

CPT violation in long baseline neutrino experiments: A three flavor analysis

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We explore possible signals of CPT violation in neutrinos in the complete three-flavor framework. Employing a systematic expansion in small parameters, we analytically estimate the CPT violating contributions to the survival probabilities of ν_μ , $\bar{\nu}_\mu$, ν_e and, $\bar{\nu}_e$. The results indicate that, in spite of the large number of CPT violating parameters, only a small number of combinations are relevant for oscillation experiments. We identify the combinations that can be constrained at the long baseline experiments, and show that their contribution to the neutrino Hamiltonian can be bounded to $\lesssim 10^{-23}$ GeV, by considering the NOvA experiment for the muon sector, and neutrino factories for the electron sector. This formalism also allows us to translate the bounds on the parameters describing non-standard interactions of neutrinos into the bounds on CPT violating quantities.

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I. INTRODUCTION

There have been theoretical suggestions that Lorentz invariance may not be an exact symmetry [1]. In such a case, even if the invariance is broken at a very high energy scale (say at the Planck scale in quantum gravity theories), the breaking is expected to leave its signature, however small, at laboratory energies. Such Lorentz violation may manifest itself as CPT violation. Indeed, in local field theories, CPT violation implies Lorentz violation [2,3].

In theories with spontaneous CPT violation [4], the Lagrangian for a fermion to the lowest order in the high scale can be written as

$$\mathcal{L} = i\bar{\psi}\partial_\mu\gamma^\mu\psi - m\bar{\psi}\psi - A_\mu\bar{\psi}\gamma^\mu\psi - B_\mu\bar{\psi}\gamma_5\gamma^\mu\psi, \quad (1)$$

where A_μ and B_μ are real numbers. The terms containing A_μ and B_μ are clearly Lorentz violating, and give rise to an effective contribution to the neutrino Lagrangian that can be parametrized as

$$\mathcal{L}_\nu^{CPTV} = \bar{\nu}_L^\alpha b_\mu^{\alpha\beta}\gamma^\mu\nu_L^\beta. \quad (2)$$

Here b_μ are four Hermitian 3×3 matrices corresponding to the four Dirac indices μ , wherein α, β are flavor indices. Then the effective Hamiltonian for ultrarelativistic neutrinos with definite momentum p is

$$\mathbb{H} \equiv \frac{\mathbb{M}\mathbb{M}^\dagger}{2p} + \mathbb{b}, \quad (3)$$

where $\mathbb{b} \equiv b_0$ and \mathbb{M} is the neutrino mass matrix in the CPT conserving limit. Following [1], we choose to work in the preferred frame in which the cosmic microwave background radiation (CMBR) is isotropic, where the rotational invariance implies no directional dependence for \mathbb{b} .

The same effective Hamiltonian can also be obtained by considering a modified dispersion relation for neutrinos, $E^2 = F(p, m)$, in the presence of Lorentz violation. This dispersion relation may be written, using rotational invariance in the CMBR frame and demanding Lorentz invariance at low energy, as [5]

$$E^2 = m^2 + p^2 + E_{\text{Pl}}f^{(1)}|p| + f^{(2)}p^2 + \frac{f^{(3)}}{E_{\text{Pl}}}|p|^3 + \dots, \quad (4)$$

where $f^{(n)}$'s are dimensionless quantities. The Planck energy E_{Pl} is introduced since it is the energy where Lorentz invariance is expected to be broken in quantum gravity. For ultrarelativistic neutrinos with fixed momentum p , the dispersion relation becomes

$$E = p + \frac{m^2}{2p} + b + \dots, \quad (5)$$

such that $b = E_{\text{Pl}}f^{(1)}/2$ is the leading CPT violating contribution. Generalizing this to three flavors leads to the same effective Hamiltonian as in (3).

The possible origin of CPT violation in the neutrino sector has been studied in the context of extra dimensions [6,7], nonfactorizable geometry [8], and nonlocal causal Lorentz invariant theories [9]. Bounds on the CPT violating parameters have been obtained in many different contexts. For example, the analyses of neutral meson mixings give $|m_{K^0} - m_{\bar{K}^0}| \lesssim 10^{-18}m_{\text{avg}}$ [10], and $|m_{B_d^0} - m_{\bar{B}_d^0}| \lesssim 1.6 \cdot 10^{-14}m_{\text{avg}}$ [11], whereas experiments on anomalous magnetic moment of muon put the bound on the anomalous frequency as $|\omega_a^{\mu^+} - \omega_a^{\mu^-}| \lesssim 10^{-23}m_\mu$ [12]. However, it is difficult to compare these bounds directly with the bounds obtained from the neutrino sector since we do not have an all-encompassing theory of CPT violation.

The formalism to analyze CPT violating effects on neutrino oscillations has been proposed for the two-flavor

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case in [1]. The CPT violating contribution to the Hamiltonian would change the effective neutrino masses, which in turn would affect the neutrino oscillation wavelengths. Henceforth, for neutrinos we shall use $p \approx E$, so that $\mathbb{M}^2/(2p) \rightarrow \mathbb{M}^2/(2E)$. The typical frequency of neutrino oscillations is $\Delta m^2/(2E)$, which can be as small as 10^{-22} GeV in the atmospheric and long baseline experiments. Since the experiments measure the oscillation frequencies to an accuracy of $\sim 10\%$, it may be naively estimated that neutrino experiments would constrain the CPT violating parameters to the order $\sim 10^{-23}$ GeV.

After LSND result [13] indicated three distinct neutrino mass squared differences when combined with the solar and atmospheric neutrino observations, it was proposed that the CPT violating effects may be large enough to make the neutrino and antineutrino spectra significantly different [14,15]. However, this scenario was found not to be viable when combined with other neutrino experiments [16], and the subsequent observation of oscillations corresponding to Δm_{\odot}^2 in antineutrinos at KamLAND [17] ruled it out. If the LSND results are ignored in the light of the negative results of MiniBooNE [18] that explore the same parameter space, CPT violation is not required to explain any neutrino oscillation data. However, the current uncertainties in the measurements of Δm_{\odot}^2 and Δm_{atm}^2 , which are $\sim 9\%$ and $\sim 14\%$ respectively [19], allow the possibility of CPT violating effects in neutrino oscillations, which may be observed or constrained at the future high precision neutrino oscillation experiments.

CPT violation in neutrino oscillations would manifest itself in the observation $P(\nu_{\alpha} \rightarrow \nu_{\beta}) \neq P(\bar{\nu}_{\beta} \rightarrow \bar{\nu}_{\alpha})$. However, when neutrinos propagate through matter, the matter effects give rise to “fake” CP and CPT violation even if the vacuum Hamiltonian is CPT conserving. These fake effects need to be accounted for while searching for CPT violation. The $\nu_{\mu} \rightarrow \nu_{\mu}$ channel was explored in this context in [20], where it was pointed out that CPT violating signals could become larger due to resonant effects. Using a two-flavor analysis, it was shown that a long baseline ($L = 735$ km) experiment with a typical neutrino factory setup can detect a difference of the eigenvalues of \mathbb{b} up to $\delta b \sim 10^{-23}$ GeV. In [21] it was shown that, using the atmospheric neutrino data at a 50 kt magnetized iron calorimeter with 1.2 T magnetic field, $\delta b \sim 3 \times 10^{-23}$ GeV should be clearly discernible in 8 years. The solar and KamLAND data gives the bound $\delta b \lesssim 1.6 \times 10^{-21}$ GeV in [22]. Ref. [23] showed that for a hierarchical neutrino mass spectrum, the upper bound for the neutrino-antineutrino mass difference that can be achieved in a neutrino factory is $|m_3 - \bar{m}_3| \lesssim 1.9 \times 10^{-4}$ eV. Global two-flavor analysis of the full atmospheric data and long baseline K2K data puts the bound $\delta b \lesssim 5.0 \times 10^{-23}$ GeV [24].

