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## Proposal for a geophysical search for dilatonic waves

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We propose a new method of searching for the composition-dependent dilatonic waves, predicted by unified theories of strings. In this method, Earth's surface-gravity changes due to translational motions of its inner core, excited by dilatonic waves, are searched for by using superconducting gravimeters. This method has its best sensitivity at the frequency of  $\sim 7 \times 10^{-5}$  Hz, which is lower than the sensitive frequencies of previous proposals using gravitational wave detectors:  $\sim 10$  to 1000 Hz. Using available results of surface-gravity measurements with superconducting gravimeters and assuming a simple Earth model, we present preliminary upper limits on the energy density of a stochastic background of massless dilatons at the low frequency. Though the results are currently limited by the uncertainty in the Earth model, this method has a potential of detecting dilatonic waves in a new window.

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#### I. INTRODUCTION

The scalar gravitational fields (or the dilatons) appear in scalar-tensor theories of gravitation and also in theories toward the unification of fundamental forces in nature, such as Kaluza-Klein, supergravity and string theories. Scalar-tensor theories must agree with general relativity within an accuracy of  $\sim 10^{-4}$ – $10^{-5}$  in the post-Newtonian limit, because of the results from the Cassini time-delay experiment [1] and the search for the Nordtvedt effect (i.e. violations of the universality of free-fall for massive bodies [2]) [3]. However, in strong gravity systems, in which the post-Newtonian approximation is not applicable, they could exhibit large observable deviations from general relativity [4], and other tests involving a strong gravity source are necessary. One such tests is to search for scalar waves from a spherical symmetric gravitational collapse of a star [5,6], which are predicted by scalar-tensor theories but not by general relativity.

In many models of unified theories, the scalar field mediates a new force (or a fifth force [7]), which leads to a violation of the universality of free-fall and/or a deviation from the inverse-square law of gravitation. Such deviations from the inverse-square law and violations of the universality of free-fall have been searched for and constrained by various experiments at different ranges (see e.g. [7–9]). However, string theory, one of the most promising theories for the unification, predicts the existence of a relic background of the dilaton, which could be a significant component of the dark matter in the present universe [10]; though the fifth-force searches indicate that the dilaton coupling to matter is very weak, the weakness of the coupling could be compensated by the high intensity of the background. Therefore, the detection of the scalar waves or a direct experimental constraint on the amplitude of the scalar

waves would play an important role for the development of the unified theories and verification of theory of gravitation.

Motivated by these predictions, strategies of detecting the scalar waves have been studied (see e.g. [5,11–19]). These analyses show that some of the predicted scalar waves could be detected by using currently operating or advanced planned ground-based gravitational wave detectors. However, no experimental results have been presented yet. Here, we propose a new approach of searching for scalar waves, using superconducting gravimeters, and we tentatively estimate upper limits on the energy density of a stochastic background of dilatonic waves. The concept of this method (described in detail later) is similar to the one of the geophysical test of the universality of free-fall [20] in the way that both methods use the Earth as the test body and superconducting gravimeters as the displacement sensor.

Dilatonic waves can interact with detectors in two ways [21]: (i) directly, through the effective dilatonic charges of the detectors, which depend on the internal composition of the detectors, and (ii) indirectly, through the geodesic coupling of the detectors to the scalar component of the metric fluctuations. In the former case, the response of the detectors to the dilatonic waves is nongeodesic [21]. The proposed method is to seek for an effect of the direct coupling.

The sensitive frequency range of ground-based gravitational wave detectors is about 10 to 1000 Hz, while it is about  $7 \times 10^{-5}$  Hz for the proposed method; a new window can be explored with this method. According to a cosmological scenario motivated by string theory, a relic background of light dilatons, with mass as small as  $\sim 10^{-19}$  eV or less (depending on the level of the flatness of the background spectrum), could be a significant component of the dark matter (see Sect. 6 of [10] and references therein). The sensitive frequency range of the

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Earth

proposed method corresponds to the mass range of  $\sim 3 \times 10^{-19}$  eV; the possible cosmological scenario could be tested with this method. However, in this paper, we focus on the massless dilaton for simplicity.

