

***WW* scattering in walking technicolor: No discovery scenarios at the CERN LHC and ILC**

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We analyze the *WW* scattering in scenarios of dynamical electroweak symmetry breaking of walking technicolor type. We show that in these theories there are large regions of the parameters space allowed by the electroweak precision data, in which unitarity is delayed at tree level up to around 3–4 TeV without the inclusion of any sub-TeV resonances. If any of these scenarios is actually realized then the new physical states are not within reach of the LHC and International Linear Collider.

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The simplest argument often used to predict the existence of yet undiscovered particles at the TeV scale comes from unitarity of longitudinal gauge boson scattering amplitudes. If the electroweak symmetry breaking sector (EWSB) is weakly interacting, unitarity implies that new particle states must show up below 1 TeV, being these spin-0 isosinglets (the Higgs boson) or spin-1 isotriplets (e.g. Kaluza-Klein modes). A strongly interacting EWSB sector can however change this picture, because of the strong coupling between the pions (eaten by the longitudinal components of the standard model gauge bosons) and the other bound states of the strongly interacting sector. An illuminating example comes from QCD. In Ref. [1] it was shown that for six colors or more, the 770 GeV ρ meson is enough to delay the onset of unitarity violation of the pion-pion scattering amplitude up to well beyond 1 GeV. Here the 't Hooft large N limit was used, however, an even lower number of colors is needed to reach a similar delay of unitarity violation when an alternative large N limit is used [2]. Scaling up to the electroweak scale, this translates in a 1.5 TeV techni ρ being able to delay unitarity violation of longitudinal gauge boson scattering amplitudes up to 4 TeV or more. Such a particle would be just beyond the reach of LHC and ILC, and a worst case scenario of no discovery of new physics in the upcoming generation of colliders could not be excluded based solely on unitarity arguments. Such a model, however, would not be realistic for other reasons: a large contribution to the S parameter [3], and large flavor changing neutral currents (FCNC) if the ordinary fermions acquire mass via an old fashioned extended technicolor sector, to mention the most relevant ones.

Walking technicolor (WT) [4–6] provides a natural framework to address these problems. In fact, walking dynamics helps suppressing FCNC without preventing extended technicolor sector from yielding realistic fermion masses. Notice that it is always possible to resort to new scalars to give mass to the ordinary fermions while having

technicolor only in the gauge sector, or even marry technicolor with supersymmetry [7–14]. Furthermore, certain WT models are in agreement with the constraints imposed by electroweak precision data [15–17], since the walking dynamics itself naturally lowers the contribution to the S parameter relative to a running theory [18]. Besides, new leptonic sectors [17] that may be needed to avoid possible Witten topological anomalies can render the overall S parameter negative. The contributions from these sectors, not gauged under the technicolor gauge group, are calculable to any order in perturbation theory. A relevant question to ask is whether a walking regime can be achieved with a sufficiently small number of fermions. In the context of $SU(N_{TC})$ gauge theories with fermions in the fundamental representation, a large number N_{TF} of techniflavors is required to achieve walking dynamics, even for small values of N_{TC} . If all technifermions have electroweak charge, this results in a large contribution to S and other electroweak parameters. Recently, however, a new class of $SU(N_{TC})$ gauge theories with fermions in higher-dimensional representations have been argued to display near-to-conformal behavior already for small values of N_{TF} and N_{TC} [19]. A complete catalogue, based on the Schwinger-Dyson approximation, of all possible $SU(N_{TC})$ walking technicolor gauge theories with fermions in a given but arbitrary representation can be found in [20]. An all order β function for any nonsupersymmetric, strongly interacting, and asymptotically free gauge theory has been suggested in [21], thereby allowing new methods for the investigation of nonsupersymmetric conformal windows. The first hints of walking or possibly conformal dynamics for gauge theories with fermions in higher dimensional representations has appeared in [22], and a first benchmark for a working model of WT has been constructed in [16]. Finally, it should be mention that in this framework unification of the standard model gauge couplings can be achieved [14].

WW scattering in walking technicolor.—In order to extract predictions in presence of a strongly interacting sector and an asymptotically free gauge theory, we make use of the time-honored Weinberg sum rules (WSR) [23], which