All the above analyses have been carried out in the two-flavor approximation. Moreover, it has been assumed in

[20,21] that the mixing angles as well as phases of the unitary matrices that diagonalize \mathbb{M} and \mathbb{b} in (3) are identical. The analysis of [22] takes the mixing angles to be identical and considers two specific values of the relative phase between the two unitary matrices. These assumptions have been made solely to simplify the analytic treatment, and do not have any physical motivation behind them. Ref. [24] analyzes the two-flavor case in its full generality, putting no extra condition on the mixing angles and the relative phase. However, a three-flavor treatment is needed in order to obtain reliable results, since a two-flavor analysis cannot account for the CP violating effects that may interfere with the identification of CPT violation. The addition of the third (electron) flavor also compels one to take care of the matter effects when neutrinos pass through the Earth.

In this article we consider the possible CPT violating effects that appear through (3), when three-neutrino oscillations are considered in their full generality. We treat the effect of the CPT violating term as a perturbation parametrized by a dimensionless auxiliary parameter $\epsilon \equiv 0.1$ and express the differences of the eigenvalues of the CPT violating \mathbb{b} matrix, the reactor angle θ_{13} , and the ratio $\Delta m_{\odot}^2/\Delta m_{\text{atm}}^2$ as some power of ϵ multiplied by $\mathcal{O}(1)$ numbers, so that a systematic expansion in ϵ can be carried out. The survival probabilities of ν_{μ} and ν_e (and their antiparticles) can then be written down as a power series in ϵ in a transparent form. This allows us to identify the combinations of CPT violating parameters that contribute to these probabilities to leading order in ϵ . We compare the signals in the channels $\nu_{\mu} \rightarrow \nu_{\mu}$, $\bar{\nu}_{\mu} \rightarrow \bar{\nu}_{\mu}$ and $\nu_e \rightarrow \nu_e$, $\bar{\nu}_e \rightarrow \bar{\nu}_e$ to estimate the extent to which these CPT violating combinations can be constrained or identified in future long baseline experiments.

Bounds have been obtained on parameters describing the non-standard interactions (NSI) of neutrinos with matter [25–30], using both oscillation and nonoscillation experiments. We explicitly show how to translate these bounds into the bounds on the CPT violating parameters in our formalism. This will allow us to compare and combine the bounds from these two approaches to restrict new physics in the neutrino sector.

In the paper, Sec. II gives the parametrization of the CPT violating part of the effective Hamiltonian in flavor basis using the perturbative expansion scheme. Sec. III and IV give the probability expressions and possible signatures in the long baseline experiments in μ and e channels, respectively. In Sec. V we summarize the current constraints on NSI parameters and translate them to the bounds on CPT violating quantities. Sec. VI summarizes our results.

II. PERTURBATIVE EXPANSION OF THE HAMILTONIAN WITH THE CPTV TERM

In the three neutrino oscillation scheme, $(\nu_e, \nu_{\mu}, \nu_{\tau})$ form the flavor basis and (ν_1, ν_2, ν_3) form the mass basis,

i.e. the basis in which $\mathbb{M}\mathbb{M}^\dagger/(2E)$ is diagonal, and these are related by

$$\nu_\alpha = [\mathbb{U}_0]_{\alpha i} \nu_i, \quad (6)$$

where $\alpha \in \{e, \nu, \tau\}$ and $i \in \{1, 2, 3\}$. Let $(\nu_1^b, \nu_2^b, \nu_3^b)$ be the basis in which \mathbb{b} is diagonal and let this basis be related to the flavor basis as

$$\nu_\alpha = [\mathbb{U}_b]_{\alpha x} \nu_x^b, \quad (7)$$

where $x \in \{1, 2, 3\}$. Both \mathbb{U}_m and \mathbb{U}_b are unitary matrices.¹

When neutrinos pass through the matter, the electron neutrinos acquire an effective potential $V_e = \sqrt{2}G_F N_e$ due to their charged current forward scattering interactions, compared to the other two flavors. Here G_F is the Fermi constant and N_e is the number density of electrons. For antineutrinos, the sign of V_e is reversed. The effective Hamiltonian in the flavor basis is

$$\begin{aligned} \mathbb{H}_f \approx & \mathbb{U}_0 \cdot \frac{\text{diag}(0, \Delta m_{21}^2, \Delta m_{31}^2)}{2E} \cdot \mathbb{U}_0^\dagger + \mathbb{H}_b \\ & + \text{diag}(V_e, 0, 0), \end{aligned} \quad (8)$$

$$[\mathbb{U}_{ij}(\theta, \delta)]_{pq} = \begin{cases} \cos\theta & p = q = i \text{ or } p = q = j \\ 1 & p = q \neq i \text{ and } p = q \neq j \\ \sin\theta e^{-i\delta} & p = i \text{ and } q = j \\ -\sin\theta e^{i\delta} & p = j \text{ and } q = i \\ 0 & \text{otherwise.} \end{cases} \quad (11)$$

Once we have redefined the phases of the lepton and neutrino wave functions to get \mathbb{U}_0 in the form (10), the basis of neutrino flavor eigenstates is completely defined. The matrix \mathbb{U}_b then needs three angles $\theta_{b12}, \theta_{b23}, \theta_{b13}$ and six phases for a complete parametrization, given by

$$\begin{aligned} \mathbb{U}_b(\{\theta_{bij}\}; \{\phi_{bi}\}; \{\alpha_{bi}\}; \delta_b) = & \text{diag}(1, e^{i\phi_{b2}}, e^{i\phi_{b3}}) \\ & \cdot \mathbb{U}_{\text{CKM}}(\{\theta_{bij}\}; \delta_b) \\ & \cdot \text{diag}(e^{i\alpha_{b1}}, e^{i\alpha_{b2}}, e^{i\alpha_{b3}}), \end{aligned} \quad (12)$$

where $\alpha_{b1}, \alpha_{b2}, \alpha_{b3}$ are the Majorana phases and will not have any contribution to \mathbb{H}_f through \mathbb{H}_b . Hence \mathbb{U}_f may be written in term of a total of six mixing angles $(\theta_{12}, \theta_{23}, \theta_{13}, \theta_{b12}, \theta_{b23}, \theta_{b13})$ and four phases $(\delta_{\text{cp}}, \delta_b, \phi_{b2}, \phi_{b3})$.