### II. THE CONCEPT

In this method, we consider the Earth's inner core as the receiver of dilatonic waves. The inner core, enclosed in its liquid outer core, is weakly coupled to the rest part of the Earth mainly by gravitational forces. When a gravitational wave impinges on the Sun and the Earth, from any direction that is not aligned with the Sun-Earth line, their proper separation oscillates. Gravitational waves couple to matter in the universal way; to first order, the propagation of gravitational waves would not cause relative motions between the inner core and the rest part of the Earth. However, when dilatonic waves pass, because of the difference in dilatonic charge (namely, the difference in their chemical composition), there would be relative motions between the inner core and the rest part of the Earth (see next section for detail). Such relative motions would result in changes of the gravitational field at the surface of the Earth. The surface-gravity changes can be searched for by superconducting gravimeters, which are the most sensitive instruments for measurements of gravity changes at low frequencies.

The dilatonic charges of the inner core and the rest part of the Earth are different because of the difference in their chemical composition (to be discussed in detail later); the inner core mainly consists of iron and nickel, while the rest part of the Earth is mainly made of lighter elements such as silicon oxides.

The response of the inner core to the dilatonic waves could be greatly enhanced when the waves have the same frequencies as the natural oscillation frequencies of the inner core.

# III. RESPONSE OF THE INNER CORE TO DILATONIC WAVES

For simplicity, we assume a simple Earth model and configuration as shown in Fig. 1. A dilatonic wave propagates along the Sun-Earth line, which connects the centers of mass of the Sun and the Earth. Their rest separation is  $L \approx 1.5 \times 10^{11}$  m.

The Earth's interior can be classified into four parts: the solid inner core, the liquid outer core, the mantle, and the crust. We assume that the solid inner core is a homogeneous sphere (with density  $\rho_{\rm ic} \simeq 1.29 \times 10^4~{\rm kg}~{\rm m}^{-3}$  and radius  $r_{\rm ic} \simeq 1.22 \times 10^6~{\rm m}$  [22]) and it is enclosed in the spherical liquid outer core (with the density of the fluid at the inner core boundary  $\rho_{\rm oc} \simeq 1.22 \times 10^4~{\rm kg}~{\rm m}^{-3}$  [22]). We do not consider any deformations (such as tidal and rotational deformations). We assume that the mantle and the crust are spherical shells with uniform densities, and

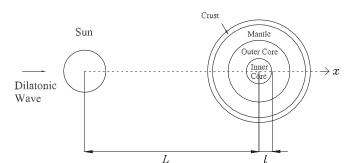


FIG. 1. A schematic cross section of the assumed configuration (not drawn to scale). As a dilatonic wave passes along the Sun and the Earth (separated by  $L \approx 1.5 \times 10^{11}$  m), the inner core oscillates along the Sun-Earth line (amplitude l). The surface-gravity changes due to the inner core oscillations are searched for by superconducting gravimeters. We do not consider the influence of the Earth's rotation on the inner core for simplicity (see text).

their centers of figures are coincident; their gravitational influence on the inner core is negligible due to Newton's shell theorem. We do not consider any electromagnetic effects. Also, we assume that the friction (the friction coefficient  $\gamma$ ) between the inner core and the outer core is proportional to the velocity of the inner core.

By applying the generalized equation of geodesic deviation (Eq. (25) of [21]), which includes the dilaton corrections to the standard equation of geodesic deviation [23], the translational motion of the inner core, relative to the rest part of the Earth, can approximately be given by

$$\ddot{l}^{i} \approx -\gamma \dot{l}^{i} - \omega_{0}^{2} l^{i} - \Delta q (L^{k} \partial_{k} + 1) \partial^{i} \phi, \tag{1}$$

where i and k indicate the space-time components of the parameters,  $\Delta q$  is the difference in dilatonic charge between the inner core and the rest part of the Earth [for the definition, see Eq. (5) below], L is the rest separation between the Sun and the Earth (see Fig. 1), and  $\phi$  is the dilaton field. The gravitational stiffness [24] and the friction coefficient are, respectively, given by

$$\omega_0^2 \approx \frac{4}{3}\pi G \frac{\rho_{\rm ic} - \rho_{\rm oc}}{\rho_{\rm ic}} \rho_{\rm oc} \approx 1.8 \times 10^{-7} \text{ s}^{-2}$$
  
 $\approx \{2\pi (4.1 \text{ h})^{-1}\}^2,$  (2)

$$\gamma \equiv \frac{6\pi\eta r_{\rm ic}}{m_{\rm ic}} \approx 2.3 \times 10^{-16} \eta \text{ s}^{-1},$$
 (3)

where  $G=6.67\times 10^{-11}~{\rm N\,m^2\,kg^{-2}}$  is the gravitational constant,  $m_{\rm ic}\approx 9.8\times 10^{22}~{\rm kg}$  is the mass of the inner core, and  $\eta$  is the effective viscosity of the outer core. The stiffness due to the Sun's tidal force and the Moon's tidal force, which act to enlarge any displacement of the inner core from the center of the Earth, is negligible.