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are statements about the vector-vector minus axial-axial vacuum polarization functions, known to be sensitive to chiral symmetry breaking. Assuming a low energy spectrum consisting of a narrow vector-vector resonance and a narrow axial-vector resonance, the first WSR reads

$$F_V^2 - F_A^2 = F_\pi^2, \quad (1)$$

where F_V (F_A) is the decay constant of the vectorial (axial) resonance, and F_π is the pion decay constant. In technicolor models $F_\pi = 1/\sqrt{2}G_F = 246$ GeV. The second WSR, unlike the first one, receives important contributions from throughout the near conformal region, and reads [16,18]

$$F_V^2 M_V^2 - F_A^2 M_A^2 = a \frac{8\pi^2}{d(\mathbf{R})} F_\pi^4, \quad (2)$$

where $d(\mathbf{R})$ is the dimension of the representation of the underlying fermions, and a is expected to be positive and $\mathcal{O}(1)$. In the case of running dynamics we obtain $a = 0$, and the second WSR recovers its familiar form. The parameter a is a nonuniversal quantity depending on the details of the underlying gauge theory. Any other approach trying to model the walking behavior will have to reduce to ours. The fact that a is positive and of order one in walking dynamics can be deduced, indirectly, from the work of Kurachi and Shrock [24]. At the onset of conformal dynamics the axial and the vector will be degenerate, i.e. $M_A = M_V = M$, using the first sum rule one finds via the second sum rule $a = d(\mathbf{R})M^2/(8\pi^2 F_\pi^2)$ leading to a numerical value of about 4–5 from the approximate results in [24]. We will however use only the constraints coming from the generalized WSRs expecting them to be less model dependent.

There is also a “zeroth” sum rule, which is nothing but the definition of the S parameter

$$S = 4\pi \left[\frac{F_V^2}{M_V^2} - \frac{F_A^2}{M_A^2} \right]. \quad (3)$$

This is the contribution to the S parameter coming solely from the new strongly coupled dynamics. Using Eqs. (1) and (2) this becomes

$$S = 4\pi F_\pi^2 \left[\frac{1}{M_V^2} + \frac{1}{M_A^2} - a \frac{8\pi^2 F_\pi^2}{d(\mathbf{R}) M_A^2 M_V^2} \right]. \quad (4)$$

The first two terms of this equation are the ordinary QCD-like contributions. The third term is negative and of the same order of the first two. Therefore, if the ρ and a_1 mesons of QCD were resonances of a walking theory, the corresponding S parameter would be considerably lower than the QCD value $S \simeq 0.3$. As mentioned before, this shows that the walking dynamics is not only important in suppressing FCNC, but also in lowering the contribution to the S parameter relative to a theory with a running coupling constant. Since QCD’s S is approximately twice its perturbative estimate, performed by computing loops of dynamical quarks, it appears safe to estimate S for any WT theory

to be of the order of the value calculated from techniquark loops. Recent computations further support the reduction of S in walking theories [25].

As a phenomenologically interesting example we consider minimal walking technicolor (MWT) [16,17,19]. This is an SU(2) gauge theory with two fermions in the adjoint representation. If these form an electroweak doublet, the perturbative contribution to the S parameter will be $S = d(\mathbf{R})/6\pi = 1/2\pi \simeq 0.16$. MWT is arguably the WT model with the lowest S . This value can be further reduced by adding to the theory new leptons (which in MWT are required to cure the Witten anomaly), with weak SU(2) mass splitting. With these ingredients the full estimates for S and T were shown to be within 1σ of the experimental expectation values for a wide portion of the parameter space [16,17,19]. This is true also for the more stringent tests based on the precise LEP parameters of Barbieri *et al.*[26] explored for WT and CT in [15]. Taking $S \in (0.1, 0.3)$ as a realistic estimate in MWT, and using Eqs. (1) and (3) allows us to take F_V and M_A (for example) as the only independent inputs, since F_π is known. In an alternative to F_V we can take the “gauge coupling” g of an effective model [16] in which the vector mesons are treated as gauge fields. F_V is given by

$$F_V^2 = \frac{2M_V^2}{g^2}. \quad (5)$$

From Eqs. (1), (3), and (5) we find

$$M_V^2 = \left(1 - \frac{g^2 S}{8\pi} \right) M_A^2 + \frac{g^2 F_\pi^2}{2}, \quad (6)$$

$$F_V^2 = \left(1 - \frac{g^2 S}{8\pi} \right) \frac{2M_A^2}{g^2} + F_\pi^2, \quad (7)$$

$$F_A^2 = \left(1 - \frac{g^2 S}{8\pi} \right) \frac{2M_A^2}{g^2}. \quad (8)$$

Equation (8) immediately gives an upper bound for g

$$g < \sqrt{\frac{8\pi}{S}}. \quad (9)$$

This is represented by the upper straight line of Fig. 3(a), for $S = 0.1$ (top), $S = 0.2$ (center), and $S = 0.3$ (bottom). Additional upper and lower bounds are given by the requirement that the vector mesons are narrow states, since this is a necessary condition for the WSR’s to hold. Above the upper left solid curve M_V is larger than $2M_A$, and the dominant $V \rightarrow A + A$ decay channel opens up, with a large contribution to Γ_V . Below the lower thin solid curve $\Gamma_V(\pi\pi)/M_V$ is larger than 0.5. Here, $\Gamma_V(\pi\pi)$ is the partial width for decays into two π ’s, which turns out to be

$$\Gamma_V(\pi\pi) = \frac{M_V^5}{96\pi g^2 F_\pi^4}. \quad (10)$$

A final constraint is the requirement that a is a positive number and $\mathcal{O}(1)$. On the lower thick curve $a = 0$, and

both WSR's are satisfied in a running regime. Below the curve a is negative. Above the curve a is positive and $\mathcal{O}(1)$, and all corresponding values of M_A and g are possible.

