Further study of an approach to the unification of gauge symmetries in theories with dynamical symmetry breaking

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We extend to larger unification groups an earlier study exploring the possibility of unification of gauge symmetries in theories with dynamical symmetry breaking. Based on our results, we comment on the outlook for models that seek to achieve this type of unification.

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I. INTRODUCTION

The origin of electroweak symmetry breaking is one of the most important outstanding questions in particle physics. One possibility is that this breaking is caused by the formation of a bilinear condensate of new fermions interacting via an asymptotically free, vectorial gauge interaction, called technicolor (TC), that becomes strong at the TeV scale [1]. To communicate the electroweak symmetry breaking to the quarks and leptons and generate masses for these fermions, one embeds this theory in a larger one, extended technicolor (ETC), containing gauge bosons that transform quarks and leptons into the new fermions, and vice versa [2,3]. These theories are subject to stringent constraints from precision electroweak measurements and measurements of, or limits on, flavor-changing neutral currents. Modern theories of this type incorporate a gauge coupling that runs slowly over an extended interval of energies to enhance quark and lepton fermion masses. Calculations indicate that this behavior can also reduce technicolor corrections to the Z and W boson propagators somewhat [4,5]; however, because of the strongly interacting nature of the relevant physics, there remain significant theoretical uncertainties in the estimates of these corrections.

A natural question that arises in considering these theories with dynamical electroweak symmetry breaking is how the technicolor gauge interaction might be unified with the gauge group of the standard model (SM), $G_{SM} = SU(3)_c \times SU(2)_w \times U(1)_Y$. In Ref. [6], a partially unified model of this type was constructed with the property that the electric charge operator is a linear combination of generators of non-Abelian gauge groups, and hence electric charge is quantized. Ideally, one would like to go further and embed the TC gauge group G_{TC}, together with G_{SM}, in a simple group, thereby relating the associated gauge couplings [7]. In Ref. [8], a study was carried out of several approaches to this type of unification.

Here, we shall extend the analysis of Ref. [8]. We consider models that are designed to unify G_{SM} with G_{TC} or a larger gauge symmetry described by a group $G_{SC} \supseteq G_{TC}$ (where SC denotes "strongly coupled"), in a simple Lie group G

$$G \supset G_{SC} \times G_{GU}. \tag{1.1}$$

A notable feature of this approach is that it incorporates a dynamical origin for the number of generations of quarks and leptons, N_{gen}. A simple group G_{GU} that contains G_{SM} has a lower bound on its rank of $rk(G_{GU}) \geq rk(G_{SM}) = 4$, and the minimal non-Abelian group that one could use for G_{SC} has rank 2. It follows that the rank of G satisfies

$$rk(G) \geq rk(G_{SC}) + rk(G_{GU}) \geq 6. \tag{1.2}$$

It is natural to focus on SU(N) groups, using $SU(N_{SC}) \supseteq SU(N_{TC})$ and

$$SU(N) \supset SU(N_{SC}) \times SU(5)_{GU}, \tag{1.3}$$

where $SU(5)_{GU}$ is the usual grand unification group [9], with

$$N = N_{SC} + 5. \tag{1.4}$$

Since the group SU(N_{SC}) involves interactions that should get strong at or above the TeV scale, it must be asymptotically free and hence non-Abelian. Since the minimal value of N_{SC} is thus 2, it follows that the minimal value of N is 7. However, the $N = 7$ case yields only two standard-model fermion generations [7]. In Ref. [8], cases up to $N = 10$ were studied, including several that satisfy the requirement of yielding $N_g = 3$ standard-model fermion generations, and some challenges for this unification program were found. Here, we shall extend this study, considering the next two higher cases, $N = 11$ and $N = 12$. Based on our findings, we discuss aspects of this approach to unification of theories with dynamical electroweak symmetry breaking.

II. GENERAL STRUCTURE OF UNIFICATION MODELS

We consider a general approach in which some SM fermion generations may arise directly from the representations of the unified group G, while the remaining ones arise indirectly, from sequential symmetry breaking of a subgroup of G at ETC-type scales. Let us denote N_{gh} and $N_{g\ell}$ as the numbers of standard-model fermion generations arising from these two sources, respectively, where the

TABLE I. Some properties of the models discussed in the text with G_{SC} and G_{SM} unified in a simple group G. Here, $G_{SC} = SU(N_{SC})$, $G_{TC} = SU(N_{TC})$, and $G_{SC} \supseteq G_{TC}$. The column marked "SCC" lists some properties of the $SU(N_{SCC})$ theory combining the $SU(N_{SC})$ and $SU(3)_c$ groups. See text for further definitions and discussion. The fermion content is indicated by the vector **n** (with subscript omitted for brevity). The notation "no sol." means that (in the dynamical framework used) there is no solution to the requirements of absence of any $SU(N)$ gauge anomaly, well-defined SM fermion generations, and $N_{gen} = 3$. The notation VGT and CGT indicate that the gauge interaction is vectorial and chiral, respectively; AF and NAF mean asymptotically free and nonasymptotically free, respectively. The $N_{(1,1)}$ is the number of electroweak-singlet neutrinos. The results up to $N = 10$ are included for comparative purposes.

