Evidence for the multiverse in the standard model and beyond

Lawrence J. Hall and Yasunori Nomura

Department of Physics, University of California, Berkeley, California 94720, USA
and Theoretical Physics Group, Lawrence Berkeley National Laboratory, Berkeley, California 94720, USA
(Received 28 December 2007; published 4 August 2008)

In any theory it is unnatural if the observed values of parameters lie very close to special values that determine the existence of complex structures necessary for observers. A naturalness probability P is introduced to numerically evaluate the degree of unnaturalness. If P is very small in all known theories, corresponding to a high degree of fine-tuning, then there is an observer naturalness problem. In addition to the well-known case of the cosmological constant, we argue that nuclear stability and electroweak symmetry breaking represent significant observer naturalness problems. The naturalness probability associated with nuclear stability depends on the theory of flavor, but for all known theories is conservatively estimated as $P_{\rm nuc} \lesssim (10^{-3}\text{–}10^{-2})$, and for simple theories of electroweak symmetry breaking $P_{\rm EWSB} \lesssim (10^{-2}\text{–}10^{-1})$. This pattern of unnaturalness in three different arenas, cosmology, nuclear physics, and electroweak symmetry breaking, provides evidence for the multiverse, since each problem may be easily solved by environmental selection. In the nuclear case the problem is largely solved even if the multiverse distribution for the relevant parameters is relatively flat. With somewhat strongly varying distributions, it is possible to understand both the close proximity to neutron stability and the values of m_e and $m_d - m_u$ in terms of the electromagnetic mass difference between the proton and neutron, $\delta_{\rm EM} \simeq 1 \pm 0.5$ MeV. It is reasonable that multiverse distributions are strong functions of Lagrangian parameters, since they depend not only on the landscape of vacua, but also on the population mechanism, "integrating out" other parameters, and on a density of observers factor. In any theory with mass scale M that is the origin of electroweak symmetry breaking, strongly varying multiverse distributions typically lead either to a little hierarchy $v/M \approx (10^{-2}\text{–}10^{-1})$, or to a large hierarchy $v \ll M$. In certain multiverses, where electroweak symmetry breaking occurs only if M is below some critical value, we find that a little hierarchy develops with the value of v^2/M^2 suppressed by an extra loop factor, as well as by the strength of the distribution. Since the correct theory of electroweak symmetry breaking is unknown, our estimate for $P_{\rm EWSB}$ is theoretical. The LHC will lead to a much more robust determination of $P_{\rm EWSB}$, and, depending on which theory is indicated by the data, the observer naturalness problem of electroweak symmetry breaking may be removed or strengthened. For each of the three arenas, the discovery of a natural theory would eliminate the evidence for the multiverse; but in the absence of such a theory, the multiverse provides a provisional understanding of the data.

DOI: 10.1103/PhysRevD.78.035001 PACS numbers: 12.60.−i

I. INTRODUCTION

The standard model, with three gauge forces and three generations of quarks and leptons, has been laboriously constructed from decades of data involving collisions of particles at ever higher energies. It represents a triumph for symmetries, but leaves many questions about nature unanswered. These questions fall into three groups. The first revolves around observations of the Universe that are not described by the standard model, such as dark matter, dark energy, baryogenesis, and inflation. Secondly, there is the question of electroweak symmetry breaking: while the standard model provides a mathematical description, it does not contain the physical description. The quadratic divergence in the Higgs mass-squared parameter implies that electroweak symmetry breaking is determined by new physics at a scale M at which the standard model is embedded into a more fundamental theory. This extraordinary behavior is termed "unnatural"—the Higgs mass-squared parameter, m_h^2, is very sensitive to changes in some parameter x of the fundamental theory, especially as M grows

$$\Delta \equiv \frac{\partial \ln m_h^2}{\partial \ln x} \sim \frac{M^2}{m_h^2} \gg 1. \tag{1}$$

The final group of questions boils down to "why the standard model?" These include why the forces are the ones we observe, why the quarks and leptons have the masses they do, and why there are not other forces and particles.

Since symmetries were the key to the standard model, it is natural to suppose that all three sets of questions will be answered by introducing further symmetries, and this has largely defined beyond standard model physics for the last three decades. Yet there are three clouds on the horizon for this symmetry approach, one for each set of questions

(i) There is no known symmetry to explain why the cosmological constant is either zero or of order the observed dark energy. It appears to take a very unnatural value.
(ii) For the standard model to be natural, M must be low; we should already have observed either signs of supersymmetry or signals of new interactions in precision electroweak observables.
(iii) Some of the standard model parameters take values in a special region that happen to yield certain nuclear properties.

It is possible that these clouds are the first indications of a breakdown in the power of symmetry to determine fundamental physics. If our universe is just one among very many in an enormous multiverse, then observed universes will be those that contain certain complex structures necessary for observers [1]. Such arguments from environmental selection can potentially solve the cosmological constant problem, and yield a statistical prediction for the dark energy in observed universes [2]. In this paper, we consider the extent to which nuclear stability and electroweak symmetry breaking provide evidence for environmental selection.

Many physicists, however, are reluctant to countenance any form of anthropic argument. Why give up on the traditional, extraordinarily successful, methods of physics? How can we hope to understand the conditions for intelligent observers when we struggle even to define what life is? Why make the extraordinary leap of postulating an extra-horizon multiverse, which has the smell of a secular form of God? In short, many believe that appeals to the environment are an escape from true science and that, in the absence of data confirming the conventional symmetry approach, it would be better to change fields than to succumb to the philosophy of anthropics.

The case of the cosmological constant demolishes these arguments. Traditional methods have not given any satisfactory understanding for why the cosmological constant is small. In contrast, the environmental argument not only explains why it must be small, but makes a statistical prediction for a nonzero value. Furthermore, this prediction requires essentially no understanding of what it takes to make an observer. Of course, the prediction does require a multiverse; but with a landscape of vacua provided by string theory [3] and generic inflation eternally creating universes [4], a multiverse no longer appears theoretically unreasonable, and its consequences can be explored under the assumption that we are typical observers [5]. Indeed, since few doubt that environmental conditions explain why we find ourselves on the Earth as opposed to elsewhere in the solar system, the resistance to environmental reasoning is hard to understand. As for any physical theory, the real issue is whether sufficient evidence for it can be found to make it convincing.

Even if there is a multiverse, and environmental selection accounts for the value of the cosmological constant, it is far from clear what the implications are for the rest of physics. One extreme possibility is that all the other questions left open by the standard model will be solved within some symmetry framework, for example, supersymmetric $SO(10)$ with flavor symmetries. In this case, the environmental solution of the cosmological constant problem simply allows us to ignore this special problem while solving everything else with symmetries—it justifies the model-building approach of the last 30 years. The other extreme is that the standard model is as far as we can get with symmetries, all of its parameters are either strongly selected by environmental effects or just reflect random typical values in the multiverse, and similarly for issues outside the standard model. Of course there are many intermediate cases between these two extremes. For example, there could be gauge coupling unification within an $SO(10)$ theory, but the flavor sector may involve sufficient scanning parameters that the electron, up and down quark masses are determined by environmental selection. The aim of this paper is to seek evidence for environmental selection both in the measured values of standard model parameters, and in electroweak symmetry breaking.