We can use these results to analyze the $\pi - \pi$ scattering. We use nonlinear realizations in which the Higgs boson is integrated out. For the corresponding tree-level invariant amplitudes and scattering formalism see Appendix A of Ref. [27] or directly Eqs. 1, 2 in [2] after having set to zero the mass of the pions. As for QCD, here we use vector mesons dominance (VMD). This allows us to treat the spin-1 fields at the tree level. Although the underlying dynamics is quite different from QCD we expect VMD to be still a useful approximation. In fact the underlying dynamics manifests itself in the different physical values assumed by the masses and couplings of the spin-1 fields. We expect our results not to be affected much by the effects of the axial vector meson in the VMD approximation, since the axial does not couple at the tree level. However, a more complete treatment will be presented elsewhere.

In Fig. 2(a), we plot the $I = 0, J = 0$ partial wave a_0^0 for $M_A = 1.5$ TeV and three different values of g . The central curve has an $a_0^0 = 0.5$ maximum, and displays the largest value of unitarity violation energy, around 3.7 TeV, for $M_A = 1.5$ TeV. The lowest curve has a slightly lower value of g , and violates unitarity just below 3.7 TeV. However, the upper curve violates unitarity at a much lower energy, around 1.9 TeV. This shows that there is a discontinuity in the plot of unitarity violation energy as a function of g , as shown in Fig. 2(b). The location of this discontinuity corresponds therefore to a "critical" value of g , g_c . While above g_c the amplitude is not sufficiently unitarized by the vector-vector meson, and a spin-0 isospin-0 state is required with a pole mass near the energy scale where unitarity is violated [1], below and near g_c the theory may very well be Higgsless. Moreover, since $M_A < M_V$ for $M_A = 1.5$ TeV and $S = 0.1$ (see Fig. 1), these states are not within the LHC or ILC reach. Of course

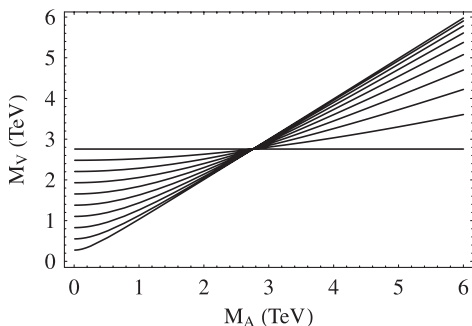


FIG. 1. M_V as a function of M_A for $S = 0.1$, and different values of g . The curve with the lowest slope corresponds to the lowest value of g compatible with the requirement $\Gamma_V/M_V \leq 0.5$. The curve with the largest slope corresponds to $g = \sqrt{8\pi/S}$: values of g above this bound give $F_A^2 < 0$, and must therefore be rejected. See below for the allowed region in the (M_A, g) plane.

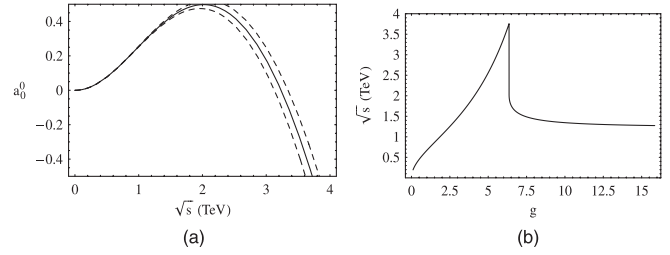


FIG. 2. (a) $I = 0, J = 0$ partial wave amplitude for $M_A = 1.5$ TeV, $S = 0.1$, and three different values of g . The central curve corresponds to the largest delay of unitarity violation, up to $\sqrt{s} \approx 3.7$ TeV. The lowest curve corresponds to a slightly lower value of g , and violates unitarity just below 3.7 TeV. The upper curve corresponds to a slightly larger value of g , but it violates unitarity at a much lower energy $\sqrt{s} \approx 1.9$ TeV. As a consequence, there is a discontinuity in the plot of unitarity violation energy as a function of g (b).