Present limits on CPT violation in the neutrino sector [20–22,24] arise from the limits on the neutrino oscillation wavelength, which in the two-flavor case gets modified as $\Delta m_{\text{atm}}^2/(2E) \rightarrow \Delta m_{\text{atm}}^2/(2E) + \delta b$. The bound on δb is therefore governed by the uncertainty in Δm_{atm}^2 , which is $\sim 10\%$. Motivated by this, we assume that

¹In Refs. [20–22,24], \mathbb{U}_0 and \mathbb{U}_b are 2×2 matrices. In addition, [20,21] take these two matrices to be identical.

where

$$\mathbb{H}_b \equiv \mathbb{U}_b \cdot \text{diag}(0, b_{21}, b_{31}) \cdot \mathbb{U}_b^\dagger. \quad (9)$$

Here b_1, b_2 and b_3 are the eigenvalues of \mathbb{b} and $b_{i1} \equiv b_i - b_1$ for $i = 2, 3$. The net spectrum of neutrino mass eigenstates is given by the eigenvalues of \mathbb{H}_f . We term the unitary matrix diagonalizing \mathbb{H}_f as \mathbb{U}_f .

A general $N \times N$ unitary matrix \mathbb{U}_N is parametrized by $N(N-1)/2$ independent real quantities (angles) and $N(N+1)/2$ independent imaginary quantities (phases). In the case of the neutrino mixing matrix, $(2N-1)$ phases can be absorbed by redefining lepton and neutrino wave functions.² Thus we can parametrize the 3×3 unitary matrix \mathbb{U}_0 by three angles θ_{12}, θ_{23} and θ_{13} and one phase δ_{cp} . We write \mathbb{U}_0 in the standard CKM parametrization as

$$\begin{aligned} \mathbb{U}_0 = & \mathbb{U}_{23}(\theta_{23}, 0) \cdot \mathbb{U}_{13}(\theta_{13}, \delta_{\text{cp}}) \cdot \mathbb{U}_{12}(\theta_{12}, 0) \\ = & \mathbb{U}_{\text{CKM}}(\{\theta_{ij}\}; \delta_{\text{cp}}), \end{aligned} \quad (10)$$

where $U_{ij}(\theta_{ij}, \delta_{ij})$ is the complex rotation matrix in the i - j plane, whose elements $[\mathbb{U}_{ij}]_{pq}$ are defined as

$$\begin{aligned} b_{21}, b_{31} \lesssim & 0.1 \times \Delta m_{\text{atm}}^2/(2E_0) \\ \approx & 0.13 \times 10^{-21} \text{ GeV}^2/E_0, \end{aligned} \quad (13)$$

where E_0 is the typical energy scale of the experiment. This may be parametrized by introducing two auxiliary quantities $\epsilon \equiv 0.1$ and $S_{E_0} \equiv 10^{-21} \text{ GeV}^2/E_0$ such that

$$b_{21} \equiv \epsilon \beta_{21} S_{E_0}, \quad b_{31} \equiv \epsilon \beta_{31} S_{E_0}. \quad (14)$$

Clearly, β_{21}, β_{31} are numbers of $\mathcal{O}(1)$ or smaller. The mixing angle θ_{13} and the ratio $\Delta m_{21}^2/\Delta m_{31}^2$ are small quantities, and may be expressed in terms of powers of ϵ as

$$\theta_{13} \equiv \epsilon \chi_{13}, \quad \Delta m_{21}^2/\Delta m_{31}^2 \equiv \epsilon^2 \zeta. \quad (15)$$

The current bounds on the mixing angles and mass squared differences [19] set

$$\chi_{13} < 1.8, \quad \zeta \sim 3.0. \quad (16)$$

The sign of ζ is positive (negative) for normal (inverted) mass ordering of neutrinos.

Using the formal representation of $\theta_{13}, \Delta m_{21}^2$ and the CPT violating parameters in terms of powers of ϵ as given

²For Majorana neutrinos, only N phases can be absorbed if \mathbb{M} needs to be kept invariant. However, $N-1$ more phases are irrelevant when \mathbb{M} only appears through the combination $\mathbb{M}\mathbb{M}^\dagger$.

in Eq. (14) and (15), the Hamiltonian \mathbb{H}_f can be expanded formally in powers of ϵ as

$$\mathbb{H}_f = \frac{\Delta m_{31}^2}{2E} \left[\mathbb{H}_f^{(0)} + \epsilon \mathbb{H}_f^{(1)} + \epsilon^2 \mathbb{H}_f^{(2)} + \mathcal{O}(\epsilon^3) \right], \quad (17)$$

where $\mathbb{H}_f^{(0,1,2)}$ are functions of all the mixing angles, phases, mass squared differences, and eigenvalues of \mathbb{b} . All the elements of $\mathbb{H}_f^{(0,1,2)}$ are of $\mathcal{O}(1)$ or smaller, and $\mathbb{H}_f^{(0)}$ has nondegenerate eigenvalues. The techniques of time independent nondegenerate perturbation theory can therefore be used to calculate the eigenvalues and eigenvectors of \mathbb{H}_f up to the required order in ϵ . These can be further used to calculate the neutrino flavor survival or conversion probabilities when neutrinos travel through matter with a constant density:

$$P_{\alpha\beta} \equiv P(\nu_\alpha \rightarrow \nu_\beta) = \left| \sum_i [\mathbb{U}_f]_{\alpha i} [\mathbb{U}_f]_{\beta i}^* \exp\left(-i \frac{\tilde{m}_i^2 L}{2E}\right) \right|^2, \quad (18)$$

where $\tilde{m}_i^2/(2E)$ are the eigenvalues of \mathbb{H}_f . This approximation is valid for neutrino propagation inside the Earth as long as neutrino trajectories do not pass through the core, and neutrino energy is not close to the θ_{13} resonance energy in the Earth matter.

III. CPT VIOLATION IN $P_{\mu\mu}$ AND SIGNATURES AT NOVA

The survival probability of muon neutrinos of energy E , after traversing a distance L through the Earth is given as

$$P_{\mu\mu} = 1 - \sin^2 2\theta_{23} \sin^2 \Delta_{31} + \epsilon \left(C_1 \Delta_0 \sin^2 2\theta_{23} \sin 2\Delta_{31} - C_2 \frac{\Delta_0}{\Delta_{31}} \sin 4\theta_{23} \sin^2 \Delta_{31} \right) + \mathcal{O}(\epsilon^2), \quad (19)$$

where we define the dimensionless quantities $\Delta_{31} \equiv \Delta m_{31}^2 L/(4E)$ and $\Delta_0 \equiv \Delta m_{31}^2 L/(4E_0)$. The first two terms in (19) are CPT conserving, and describe oscillations with frequency governed by Δm_{atm}^2 . The subleading contribution at $\mathcal{O}(\epsilon)$ is CPT violating. The quantities $C_{1,2}$ are given by

$$\begin{aligned} C_1 &= B_1 \cos 2\theta_{23} - B_2 \sin 2\theta_{23}, \\ C_2 &= B_1 \sin 2\theta_{23} + B_2 \cos 2\theta_{23}, \end{aligned} \quad (20)$$

where

$$\begin{aligned} B_1 &= (\mathbb{H}_{b22} - \mathbb{H}_{b33})/(\epsilon S_{E_0}), \\ B_2 &= 2 \operatorname{Re}(\mathbb{H}_{b23})/(\epsilon S_{E_0}). \end{aligned} \quad (21)$$