The cosmological constant illustrates an additional crucial point: environmental selection predicts parameters that are, from the conventional viewpoint of symmetries, *unnatural*. This is a key point, because so much effort of the last 30 years has been expended in trying to understand how new symmetries could naturally lead to an unnatural effective low energy theory, namely, the standard model. Perhaps the unnaturalness is a hint that symmetries are not the answer. It was realized some time ago that if only the mass parameter of the standard model scans, then environmental selection could solve the gauge hierarchy problem [6]. This analysis was extended to the two Higgs doublet case in [7]. In the context of supersymmetry, differing assumptions about scanning parameters and their distributions could lead to a large hierarchy [8] or a small hierarchy [9]. These papers both illustrate that environmental selection leads to unnaturalness, and that measurements at the LHC could experimentally confirm the presence of unnaturalness in electroweak symmetry breaking. It is an open question whether the landscape favors a low or high supersymmetry breaking scale [10].

The first part of this paper, Secs. II to V, are devoted to a consideration of the concept of naturalness, and its application to issues of nuclear stability. We are motivated by the belief that environmental selection may lead to precise predictions, and that it is important to evaluate the numerical significance of such predictions. We often hear it said that, in nuclear physics and elsewhere, nature exhibits "amazing coincidences" that give rise to life. How can these be precisely evaluated? In Sec. II, we formulate the concept of naturalness in a very general way, so that in Sec. III we are able to identify naturalness problems associated with the existence of complex structures in the universe. In Sec. IV, we study the stability of neutrons, deuterons, and complex nuclei in the parameter space of the standard model, identifying how close our universe is to these stability boundaries. In particular, we find a closeness to the neutron stability boundary that has not previously been elucidated. The naturalness problems associated with the closeness to these stability boundaries are evaluated numerically in a variety of theories of flavor in Sec. V. We stress that the evidence for unnaturalness depends on the ensemble of theories being considered, but we find that there is an irreducible amount of unnaturalness no matter what the theory of flavor.

In the second half of the paper, we investigate the consequences that follow if this unnaturalness arises from environmental selection in a multiverse. It is here that the real utility of our new formulation of naturalness becomes apparent. In Sec. VI, we argue that problems of unnaturalness are indeed solved by environmental selection, and show how the amount of unnaturalness is connected to the probability distribution of the multiverse. We argue that evidence for environmental selection will increase if symmetry arguments fail to solve an accumulation of naturalness problems. In the case of nuclear stabilities, we show in Sec. VII how this leads to multiverse predictions for the masses of the electron, up and down quarks. In Sec. VIII, we embed the standard model Higgs sector in a generic theory of electroweak symmetry breaking at mass scale M. Allowing parameters of this sector to scan, as well as parameters of the standard model, we consider environmental selection from nuclear stability boundaries. We find that sharply varying multiverse distribution functions generically lead to both large and little hierarchies between M and the scale v of electroweak symmetry breaking. From the viewpoint of the multiverse, a discovery at the LHC that electroweak symmetry breaking requires some amount of fine-tuning would not be surprising. A similar analysis on electroweak symmetry breaking is performed in Sec. IX, assuming that the relevant boundaries are the phase boundary of electroweak symmetry breaking rather than the nuclear stability boundaries. We again find that unnaturalness in electroweak symmetry breaking is expected for sharply varying multiverse distribution functions. In Sec. X, we study connections between environmental selection for the cosmological constant and for electroweak symmetry breaking. For example, we argue that the driving force on the multiverse for unnaturalness in electroweak symmetry breaking could arise from the distribution for the cosmological constant, the connection being through weakly interacting massive particle (WIMP) dark matter. Finally, our conclusions are given in Sec. XI.

II. NEW DEFINITION OF NATURALNESS PROBLEMS

The concept of naturalness has often been the driving force for finding fundamental mechanisms in nature. Arguments for naturalness are often phrased in terms of the sensitivity of low energy standard model parameters c_i to variations of the parameters in the more fundamental theory a_j. A simple measure of naturalness is then given by $\Delta \equiv |\partial \ln c_i / \partial \ln a_j|$, with a large value of Δ signaling a lack of naturalness [11]. This definition, however, could miss some of the important aspects of the naturalness problem. Here we introduce a new, quantitative definition of naturalness that can be applied in much more general situations. In particular, this allows us to identify certain classes of naturalness problems that have not been quantitatively defined. Our definition is also free from some of the problems existing in the simplest definition of naturalness based on the sensitivity of parameters.

First of all, it is very important to notice that in talking about naturalness, we are dealing, either explicitly or implicitly, with an ensemble in which parameters of the theory are varied according to some definite distribution. Consider, for example, that the Higgs mass-squared parameter m_h^2 is given by the difference of two mass scales of order the fundamental scale $m_h^2 = M_1^2 - M_2^2$. We say that the theory is unnatural if $|m_h^2| \ll M_1^2, M_2^2$. This statement, however, already assumes that it is unlikely for M_1 and M_2 to be very close or for both to be very small; more specifically, the distribution of possible values for M_1 and M_2 is assumed to be almost structureless in the M_1–M_2 (or M_1^2–M_2^2) plane. This illustrates that the concept of naturalness is closely related to the distribution of parameters in an ensemble.

A particular member of an ensemble is unnatural if it has parameters very close to special values that are not explained by the symmetries of the theory. The parameters are special if some physical property arises that is *not* a generic feature of the members in the ensemble, or if they separate regions of parameter space that have differing generic features. Depending on the property considered, we encounter various classes of naturalness problems, whose solutions could point to various different mechanisms in nature. With this definition, the degree of unnaturalness is given by how close the parameters are to the special values, which generically form a special hypersurface in multidimensional parameter space. As we will see below, we can quantify this degree in terms of the distribution of parameters within the ensemble. In this section, after presenting a new definition of naturalness, we apply it to well-known situations. In the next section, we use it to introduce new types of naturalness problems.

A. Definition

Let us start describing our precise definition of naturalness by introducing the distribution function $f(x)$ for an ensemble. The function is defined such that the number of members with parameters x_i ($i = 1, \cdots, N$) taking a value between x_i and $x_i + dx_i$ is given by

$$d\mathcal{N} = f(x_1, x_2, \cdots, x_N) dx_1 dx_2 \cdots dx_N. \tag{2}$$

Here, x_i represent continuous parameters of the theory, e.g., masses and coupling constants.[1] The overall normalization of f becomes relevant if we are interested in the total number of members, which will be the case when we discuss relative likelihoods of different theories. For the present purpose of discussing naturalness of a given theory, however, the overall normalization of f is not important.

The distribution function depends on the choice of x_i. What variables should we choose as x_i? In general we can choose any variables as x_i, depending on the context. To discuss naturalness of the low energy theory, however, it is often most convenient to take "fundamental" parameters, such as masses and coupling constants of the ultraviolet theory, as x_i. Now, suppose we have only one such variable x. If the observed value of x, x_o, is very close to a special value $\bar{x}$, the level of unnaturalness is given by the following "naturalness probability":

$$P = \left| \frac{\int_{\bar{x}}^{x_o} f(x) dx}{\int_{x_{\rm min}}^{x_{\rm max}} f(x) dx} \right|, \tag{3}$$

where $x_{\rm min} \le x \le x_{\rm max}$ gives the range of x values in the theory under consideration.[2] Here, we have included only members on one side of the special point, but depending on the situation, a factor of 2 should be added to the numerator to include members on both sides of the special point. With this definition, $P \ll 1$ signals the existence of a naturalness problem.

The form of the distribution function is modified if we redefine the variable x. For example, if $f(x) = 1/x$ for a variable x, the change of the variable $x' = \ln x$ can make the distribution function flat for x': $f(x') = 1$. It is often convenient to go to the basis in which the distribution function is constant. In that basis, the naturalness probability of Eq. (3) becomes

$$P = \left| \frac{x_o - \bar{x}}{x_{\rm max} - x_{\rm min}} \right|, \tag{4}$$

which is simply given by the distance between x_o and $\bar{x}$ divided by the available parameter space.[3] As should be the case, this quantity becomes smaller as x_o approaches $\bar{x}$.