N	N_{SCC}	N_{SC}	N_{TC}	$N_{g\ell}$	N_{gh}	**n**	SCC	$N_{(1,1)}$
7	5	2	2	0	3	no sol.	—	—
8	6	3	3	0	3	(0,2,0,0,1,0,3)	VGT, AF	1
8	6	3	2	1	2	no sol.	—	—
9	7	4	4	0	3	no sol.	—	—
9	7	4	3	1	2	(0,1,0,1,0,1,0,1)	VGT, AF	1
9	7	4	2	2	1	no sol.	—	—
10	8	5	5	0	3	(0,0,0,3,0,0,3,0,0)	CGT, NAF	0
10	8	5	4	1	2	(0,0,0,2,0,0,2,0,0)	CGT, NAF	0
10	8	5	3	2	1	(0,0,0,1,0,0,1,0,0)	CGT, AF	0
10	8	5	3	2	1	(0,0,0,1,1,0,1,0,0)	CGT, NAF	2
10	8	5	2	3	0	(0,0,0,0,1,0,0,0,0)	VGT, AF	2
11	9	6	6	0	3	no sol.	—	—
11	9	6	5	1	2	(0,0,0,0,0,0,3,0,0,2,0,2)	iia, CGT, NAF	3
11	9	6	5	1	2	(0,1,0,1,0,1,0,1,0,1)	iib, VGT, NAF	1
11	9	6	5	1	2	(0,2,0,2,0,5,0,0,0,0)	iic, CGT, NAF	5
11	9	6	4	2	1	no sol.	—	—
11	9	6	3	3	0	(0,0,0,0,1,1,0,0,0,0)	i, VGT, NAF	2
12	10	7	4	3	0	(0,0,0,0,0,1,0,0,0,0,0)	ia, VGT, NAF	0
12	10	7	4	3	0	(0,0,0,0,1,0,1,0,0,0,0)	ib, VGT, NAF	2
12	10	7	7	0	3	(0,1,0,1,0,0,0,0,2,0,2)	ii, CGT, NAF	0

subscripts gh and $g\ell$ refer to generations from the representation content of the high-scale symmetry group and from the lower-scale breaking. The sum of these satisfies

$$N_{gen} = 3 = N_{gh} + N_{g\ell}. \tag{2.1}$$

At this stage, the number $N_{g\ell}$ is only formal; that is, we construct a model so that, *a priori*, it can have the possibility that a subgroup of G such as G_{SC} might break in such a manner as to peel off $N_{g\ell}$ SM fermion generations. However, we must examine for each model whether this breaking actually occurs; this will be discussed further below.

We next explain our procedure for analyzing the models; for further details, the reader is referred to Ref. [8]. The fermion representations are determined by the structure of the fundamental representation, which we take to be

$$\psi_R = \begin{pmatrix} (N^c)^\tau \\ d^a \\ -e^c \\ \nu_e^c \end{pmatrix}_R , \tag{2.2}$$

where d, e, and ν are generic symbols for the fermions with these quantum numbers. Thus, the indices on ψ_R are ordered so that the indices in the SC set, which we shall denote τ, take on the values $\tau = 1, \ldots N_{SC}$ and then the remaining five indices are those of the 5_R of $SU(5)_{GU}$, including the color index a on d^a. The components of N_R^c transform according to the fundamental representation of $SU(N_{SC})$, are singlets under $SU(3)_c$ and $SU(2)_w$, and have zero weak hypercharge and hence also zero electric charge. This structure is concordant with the direct product in Eq. (1.1) and the corresponding commutativity property $[G_{SC}, G_{GU}] = 0$, and hence $[G_{TC}, G_{GU}] = 0$. (Recent discussions of models with higher-dimensional representations of G_{TC} include [10]; some other approaches to unification of G_{TC} with SM gauge symmetries and gauge coupling unification in the SM itself include [11].)