The quantities B_1 and B_2 depend only on \mathbb{b} and \mathbb{U}_b as

$$B_1 = [X \cos 2\theta_{b23} - Y \sin 2\theta_{b23} \cos \delta_b], \quad (22)$$

$$B_2 = -[X \sin 2\theta_{b23} \cos \delta \phi + Y \cos 2\theta_{b23} \cos \delta \phi \cos \delta_b + Y \sin \delta \phi \sin \delta_b], \quad (23)$$

wherein

$$X = \beta_{21} \cos^2 \theta_{b12} - \beta_{31} \cos^2 \theta_{b13} - \beta_{21} \sin^2 \theta_{b12} \sin^2 \theta_{b13}, \quad (24)$$

$$Y = \beta_{21} \sin 2\theta_{b12} \sin \theta_{b13}. \quad (25)$$

The phases ϕ_{bi} only appear through the combination $\delta \phi = \phi_{b2} - \phi_{b3}$. The corresponding quantity $P_{\bar{\mu}\bar{\mu}}$ for antineutrinos can be obtained simply with the substitution $\epsilon \rightarrow -\epsilon$ in (19). The terms involving matter effects as well as CP violation are suppressed due to θ_{13} and Δm_{\odot}^2 , and appear only at $\mathcal{O}(\epsilon^2)$ and $\mathcal{O}(\epsilon^3)$ respectively. CPT violation can thus be cleanly extracted from the asymmetry

$$P_{\mu\mu} - P_{\bar{\mu}\bar{\mu}} = 2\epsilon \left(C_1 \Delta_0 \sin^2 2\theta_{23} \sin 2\Delta_{31} - C_2 \frac{\Delta_0}{\Delta_{31}} \sin 4\theta_{23} \sin^2 \Delta_{31} \right) + \mathcal{O}(\epsilon^2) \quad (26)$$

if it is indeed of the magnitude allowed by the current bounds. However, one needs to be away from the θ_{13} resonance, which for the Earth matter density occurs for $E_{\text{res}} \approx 5\text{--}10$ GeV, since the enhanced value of θ_{13} makes the expansion in powers of ϵ invalid.

Equation (19) demonstrates that, though the CPT violating parameter space consists of three angles, three phases and two eigenvalue differences β_{21}, β_{31} , the effective CPT violating contribution to the probability $P_{\mu\mu}$ is much simpler and depends only on two combinations of these parameters, B_1 and B_2 , to leading order. A consequence of this result is that measurements in the muon channel can only put bounds on the two effective parameters B_1 and B_2 , and not separately on the angles, phases or eigenvalue differences.

Since $\theta_{23} \approx \pi/4$, the C_1 term in (26) dominates over the other. Accounting for the $1/L^2$ fall-off of the neutrino flux, the signal due to this term is optimized when L takes its minimum value that is able to satisfy $\sin 2\Delta_{31} \approx 1$. This calls for a (relatively) low energy experiment with $L/E \sim 240$ km/GeV. The NOvA experiment [31,32] with its $L = 812$ km baseline and the NuMI beam energy $E \approx 0.5\text{--}4.0$ GeV satisfies these criteria, and hence is well suited to look for CPT violation. The energy range of NOvA is completely below the θ_{13} resonance energy, so the contamination from CP violating θ_{13} contributions to $P_{\mu\mu} - P_{\bar{\mu}\bar{\mu}}$ is minimal. We take E_0 for NOvA to be 1 GeV, so that $S_{E_0} = 10^{-21}$ GeV.

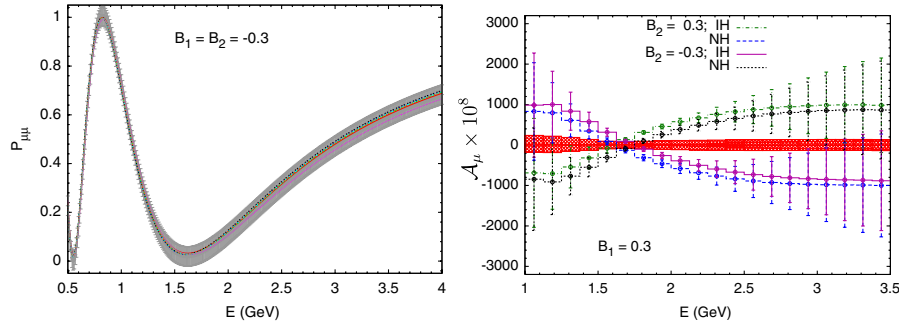


FIG. 1 (color online). The left panel compares the analytical and numerical results for eight randomly chosen combinations of CPT violating parameters (thin lines) that correspond to $B_1 = B_2 = -0.3$. The gray (shaded) band is the analytic expression plotted with an energy independent error of ± 0.04 . We choose normal hierarchy, $\Delta m_{\odot}^2 = 7.92 \times 10^{-5} \text{ eV}^2$, $\theta_{12} = 34.08^\circ$, $\Delta m_{\text{atm}}^2 = 2.6 \times 10^{-3} \text{ eV}^2$, $\theta_{23} = 42.13^\circ$, $\theta_{13} = 0.089$ and $\delta_{\text{cp}} = 0$. The right panel shows $\mathcal{A}_\mu(E)$ for 4 years of running at NOvA with each μ^+ and μ^- , with an incident flux of $10^{21} \text{ pot yr}^{-1}$. The errors shown are only statistical. The central red (hashed) band shows the contribution in absence of CPT violation when the parameters are varied over their current 2σ ranges.

We demonstrate the validity and limitations of the analytic expression (19) in the left panel of Fig. 1, where $P_{\mu\mu}$ is plotted as a function of energy for the NOvA baseline for the current best-fit values of Δm_{\odot}^2 , Δm_{atm}^2 , θ_{12} , θ_{23} and θ_{13} over the NOvA energy range. We choose normal mass ordering and B_1, B_2 with opposite signs, which is observed to be one of the worst case situations while comparing analytical results with numerical ones. We choose $B_1 = B_2 = -0.3$ and take eight randomly chosen sets of CPT violating parameters that correspond to these values of B_1 and B_2 . The plot shows that an energy independent error of ± 0.04 in $P_{\mu\mu}$ can account for the error due to neglecting terms of $\mathcal{O}(\epsilon^2)$ or higher, in the whole energy regime of interest.³

We choose a typical NOvA setup [31,32], with the NuMI beam directed towards a 0.5 kt “near” detector placed 1 km away, and a 25 kt “far” detector at a distance of 812 km. The detector is assumed to be able to identify lepton charges. The neutrino propagation through the Earth is implemented using a 5-density model of the Earth, where the density of each layer has been taken to be the average of the densities encountered by the neutrinos along their path in that layer with the Preliminary Reference Earth Model (PREM) profile [33]. We take care of the detector characteristics using the General Long Baseline Experiment Simulator (GLoBES) [34,35]. The cross-sections used are taken from [36,37], and the simulation includes an energy resolution of $\sigma_E = 10\%\sqrt{E}$, an overall detection efficiency of 80% for all charged leptons, as well as additional energy dependent post-efficiencies that are taken care of bin-by-bin *a la* GLoBES. We assume perfect lepton charge identification, and neglect any error due to wrong sign leptons produced from the oscillations of the antiparticles.

³A part of this systematic error should be cancelled out when we concentrate on $P_{\mu\mu} - P_{\bar{\mu}\bar{\mu}}$, however we choose to use a more conservative estimate of errors.

