by using a MCMC method. For this purpose, we follow the approach of Ref. [29].

Here we briefly explain the method of Ref. [29]. Observed anisotropies can be expanded in spherical harmonics and their power spectra of the coefficients a^P_{lm} are composed of signal parts $C_l^{PP'}$ and noise parts $N_l^{PP'}$:

$$\langle a^{P*}_{lm} a^{P'}_{l'm'} \rangle = (C_l^{PP'} + N_l^{PP'})\delta_{ll'}\delta_{mm'}, \quad (9)$$

where PP' represents three pairs of maps, TT, EE, and TE. The signal parts are computed from a fiducial cosmology. We assume the cosmological parameters of the WMAP5 mean values for a power-law ΛCDM model as a fiducial model. As for the noise power spectra, we assume a Gaussian beam and a spatially uniform Gaussian white noise. $N_l^{PP'}$ are given as the combined effects from these and can be approximated as

$$N_l^{PP'} = \delta_{PP'}(\theta_{\rm FWHM}\sigma^P)^2 \exp\left[l(l+1)\frac{\theta^2_{\rm FWHM}}{8\ln 2}\right], \quad (10)$$

where $\theta_{\rm FWHM}$ is the full width at half maximum of the Gaussian beam and σ^P is the root mean square of the instrumental noise. For the expected data from the Planck experiment, we use three frequency channels at 100, 143, and 217 GHz. We adopt the following values for the instrumental parameters [29]: ($\theta_{\rm FWHW}$[arcmin], $\sigma_T[\mu{\rm K}]$, $\sigma_P[\mu{\rm K}]$) = (9.5, 6.8, 10.9), (7.1, 6.0, 11.4), and (5.0, 13.1, 26.7) for $\nu = 100$, 143, and 217 GHz, respectively. We assume other frequency channels are used to remove foregrounds and they are ideally removed.

Since the anisotropies from both signal and noise are Gaussian distributed, the likelihood function of the data $\mathbf{a} = \{a^{T,E}_{lm}\}$ for a theoretical model with parameters $\mathbf{\Theta} = \{\theta_i\}$ is given by

$$\mathcal{L}(\mathbf{a}|\mathbf{\Theta}) \propto \frac{1}{\sqrt{\bar{C}(\mathbf{\Theta})}} \exp\left(-\frac{1}{2}\mathbf{a}^*[\bar{C}(\mathbf{\Theta})]^{-1}\mathbf{a}\right), \quad (11)$$

where $\bar{C}(\mathbf{\Theta})$ is a covariance matrix of the theoretical data. Denoting a covariance matrix of mock data as $\hat{C}$, the effective χ^2 is given as

$$\chi^2_{\rm eff} = \sum_l (2l+1) f_{\rm sky}\left[\ln\frac{|\bar{C}_l|}{|\hat{C}_l|} + \hat{C}_l\bar{C}_l^{-1} - 2\right]. \quad (12)$$

We take $f_{\rm sky} = 0.65$ as the expected sky coverage for the Planck experiment. The factor $(2l+1)f_{\rm sky}$ represents the effective number of independent moments obtained from the observation. For the MCMC analysis, we include the data up to $l = 2500$.

Before presenting our results, here we comment on possible contributions from the thermal and kinetic SZ effect. We assume that the thermal SZ effect can be precisely estimated from the other lower frequency channels of the Planck survey than those used in our analysis, and can be removed ideally. The contribution from the kinetic SZ effect on CMB anisotropy depends on the details of the reionization process. For a somewhat conventional scenario, as argued in [30], it would be only about a few percent in the range of multipoles we make use of, $l \leq 2500$, and also sufficiently smaller than the expected instrumental noise for the Planck survey. Reference [31] argued that "patchy" reionization would make it significantly larger but they found that the shape of the power spectrum due to the kinetic SZ effect does not depend much on the reionization model. Then, they concluded that its effect on the determination of the cosmological parameters can be neglected by marginalizing over the amplitude of the kinetic SZ power spectrum. We thus neglect the kinetic SZ effect here.