We next specify the fermion representations of $G = SU(N)$. In the following, we shall usually write the fermion fields as left handed. In order to avoid fermion representations of $SU(3)_c$ and $SU(2)_w$ other than those experimentally observed, namely, singlets and fundamental or conjugate fundamental representations, we restrict the fermions to transform as k fold totally antisymmetrized products of the fundamental or conjugate fundamental representation of SU(N); these are denoted as $[k]_N$ and $[\bar{k}]_N = \overline{[k]}_N$. A set of (left-handed) fermions $\{f\}$ transforming under G is thus given by

$$\{f\} = \sum_{k=1}^{N-1} n_k [k]_N \tag{2.3}$$

where n_k denotes the multiplicity (number of copies) of each representation $[k]_N$. We use a compact vector notation $\mathbf{n} \equiv (n_1, \ldots, n_{N-1})_N$. If $k = N - \ell$ is greater than the integral part of $N/2$, we shall work with $[\bar{\ell}]_N$ rather than $[k]_N$; these are equivalent with respect to SU(N).

An acceptable model should satisfy the following requirements: (i) the contributions from various fermions to the total SU(N) gauge anomaly must cancel each other, yielding zero gauge anomaly; (ii) the resultant TC-singlet, SM-nonsinglet left-handed fermions must comprise a well-defined set of generations, i.e., must consist of $N_{\rm gen} = 3$ copies of $[(1, \bar{5})_L + (1, 10)_L]$, where the first number in parentheses signifies that these are singlets under $G_{\rm TC}$ and the second number denotes the dimension of the $SU(5)_{\rm GU}$ representation; and (iii) in order to account for neutrino masses, one needs to have TC-singlet, electroweak-singlet neutrinos to produce Majorana neutrino mass terms that can drive an appropriate seesaw [12,13]. Here, these are also singlets under $SU(5)_{\rm GU}$.

As another requirement, (iv), the ETC gauge bosons should have appropriate masses, in the range from a few TeV to 10^3 TeV, so as to produce acceptable SM fermion masses. This requirement cannot be satisfied if G breaks directly to the direct product group $G_{\rm TC} \times G_{\rm SM}$ at the unification scale $M_{\rm GU}$ as in early approaches to TC unification [14]. The requirement could be satisfied if the breaking of G at $M_{\rm GU}$ would leave an invariant subgroup $SU(2)_w \times G_{\rm SCC}$, where

$$SU(N_{\rm SCC}) \supset SU(N_{\rm SC}) \times SU(3)_c, \tag{2.4}$$

with

$$N_{\rm SCC} = N_{\rm SC} + N_c = N_{\rm SC} + 3. \tag{2.5}$$

Here, SCC stands for the the SC group together with the color group. As the energy scale decreases, this intermediate symmetry $G_{\rm SCC}$ should break at ETC scales, eventually yielding the residual exact symmetry group $SU(2)_{\rm TC} \times SU(3)_c$. This can occur naturally if the SCC gauge interaction is chiral and asymptotically free; as the energy scale decreases and the SCC gauge coupling increases, it can thus trigger the formation of a fermion condensate which self-breaks $G_{\rm SCC}$. This type of process in which a strongly coupled chiral gauge interaction self-breaks via formation of a fermion condensate has been termed "tumbling" [15]. Further requirements are that (v) if $N_{\rm SC} > N_{\rm TC}$, there should be a mechanism to break $SU(N_{\rm SC})$ to $SU(N_{\rm TC})$; (vi) the TC interaction should be vectorial and asymptotically free, so that the TC gauge coupling gets large as the energy scale decreases to the TeV scale, triggering the formation of a technifermion condensate for EWSB; and (vii) the residual $SU(3)_c$ color group should be asymptotically free.

Let us define a $(N-1)$-dimensional vector whose components are the values of the anomaly $A([k]_N)$ with respect to SU(N), $\mathbf{a} = (A([1]_N), \ldots, A([N-1]_N))$. Then the constraint that there be no G gauge anomaly is the condition

$$\mathbf{n} \cdot \mathbf{a} = 0. \tag{2.6}$$

This is a diophantine equation for the components of the vector of multiplicities $\mathbf{n}$, subject to the constraint that the components n_k are non-negative integers (as well as additional constraints discussed below).

It is convenient to display the transformation property of a fermion representation of G with respect to the subgroups $G_{\rm SC}$ and $SU(5)_{\rm GU}$ by the notation $(\mathcal{R}_{\rm SC}, \mathcal{R}_{\rm GU})$. The number of (left-handed) fermions that transform as singlets under $G_{\rm SC}$ and $\bar{5}$'s of $SU(5)_{\rm GU}$ is

$$N_{(1,\bar{5})} = n_{N_{\rm SC}+4} + n_4, \tag{2.7}$$

and the number of (left-handed) fermions that transform as singlets under $G_{\rm SC}$ and 10's of $SU(5)_{\rm GU}$ is

$$N_{(1,10)} = n_2 + n_{N_{\rm SC}+2}. \tag{2.8}$$

Hence, the requirement that the left-handed SC-singlet, SM-nonsinglet fermions comprise equal numbers of $(1, \bar{5})$ and $(1, 10)$'s implies the condition