In the right panel of Fig. 1, we plot the asymmetry

$$\mathcal{A}_\mu(E) \equiv \frac{N_\mu^{\text{far}}(E)}{N_\mu^{\text{near}}(E)} - \frac{\bar{N}_\mu^{\text{far}}(E)}{\bar{N}_\mu^{\text{near}}(E)}, \quad (27)$$

where N_ℓ (\bar{N}_ℓ) is the number of ℓ^- (ℓ^+) observed at the near or far detector. Here the events observed in the near detector act as a normalizing factor, and help in canceling out the systematic errors due to fluxes, cross sections and efficiencies in each energy bin. Note that modulo these factors, \mathcal{A}_μ is equivalent to $(P_{\mu\mu} - P_{\bar{\mu}\bar{\mu}})$ multiplied by a geometric factor of $(L_{\text{near}}/L_{\text{far}})^2$. For plotting, we have considered a running time of 4 years with each of μ^+ and μ^- , with an incident flux of 10^{21} pot (protons on target) per year.

The right panel of Fig. 1 illustrates salient features of the CPT violating contribution to $\mathcal{A}_\mu(E)$. The central band corresponds to possible signals in the absence of any CPT violating contributions, where we have varied θ_{23} , θ_{13} and δ_{cp} over the currently allowed 2σ ranges and have allowed for normal as well as inverted mass ordering. We fix Δm_{\odot}^2 , Δm_{atm}^2 and θ_{12} at their current best-fit values, since variation with these parameters is not expected to be significant. For illustrating the signal in the presence of CPT violation, we choose $\delta_{\text{cp}} = 0$, $|B_{1,2}| = 0.3$, and fix θ_{23} and θ_{13} at their best-fit values. The figure shows that \mathcal{A}_μ depends on both the magnitude and relative sign of B_1, B_2 and also on the mass ordering. It can be shown that the effect of changing sign of $B_{1,2}$ is the same as changing the mass ordering, as expected from Eq. (26) when $\theta_{23} \approx \pi/4$.

In order to estimate the possibility of identifying the CPT violating contributions from the experimental signals, with the current uncertainties in the standard three neutrino oscillation parameters, we display the confidence level contours in Fig. 2. In the left panel of Fig. 2, we have marginalized over all the standard neutrino oscillation parameters. It shows that B_2 can be bounded from NOvA observations to the extent

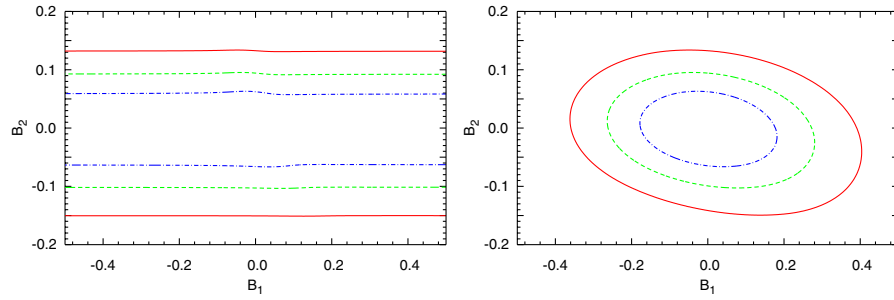


FIG. 2 (color online). Confidence level contours in the B_1 - B_2 plane. The red (solid), green (dashed) and blue (dash-dotted) curves give the 3σ , 2σ and 1σ contours, respectively. An energy independent error of ± 0.04 on $P_{\mu\mu}$ has been taken into account. Both the figures use $\theta_{12} = 34.08^\circ$, $\Delta m_{\text{atm}}^2 = 2.6 \times 10^{-3} \text{ eV}^2$, $\theta_{13} = 0.089$ and $B_1 = B_2 = 0$ as the input values. The left figure marginalizes over θ_{23} , Δm_{atm}^2 and δ_{cp} . The right figure uses $\theta_{23} = 42.13^\circ$.

$$|B_2| \lesssim 0.1 \quad (2\sigma). \quad (28)$$

The data are relatively insensitive to B_1 . This is expected from the analytic expression in Eq. (20) and (26): since $\theta_{23} \approx \pi/4$, the terms containing B_1 are highly suppressed. However, if θ_{23} is known accurately and differs from $\pi/4$, the sensitivity to B_1 is restored. This becomes clear from the right panel of Fig. 2, where we have marginalized over all parameters except θ_{23} , keeping θ_{23} fixed at a nonmaximal value of 42.13° . In such a case, B_1 can be constrained to

$$|B_1| \lesssim 0.28 \quad (2\sigma). \quad (29)$$

Accurate measurement of the deviation of θ_{23} from its maximal value [38] is essential for the above bound on B_1 . The bound on B_2 , however, does not depend on the improvement in the measurement of any other quantity. A similar analysis performed for inverted hierarchy gives virtually identical results.

The limits obtained in (28) and (29) are bounds on specific combinations of elements of \mathbb{H}_b . They imply

$$\mathbb{H}_{b22} - \mathbb{H}_{b33} = \epsilon |B_1| S_{E_0} \lesssim 10^{-23} \text{ GeV}, \quad (30)$$

$$2 \text{Re}(\mathbb{H}_{b23}) = \epsilon |B_2| S_{E_0} \lesssim 10^{-23} \text{ GeV}. \quad (31)$$

In the limit $\theta_{b23} = \pi/4$, $\beta_{21} = \theta_{b13} = \delta\phi = 0$, the quantity B_2 in fact reduces to δb , the quantity bounded in the two-flavor analysis [20,21]. Our three neutrino analysis thus identifies the quantity that can be constrained, and also demonstrates that the constraints can be quantified in a clean manner at a low energy long baseline experiment like NOvA.

IV. CPT VIOLATION IN P_{ee} AND SIGNATURES AT A NEUTRINO FACTORY

The survival probability for an electron neutrino traveling through a uniform matter density, in the presence of CPT violation, is given by

$$\begin{aligned} P_{ee} = & 1 - 4\epsilon^2 \chi_{13}^2 \left(\frac{\Delta_{31}}{\Delta_e - \Delta_{31}} \right)^2 \sin^2(\Delta_e - \Delta_{31}) - 4\epsilon^2 \chi_{13} [\text{Re}(S e^{i\delta_{\text{cp}}})] \frac{\Delta_{31} \Delta_0}{(\Delta_e - \Delta_{31})^2} \sin^2(\Delta_e - \Delta_{31}) \\ & - \epsilon^2 P_1 \cos \delta_{\text{cp}} \frac{\Delta_0^2}{4\Delta_e^2} \frac{2\Delta_e^2 - 2\Delta_e \Delta_{31} + \Delta_{31}^2}{(\Delta_e - \Delta_{31})^2} + \epsilon^2 (P_2 \cos 2\theta_{23} + P_3 \sin 2\theta_{23}) \cos \delta_{\text{cp}} \frac{\Delta_0^2}{4\Delta_e^2} \frac{\Delta_{31}(-2\Delta_e + \Delta_{31})}{(\Delta_e - \Delta_{31})^2} \\ & + \epsilon^2 \left[P_4 \frac{\Delta_0^2}{2\Delta_e^2} \cos 2\Delta_e - |S|^2 \frac{2\Delta_0^2}{(\Delta_e - \Delta_{31})^2} \cos(2\Delta_e - 2\Delta_{31}) \right] + \mathcal{O}(\epsilon^3), \end{aligned} \quad (32)$$

where $\Delta_e \equiv V_e L/2$ and recall that $\theta_{13} = \epsilon \chi_{13}$. The CPT violating quantities appearing in (32) can be expressed in terms of two complex quantities Q and R :