Now we discuss a future constraint on Y_p from the Planck experiment. In Fig. 3, a one-dimensional marginal-

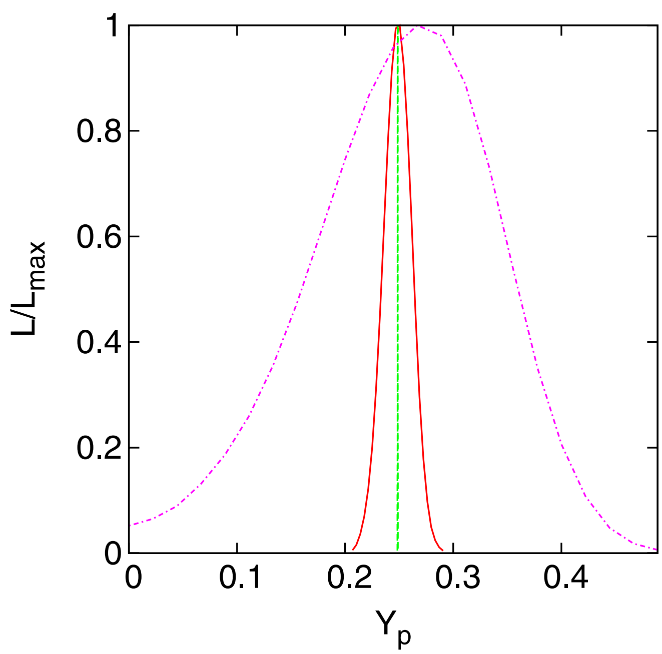

FIG. 3 (color online). One-dimensional marginalized distributions of Y_p. Shown are the distributions from the Planck experiment for the cases with no priors on Y_p (red solid line) and assuming the BBN relation (green dashed line). For reference, the distribution for the case with no priors on Y_p using current CMB data is also shown (dash-dotted magenta line).

TABLE III. Mean values and 68% errors from the Planck experiment for some assumptions on Y_p.

Parameters	Y_p free	SBBN $Y_p(\omega_b)$	$Y_p = 0.24$
ω_b	$0.022\,73^{+0.000\,24}_{-0.000\,25}$	$0.022\,73^{+0.000\,17}_{-0.000\,17}$	$0.022\,61^{+0.000\,16}_{-0.000\,17}$
ω_c	$0.1098^{+0.0015}_{-0.0014}$	$0.1099^{+0.0015}_{-0.0014}$	$0.1100^{+0.0013}_{-0.0016}$
θ_s	$1.040\,63^{+0.000\,57}_{-0.000\,61}$	$1.040\,61^{+0.000\,37}_{-0.000\,36}$	$1.040\,31^{+0.000\,34}_{-0.000\,38}$
τ	$0.0879^{+0.0054}_{-0.0062}$	$0.0880^{+0.0055}_{-0.0060}$	$0.0871^{+0.0049}_{-0.0061}$
n_s	$0.9627^{+0.0079}_{-0.0085}$	$0.9631^{+0.0046}_{-0.0042}$	$0.9580^{+0.0042}_{-0.0044}$
$\ln(10^{10}A_s)$	$3.064^{+0.011}_{-0.013}$	$3.065^{+0.010}_{-0.013}$	$3.061^{+0.010}_{-0.012}$
Y_p	$0.248^{+0.014}_{-0.011}$	$0.248\,586^{+0.000\,078}_{-0.000\,076}$	...
Ω_m	$0.2567^{+0.0080}_{-0.0086}$	$0.2565^{+0.0073}_{-0.0080}$	$0.2587^{+0.0074}_{-0.0083}$
H_0	$71.88^{+0.80}_{-0.85}$	$71.92^{+0.78}_{-0.66}$	$71.61^{+0.73}_{-0.72}$