We ignore the influence of the Earth's rotation on the inner core. The drag associated with the Earth's rotation takes its maximum value when it is balanced with the Coriolis force ( $\sim 2m_{\rm ic}\omega_R\omega_0l_0$ , where  $\omega_R\approx 2\pi(24\,{\rm h})^{-1}$  is the angular frequency of the Earth's rotation and  $l_0$  is a nominal magnitude of the inner-core displacement), which is about 30% of the gravitational restoring force ( $m_{ic}\omega_0^2l_0$ ); for the liquid outer core, the drag would be less than the maximum value. The central force should be less than 2% of the gravitational restoring force.

The last term of Eq. (1) is due to the direct coupling to the gradients of the dilaton field  $\phi$ .  $\Delta q$  is the difference in dilatonic charge between the inner core and the rest part of the Earth. For ordinary matter, the dilatonic charge can be obtained by summing over all the components n of the matter [21]:  $q = \frac{\sum_n m_n q_n}{\sum_n m_n}$ ; q is defined as the relative strength of scalar to gravitational forces. For a body (mass M) composed by B barions with mass  $m_b$  and (dimensionless) fundamental charge  $q_b$ , and  $Z(\cong B)$  electrons with mass  $m_e$  and (dimensionless) fundamental charge  $q_e$ , we obtain (Eq. (22) of [21])

$$q \simeq \frac{Bm_b q_b}{M} \equiv \frac{B}{\mu} q_b, \tag{4}$$

where  $\mu = \frac{M}{m_b}$  is the mass of the body in units of baryonic masses. Therefore,  $\Delta q$  can be given by

$$\Delta q \simeq \left\{ \left( \frac{B}{\mu} \right)_{\text{ic}} - \left( \frac{B}{\mu} \right)_{\text{roe}} \right\} q_b \equiv \Delta \left( \frac{B}{\mu} \right) q_b,$$
 (5)

where ic and rpe denote the inner core and the rest part of the Earth, respectively. The magnitude of  $\Delta(\frac{B}{\mu})$  is about  $10^{-3}$  or less (see, e.g., Table 2.1 of [7]). The upper limit on  $q_b^2$  is determined to  $\sim 1.5 \times 10^{-10}$  by tests of the equivalence principle [9]. Therefore, we obtain  $\Delta q \lesssim 1 \times 10^{-8}$ .

When a dilatonic wave propagates along the Sun-Earth line, the spectrum of the displacement along the line is given by

$$S_l(f_0) = \frac{(\Delta q)^2 (L^2 \kappa^2 + 1) \kappa^2 c^4}{4\omega_0^2 \gamma^2} S_h(f_0)$$
 (6)

at the resonant frequency:  $f_0 \equiv \omega_0/2\pi \approx 6.8 \times 10^{-5}$  Hz, where  $S_h(f)$  is the power spectrum of the strain due to massless dilatonic waves:  $h \equiv 2\phi/c^2$ ;  $\kappa$  is the wave number:  $\kappa = \omega_0/c$ ; and c is the speed of light.

The intensity of a stochastic background of massless dilaton can be characterized by the dimensionless energy density [10]:  $\Omega_{\rm DW}(f) \equiv (1/\rho_c)(d\rho_{\rm DW}/d\log f)$ , where  $\rho_c = 3H_0^2c^2(8\pi G)^{-1}$  is the present value of the critical energy density for closing the Universe;  $H_0$  is the Hubble expansion rate:  $H_0 = 3.2 \times 10^{-18} h_{100} \ {\rm s}^{-1}$ ;  $h_{100}$ , ranging from  $\sim$ 0.5 to 1, is a dimensionless factor to account for different values of  $H_0$ .

The power spectrum of the strain is related to the dimensionless energy density by the following expression [18]:

$$\Omega_{\rm DW}(f) = \frac{\pi^2 f^3}{3H_0^2} S_h(f) \tag{7}$$

for massless dilaton. Here the stochastic dilatonic background is assumed to be isotropic, unpolarized, and stationary [18].