$$Q \equiv Q_1 + iQ_2 = -\frac{1}{2} \cos \theta_{b13} e^{-i\phi_{b3}} \left\{ \beta_{21} \sin 2\theta_{b12} \sin \theta_{b23} - 2(\beta_{31} - \beta_{21} \sin^2 \theta_{b12}) \sin \theta_{b13} \cos \theta_{b23} e^{-i\delta_b} \right\}, \quad (33)$$

$$R \equiv R_1 + iR_2 = \frac{1}{2} \cos \theta_{b13} e^{-i\phi_{b2}} \left\{ \beta_{21} \sin 2\theta_{b12} \cos \theta_{b23} + 2(\beta_{31} - \beta_{21} \sin^2 \theta_{b12}) \sin \theta_{b13} \sin \theta_{b23} e^{-i\delta_b} \right\}, \quad (34)$$

where Q_i and R_i are real numbers. In terms of Q and R , the CPT violating parameters S and $P_{1,2,3,4}$ in (32) may be written as

$$S = Q \cos\theta_{23} + R \sin\theta_{23}, \quad (35)$$

$$\begin{aligned} P_1 &= |Q|^2 + |R|^2, & P_2 &= |Q|^2 - |R|^2, \\ P_3 &= 2 \operatorname{Re}(QR^*), & P_4 &= P_1 - |S|^2. \end{aligned} \quad (36)$$

In Eq. (32), the first two terms are the expression for P_{ee} with a nonzero θ_{13} when CPT is conserved, while all the other terms are CPT violating contributions. There are no $\mathcal{O}(\epsilon)$ terms. Both the θ_{13} correction as well as the CPT violating contributions appear at $\mathcal{O}(\epsilon^2)$. These also include terms that get contributions from both θ_{13} and CPT violating parameters. The probability P_{ee} depends only on two complex combinations Q and R of CPT violating parameters, so this channel can put bounds only on these two parameters.

Note that the coefficients of $\mathcal{O}(\epsilon^2)$ in (32) contain terms proportional to $(\Delta_{31}/\Delta_e)^2$, which should be small for the ϵ -expansion to be under control. Therefore, the expression (32) is valid only when $\Delta_e \gtrsim \Delta_{31}$, which happens at energies above the θ_{13} resonance energy. In order to get significant effects at large energies, one also needs long baselines. Both these conditions would be satisfied at a neutrino factory with an energy range 10–50 GeV and a baseline ~ 3000 km. If we restrict ourselves to energies well above the θ_{13} resonance energy ≈ 5 –10 GeV, even the CPT conserving θ_{13}^2 contribution is suppressed, so that the CPT violating contribution can be more cleanly identified. For the neutrino factory, we can set the typical energy $E_0 = 10$ GeV, so that $S_{E_0} = 10^{-22}$ GeV.

In Fig. 3 we demonstrate the validity and limitations of the analytic probability expression (32), where we choose the mixing parameters Δm_{\odot}^2 , Δm_{atm}^2 , θ_{12} , θ_{23} , θ_{13} to have their best-fit values [19]. We choose normal mass hierarchy, and fix $\delta_{\text{cp}} = 0$. This is observed to be one of the

worst case situations while comparing the analytical expressions with the numerical ones. For the CPT violating part we choose $Q_1 = Q_2 = R_1 = R_2 = 1.0$, and six random choices of the elements of \mathbb{b} that map to these values of Q_i s and R_i s. It is seen that an energy independent error of ± 0.015 on P_{ee} can account for the error due to neglecting higher order terms in ϵ over the whole energy range of interest.

To demonstrate the capability of a typical neutrino factory setup for identifying the CPT violating contributions, we define the asymmetry

$$\mathcal{A}_e(E) \equiv \frac{N_e^{\text{far}}(E)}{N_e^{\text{near}}(E)} - \frac{\bar{N}_e^{\text{far}}(E)}{\bar{N}_e^{\text{near}}(E)} \quad (37)$$

in a typical neutrino factory setup [39] with a 50 GeV muon beam directed to a 0.5 kt near detector 1 km away, and a 50 kt far detector 3000 km away. The detectors are assumed to be capable of identifying lepton charges. The number of useful muons in the storage ring is taken to be 1.066×10^{21} , which corresponds to approximately two years of running with μ^- and μ^+ each at the neutrino factory, using the NuFact-II parameters in [40]. The simulation includes an energy resolution of $\sigma_E/E = 15\%$, and an overall detection efficiency of 75% for all charged leptons. Earth matter effects, interaction cross-sections and post-efficiencies are taken care of in the same way as was done in the case of NOvA. We assume perfect lepton charge identification, and neglect any error due to wrong sign leptons produced from the oscillations of the antiparticles. GLoBES is used to get the energy variation of the asymmetry $\mathcal{A}_e(E)$ as shown in the right panel of Fig. 3. The figure indicates that it will be possible to discern the CPT violating contributions from the background of CPT conserving contributions if, for example, $Q_i = 0.8$ and $R_i = 0.5$. Note that \mathcal{A}_e is approximately equivalent to $(P_{ee} - P_{\bar{e}\bar{e}})$ multiplied by a geometric factor of $(L_{\text{near}}/L_{\text{far}})^2$.

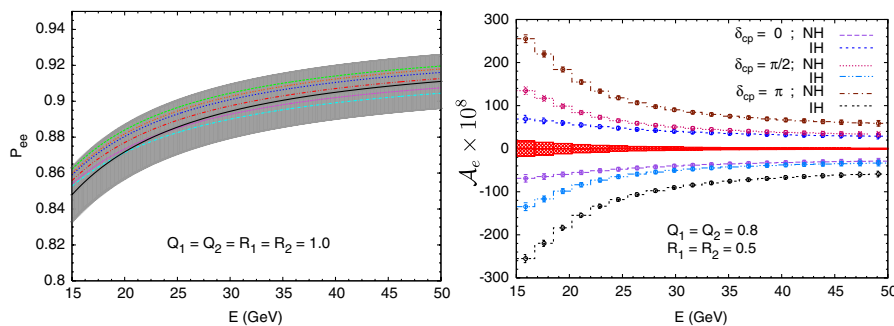


FIG. 3 (color online). In the left panel, the black (solid) line is the analytic expression, and the gray (shaded) band corresponds to the analytic value with an error ± 0.015 . The lines in six different colors (symbols) are for six random sets of CPT violating parameters with $Q_1, Q_2, R_1,$ and R_2 fixed at 1.0. We choose normal ordering, and the values of $\Delta m_{\odot}^2, \theta_{12}, \Delta m_{\text{atm}}^2, \theta_{23}, \theta_{13}$ and δ_{cp} the same as that in Fig. 1. The neutrinos traverse through the Earth for $L = 3000$ km before being detected. In the right panel, the central band shows the contribution in absence of CPT violation when the parameters are varied over their current 2σ ranges. The counts are for 2 years of running of neutrino factory with each of e^+ and e^- . The errors are only statistical.