ized likelihood for Y_p is shown. For comparison, we also plot the constraint from current observations. We can expect that the uncertainty for Y_p at 68% C.L. becomes as $\Delta Y_p \sim 10^{-2}$, which is 10 times smaller than that from current data (see Table III below). Since the Planck experiment can measure the CMB power spectrum at higher multipoles very precisely, the effects of damping due to Y_p can be well probed. It should also be noted that since likelihood functions for Y_p and other cosmological parameters have almost the Gaussian form, our results here using the MCMC approach are almost the same as those obtained by the Fisher matrix analysis which has already been done in Refs. [10,12]. Thus we found that the Fisher matrix analysis can give a good estimate for Planck data for the parameter set we assumed here. The results here are consistent with those given in Refs. [29,32] in which a forecast on Y_p is investigated using a MCMC analysis too.

Next we discuss the effects of prior on Y_p on the constraints on other cosmological parameters in the Planck experiment. As mentioned in the Introduction, when one

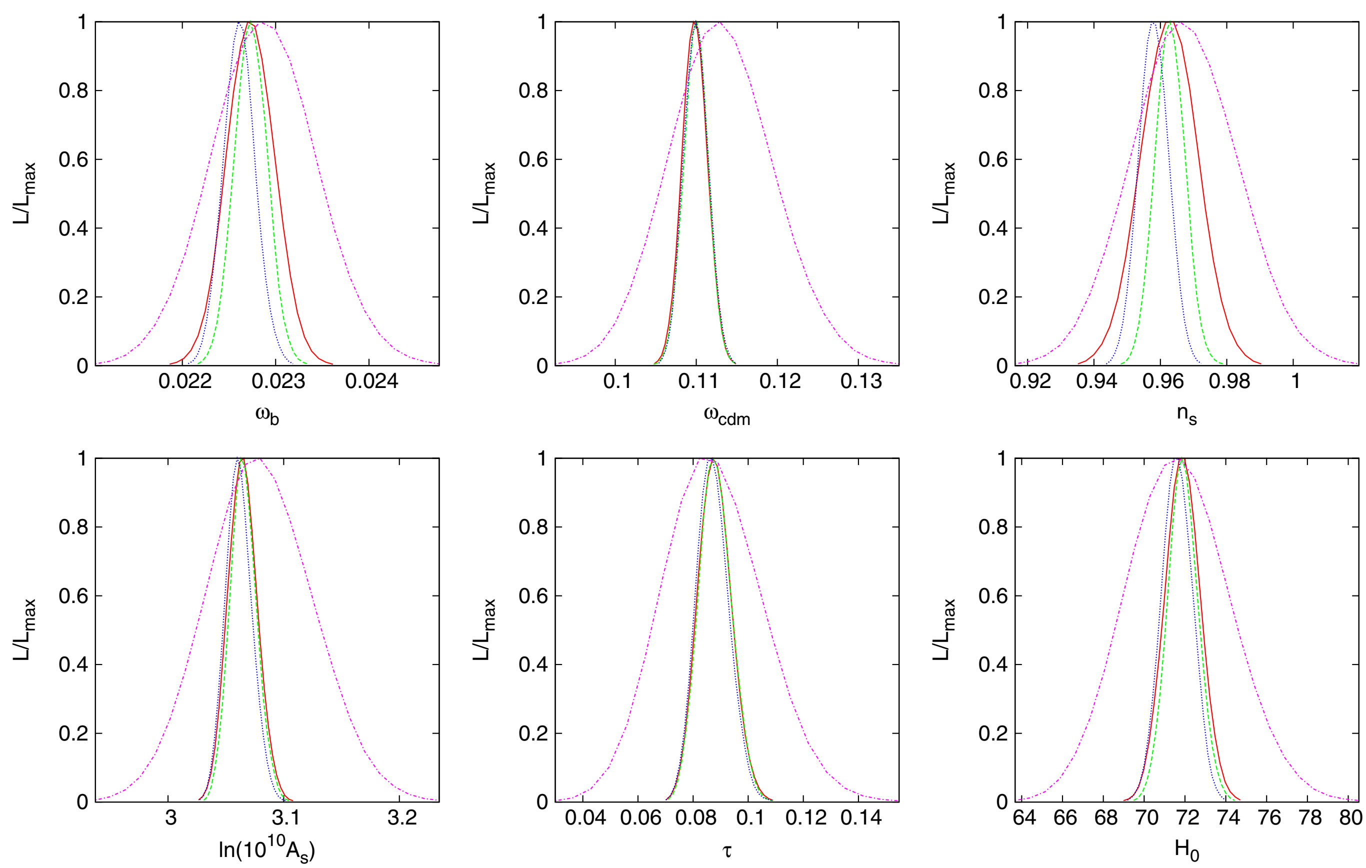

FIG. 4 (color online). One-dimensional marginalized distributions of ω_b, ω_c, n_s, A_s, τ, and H_0, using the same data as in Fig. 3. Additionally, the distributions from the Planck experiment for the case with fixing $Y_p = 0.24$ are also shown (dotted blue line).