$$n_{N_{\rm SC}+4} + n_4 = n_2 + n_{N_{\rm SC}+2}. \tag{2.9}$$

The number of SM fermion generations N_{gh} produced by the representations of G is given by either side of this equation

$$N_{gh} = n_2 + n_{N_{\rm SC}+2}. \tag{2.10}$$

The remaining $N_{g\ell}$ generations of SM fermions arise via the breaking of $G_{\rm SC}$. Electroweak-singlet neutrinos, arise, in general, from two sources: (i) $[N_{\rm SC}]_N$, when all of the $N_{\rm SC}$ indices take values in $SU(N_{\rm SC})$; and (ii) $[5]_N$, when all of the indices take values in $SU(5)_{\rm GU}$. In the special case $N_{\rm SC} = 5$, these each contribute. Hence,

$$N_{(1,1)} = n_{N_{\rm SC}} + n_5. \tag{2.11}$$

Electroweak-singlet neutrinos arise from fermions that are singlets under both $G_{\rm SC}$ and $SU(5)_{\rm GU}$; there are $N_{(1,1)} = n_{N_{\rm SC}} + n_5$ of these.

With the envisioned sequential breaking of $G_{\rm SCC}$ and $G_{\rm SC}$ that would produce the $N_{g\ell}$ SM fermion generations, one has $N_{g\ell} = N_{\rm SCC} - (N_{\rm TC} + N_c)$, and

$$N_{g\ell} = N_{\rm SC} - N_{\rm TC}. \tag{2.12}$$

The requirement that there be no (left-handed) fermions transforming as singlets under $SU(N_{\rm SC})$ and in an exotic manner, as 5's or $\overline{10}$'s of $SU(5)_{\rm GU}$ is satisfied if

$$n_1 = 0, \qquad n_{N_{SC}+1} = 0 \tag{2.13}$$

and

$$n_3 = 0, \qquad n_{N_{SC}+3} = 0, \tag{2.14}$$

respectively.

III. $N_{SC} = 6$, $G = SU(11)$

We next proceed to analyze the new models, and first consider the case where $N_{SC} = 6$, so that $N = N_{SC} + 5 = 11$ and $\mathbf{n} = (n_1, \ldots, n_{10})_{11}$. With $N_{gh} + N_{g\ell} = N_{gen} = 3$ and $N_{SC} - N_{TC} = N_{g\ell}$, one has, *a priori*, four possibilities for the manner in which the SM fermion generations arise, as specified by $(N_{gh}, N_{g\ell}, N_{TC})$, namely, (3, 0, 6), (2, 1, 5), (1, 2, 4), and (0, 3, 3). However, as we shall show, only the cases with $N_{gh} = 0$ and $N_{gh} = 2$ are actually allowed by the various constraints. This SU(11) model was not studied in Ref. [8], because it does not allow one to use the preferred, minimal value $N_{TC} = 2$. This latter value is preferred in order to minimize technicolor corrections to precisely measured electroweak quantities, and because it makes possible a mechanism to produce light neutrino masses [6,12,13]. However, if one takes into account the fact that quasiconformal behavior in the technicolor theory can reduce the technicolor corrections to the Z and W boson propagators, the effect of the larger value of N_{TC} might not be too serious. The conditions that the theory should not contain any 5_L or $\overline{10}_L$ yield

$$n_1 = n_3 = n_7 = n_9 = 0, \tag{3.1}$$

and Eq. (2.9) is

$$N_{gh} = n_2 + n_8 = n_4 + n_{10}. \tag{3.2}$$

The condition of zero gauge anomaly, Eq. (2.6), is

$$7(n_2 + 4n_4 + 2n_5 - 2n_6) - 20n_8 - n_{10} = 0. \tag{3.3}$$

For a given value of $N_{gh} = 3 - N_{g\ell}$, these are three nondegenerate linear equations for the six quantities n_2, n_4, n_5, n_6, n_8, and n_{10}. The solution entails the relation

$$n_5 = n_6 + \frac{1}{14}(27n_8 + 29n_{10}) - \frac{5}{2}N_{gh}. \tag{3.4}$$

A necessary condition for an acceptable solution is thus that

$$27n_8 + 29n_{10} - 35N_{gh} = 0 \mod 14. \tag{3.5}$$

Let r be a non-negative integer. We find two classes of such solutions: (i) $N_{gh} = 0$, $n_8 = n_{10} = r$, and hence, from Eq. (3.4), $n_5 = n_6 + 4r$; (ii) $N_{gh} = 2$, $n_8 = n_{10} = r$, and hence $n_5 = n_6 + 4r - 5$.