### IV. ESTIMATION OF THE UPPER LIMITS

Coriolis acceleration splits the inner core oscillation into a triplet of periods (the Slichter triplet [25]). To determine physical properties of the core, the three translational mode signals (prograde equatorial, axial, retrograde equatorial) have been searched for in geophysics (e.g. [26–33]), but the detection has not been confirmed yet. We use a recent analysis by Rosat *et al.* [31] to estimate the upper limit on the magnitude of the inner-core displacement.

Rosat *et al.* examined gravity data obtained over the year 2001 from global observatories (in Canada, Australia, Japan, France, and South Africa) by applying a multistation stacking method [31]. They found no significant peaks that could originate from the translational motions of the inner core. The average noise level and the standard intervals were  $(4.2 \pm 1.6) \times 10^{-12}$  m s<sup>-2</sup> Hz<sup>-1/2</sup> at  $\sim$ 7 ×  $10^{-5}$  Hz in their analysis (Fig. 12(b) of [31]). This indicates that the upper limit on the amplitude of each translational mode is  $\delta g = 5.8 \times 10^{-12}$  m s<sup>-2</sup> Hz<sup>-1/2</sup> at the frequency. This upper limit approximately corresponds to a displacement of the inner core:

$$x \approx \frac{r_e^3}{2Gm_{ic}} \delta g \approx 0.11 \text{ mm Hz}^{-1/2}, \tag{8}$$

where  $r_e = 6371$  km [22] is the mean radius of the Earth. We assume that x represents the upper limit on the typical magnitude of the displacement. With the value of x and Eq. (6), we obtain the upper limit on the strain spectrum:

$$\sqrt{S_h(f_0)} \le \frac{2\gamma}{\Delta q \cdot c\sqrt{L^2 \omega_0^2/c^2 + 1}} \sqrt{S_l(f_0)}$$

$$\approx 1.5 \times 10^{-20} \eta \text{ Hz}^{-1/2}.$$
(9)

From Eq. (7) and the upper limit on the strain spectrum, we obtain

$$\Omega_{\rm DW}(f_0)h_{100}^2 < 2.2 \times 10^{-17} \eta^2.$$
(10)

The effective viscosity  $\eta$  is not well determined and estimates from various methods vary from  $\sim \! 10^{-3}$  Pa s to  $\sim \! 10^{12}$  Pa s [34]. The Reynolds number (  $\equiv \rho_{\rm oc} v r_{\rm ic}/\eta$ , where v is the velocity of the inner core) is less than unity when  $\eta$  is larger than  $\sim \! 7 \times 10^2$  Pa s and the amplitude of oscillations  $x (\equiv v/\omega_0)$  is 0.11 mm. We obtain  $\Omega_{\rm DW}(f_0)h_{100}^2 < 1 \times 10^{-11}$  when the Reynolds number is unity and  $\Omega_{\rm DW}(f_0)h_{100}^2 < 2 \times 10^7$  in the worst case when

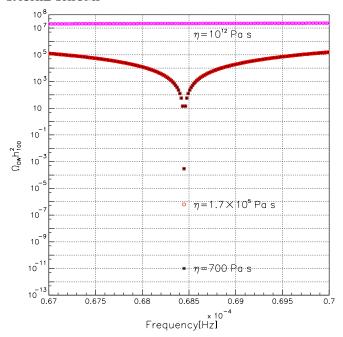


FIG. 2 (color online). Expected upper limits on  $\Omega_{\rm DW}h_{100}^2$  for different values of  $\eta$ . Asterisks, circles, and diamonds indicate expected upper limits for  $\eta = 700$  Pas (when the Reynolds number is unity),  $1.7 \times 10^5$  Pas (the upper bound estimated from nutation data [35]), and  $10^{12}$  Pas (the largest estimate given in [34]), respectively (see text).

the effective viscosity is as large as  $10^{12}$  Pa s. When we use a recent estimate of an upper bound from nutation data [35],  $\eta \leq 1.7 \times 10^5$  Pa s, we obtain  $\Omega_{\rm DW}(f_0)h_{100}^2 < 6 \times 10^{-7}$ . The expected upper limits for those different values of  $\eta$  are shown in Fig. 2.

### V. DISCUSSION

By assuming the simple Earth model and using available geophysical results, we have estimated the upper limits on the dimensionless energy density of the stochastic background of massless dilaton. As one can see in the relation (10), the upper limits largely depend on the magnitude of the effective viscosity of the outer core, which is poorly constrained by experiments. Identification of the magnitude of the friction coefficient is crucial to this method.