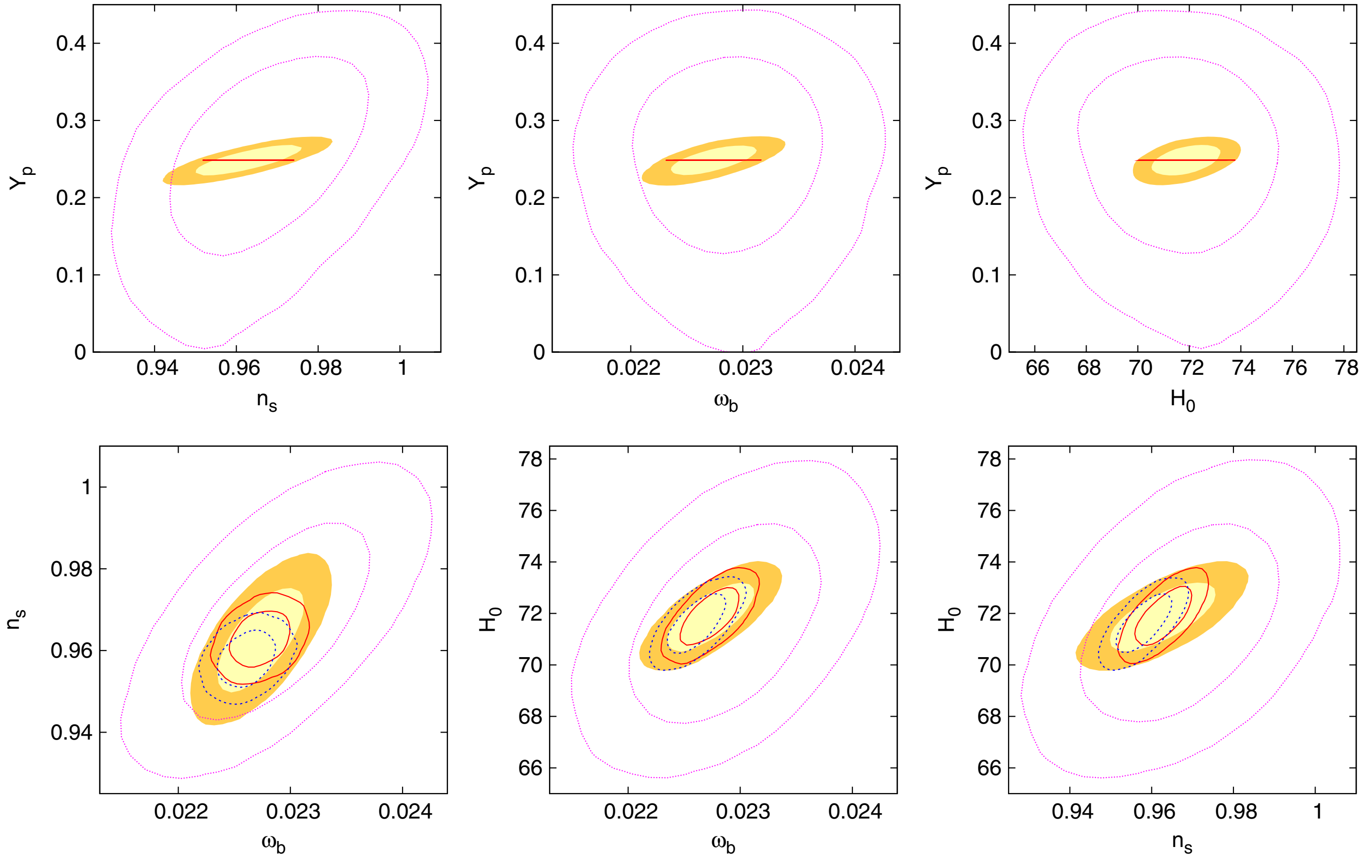

FIG. 5 (color online). Two-dimensional marginalized constraints in several combinations of cosmological parameters. Shown are a Planck forecast on the constraints for the cases with no prior on Y_p (orange and yellow shaded regions), assuming the BBN relation (solid red line) and fixing $Y_p = 0.24$ (blue dotted line). For reference, the constraint for the case with no prior on Y_p using current CMB data is also shown (dash-dotted magenta line).

tries to constrain some cosmological parameters from CMB, the value of Y_p is fixed to be 0.24 in most of the analysis. In the previous section, we showed that, when we use current cosmological data, the fixing of $Y_p = 0.24$ does not affect much the constraints on other cosmological parameters since the value of Y_p itself is not constrained well. However, as just shown above, the Planck experiment can measure the value of Y_p much more precisely, thus we should study the effects of the assumption of Y_p when we constrain other parameters. For this purpose, we made MCMC analyses for three cases: (i) Y_p is not fixed but varied freely, (ii) Y_p is fixed as $Y_p = 0.24$, and (iii) Y_p is regarded as a function of ω_b via the standard BBN calculation. For case (iii), we relate the value of Y_p to ω_b by the fitting formula given in [33], which we refer to as the "BBN relation" in the following.