The naturalness probability of Eqs. (3) and (4) can be extended to the case of multiple parameters x_i. The precise definition depends on the dimensionality of x_i and the special surface consisting of $\bar{x}_i$. If the codimension of the special surface is 1, we can choose x to be a linear combination of x_i perpendicular to the surface, and then use the definition of Eq. (4) (in the basis where the distribution function is constant). In the case that the codimension is higher, we must use an appropriate generalization of Eqs. (3) and (4) defined using multidimensional volumes, rather than simple distances. A useful definition, in the basis where $f(x_i)$ is constant, is

$$P = \left| \frac{c_n \{\sum_{a=1}^n (x_{a,o} - \bar{x}_a)^2\}^{n/2}}{\prod_{a=1}^n (x_{a,\rm max} - x_{a,\rm min})} \right| = \frac{v_n}{V_n}, \tag{5}$$

where $a = 1, \cdots, n$ runs over variables whose observed values $x_{a,o}$ are close to the special values, $\bar{x}_a$, and $c_n = \pi^{n/2}/\Gamma(n/2 + 1)$ is the volume of the unit ball in n dimensions. Here, the expression in the numerator v_n is a measure of the accidentally small volume of parameter space required for $x_{a,o}$ to be close to $\bar{x}_a$, while that in the denominator V_n is the total volume of parameter space available. The restriction of the variables x_a to those with $|(x_{a,o} - \bar{x}_a)/(x_{a,\rm max} - x_{a,\rm min})| \ll 1$, e.g., $\lesssim 1/5$, is important to avoid obtaining $P \ll 1$ simply as a result of high dimensionality of the parameter space. In the case that the special surface in question arises as an intersection of codimension 1 surfaces, we may restrict the volume of the numerator to be one side of the codimension 1 surfaces, depending on the situation. This definition reduces to that of Eq. (4) in the special case of $n = 1$.

The definition of Eq. (5) is illustrated for a 2-dimensional parameter space (x_1, x_2) in Fig. 1(a). The curve C represents special values of these parameters corresponding to some physical phenomenon, and the curve C' similarly represents special values for some other phenomenon. If the observed values $(x_{1,o}, x_{2,o})$, denoted by the dot, are close to the intersection point $(\bar{x}_1, \bar{x}_2)$, the naturalness probability is given by Eq. (5), with $a = 1, 2$. The numerator represents the area of the shaded region, and the denominator represents a much larger area corresponding to the range of the parameters. Here, we have restricted the area of the numerator to one side (the lower side) of the curves C and C', anticipating an application in later sections.

[1] In principle, we can define a distribution function that also has discrete labels representing, e.g., particle content and symmetries of the theories. Here, we consider a distribution function for each theory that has a definite symmetry, matter content, number of spacetime dimensions, and so on, and restrict its arguments x_i to be continuous parameters associated with that particular theory.

[2] With $f(x) = 0$ for $x < x_{\rm min}$ and $x > x_{\rm max}$, the range of the integration in the denominator can be taken from $-\infty$ to $+\infty$. Practically, if $f(x)$ has a sharp dropoff, it is useful to restrict the range of the variable by a sharp cutoff, as $x_{\rm min}$ and $x_{\rm max}$ in Eq. (3).

[3] This interpretation is also possible for a general distribution function $f(x)$ if we consider $f(x)$ to be a sort of metric, or volume factor, in parameter space.

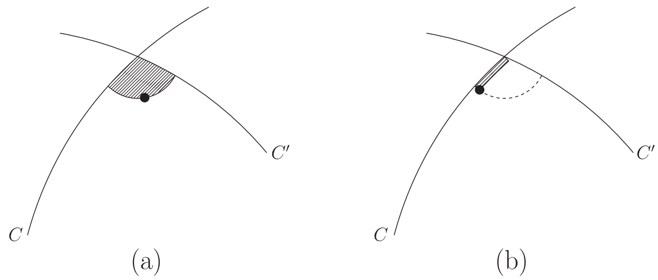

FIG. 1. Illustrations of the definition of the naturalness probability P in multidimensional parameter space.

In Fig. 1(b), it is apparent that the observed point is much closer to C than C'; but the definition of Eq. (5) does not capture the additional unnaturalness associated with this. (The area of the numerator in this expression is represented by the dotted line.) In this situation, we must first consider the naturalness probability P_C associated with C, using the definition of Eq. (4), where the axis x is taken normal to C and passing through the observed point at $x = x_o$. We then take the coordinate x' along the curve C and consider the naturalness probability $P_{C'}$ associated with C' in this coordinate. The resulting naturalness probability is then $P = P_C P_{C'}$, which is given by the area of the shaded region in the figure divided by that corresponding to the range of the parameters. This tends to zero as the observed point approaches the curve C, as expected. In general, if we find several unnatural features that have a hierarchy in their degrees of unnaturalness, we can obtain the correct estimate for the naturalness probability P by considering it as a product of several naturalness probabilities, each associated with an unnaturalness of some fixed degree. The decomposition of P can be made along the lines presented here.

B. Illustrations

Let us now illustrate the use of our definition in the case of the conventional gauge hierarchy problem. Consider that the standard model is embedded into the fundamental theory at a high scale of $M_1 \approx M_2 \approx M_*$ and that the Higgs mass-squared parameter is given in terms of M_1 and M_2 by $m_h^2 = M_1^2 - M_2^2$. In general, we expect that the distribution function is roughly flat in terms of M_i ($i = 1, 2$) and that the range of parameters is $0 \leq M_i \leq O(M_*)$.[4] In Fig. 2, we plot the contour of m_h^2 in the M_1–M_2 plane in units of M_*. We clearly see that there is a special line in this plane, $M_1 = M_2$, where $m_h^2 = 0$. The physical phenomenon making this line special is that members very close to this line have a low energy effective field theory that contains a scalar excitation whose mass is hierarchically smaller than M_*. For $M_* \approx 10^{18}$ GeV, the observed value of $m_h^2 \simeq -O(100\ \text{GeV})^2$ is denoted by the little dot for an arbitrary value of $M_1 = 1.2M_*$. We find that it is located very close to the special line compared with the expected range of M_i. In fact, this is always the case regardless of the value of M_1 we choose, and is a manifestation of the lack of naturalness in the standard model. The problem is that the special line with $m_h^2 = 0$ is *not* special in terms of the symmetry structure of the theory. The points with $m_h^2 = 0$ are no more symmetric than those with $m_h^2 \neq 0$. By choosing a variable x to be a linear combination of M_1 and M_2 perpendicular to the special line, and then using the definition of Eq. (4), we obtain $P \approx 10^{-32}$. A similar analysis can also be made for the cosmological constant problem. In general, the cosmological constant Λ is given by the sum of terms of order M_*^4 taking the form of $M_i^2 M_j^2$. The observed value of Λ is then very close to the special hypersurface with $\Lambda = 0$, and the points with $\Lambda = 0$ are no more symmetric than those with $\Lambda \neq 0$. The physical property making this hypersurface special is that members very close to this hypersurface allow an arbitrary large observable universe. From Eq. (4), the degree of unnaturalness we obtain in this case is $P \approx 10^{-120}$.