We first consider solutions of class (i). These have $N_{g\ell} = 3$ and $N_{TC} = 3$. Now $N_{gh} = n_2 + n_8 = n_4 + n_{10} = 0$, which implies that $r = 0$, $n_2 = n_8 = n_4 = n_{10} = 0$, and $n_5 = n_6 = s$, where s is some positive integer. The resultant vector $\mathbf{n}$ is

$$\text{class (i)}: \mathbf{n} = (0, 0, 0, 0, s, s, 0, 0, 0, 0). \tag{3.6}$$

The minimal choice would be $s = 1$, but for generality, we shall keep s arbitrary. Since $[6]_{11} \approx [\bar{5}]_{11}$, this SU(11) theory has left-handed chiral fermion content

$$s\{[5]_{11} + [\bar{5}]_{11}\} \tag{3.7}$$

and thus is vectorial. Consequently, the fermion content with respect to the subgroups $SU(9)_{SCC}$ and $SU(6)_{SC}$ is also vectorial. With respect to the subgroup

$$SU(2)_w \times SU(9)_{SCC}, \tag{3.8}$$

the $[5]_{11}$ representation transforms as

$$[5]_{11} = (1, [\bar{4}]_9) + (2, [4]_9) + (1, [3]_9), \tag{3.9}$$

where we use the $[k]_9$ notation for the representations of $SU(9)_{SCC}$ and the well-known dimensions to label the representations of $SU(2)_w$. The total fermion content with respect to the subgroup (3.8) is comprised of s copies of Eq. (3.9) and its conjugate. We recall the requirement that the SCC and SC interactions should be asymptotically free. For a given gauge group G_j with gauge coupling g_j and $\alpha_j = g_j^2/(4\pi)$, the evolution of the gauge couplings as a function of the momentum scale μ is given by the beta function $\beta_j = d\alpha_j/dt = -b_0^{G_j}\alpha_j^2/(2\pi) + O(\alpha_j^3)$, where $t = \ln\mu$. We find that the $SU(9)_{SCC}$ gauge interaction is nonasymptotically free. Here and below, for comparative purposes, it will be useful to give the actual coefficients. We calculate

$$b_0^{SU(9)_{SCC}} = 3(11 - 28s) \qquad (\text{class i}), \tag{3.10}$$

which is negative for any value $s \geq 1$. With respect to the subgroup

$$SU(6)_{SC} \times SU(5)_{GU}, \tag{3.11}$$

the $[5]_{11}$ representation transforms as

$$[5]_{11} = (1, 1) + ([1]_6, \bar{5}) + ([2]_6, \overline{10}) + ([3]_6, 10) + ([\bar{2}]_6, 5) + ([\bar{1}]_6, 1), \tag{3.12}$$

where, aside from the overall singlet (1, 1), we use the $[k]_6$ notation for the representations of $SU(6)_{SC}$ and the well-known dimensions to label the representations of $SU(5)_{GU}$. The fermion content of this model with respect to the subgroup is the sum of s copies of Eq. (3.12) and its conjugate. The $SU(6)_{SC}$ gauge interaction is not asymptotically free; the leading coefficient of its beta function is

$$b_0^{SU(6)_{SC}} = 2(11 - 42s) \quad (\text{class i}), \tag{3.13}$$

which is negative for any $s \geq 1$. This disfavors the model.

We next consider models of class (ii). These have $N_{g\ell} = 1$ and $N_{TC} = 5$. The relations $N_{gh} = n_2 + n_8 = n_4 + n_{10} = 2$, together with the assignment $n_8 = n_{10} = r$ imply that

$$n_2 = n_4 = 2 - r. \tag{3.14}$$

We thus have three subclasses of solutions, namely, (iia) $r = 2$, whence $n_2 = n_4 = 0$ and $n_5 = n_6 + 3$; (ii.b) $r = 1$, whence $n_2 = n_4 = 1$ and $n_5 = n_6 - 1$; and (ii.c) $r = 0$, whence $n_2 = n_4 = 2$ and $n_5 = n_6 - 5$. Minimal choices in each of these three subclasses have the following **n** vectors (see Table I):

$$\text{(iia): } \mathbf{n} = (0, 0, 0, 0, 3, 0, 0, 2, 0, 2), \tag{3.15}$$

$$\text{(iib): } \mathbf{n} = (0, 1, 0, 1, 0, 1, 0, 1, 0, 1), \tag{3.16}$$

$$\text{(iic): } \mathbf{n} = (0, 2, 0, 2, 0, 5, 0, 0, 0, 0). \tag{3.17}$$

The fermions of set (iia) transform, with respect to the subgroup (3.8), according to

$$3[5]_{11} = 3\{(1, [\bar{4}]_9) + (2, [4]_9) + (1, [3]_9)\}, \tag{3.18}$$

$$2[\bar{3}]_{11} = 2\{(1, [\bar{3}]_9) + (2, [\bar{2}]_9) + (1, [\bar{1}]_9)\}, \tag{3.19}$$

$$2[\bar{1}]_{11} = 2\{(1, [\bar{1}]_9) + (2, 1)\}. \tag{3.20}$$

With the $SU(2)_w$ couplings small, the nonsinglet $SU(9)_{SCC}$ fermion content is thus

$$\{f\} = 4[\bar{1}]_9 + 4[\bar{2}]_9 + 3[3]_9 + 2[\bar{3}]_9 + 6[4]_9 + 3[\bar{4}]_9. \tag{3.21}$$