Now we show a one-dimensional marginalized likelihood for ω_b, ω_c, n_s, A_s, τ, and H_0 in Fig. 4. In the figure, three cases (i), (ii), and (iii) are depicted. In Table III, the mean values and errors at 68% C.L. are shown for representative parameters. By looking at Fig. 4, some features can be noticed. For ω_c, A_s, and τ, the effects of the prior on Y_p are very small even with the precision of the Planck experiment. However, for ω_b, n_s, and H_0, marginalized distributions are changed depending on the prior on Y_p. This tendency can also be seen by reading the errors at 68% C.L. from Table III. For ω_b, n_s, and H_0, when we assume the BBN relation or fix the value of the helium abundance as $Y_p = 0.24$, the errors are reduced to some extent, which clearly indicates that the assumption of Y_p can affect the determination of other cosmological parameters. Furthermore, for these parameters, when we fix $Y_p = 0.24$, the central values differ from the fiducial values by about the uncertainties at 68% C.L.[4] Therefore, in the Planck era, we advocate varying the value of Y_p freely in the cosmological parameter estimation for a conservative constraint, or, if we would like to do the cosmological parameter estimation in the framework of the standard cosmology, we should impose the BBN relation.

To see how these parameters are correlated with Y_p, 2D marginalized contours may be useful, which are shown in

[4]A similar analysis was done in Ref. [32] recently and their results are consistent with ours.