There are a number of active researches going on to determine the effective viscosity and other physical parameters of the Earth's interior. Some of them are laboratory measurements of the viscosity at high pressures and temperatures [36], studies for neutrino oscillation tomography of the Earth's interior [37], and coincidence measurements with a laser strainmeter system and a superconducting gravimeter at the Kamioka Observatory in Japan [38]. With these researches employing new technologies, one can expect that our knowledge on the effective viscosity and dynamics of Earth's interior will be improved significantly in the near future.

The dimensionless energy density of a massless background is constrained to  $\Omega h_{100}^2 \lesssim 10^{-5}$ , in the frequency range of our interest, by the nucleosynthesis and resent measurements of the cosmic microwave background [39]. From the relation (10), one can see that, if the effective viscosity is smaller than about  $7 \times 10^5$  Pas, the current sensitivity of this method is sufficient to reach the limit imposed by the astrophysical observations. As for the relic background of light dilatons, mentioned in the introduction, the intensity of the background is not constrained by the astrophysical observations and could compensate the weakness of the dilaton coupling [10]. The possibility of testing the cosmological scenario has to be discussed in the future.

We have assumed that the dilatonic waves are isotropic. However, if they are not isotropic, the magnitude of the inner-core displacement would depend on the location of the observatories. Such location dependence could be investigated through the Global Geodynamics Project network (GGP [40]) of superconducting gravimeters.

Though this method is currently limited by the uncertainty in the Earth model, it is in principle sensitive to any composition-dependent waves at low frequencies. Other possible sources of such waves and their influence on the inner core have to be investigated.

Geophysical sources of the excitation of translational motions of the inner core are not well known. Possible sources of the excitation are, for example, large earth-quakes [41], some dynamic flows [42], and magnetohydrodynamic processes in the core [43]. The effects of earthquakes are thought to be very small [41]. If the translational motions are experimentally confirmed, all possible sources of the excitation have to be considered carefully before estimating the amplitude of the dilatonic waves.

The sensitivity could be improved, for instance, by extracting correlated signals from the GGP network and applying an advanced data analysis method [44]. Also, the measurement sensitivity could be improved by further studies.

### VI. CONCLUSIONS

We have proposed a new method of searching for dilatonic waves. In this method, Earth's surface-gravity changes due to translational motions of the inner core, excited by dilatonic waves, are searched for by using superconducting gravimeters. The main merits of this method may be as follows: (1) unlike the previous proposals, which intend to use gravitational wave detectors, this method employs the geophysical approach with superconducting gravimeters, (2) the sensitive frequency range is low ( $\sim 7 \times 10^{-5}$  Hz), in comparison with the previous proposals ( $\sim 10$  to 1000 Hz), (3) it is devoted to searching for the direct coupling of dilatonic waves to matter, and (4) it has a potential to search for anisotropy in composition-dependent waves, by observing anisotropy

in the oscillation direction of the inner core through the GGP network. The major obstacle of this method is its dependence on the Earth model. This obstacle would diminish in time with the progress on understanding of the Earth's interior.

Assuming the simple Earth model and configuration, we have obtained the preliminary upper limits on the stochastic massless dilatonic background at the frequency of  $f_0 \approx 7 \times 10^{-5}$  Hz. In the worst case when the effective viscosity  $(\eta)$  is as large as  $10^{12}$  Pa s, the upper limit on the dimensionless energy density  $[\Omega_{\rm DW}(f_0)h_{100}^2]$  is  $2 \times 10^7$ . When the Reynolds number is about unity  $(\eta \approx 7 \times 10^2 \ {\rm Pa\,s})$ , the upper limit is  $1 \times 10^{-11}$ . With the recent estimate of the upper bound from nutation data,  $\eta \leq 1.7 \times 10^{-5}$ , we obtain  $\Omega_{\rm DW}(f_0)h_{100}^2 \leq 6 \times 10^{-7}$ .

This method is in principle sensitive to any compositiondependent waves at low frequencies. Other possible sources of such waves have to be investigated. We have focused on the massless dilaton. The possibility of searching for the relic background of light dilatons, predicted as a scenario in string cosmology, has to be discussed in the future

The sensitivity of this method could be increased with further studies, such as a development of coincidence measurements through the GGP network. With the increased sensitivity and refined Earth model, we could test predictions of unified theories and cosmological models in a new window.

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