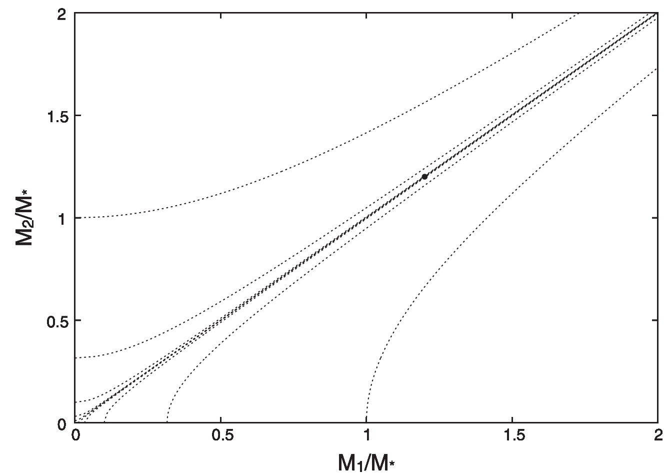

FIG. 2. The contour of $m_h^2/M_*^2 = \pm 1, \pm 0.1, \pm 0.01, \cdots$ in the M_1–M_2 plane. The special line of $m_h^2 = 0$ is visible at $M_1 = M_2$. The observed value of $M_2 = M_1 + O(10^{-32})$ is denoted by the little dot for an arbitrary value of $M_1 = 1.2M_*$.

Here, we note that our definition does not suffer from the problem existing in the simplest definition of naturalness based on the logarithmic derivative $\Delta \equiv |\partial \ln c_i / \partial \ln a_j|$. Suppose that a dimensionful parameter μ is given by two dimensionless constants g^2 and b as $\mu = M_* e^{-8\pi^2/g^2 b}$, where M_* is the fundamental scale, and that a natural range for g^2 and b in the fundamental theory is $g^2 b = O(0.1)$. We then naturally obtain $\mu/M_* = e^{-O(1000)}$; there is nothing unnatural with this. The simplest definition, however, gives $\Delta = |\partial \ln\mu / \partial \ln g^2| = O(1000)$, signaling (incor-

[4]Here, we have assumed that M_i are mass parameters associated with "fermions" (including mass parameters in supersymmetric theories). The argument, however, is not affected if they are associated with scalars, in which case the distribution is expected to be roughly flat in M_i^2 with the range $-O(M_*^2) \leq M_i^2 \leq O(M_*^2)$.

rectly) the existence of unnaturalness. Our definition does not lead to such a fake signal, since any point with $g^2 b = O(0.1)$ is generic—there is no special value of $g^2 b$ that can be singled out as $\bar{x}$ in Eqs. (3) and (4).

We now see how theories beyond the standard model solve the gauge hierarchy problem in our language. As we have seen, the standard model embedded into more fundamental theory at $M_* \approx 10^{18}$ GeV leads to the probability of having a small weak scale $v \approx 100$ GeV to be $P \approx (v/M_*)^2 \approx 10^{-32}$. This is because the Higgs mass-squared parameter m_h^2 is given by the sum of various contributions M_i^2 of order M_*^2, with the distribution function expected to be roughly flat in M_i (or M_i^2). The distribution function, however, varies from one theory to another. For example, if the weak scale arises from dimensional transmutation associated with some gauge coupling g becoming strong, as in technicolor [12] and certain supersymmetric theories [13], then $v \approx M_* \exp(-8\pi^2/|b|g_*^2)$. Here, b is the one-loop beta function coefficient for the new gauge force and $g_* = g(M_*)$. The distribution function is expected to be roughly flat in the variable g_*^2,[5] giving an approximately flat distribution for $1/\ln(v/M_*)$. To illustrate this situation, we plot in Fig. 3 the contour of $m_h^2 = M_1^2 - M_2^2$ with $M_1 = M_* \exp(-8\pi^2/|b|g_*^2)$ and $M_2/M_1 \equiv r$ in the $|b|g_*^2$–r plane. This can be regarded as a simplified model for the situation in, e.g., dynamical supersymmetry breaking. Typically, there is considerable uncertainty in the range of $|b|g_*^2$. Here, we take $0 \leq |b|g_*^2 \leq O(10)$. As is seen from the figure, the observed value of v can correspond to a completely generic (not special) point in the plane, which is indicated by the little dot for an arbitrary value of $r = 1.4$. This, therefore, provides a solution to the gauge hierarchy problem; see Fig. 2 for comparison. The important point here is that different theories can lead to very different distribution functions $f(x)$ for physical parameters x_i. A distribution flat in v^2 is the origin of the gauge hierarchy problem, while one flat in $\ln v$ (or $1/\ln v$) provides the solution. The situation with the cosmological constant is similar, although the problem is much more severe because all known theories give distributions that are essentially flat in Λ, while none are known that are flat in $\ln\Lambda$ (or $1/\ln\Lambda$).

C. Generality of the framework

So far, we have illustrated our definition of naturalness by looking at naturalness problems that have a conventional description: the observed values of parameters are (very) close to special values where the sensitivity of a low energy parameter to high energy ones diverges, for example, $\partial \ln m_h^2/\partial \ln M_i$, $\partial \ln\Lambda/\partial \ln M_i \rightarrow \infty$. This is one class of naturalness problems, which we may call "fine-

[5]The conclusion below does not change if the distribution is flat in $1/g_*^2$, g_*, and so on. The distribution, however, could affect arguments on certain naturalness problems; see Sec. IV.

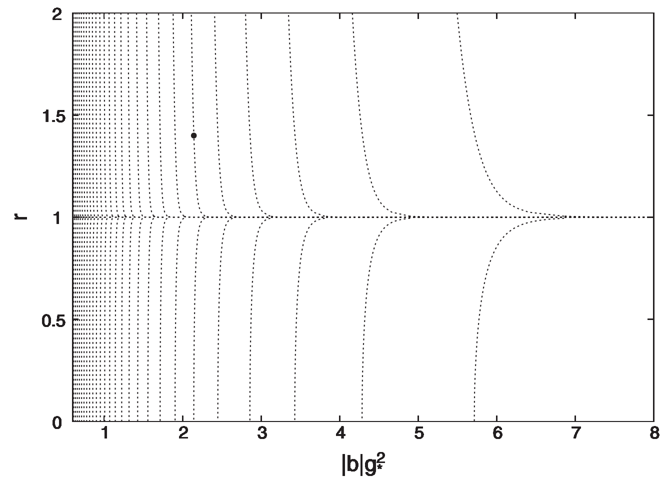

FIG. 3. The contour of $m_h^2/M_*^2 = 10^{-4n}$ with $n = 3, 4, 5, \cdots$ in the $|b|g_*^2$–r plane. The observed value of $v \approx \sqrt{-m_h^2} \approx 100$ GeV can correspond to a completely generic point in the plane, which is indicated by the little dot for an arbitrary value of $r = 1.4$.

tuning naturalness problems." The real power of our formalism, however, lies in the fact that we can discuss many different types of naturalness problems *in a unified manner*, simply by extending what we mean by "special values" for parameters. Any physical property that is *not* a generic feature of the members in the ensemble is a candidate for identifying special values. There can be many classes of naturalness problems, depending on the property considered. The closeness of the observed values to the special values, signaled by $P \ll 1$, can then be used as evidence for a new mechanism to understand such accidents.