TABLE IV. Comparison for the cases with and without uncertainties from the recombination process being considered. In the first two columns Y_p is treated as a free parameter and in the latter two columns the SBBN relation is assumed.

	Y_p free		SBBN $Y_p(\omega_b)$	
Parameters	$F_{\rm H}$, $b_{\rm He}$ fixed	$F_{\rm H}$, $b_{\rm He}$ free	$F_{\rm H}$, $b_{\rm He}$ fixed	$F_{\rm H}$, $b_{\rm He}$ free
ω_b	$0.022\,73^{+0.000\,24}_{-0.000\,25}$	$0.2273^{+0.000\,25}_{-0.000\,24}$	$0.022\,73^{+0.000\,17}_{-0.000\,17}$	$0.002\,273^{+0.000\,16}_{-0.000\,18}$
ω_c	$0.1098^{+0.0015}_{-0.0014}$	$0.1098^{+0.0014}_{-0.0016}$	$0.1099^{+0.0015}_{-0.0014}$	$0.1098^{+0.0014}_{-0.0016}$
θ_s	$1.040\,63^{+0.000\,57}_{-0.000\,61}$	$1.040\,66^{+0.000\,60}_{-0.000\,61}$	$1.040\,61^{+0.000\,37}_{-0.000\,36}$	$1.040\,64^{+0.000\,36}_{-0.000\,39}$
τ	$0.0879^{+0.0054}_{-0.0062}$	$0.0882^{+0.0052}_{-0.0062}$	$0.0880^{+0.0055}_{-0.0060}$	$0.0880^{+0.0050}_{-0.0064}$
n_s	$0.9627^{+0.0079}_{-0.0085}$	$0.9628^{+0.0091}_{-0.0081}$	$0.9631^{+0.0046}_{-0.0042}$	$0.9626^{+0.0046}_{-0.0050}$
$\ln(10^{10}A_s)$	$3.064^{+0.011}_{-0.013}$	$3.064^{+0.013}_{-0.014}$	$3.065^{+0.010}_{-0.013}$	$3.064^{+0.012}_{-0.012}$
Y_p	$0.248^{+0.014}_{-0.011}$	$0.249^{+0.013}_{-0.012}$	$0.248\,586^{+0.000\,078}_{-0.000\,076}$	$0.248\,583^{+0.000\,087}_{-0.000\,058}$
Ω_m	$0.2567^{+0.0080}_{-0.0086}$	$0.2564^{+0.0081}_{-0.0088}$	$0.2565^{+0.0073}_{-0.0080}$	$0.2565^{+0.0080}_{-0.0081}$
H_0	$71.88^{+0.80}_{-0.85}$	$71.91^{+0.86}_{-0.82}$	$71.92^{+0.78}_{-0.66}$	$71.89^{+0.75}_{-0.75}$

Fig. 5. From this figure, we can see that Y_p and these parameters ω_b, n_s, and H_0 are positively correlated. Positive correlations of Y_p with n_s mainly come from the degeneracy at higher multipoles where the effect of the diffusion damping is significant. From Eqs. (5)–(8), the responses of H_i for $i \geq 2$ to the change of Y_p and n_s are opposite sign, which indicates that the correlation between these are positive. Correspondingly, ω_b and Y_p become positively correlated because of the positive correlation between ω_b and n_s which can be read off, in particular, from Eq. (4). For the correlation of Y_p with H_0, it should be noticed that the position of the first peak can be significantly affected by changing n_s and H_0. By increasing the value of Y_p, the diffusion damping suppresses the power on small scales. To compensate this effect to fit the data well, increasing n_s can enhance the power. Because of the change of n_s, the first peak position is in turn also shifted. However, as is already well determined by WMAP, the position of the first peak should be tuned to the right position to fit the data well. This can be done by changing H_0 as can be seen from Eq. (3). In fact, the change of Y_p itself can also shift the peak position. However the direct effect of Y_p on l_1 is very small compared to other quantities.