Hence, the $SU(9)_{SCC}$ sector is a chiral gauge theory. If the $SU(9)_{SCC}$ gauge interaction were asymptotically free and hence increased as the energy scale decreased below M_{GU}, one could proceed to the next step and analyze self-breaking condensate formation in the theory. However, we find that the $SU(9)_{SCC}$ interaction is nonasymptotically free, having a leading coefficient of its beta function equal to

$$b_0^{SU(9)_{SCC}} = -\frac{353}{3} \qquad \text{(class iia)}. \tag{3.22}$$

With respect to the subgroup (3.11), the (left-handed chiral) fermions of the set (iia) decompose according to

$$3[5]_{11} = 3\{(1, 1) + ([1]_6, \bar{5}) + ([2]_6, \overline{10}) + ([3]_6, 10) + ([\bar{2}]_6, 5) + ([\bar{1}]_6, 1)\}, \tag{3.23}$$

$$2[8]_{11} \approx 2[\bar{3}]_{11} = 2\{(1, 10) + ([\bar{1}]_6, \overline{10}) + ([\bar{2}]_6, \bar{5}) + ([\bar{3}]_6, 1)\}, \tag{3.24}$$

and

$$2[10]_{11} \approx 2[\bar{1}]_{11} = 2\{([\bar{1}]_6, 1)) + (1, \bar{5})\}. \tag{3.25}$$

With the $SU(5)_{GU}$ couplings small, the nonsinglet left-handed fermions transform according to the following $SU(6)_{SC}$ representations:

$$\{f\} = 15[1]_6 + 25[\bar{1}]_6 + 30[2]_6 + 25[\bar{2}]_6 + 32[3]_6], \tag{3.26}$$

where we have used the fact that $[3]_6$ is equivalent to $[\bar{3}]_6$. Hence, the $SU(6)_{SC}$ gauge interaction is chiral. However, this class of models is disfavored because the $SU(6)_{SC}$ gauge interaction is not asymptotically free; the leading coefficient of the beta function is

$$b_0^{SU(6)_{SC}} = -\frac{386}{3} \qquad \text{(class iia)}. \tag{3.27}$$

Hence, the $SU(6)_{SC}$ gauge coupling gets smaller rather than larger as the energy scale decreases from high values, precluding the possibility of condensate formation and self-breaking of $SU(6)_{SC}$ to extract the $SU(5)_{SC}$ group and a $N_{g\ell} = 1$ generation of SM fermions.

We next consider the subclass (iib). The fact that an $SU(N)$ gauge theory with odd $N \geq 5$ and left-handed fermion content given by $n_i = 0$ for $i = 1, 3, \ldots, N-2$ and $n_i = 1$, $i = 2, 4, \ldots, N-1$ is anomaly free was shown in [16]. With respect to the subgroup (3.8), the fermions for this class decompose according to

$$[2]_{11} = (1, [2]_9) + (2, [1]_9) + (1, 1), \tag{3.28}$$

$$[4]_{11} = (1, [4]_9) + (2, [3]_9) + (1, [2]_9), \tag{3.29}$$

$$[6]_{11} \approx [\bar{5}]_{11} = (1, [4]_9) + (2, [\bar{4}]_9) + (1, [\bar{3}]_9), \tag{3.30}$$

$$[8]_{11} \approx [\bar{3}]_{11} = (1, [\bar{3}]_9) + (2, [\bar{2}]_9) + (1, [\bar{1}]_9), \tag{3.31}$$

$$[10]_{11} \approx [\bar{1}]_{11} = (1, [\bar{1}]_9) + (2, 1). \tag{3.32}$$

With the $SU(2)_w$ couplings small, the nonsinglet $SU(9)_{SCC}$ fermion sector is then

$$\{f\} = 2\{[1]_9 + [\bar{1}]_9 + [2]_9 + [\bar{2}]_9 + [3]_9 + [\bar{3}]_9 + [4]_9 + [\bar{4}]_9\}. \tag{3.33}$$