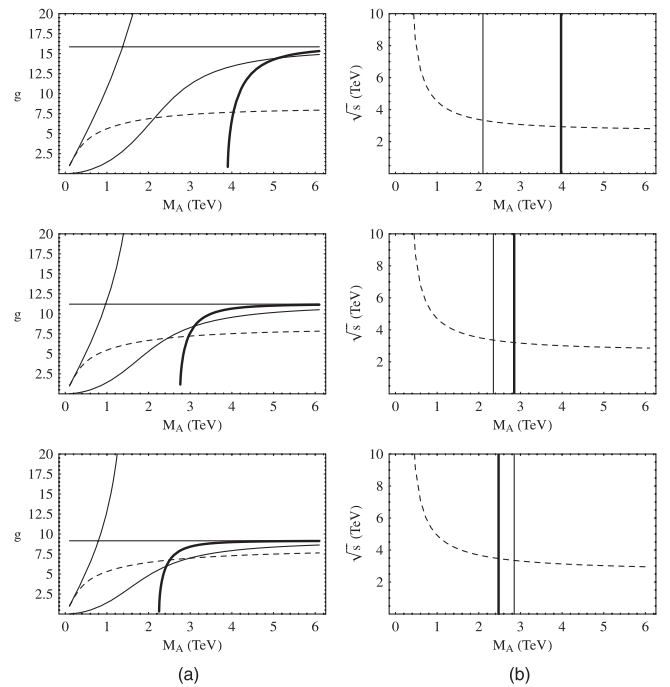


FIG. 3. From top to bottom these figures correspond to $S = 0.1, S = 0.2$, and $S = 0.3$. (a) The dashed line corresponds to g_c as a function of M_A . The thin solid lines give bounds on g and M_A coming from self-consistency (no imaginary numbers) and the requirement that the vector-vector meson is a narrow state, as explained in the text. The thick solid line corresponds to $a = 0$: along this curve both WSR's are satisfied in a running regime, while below the curve $a < 0$, and the corresponding values of g and M_A must be rejected. This curve is hit by g_c , with a narrow width, for $S = 0.3$, but not for $S \leq 0.2$, proving that QCD-like theories are only unitarized, at the tree level, by vector mesons for large values of S or, which is the same, a large number of colors. (b) Unitarity violation scale along $g = g_c$. To the right of the vertical thin line Γ_V/M_V becomes greater than 0.5. To the right of the thick vertical line a becomes negative.

this argument does not exclude the presence of light states, but shows clearly that a worst case scenario of no detection of new particles cannot be excluded, as it is generally expected in models with a weak EWSB sector. It should be noticed that $g_{V\pi\pi}$ here is expected to be stronger than the corresponding coupling in Higgsless models from (deconstructed) extra dimension [28,29]. In the latter, there is one mass scale, and the analogue of $g_{V\pi\pi}$ is constrained by unitarity and the standard model boson masses. In our effective theory there are two mass scales, F_π and the mass of the vector mesons, and $g_{V\pi\pi}$ is therefore unconstrained. Unitarity requires a larger $g_{V\pi\pi}$ for larger vector meson masses.

Notice that for small values of g the theory seems to lose unitarity rather soon, even below the standard 1.2 TeV bound of the chiral Lagrangian. This seemingly unreasonable result comes from holding M_A fixed, in which case taking small values of g corresponds to taking large values of $g_{V\pi\pi}$. In this limit, and for relatively small values of M_V , the interaction between the vector-vector meson and the pions becomes too strong, and the model quickly loses unitarity. Another way to see this comes from Eq. (5): taking the $g \rightarrow 0$ limit with M_V fixed gives $F_V \rightarrow \infty$. It would be more physical to let M_V rapidly approach zero at the same time, in such a way that $F_V \rightarrow 0$ as well. However, here we only focus on the phenomenologically

relevant regions of the parameter space, in which the vector mesons are expected to be at least in the 10^2 GeV range.

In Fig. 3(a), the dashed curve gives g_c as a function of M_A . Below and in the vicinity of this curve unitarity violation is delayed to higher energy scales, and a spin-0 isospin-0 state is not needed with mass below 1 TeV. Notice that for $S = 0.1$ (top) and $S = 0.2$ (center) g_c is only attained in the running regime, i.e. $a = 0$, when $\Gamma_V/M_V > 0.5$, while for $S = 0.3$ one still has $\Gamma_V/M_V < 0.5$. This is in agreement with the results of Refs. [1,2], and shows that QCD-like theories are unitarized by the vector meson only for large values of S . In Fig. 3(b), the unitarity violation energy for $g = g_c$ is plotted as a function of M_A . To the right of the vertical thin line Γ_V/M_V becomes greater than 0.5. To the right of the vertical thick line a becomes negative. Notice how the thin line is first hit by the dashed curve for $S = 0.1$ and $S = 0.2$, while the thick line is hit first for $S = 0.3$.

We have shown that there are regions in the parameters space of phenomenologically viable models of dynamical EWSB according to which unitarity is delayed up to around 34 TeV without the inclusion of any sub-TeV resonances. If our scenario is realized in nature, LHC and ILC may not be able to directly observe any new physical state.

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