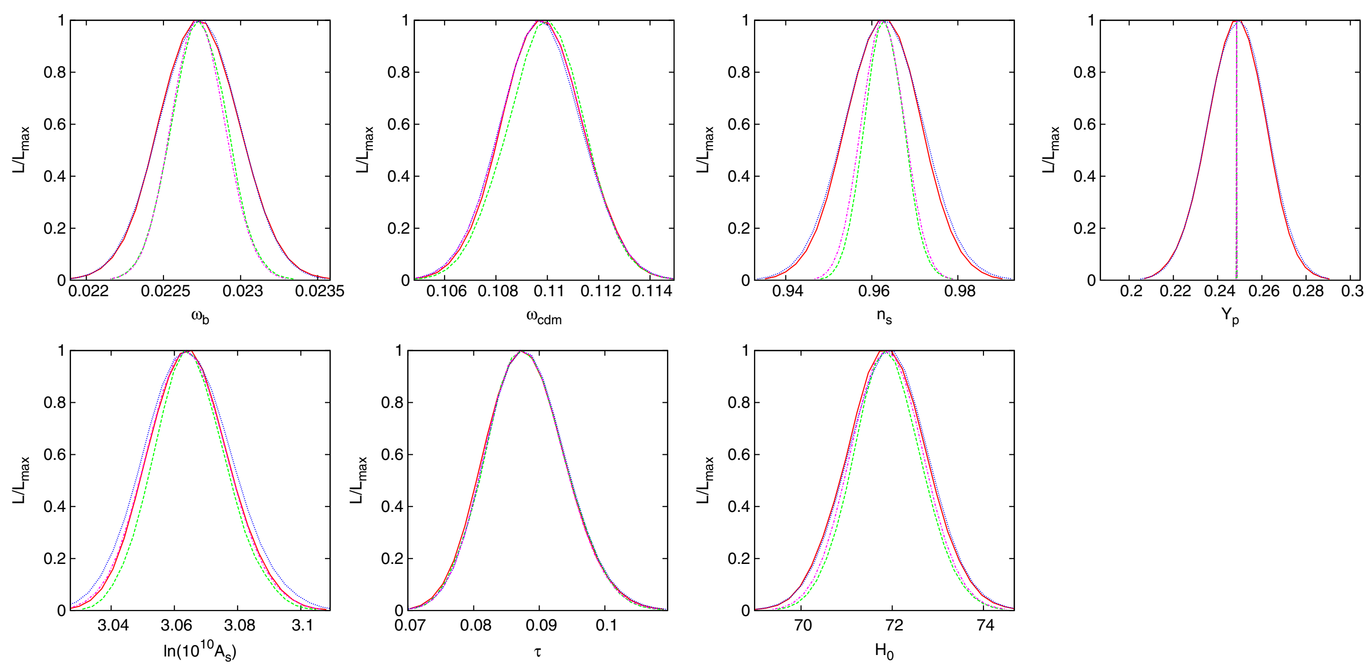

FIG. 6 (color online). Comparison for the cases with and without uncertainties from the recombination process being considered. Shown are the cases with Y_p being treated as a free parameter and the fudge factors being fixed to the standard values (red solid lines), Y_p being treated as a free parameters and the fudge factors being treated as free parameters (blue dotted lines), Y_p from the SBBN relation and the fudge factors being fixed to the standard values (green dashed lines), and Y_p from the SBBN relation and the fudge factors being treated as free parameters (magenta dot-dashed lines).