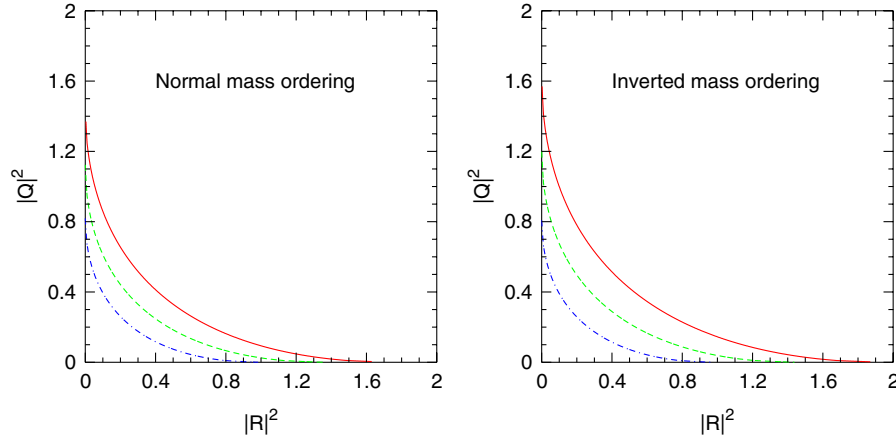


FIG. 4 (color online). Confidence level contours on $|Q|^2$ - $|R|^2$ plane. The red (solid), green (dashed) and blue (dash-dotted) curves give the 3σ , 2σ and 1σ contours, respectively. An energy independent error of ± 0.015 on P_{ee} has been taken into account. We use the same Δm_{21}^2 , θ_{12} , $|\Delta m_{\text{atm}}^2|$, θ_{23} and θ_{13} input values as in Fig. 1. The additional input values are $\delta_{\text{cp}} = \pi/3$ and $|Q|^2 = |R|^2 = 0$. All the parameters other than $|Q|^2$ and $|R|^2$ are marginalized over in the analysis.

The magnitude of $\mathcal{A}_e(E)$ depends on Q_1 , Q_2 , R_1 , R_2 , δ_{cp} and mass ordering. To estimate the possibility of identifying any CPT violating signal in spite of our current lack of knowledge about the standard oscillation parameters in the CPT conserving case, we display the confidence level contours in Fig. 4. We have chosen the best-fit values of Δm_{21}^2 , θ_{12} , $|\Delta m_{\text{atm}}^2|$, θ_{23} and θ_{13} [19] as the input values. Since we do not have any information about δ_{cp} , we choose the input value of δ_{cp} in the range that is observed to give the most conservative bound on $|Q|$ and $|R|$. From the left panel of Fig. 4, for the normal mass ordering the bounds obtained are

$$|Q|^2 \lesssim 1.1, \quad |R|^2 \lesssim 1.35 \quad (2\sigma), \quad (38)$$

while the right panel with inverted mass ordering gives

$$|Q|^2 \lesssim 1.2, \quad |R|^2 \lesssim 1.4 \quad (2\sigma). \quad (39)$$

It is observed that if the actual value of θ_{23} is smaller, the $|Q|^2$ bound decreases and the bound on $|R|^2$ becomes larger. The reverse is true when θ_{23} value is higher than the current best-fit value. This is true for both the mass orderings.

The bounds on $|Q|^2$ and $|R|^2$ translate to

$$|\mathbb{H}_{b13}| = \epsilon |Q| S_{E_0} \lesssim 10^{-23} \text{ GeV}, \quad (40)$$

$$|\mathbb{H}_{b12}| = \epsilon |R| S_{E_0} \lesssim 10^{-23} \text{ GeV}. \quad (41)$$

The reach of \mathcal{A}_e for the CPT violating observables is thus similar to that of \mathcal{A}_μ as obtained in Sec. III. However, note that the actual combinations of elements of \mathbb{H}_b constrained by the muon and electron channels are quite different.

V. CONSTRAINTS FROM BOUNDS ON NON-STANDARD INTERACTIONS

In the presence of NSI of neutrinos with matter, the effective Hamiltonian in the three-flavor basis becomes

$$\mathbb{H}_f \approx \mathbb{U}_0 \cdot \frac{\text{diag}(0, \Delta m_{21}^2, \Delta m_{31}^2)}{2E} \cdot \mathbb{U}_0^\dagger + V_e \epsilon_{\text{NSI}} + \text{diag}(V_e, 0, 0), \quad (42)$$

where ϵ_{NSI} is a 3×3 matrix

$$\epsilon_{\text{NSI}} = \begin{pmatrix} \epsilon_{ee} & \epsilon_{e\mu} & \epsilon_{e\tau} \\ \epsilon_{e\mu}^* & \epsilon_{\mu\mu} & \epsilon_{\mu\tau} \\ \epsilon_{e\tau}^* & \epsilon_{\mu\tau}^* & \epsilon_{\tau\tau} \end{pmatrix} \quad (43)$$

that parametrizes the NSI interactions. The factor of V_e multiplying ϵ_{NSI} represents that the net NSI strength depends on the density of matter. The Hamiltonian for the antineutrinos will be obtained just by $V_e \rightarrow -V_e$ and $\epsilon_{\alpha\beta} \rightarrow \epsilon_{\alpha\beta}^*$.

Since CPT violation necessarily implies NSI, the bounds on the NSI violating parameters $\epsilon_{\alpha\beta}$ would restrict CPT violating parameters as well. In order to see the exact correspondence, note that the oscillation experiments are sensitive to only differences in the eigenvalues of the Hamiltonian, and not to the absolute eigenvalues. Therefore, the part of the NSI relevant for oscillation experiments is only

$$\mathbb{H}_{\text{NSI}} \equiv \epsilon_{\text{NSI}} - \epsilon_{ee} \mathbb{1}, \quad (44)$$

where $\mathbb{1}$ is the identity matrix. Then the comparison of Eqs. (8) and (42) implies that the mapping

$$\mathbb{H}_b \iff \mathbb{H}_{\text{NSI}} \quad (45)$$

would allow us to translate the results from one parametrization to the other. Note that there is a difference be-

tween the two sources of nonstandard physics under consideration. Whereas \mathbb{H}_{NSI} is proportional to the matter density, \mathbb{H}_b is independent of it. However, as long as the matter density relevant for the experiments restricting \mathbb{H}_{NSI} is known and is almost a constant, the CPT violating contributions may be mimicked by the NSI ones. Therefore, the bounds on $\epsilon_{\alpha\beta}$ from the NSI analysis can be translated to the bounds on the elements of \mathbb{H}_b in the CPT parametrization.

Note that the bounds obtained from the CPT analysis cannot be applied to the NSI bounds, since there can be sources of NSI that are CPT conserving. If an experiment is sensitive to the variations of matter density along the neutrino path, it will be able to separate the NSI contributions from the CPT violating ones.

A two-flavor analysis of the atmospheric neutrino data combined with the MACRO data [25] and K2K data [26] yields

$$\begin{aligned} -0.05 \leq \epsilon_{\mu\tau} \leq 0.04 \quad (99\% \text{C.L.}) \Rightarrow \\ |\mathbb{H}_{b23}| \leq 10^{-23} \text{ GeV}, \end{aligned} \quad (46)$$

where we have assumed an average density of 4.5 g/cc inside the Earth. This bound is comparable to what would be obtained using long baseline experiments as described in Sec. IV.

The neutrino scattering experiments CHARM and NuTeV mainly constrain the NSI couplings of ν_μ , and give [27,41,42]

$$|\epsilon_{e\mu}| \leq 10^{-3} \Rightarrow |\mathbb{H}_{b23}| \leq 1.1 \times 10^{-25} \text{ GeV}, \quad (47)$$

where we take the average Earth matter density to be 2.7 g/cc. This constraint is extremely strong, and would imply $|R| \approx 0$, thus simplifying the analysis of Sec. IV. These experiments also bound

$$|\epsilon_{\mu\mu}| \leq 10^{-2}, \quad (48)$$

which by itself does not put any constraints on the CPT violating parameters since only the differences between the diagonal elements of new physics Hamiltonian are relevant.