One application of this general idea is to use naturalness arguments as evidence for the presence of some symmetry beyond the standard model. For this purpose it is often convenient to take parameters of the low energy theory to be x_i. Imagine that the observed values $x_{i,o}$ of N dimensionless standard model parameters x_i ($i = 1, \cdots, N$) take values close to the special surface that defines a symmetry relation

$$S(x_i) = 0. \tag{6}$$

If the symmetry were in fact absent, nature would be described by some member of an ensemble giving the x_i parameters distributed according to $f(x_i)$, which is not generically concentrated on the surface of Eq. (6). After redefining the parameters to make the distribution function flat, we can introduce an axis x normal to the symmetry surface that passes through the observed point at $x_i = x_{i,o}$. The degree of unnaturalness is then given by Eq. (4). The closer the observed point is to the special value $\bar{x}$ on the symmetry surface, the less probable it becomes that nature is described by this ensemble, i.e., by the theory without the symmetry. A small value for P can thus be taken as evidence that the symmetry is indeed present in nature, at least in an approximate form.[6] It could be that $x_o - \bar{x}$ is dominated by the experimental uncertainty. In this case, improved experiments may provide further evidence for the symmetry. A well-known example is provided by a grand unified symmetry, such as $SU(5)$, that gives a symmetry relation between the standard model gauge coupling constants $S(g_1^2, g_2^2, g_3^2) = 0$ [14]. If in fact there is no unified symmetry, the observed values of α, α_s, and $\sin^2\theta_W$ give a naturalness probability of $P \approx 0.1$ in an ensemble without supersymmetry, and $P \approx 0.01$ in an ensemble with supersymmetry [15]. Here, α, α_s, and θ_W are the fine structure constant, the effective QCD coupling, and the Weinberg angle, respectively. This lack of naturalness can then be regarded as evidence for some form of a unified symmetry in theories with supersymmetry.

In the next section, we introduce classes of naturalness problems that arise from the existence of complex structures. We will see that the formalism developed here elucidates the identification of these naturalness problems. As in other naturalness problems, successful solutions to these problems could lead us to find new mechanisms or dynamics in nature, which we consider in later sections.

III. COMPLEXITY AND OBSERVER NATURALNESS PROBLEMS

The definition of naturalness introduced in the previous section allows us to identify new classes of naturalness problems. A member in the ensemble is unnatural if it has parameters unusually close to "special" values; but clearly there are many reasons that parameters could be "special." We frequently stress special values that lead to a large hierarchy of mass scales, but in this section we consider special values that lead to the existence of relatively long-lived complex structures, such as nuclei, stars, and galaxies.

A. Complexity naturalness problem

As the parameters x_i vary, moving from one member of an ensemble to another, suppose that a physical threshold is crossed that is crucial for the existence of some complex structure. This defines a special surface in the parameter space

$$C(x_i) = 1, \tag{7}$$

which divides the volume of parameter space into two regions, one that supports the complex structure $C(x_i) < 1$ and one that does not $C(x_i) > 1$. In general, this surface is not one with enhanced symmetry. Therefore, a member in the ensemble having parameters unusually close to this surface has a "complexity naturalness problem." The degree of unnaturalness is given numerically by Eq. (3), where the single variable x is normal to the "complexity surface" of Eq. (7), where it takes the value $\bar{x}$, and the particular unnatural member in question has a nearby value for this parameter, x_o.

One caveat is that we have assumed that the physical threshold relevant for the existence of the complex structure is sharp in the parameter space. This can be verified in any particular case, and is certainly true in the examples discussed in the next section. For example, the stability of any particular nucleus gives a sharp boundary corresponding to values of the coupling strengths and quark masses that lead to a surface of zero binding energy. In general, a lack of sharpness is due to the time evolution of complex structures in universes corresponding to the different members of an ensemble. As parameters vary from one member to another, complex structures could gradually become less stable. For example, the stability of large scale structure is not completely sharp—as the cosmological constant is gradually increased only the regions with larger statistical fluctuations in the density perturbations are able to collapse. Still, the relevant parameter space for the cosmological constant spans over 100 orders of magnitude, and over this space the transition for the existence of large scale structure is very sharp.

Another caveat is the hidden assumption that these complexity surfaces are not distributed so densely throughout the entire parameter space that a typical member in the ensemble is expected to be close to one or more surfaces. There may indeed be many complexity surfaces; for example, in the standard model there are several hundred relatively stable nuclei, each with its own complexity surface in an ensemble that contains the standard model. However, if the relevant parameters—the Yukawa couplings y, the weak scale v, and the QCD scale $\Lambda_{\rm QCD}$—vary by many orders of magnitudes in the ensemble, then these complexity surfaces will all be tightly clustered in a "complexity zone" $yv \sim \alpha\Lambda_{\rm QCD} \sim O(0.01)\Lambda_{\rm QCD}$. A complexity naturalness problem now arises, because this zone is itself small compared with the entire volume of parameter space. Most members in the ensemble lie in voids far from the complexity zone, and the closeness of the observed parameters to one or more of the complexity surfaces implies that the member describing our universe lies in a special region. There may be several complexity zones for nuclear physics; for example, ones with four, five, or six quark flavors lighter than the QCD scale, but in certain parameter directions each zone will be small. In the case of a single variable, the degree of unnaturalness can be taken to be $P = \Delta x_z/(x_{\rm max} - x_{\rm min})$ where Δx_z is the width of the complexity zone. If the density of complexity surfaces is very high it might be that a member in the complexity zone is very close to some surface; but this could be

[6]Here, we assume that conversions from low energy parameters to high energy ones, e.g., through renormalization group evolution, do not significantly affect the size of P. This is generically a good assumption.

a reflection of the density of the surfaces rather than any additional unnaturalness beyond that of being in the complexity zone.

In summary, a theory possesses a complexity naturalness problem if, in the parameter space of an ensemble, the member describing our universe lies very close to a surface corresponding to a physical threshold that allows the existence of some relatively long-lived complex structure. For the problem to exist, there should not be many surfaces of a similar character distributed densely and almost uniformly over the parameter space; otherwise, the closeness to one of these surfaces would simply be a generic phenomenon for the members in the ensemble.

B. Observer naturalness problem

Now, we introduce another, closely related, naturalness problem. Complex structures are required for the existence of "observers." This implies that some complexity boundaries may also act as boundaries that divide the parameter space of the ensemble into those members that may support certain observers and those that cannot. These boundaries are harder to define than general complexity boundaries, since we are unable to give a precise definition of an observer. Moreover, the parameter region may not simply be divided into the two regions "with" and "without" observers—the expectation value for the number of observers may in general be a complicated function over the parameter space, with the value significantly varying across a complexity boundary. Nevertheless, since the changes of the expectation value across some of these boundaries are expected to be drastic, caused by drastic changes of complex structures, for certain purposes we may approximate this function to be steplike. This leads to the concept of the "observer boundary"

$$O(x_i) = 1, \tag{8}$$

which divides the parameter space into one "with" observers $O(x_i) < 1$ and one "without" $O(x_i) > 1$. In general, there are many complexity boundaries $C_a(x) = 1$ that are relevant for the existence of observers. The region of parameter space that allows observers $\mathcal{O}$ is then given by the common set of $C_a(x) < 1$ for all a, and the observer boundary $O(x_i) = 1$ is the border of this region. As more boundaries are added as relevant ones for observers, the region $\mathcal{O}$ shrinks, but the number of observer boundaries does not generically increase. This is the crucial difference between complexity and observer boundaries. For illustration of these boundaries in an example of 2-dimensional space, see Fig. 4.