Hence, although the SU(11) gauge interaction is chiral, the $SU(9)_{SCC}$ gauge interaction is vectorial. Even if the $SU(9)_{SCC}$ interaction were asymptotically free, this vectorial property would disfavor this class of models, because it would not self-break. The $SU(9)_{SCC}$ interaction is actually not asymptotically free; we calculate that

$$b_0^{SU(9)_{SCC}} = -\frac{157}{3} \qquad \text{(class ii.b)}. \tag{3.34}$$

With respect to the subgroup (3.11), the fermion decompose according to

$$[2]_{11} = (1, 10) + ([1]_6, 5) + ([2]_6, 1), \tag{3.35}$$

$$[4]_{11} = (1, \bar{5}) + ([1]_6, \overline{10}) + ([2]_6, 10) + ([3]_6, 5) + ([\bar{2}]_5, 1), \tag{3.36}$$

and

$$[6]_{11} \approx [\bar{5}]_{11} = (1,1) + ([\bar{1}]_6, 5) + ([\bar{2}]_6, 10) + ([\bar{3}]_6, \overline{10}) + ([2]_6, \bar{5}) + ([1]_6, 1), \quad (3.37)$$

with the decompositions of $[8]_{11} \approx [\bar{3}]_{11}$ and $[10]_{11} \approx [\bar{1}]_{11}$ given above. With the $SU(5)_{GU}$ couplings small, the nonsinglet fermion content under $SU(6)_{SC}$ is

$$16\{[1]_6 + [\bar{1}]_6 + [2]_6 + [\bar{2}]_6 + [3]_6\}. \quad (3.38)$$

We find that the $SU(6)_{SC}$ gauge interaction for this set is not asymptotically free, with a leading coefficient of its beta function equal to

$$b_0^{SU(6)_{SC}} = -\frac{190}{3} \qquad \text{(class iib)}. \quad (3.39)$$

This disfavors this class of models.

We have analyzed the class (iic) in a similar manner. Decomposing the fermion representations with respect to the subgroup and cataloguing the resultant $SU(9)_{SCC}$ content, we obtain the following nonsinglet $SU(9)_{SCC}$ fermions:

$$\{f\} = 4[1]_9 + 4[2]_9 + 4[3]_9 + 5[\bar{3}]_9 + 7[4]_9 + 10[\bar{4}]_9. \quad (3.40)$$

Hence, the $SU(9)_{SCC}$ gauge theory is chiral. However, we find that the $SU(9)_{SCC}$ gauge interaction is nonasymptotically free, with

$$b_0^{SU(9)_{SCC}} = -239 \qquad \text{(class iic)}. \quad (3.41)$$

Decomposing the fermion representations with respect to the subgroup (3.11), and cataloguing the resultant $SU(6)_{SC}$ content, we find the $SU(6)_{SC}$ theory is chiral, but not asymptotically free, with

$$b_0^{SU(6)_{SC}} = -250 \qquad \text{(class iic)}. \quad (3.42)$$

For the same reasons as were given above, this model is thus disfavored as a promising candidate for unification.

IV. $N_{SC} = 7, G = SU(12)$

We next study the case where $N_{SC} = 7$, so that $N = N_{SC} + 5 = 12$ and $\mathbf{n} = (n_1, \ldots, n_{11})_{12}$. With $N_{gh} + N_{g\ell} = N_{gen} = 3$ and $N_{SC} - N_{TC} = N_{g\ell}$, one has, *a priori*, four possibilities for the manner in which the SM fermion generations arise, as specified by $(N_{gh}, N_{g\ell}, N_{TC})$, namely, (3, 0, 7), (2, 1, 6), (1, 2, 5), and (0, 3, 4). The conditions and that the theory should not contain any 5_L or $\overline{10}_L$ yield

$$n_1 = n_3 = n_8 = n_{10} = 0, \quad (4.1)$$

and Eq. (2.9) is

$$N_{gh} = n_2 + n_9 = n_4 + n_{11}. \quad (4.2)$$

The condition of zero gauge anomaly, Eq. (2.6), is

$$8n_2 + 48n_4 + 42(n_5 - n_7) - 27n_9 - n_{11} = 0. \quad (4.3)$$

For a given value of $N_{gh} = 3 - N_{g\ell}$, these are three linear equations for the seven quantities n_2, n_4, n_5, n_6, n_7, n_9, and n_{11}. The solution implies the relations

$$n_4 = \frac{1}{7}[6(-n_5 + n_7) + 5n_9 - N_{gh}], \quad (4.4)$$

and

$$n_{11} = \frac{1}{7}[6(n_5 - n_7) - 5n_9 + 8N_{gh}]. \quad (4.5)$$

If $N_{gh} = 0$, then $n_4 = -n_{11}$, so the only allowed values are $n_4 = n_{11} = 0$. It follows that $n_2 = n_9 = 0$ also, and, substituting these values into Eqs. (4.4) and (4.5), one obtains $n_5 = n_7$. Thus, this class of solutions, which we denote as (i), has an $\mathbf{n}$ vector equal to