Finally, we briefly discuss how our arguments above are affected by considering the uncertainties arising from the recombination process. Several authors have claimed that the uncertainties in the theory of recombination make some effects on the CMB power spectra and the determination of the cosmological parameters from CMB [34,35]. Although some detailed studies of the recombination modeling have been done [36–41], more developments of the recombination modeling may be needed [35]. Since the primordial helium abundance, which we are studying in this paper, directly affects the recombination history, it may be interesting to check how the uncertainties in the recombination modeling affect the determination of Y_p and other cosmological parameters in the future Planck experiment. For this purpose, we treat two numerical parameters, the so-called fudge factors $F_{\rm H}$ and $b_{\rm He}$ used in RECFAST [35,42,43] as free parameters to represent the uncertainties of the recombination modeling. $F_{\rm H}$ is introduced to fit the hydrogen recombination rate in the three-level approximation to the result from multilevel calculations. $b_{\rm He}$ is a fitting parameter for the recombination rate of HeI. We repeated a MCMC analysis including $F_{\rm H}$ and $b_{\rm He}$ and marginalized over these parameters with the top-hat priors given in Table I, which are very conservative ones. In Table IV we summarize the resultant constraints on the cosmological parameters from expected Planck data. We also show the probability distributions for several cosmological parameters in Fig. 6. Table IV shows that, even if we adopt a very conservative prior on the fudge factors, the uncertainties of the recombination modeling which is represented by $F_{\rm H}$ and $b_{\rm He}$ do not significantly affect the determination of cosmological parameters in the Planck era. (Errors for some parameters are changed at most by 10%.) Therefore, we can say at least that the theoretical uncertainties of the recombination process which are discussed recently do not affect our previous discussions. However, since our analysis is limited, we need more understanding of the recombination process and more detailed analysis in order to reduce systematic errors in the helium estimation from future CMB data.

V. SUMMARY

We studied the constraint on Y_p and the effects of the priors for Y_p on constraining other cosmological parameters using current CMB data from WMAP5, ACBAR, BOOMERANG, and CBI, and also from the future Planck experiment. After briefly reviewing the effects of Y_p on CMB, we studied current constraints on the primordial helium abundance. We obtained the current limit on Y_p from WMAP5 alone as $Y_p \leq 0.44$ at 95% C.L., which is improved to be $Y_p = 0.25^{+0.10(+0.15)}_{-0.07(-0.17)}$ at 68% (95%) C.L. by adding the data of ACBAR, BOOMERANG, and CBI around the damping tail. We have also considered how the prior of Y_p can affect the constraints on other cosmological parameters using currently available data. We found that, at the present precision level of CMB measurements, the prior on Y_p has little effect for determinations of other cosmological parameters.

We have also investigated the future constraint from the Planck experiment. By performing a MCMC analysis, we derived an expected error for the helium abundance from the future Planck experiment and found that it will be well measured with the accuracy of $\Delta Y_p \sim 10^{-2}$ (68% C.L.) in the Planck experiment, which is 10 times smaller values compared with current data. Furthermore, it may be interesting to notice that this precision is comparable to that obtained by HII region observations. As for the effects of the prior on Y_p on the determination of other cosmological parameters, we found that, with the precision of Planck, the assumption on Y_p can affect the constraints on other cosmological parameters such as ω_b and n_s. In this respect, the prior on Y_p can be important for determining the other parameters. In addition, the constraint on Y_p from CMB itself can be an independent test from other methods such as using the HII region observations.

In the near future, we can have more precise measurements of CMB. Such upcoming data would give us more precise information of the primordial helium abundance. At the same time, it is necessary to study the effects of the helium abundance more rigorously in order to extract information of other cosmological parameters.

ACKNOWLEDGMENTS

This work is supported in part by the Sumitomo Foundation (T. T.) and the Grant-in-Aid for Scientific Research from the Ministry of Education, Science, Sports, and Culture of Japan, No. 18840010 (K. I.) and No. 19740145 (T. T.). T. S. would like to thank the Japan Society for the Promotion of Science for financial support.

Note added.—While we were finishing the present work, Ref. [32] appeared on the arXiv, which has some overlap with our analysis on the constraints from the future Planck experiment.

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