While the identification of observer boundaries suffers from some ambiguities, there are certain advantages to focusing on these boundaries rather than on general complexity boundaries. In general, there can be several disconnected regions $\mathcal{O}_I$ that can support observers. However, since the existence of observers undoubtedly requires certain complex structures, we can be convinced relatively easily that these regions are not distributed densely throughout the entire parameter space. For example, if we take the existence of stable complex nuclei to be one of the required conditions for observers, then we find that almost the entire region with $v/\Lambda_{\rm QCD} \gtrsim 10^4$, with the other parameters of the standard model fixed, is outside $\mathcal{O}_I$ [6], regardless of the existence of any other physical thresholds. Increasing the size of the up or down Yukawa coupling more than a factor of a few, with the other parameters fixed, also leads to a sterile universe in which both proton and neutron cannot be stable inside nuclei. The situation is similar for other conditions. For example, the requirement for the existence of complex structures in the universe, such as galaxies, makes the entire region with $\Lambda/Q^3 T_{\rm eq}^4 \gtrsim 1$ outside $\mathcal{O}_I$ [2], where Q is the primordial scale of density perturbations and $T_{\rm eq}$ the temperature of matter-radiation equality. These imply that the observer regions $\mathcal{O}_I$ fill only a small fraction of the parameter space. They are small islands in the entire parameter space of the ensemble, and so the observer boundaries are not distributed densely and uniformly over the parameter space.

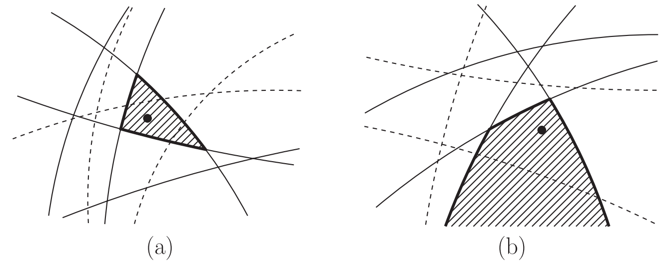

FIG. 4. Characteristic situations for complexity and observer boundaries in 2-dimensional parameter space. Complexity boundaries that are and are not relevant for the existence of observers are depicted by solid and dashed lines, respectively. The shaded region indicates an observer region $\mathcal{O}$, which (a) may or (b) may not be a small region around the observed point, which is denoted by the dot. The observer boundary $O(x_i) = 1$ is given by the border of $\mathcal{O}$ and is represented by the thick solid line.

The sparseness of $\mathcal{O}_I$ allows us to focus on the region $\mathcal{O}$ ($\subset \{\mathcal{O}_I\}$) that contains our observed point. The unusual closeness of the observed parameters to the relevant observer boundary (the boundary of $\mathcal{O}$) then signals unnaturalness, raising an "observer naturalness problem." The degree of unnaturalness can be quantified using the definition given in the previous section. The estimate for the degree will be conservative if we use only crucial requirements for the existence of observers, such as the existence of nuclei, stars, and galaxies, since this will give a large allowed region for observers, $\mathcal{O}$. Imposing more and more subtle conditions, such as the existence of carbon, oxygen, and so on, will only decrease $\mathcal{O}$ and therefore increase the degree of unnaturalness. A relevant question is whether we find an observer naturalness problem in the standard model and beyond, keeping only relatively robust conditions. This question will be addressed in the next section.

IV. EVIDENCE FOR OBSERVER NATURALNESS PROBLEMS

In this section we study if there exist observer naturalness problems in the standard model and beyond. In general, it is easier to study the existence of an observer naturalness problem than that of a complexity naturalness problem, since it is easier there to see that the relevant boundaries are *not* uniformly and densely distributed over the parameter space. In order to identify and quantify the observer naturalness problem reliably, we take the following approach. We first list complexity boundaries that are candidates for composing the observer boundary, in the relevant parameter space of the standard models of particle physics and cosmology. These boundaries have effects of various degrees on the environment, and we take only the ones giving fairly drastic effects. This leads to a large observer volume $\mathcal{O}$ and hence to a *conservative* estimate for the degree of an observer naturalness problem. The actual degree of unnaturalness could only be more severe if other complexity boundaries are important for the development of observers, since this would reduce $\mathcal{O}$.

A. The relevant parameters

Let us consider some ensemble that leads at low energies to the standard model of particle physics (taken to include neutrino masses) and to the standard cosmological model. Different members of this ensemble give different values for the parameters of the standard model $x_{\rm SM}$ and cosmology $x_{\rm cos}$. Some members may give values for this set $x = \{x_{\rm SM}, x_{\rm cos}\}$ that are so distant from the observed values x_o that the corresponding physics and astrophysics is completely different from that observed—for example, if five quark flavors were lighter than the QCD scale, or if the baryon asymmetry were of order unity. Here, we restrict our discussion to a subset of the ensemble in which variations in the parameters about x_o are limited. In fact, we are interested in determining whether small deviations in x from x_o could dramatically change certain relatively stable complex structures (atoms, stars ...) that we observe.

The physics and astrophysics of these complex structures depend on only a subset of x; for example, variations in the bottom quark mass, or its mixing to the charm quark, by a factor of 2 have no effect on the complex structures of interest. Hence, we now restrict x to include only

$$x_{\rm SM}\colon\ \alpha, \alpha_s, y_u, y_d, y_t, y_e, \lambda_h, m_h^2, \tag{9}$$

and

$$x_{\rm cos}\colon\ M_{\rm Pl}, T_{\rm eq}, Q, \eta_B, \Lambda. \tag{10}$$

Here, $y_{u,d,t,e}$ are the Yukawa couplings for the up, down and top quarks and the electron, λ_h the Higgs-quartic coupling, m_h^2 the Higgs mass-squared parameter, $M_{\rm Pl}$ the reduced Planck mass, and η_B the baryon asymmetry. In this 13-dimensional parameter space there are special surfaces that represent complexity boundaries

$$C_A(x) = 1, \tag{11}$$

such that the complex structure A differs drastically from one side of the boundary to the other.

Several comments are in order for our choice of the parameter set x.

(i) λ_h and m_h^2 can be traded for the electroweak vacuum expectation value, v and the Higgs boson mass. We include λ_h and y_t, because they typically play an important role in electroweak symmetry breaking. For example, in the standard model there is a boundary corresponding to the existence of a electroweak symmetry breaking vacuum with $v \ll M_{\rm Pl}$ [16]

$$C_v(\lambda_h, y_t) = 1. \tag{12}$$

(ii) The $SU(2)$ gauge coupling of the standard model is omitted from Eq. (9), since charged current weak interactions at low energies are described by v. Neutral current, and therefore the weak mixing angle, play little role on relevant complex structures.

(iii) We have omitted the strange quark Yukawa coupling y_s for simplicity. It does play a role in nuclear physics.

(iv) The only parameter from the lepton sector is y_e. Other lepton sector parameters could affect η_B via leptogenesis [17]. Since the source of η_B is unknown, we prefer to list it as an independent cosmological parameter.

(v) The 13-dimensional parameter set could be further reduced, since only certain combinations appear in C_A of Eq. (11). For example, only dimensionless combinations appear, so that $M_{\rm Pl}$ could be removed by using it as the unit of mass. For the complexity boundaries arising from atomic and nuclear structures, v appears only in the combinations $m_f \equiv y_f v$ ($f = u, d, e$) as fermion masses, so that for these boundaries $x_{\rm SM}$ can be reduced to

$$x'_{\rm SM}\colon\ \alpha, \frac{m_u}{\Lambda_{\rm QCD}}, \frac{m_d}{\Lambda_{\rm QCD}}, \frac{m_e}{\Lambda_{\rm QCD}}, \tag{13}$$

where $\Lambda_{\rm QCD}$ is the QCD scale, which takes a value of ≈ 100 MeV in our universe. For naturalness arguments, however, we often prefer to use the more basic set of Eq. (9).

(vi) In a theory with two Higgs doublets, such as the minimal supersymmetric standard model, the basic set of Eq. (9) should be expanded. For most purposes, we simply have to replace the electroweak vacuum expectation value v by two vacuum expectation values; for type-II two Higgs doublet theories, we have v_u for the up-type Higgs doublet and v_d for the down-type Higgs doublet.