$$\mathbf{n} = (0, 0, 0, 0, s, t, s, 0, 0, 0, 0), \quad (4.6)$$

where s and t are non-negative integers. Since $[6]_{12} \approx [\bar{6}]_{12}$ and $[5]_{12} \approx [\bar{7}]_{12}$, this SU(12) theory is vectorial, and hence so are resultant $SU(10)_{SCC}$ and $SU(5)_{SC}$ theories. Hence, even if the SCC and SC interactions were asymptotically free (which they are not), these sectors would not self-break via condensate formation as would be necessary in order to extract the TC theory and the SM fermion generations. In order to minimize the number of fermions in an effort to maintain asymptotic freedom, we consider the two minimal classes (cases), (ia) $s = 0$, $t = 1$; and (ib) $s = 1$, $t = 0$. We find that

$$b_0^{SU(10)_{SCC}} = -\frac{142}{3} \qquad \text{(class (ia))} \quad (4.7)$$

and

$$b_0^{SU(10)_{SCC}} = -\frac{310}{3} \qquad \text{(class (ib))}, \quad (4.8)$$

which disfavors these cases from further consideration.

Among other solutions, we focus on one that minimize the fermion content in an effort to preserve asymptotic freedom. We find cases with minimal $\mathbf{n}$ vectors for $N_{gh} = 3$. Among these, the minimal one has

$$\text{(ii)}: \mathbf{n} = (0, 1, 0, 1, 0, 0, 0, 0, 2, 0, 2). \quad (4.9)$$

We find that this yields a chiral $SU(10)_{SCC}$ gauge interaction, as desired, but the $SU(10)_{SCC}$ sector is not asymptotically free:

$$b_0^{SU(10)_{SCC}} = -\frac{112}{3} \qquad \text{(class (ii))}. \quad (4.10)$$

We have found similar nonasymptotically free SCC sectors for other solutions for this $N_g = 3$ case, and also for cases with $N_g = 1, 2$. Our results suggest that nonasymptotically free SCC and SC sectors appear to be a generic problem with models having unification groups SU(N) with $N \geq 11$.

V. DISCUSSION AND CONCLUSIONS

On the basis of our results, we can infer some generalizations concerning this type of approach to unification of gauge symmetries in theories with dynamical symmetry breaking. We first recall some findings from Ref. [8] for SU(N) models with N up to 10. In that study, several cases were found that satisfied the various necessary conditions listed above, including anomaly cancellation, potential for $N_g = 3$ standard-model fermion generations, absence of SC-singlet fermions with exotic SM quantum numbers, etc., and for which the $G_{\rm SCC}$ gauge interaction was asymptotically free. However, in many of these cases, this SCC gauge symmetry is vectorial, so that as the energy scale decreases from $M_{\rm GU}$, the SCC interaction eventually becomes strong, confines, and produces a bilinear fermion condensate, but this condensate is invariant under $G_{\rm SCC}$, so this group does not self-break, as is necessary to peel off the SC and color groups, and eventually the TC group. One model with $G = {\rm SU}(10)$ and fermion content specified by $\mathbf{n} = (0, 0, 0, 1, 0, 0, 1, 0, 0)_{10}$ yielded an asymptotically free chiral gauge sector for $G_{\rm SCC}$, but the condensate formation via the most attractive channel did not produce an acceptable low-energy theory.

In the present work, we have searched for more promising models by examining higher values of N, including $N = 11$ and $N = 12$. Here, we have encountered a problem that was already present for a number of the models considered in Ref. [8] with $N \leq 10$, namely, the property that the models contain sufficiently many fermions that $G_{\rm SCC}$ is not asymptotically free. This feature tends to preclude the desired scenario in which the SU($N_{\rm SCC}$) group would become strongly coupled as the energy scale decreases below $M_{\rm GU}$ and would self-break via formation of fermion condensates to separate out the ${\rm SU}(3)_c$ and SU($N_{\rm SC}$) groups, and thus the SU($N_{\rm TC}$) group. This appears to be a generic problem. Thus, the necessary conditions stipulated above, in their entirety, constitute a significant challenge for a viable unification model.

Although our results are somewhat negative, the knowledge that we have gained concerning models embodying the present type of approach is useful for continuing efforts to construct theories that could unify the standard-model gauge symmetries with gauge interactions that would become strong on the TeV scale and cause dynamical electroweak symmetry breaking. One may anticipate that data from the CERN Large Hadron Collider, soon to go into operation, will elucidate the question of the origin of electroweak symmetry breaking. If there is evidence that this symmetry breaking is dynamical, it will be interesting to pursue further the goal of higher unifcation addressed here.

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