Using the bounds on $\epsilon_{\mu\beta}$ stated above, [28–30] analyzed the possibility of constraining ϵ_{ee} , $\epsilon_{e\tau}$ and $\epsilon_{\tau\tau}$ in MINOS experiment assuming $\epsilon_{e\mu} = \epsilon_{\mu\mu} = \epsilon_{\mu\tau} = 0$. This effectively two-neutrino analysis leads to

$$\begin{aligned} |\epsilon_{e\tau}| \leq 2.9 \quad (99\% \text{C.L.}) \Rightarrow \\ |\mathbb{H}_{b13}| \leq 3.2 \times 10^{-22} \text{ GeV}, \end{aligned} \quad (49)$$

which will be improved significantly at the neutrino factory with the $\nu_e \rightarrow \nu_e$ channel, as described in Sec. IV.

The 99% C.L. bounds on the diagonal NSI elements, given the initial assumption of $\epsilon_{\mu\mu} = 0$, translate as [29]

$$-0.4 \leq \epsilon_{\tau\tau} \leq 4.5 \Rightarrow$$

$$-0.5 \times 10^{-22} \text{ GeV} < \mathbb{H}_{b22} - \mathbb{H}_{b33} < 5.0 \times 10^{-22} \text{ GeV}, \quad (50)$$

$$-1.0 \leq \epsilon_{ee} \leq 0.9 \Rightarrow |\mathbb{H}_{b22}| < 10^{-22} \text{ GeV}. \quad (51)$$

Here, we have taken an average matter density of 2.7 g/cc, which is relevant for the MINOS baseline of 732 km. As seen in Sec. III, NOvA will be able to constrain $\mathbb{H}_{b22} - \mathbb{H}_{b33}$ to a much better accuracy. The channels we have considered are rather insensitive to the absolute value of $|\mathbb{H}_{b22}|$.

Current and future long baseline experiments like OPERA and T2KK are expected to improve the bounds on NSI parameters [43,44], and hence indirectly, those on the CPT violating parameters. Data from a future galactic supernova will also contribute to constraints on NSI parameters [45], but converting them to bounds on CPT violation will not be straightforward since the situation cannot be approximated with a constant matter density.

VI. SUMMARY AND CONCLUSIONS

We have calculated possible CPT violating contributions to neutrino masses and mixings in the complete three-flavor analysis. Parametrizing the leading CPT violating effects by a Hermitian matrix \mathbb{b} that adds to the effective neutrino Hamiltonian, we have developed a framework based on the perturbative expansion in a small auxiliary parameter $\epsilon \equiv 0.1$. It involves expanding the elements of \mathbb{H}_b (the matrix \mathbb{b} in the flavor basis), the ratio $\Delta m_{\text{O}}^2 / \Delta m_{\text{atm}}^2$, and θ_{13} as powers of ϵ multiplied by $\mathcal{O}(1)$ numbers. This allows us to treat the CPT violating \mathbb{b} contributions in all generality, while keeping the analytical expressions simple and transparent. Though the complete parametrization of \mathbb{b} involves three eigenvalues, three mixing angles, and six phases, we show that only certain combinations appear in the survival probabilities of muon and electron neutrinos, so that the analysis needs to concentrate only on limiting those combinations.

The survival probabilities of ν_μ and $\bar{\nu}_\mu$ to $\mathcal{O}(\epsilon)$ involve only two combinations of elements of \mathbb{H}_b , viz. the real parameters $B_1 \propto \mathbb{H}_{b22} - \mathbb{H}_{b33}$ and $B_2 \propto \text{Re}(\mathbb{H}_{b23})$. Formally, the CPT violating contribution due to these terms is of a higher order than the CP violating contribution in the CPT conserving limit. The contribution due to B_1 vanishes when θ_{23} is maximal, so that a deviation of θ_{23} needs to be established in order to put any bounds on this parameter. The other quantity B_2 may be constrained to be $|B_2| \leq 0.1$ with 4 years of running with ν_μ and $\bar{\nu}_\mu$ each at NOvA with an incident flux of $10^{21} \text{ pot yr}^{-1}$ at 2σ . This would correspond to bounds on $\mathbb{H}_{b22} - \mathbb{H}_{b33}$ and $\text{Re}(\mathbb{H}_{b23})$ of the order 10^{-23} GeV . Note that though the constraints that we have obtained are of the same order as those obtained in earlier studies [20,21], our analysis identifies

the exact combination of elements of \mathbb{H}_b , and hence \mathbb{b} , that these bounds apply to.

The CPT violating contribution to the survival probability of ν_e and $\bar{\nu}_e$ appears only at $\mathcal{O}(\epsilon^2)$, and hence is expected to be more difficult to extract. We isolate two different combinations of elements of \mathbb{b} , *viz.* the complex parameters $Q \propto \mathbb{H}_{b13}$ and $R \propto \mathbb{H}_{b12}$, that govern this contribution and numerically analyze the feasibility of extracting them. We demonstrate that for $|Q|, |R| \gtrsim 1.2$, it may be possible to ascertain the presence of CPT violation at 2σ at a neutrino factory with a detector at $L = 3000$ km that can distinguish ν_e from $\bar{\nu}_e$, within 4 years. This corresponds to bounds on \mathbb{H}_{b12} and \mathbb{H}_{b13} of the order 10^{-23} GeV. Note that the exact combinations of elements of \mathbb{H}_b that are constrained by the muon and electron channels are quite different.

The CPT violating observables \mathcal{A}_μ and \mathcal{A}_e in this paper are the same as those considered in [46] for disentangling the signals of sterile neutrinos. The energy dependence of the signatures of CPT violation and sterile neutrinos, however, is different and these two new physics signatures may be disentangled with a combined analysis.

The constraints obtained on the NSI parameters through oscillation and nonoscillation experiments can be translated to bounds on elements of \mathbb{H}_b . We find that the bound on $|\mathbb{H}_{b12}|$ implied by the NSI constraints is much stronger than the expected reach of even neutrino factories, whereas

the bound on $|\mathbb{H}_{b23}|$ is comparable to the one expected at NOvA. On the other hand, $|\mathbb{H}_{b13}|$ and the difference $\mathbb{H}_{b22} - \mathbb{H}_{b33}$ will be much better constrained by the long baseline experiments. NSI analyses give a constraint on the absolute value of \mathbb{H}_{b22} , to which the channels we have considered are rather insensitive.

In this paper, we have confined ourselves to low energies ($E < 5$ GeV) for the muon channel and high energies ($E > 15$ GeV) for the electron channel. This allowed us to cleanly isolate certain combinations of elements of \mathbb{H}_b , *viz.* two real quantities $\mathbb{H}_{b22} - \mathbb{H}_{b33}$, $\text{Re}(\mathbb{H}_{b23})$ through the muons and two complex quantities \mathbb{H}_{b12} , \mathbb{H}_{b13} through the electrons. A more exhaustive analysis that uses the complete energy range and the long baseline as well as the atmospheric neutrino data may lead to constraints on other combinations of elements of \mathbb{H}_b . However, it is not clear if it can be achieved through a clean analytic treatment.

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