B. The relevant complex structures

A crucial question for the complexity boundaries is what are the relevant complex structures. Our interest in a particular boundary depends on how important the corresponding complex structure is for explaining the structure of the physical world. Consider two extremes. The size of the cosmological constant, relative to the matter density of the universe when density perturbations become nonlinear, leads to a boundary that determines whether large scale structure forms

$$\frac{\Lambda}{Q^3 T_{\rm eq}^4} \approx 1. \tag{14}$$

A member on one side of the boundary will lead to formation of galaxies, while one on the other side leads to an inflating universe of isolated elementary particles. This clearly has important effects on the basic structure of the universe. As a second extreme example, consider the complex mesons with a b quark constituent. As the b quark mass m_b is increased above the W boson mass m_W the b quark decays so rapidly that B mesons cease to exist. This boundary of $m_b/m_W \approx 1$, however, has a negligible effect on our environment, which is why we did not include the b-quark Yukawa coupling y_b in $x_{\rm SM}$.

Somewhat arbitrarily, we divide the relevant complexity boundaries into three classes according to how dramatic the environmental change is across the boundary:

(i) *Catastrophic* boundaries change our universe into one that is essentially unrecognizable. In addition to Eqs. (12) and (14), we would include the case that electroweak symmetry is broken dominantly by the Higgs potential

$$\frac{v}{\Lambda_{\rm QCD}} \approx 1. \tag{15}$$

This is required so that the baryon asymmetry of the universe is not washed out by the sphaleron effects [18]. We also consider that the absence of any complex nuclei is catastrophic. In the simplified case that the only parameter that is varied from its standard model value is v, this boundary is [6]

$$\frac{v}{10^4 \Lambda_{\rm QCD}} \approx 1. \tag{16}$$

For v larger than this boundary, the only stable nucleus is either $p = uud$ or $\Delta^{++} = uuu$.

(ii) *Violent* boundaries separate members where a crucial complex structure of our universe is absent. For example, across the boundary

$$\frac{m_n}{m_p + m_e} = 1, \tag{17}$$

where m_p and m_n are the masses for the proton and neutron, the neutron becomes stable and hydrogen unstable. Such a neutron-stable world would not have dense astrophysical objects fueled by nuclear energy release, such as main-sequence stars in our universe [19]. Another example of violent boundaries is that of vanishing deuteron binding energy

$$B_D(m_u, m_d, \Lambda_{\rm QCD}, \alpha) = 0. \tag{18}$$

Across this boundary, no nuclei form during big bang nucleosynthesis, so that the universe protonizes (or neutronizes). Stars could only burn via exotic triple proton reactions, with extremely high central densities.

(iii) *Substantial* boundaries separate members where a crucial complex structure of our universe is drastically changed. For example, across certain boundaries stars may exist but are very different from those we see. For example, if the pp reaction $pp \to De^+\nu$ is not available to ignite stars, then protostars would collapse to a higher temperature before igniting via the pep reaction, $ppe^- \to D\nu$. A more substantial change to stars occurs if the deuteron is beta unstable $D \to ppe^-\bar{\nu}$; stars could still burn, but only by using the helium produced during big bang nucleosynthesis. The existence of a stable diproton, the ^{2}He nucleus, would also change stellar nuclear reactions, shortening lifetimes of hydrogen burning stars. Changes of some nuclear energy levels, controlled by quark masses and α, could also lead to substantial changes in the abundance of various nuclear species, such as carbon and oxygen.

Each boundary gives a surface of special values for the parameters $\bar{x}$, which does not correspond to a surface of an enhanced symmetry. Among the ones listed, we find that most boundaries are clustered around a zone $y_{u,d,e} v \sim \alpha \Lambda_{\rm QCD} \sim O(0.01)\Lambda_{\rm QCD}$, where our universe also resides. This strongly suggests the existence of a complexity naturalness problem. Here, however, we focus more on the observer naturalness problem, which arises if the member describing our universe lies unusually close to the observer boundary. There are certain ambiguities in identifying which of the complexity boundaries compose the observer boundary. An important point, however, is that by using only the boundaries that certainly have disastrous effects on the environment, we can be on the conservative side in evaluating the existence of an observer naturalness problem. For this reason, we take only boundaries that are catastrophic or violent, rather than just substantial, to compose our observer boundary. We also select only the boundaries that allow us to derive reasonably accurate values for $\bar{x}$, allowing a reliable estimate of the naturalness probability P.

C. Stability boundaries for neutrons, deuterons, and complex nuclei

Following work by others, we focus on the boundaries across which we lose complex nuclei, Eq. (16), neutron instability, Eq. (17), and the deuteron, Eq. (18). We take y_u, y_d, y_e, $v/\Lambda_{\rm QCD}$, and α to be our parameters x_i. The choice is motivated by the expectation that the relation of these parameters to those in the ultraviolet theory is relatively direct.

We now represent the boundaries of Eqs. (16)–(18) in terms of the deviations of x_i from the observed values $x_{i,o}$. Let us first discuss the neutron stability boundary of Eq. (17). The instability of a neutron, or equivalently the stability of hydrogen, requires

$$m_n - m_p - m_e > 0, \tag{19}$$

where we have neglected the neutrino mass. The neutron-proton mass difference $m_n - m_p$ arises from both the strong isospin violating effect, δ_{d-u}, and the electromagnetic contribution to the proton mass, $\delta_{\rm EM}$: $m_n - m_p = \delta_{d-u} - \delta_{\rm EM}$. In our universe, $\delta_{d-u} \simeq 2.26 \pm 0.51$ MeV [20] and $\delta_{\rm EM} = \delta_{d-u} - (m_n - m_p) \simeq (2.26 \pm 0.51) - 1.29$ MeV. To a first approximation, these quantities scale as $\delta_{d-u} \propto m_d - m_u$ and $\delta_{\rm EM} \propto \alpha$, so that

$$m_n - m_p \simeq \frac{m_d - m_u}{m_{d,o} - m_{u,o}}(2.26~{\rm MeV}) - \frac{\alpha}{\alpha_o}(0.97~{\rm MeV}). \tag{20}$$

Here, we have taken the central value for δ_{d-u}, and the variables with and without the subscript o represent, respectively, the values in our observed universe and those in an arbitrary member of the ensemble. In terms of the parameters x_i, Eq. (19) can be written as

$$\frac{1}{0.77}\left(\frac{y_d - y_u}{y_{d,o} - y_{u,o}} - 0.23\frac{y_e}{y_{e,o}}\right)\frac{v/\Lambda_{\rm QCD}}{(v/\Lambda_{\rm QCD})_o} > 0.56\frac{\alpha}{\alpha_o}. \tag{21}$$

Here, we have neglected a small dependence of $\Lambda_{\rm QCD}$ on v as well as logarithmic evolution of the Yukawa couplings. For two Higgs doublet theories,[7] the Yukawa couplings in Eq. (21) should be replaced as

$$y_u \to y_u \sin\beta, \qquad y_d \to y_d \cos\beta, \qquad y_e \to y_e \cos\beta, \tag{22}$$

$$y_{u,o} \to y_{u,o} \sin\beta_o, \qquad y_{d,o} \to y_{d,o} \cos\beta_o, \qquad y_{e,o} \to y_{e,o} \cos\beta_o, \tag{23}$$

where $v \equiv \sqrt{v_u^2 + v_d^2}$ and $\tan\beta \equiv v_u/v_d$.

[7] Here and below, we assume type-II two Higgs doublet models when we discuss two Higgs doublet theories.

We next discuss the boundary of Eq. (18). This boundary corresponds to the stability of a deuteron under strong interactions, which requires

$$B_D = m_p + m_n - m_D > 0, \tag{24}$$

where m_D is the deuteron mass. The binding energy B_D depends on the quark masses as

$$B_D - B_{D,o} = -a\left(\frac{m_u + m_d}{m_{u,o} + m_{d,o}} - 1\right), \tag{25}$$

where $B_{D,o} \simeq 2.2$ MeV is the observed deuteron binding energy. The parameter a is uncertain, but it is estimated in Ref. [6] to be $a \simeq (1.3\text{–}5.5)$ MeV using models of nucleon binding. This allows us to write Eq. (24) in terms of x_i as

$$\frac{y_u + y_d}{y_{u,o} + y_{d,o}}\frac{v/\Lambda_{\rm QCD}}{(v/\Lambda_{\rm QCD})_o} < 1 + \frac{2.2~{\rm MeV}}{a}. \tag{26}$$

For two Higgs doublet theories, the Yukawa couplings must be replaced as in Eqs. (22) and (23).

We finally consider the condition for the existence of complex nuclei. The stability of complex nuclei requires the energy release for the β decay $n \to pe^-\bar{\nu}$ to be smaller than the binding energy of nuclei per nucleon $E_{\rm bin}$:

$$m_n - m_p - m_e \lesssim E_{\rm bin}, \tag{27}$$

where we have neglected the neutrino mass. Precisely speaking, $E_{\rm bin}$ varies with a nucleus, receiving contributions both from nuclear forces and the Coulomb repulsion between protons. To a first approximation, however, $E_{\rm bin}$ can be regarded as the same for all nuclei and being controlled purely by $\Lambda_{\rm QCD}$, taking the value of $E_{\rm bin} \simeq 8$ MeV in our universe.[8] Substituting this into Eq. (27), and using Eq. (20), we obtain

$$\frac{1}{0.77}\left(\frac{y_d - y_u}{y_{d,o} - y_{u,o}} - 0.23\frac{y_e}{y_{e,o}}\right)\frac{v/\Lambda_{\rm QCD}}{(v/\Lambda_{\rm QCD})_o} \lesssim 4.6 + 0.56\frac{\alpha}{\alpha_o}. \tag{28}$$

The expression should be modified according to Eqs. (22) and (23) for two Higgs doublet theories.

D. Observer naturalness problem in the standard model and beyond

The region inside our observer boundary, $\mathcal{O}$, is defined by Eqs. (21), (26), and (28), with modification by Eqs. (22) and (23) for two Higgs doublet theories. To visualize the overall shape of this region, in Fig. 5 we depict the three boundaries of Eqs. (21), (26), and (28) in m_u–m_d–m_e space. The plots in the left column adopt a logarithmic

[8] We neglect the dependence of $E_{\rm bin}$ on m_u and m_d, as this will not affect our conclusions.

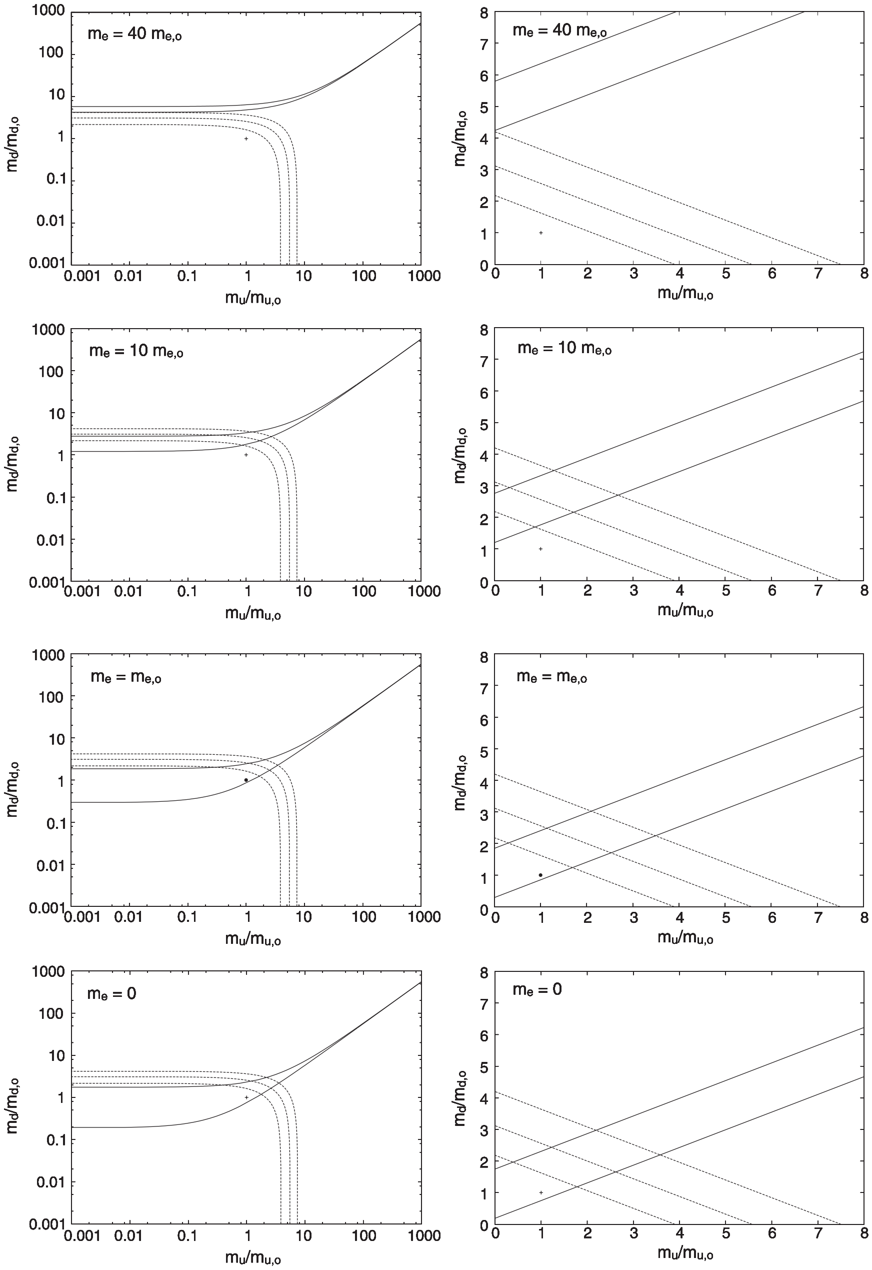

FIG. 5. The location of the observer boundary in m_u–m_d–m_e space. The neutron and complex nuclei boundaries of Eqs. (21) and (28) are depicted by solid lines (below and above, respectively). Dashed lines represent the deuteron boundary of Eq. (26) (for $a = 5.5$, 2.2, and 1.3 MeV from below). The observed point is represented by little dots.

scale in m_u and m_d, while those in the right a linear scale in m_u and m_d. The fine structure constant is fixed to be $\alpha = \alpha_o$, and we have used the leading-order chiral perturbation value of $m_{u,o}/m_{d,o} = 0.56$ to draw these plots. (The qualitative features of the plots are not affected if we vary δ_{d-u} and $m_{u,o}/m_{d,o}$ in the range of $\delta_{d-u} \simeq 2.26 \pm 0.51$ MeV [20] and $m_{u,o}/m_{d,o} = 0.3$–0.6 [21].) From the figure, we find that the observed parameters are close to the three boundaries, especially when viewed on a logarithmic scale. In particular, they are very close to the neutron stability bound of Eq. (19). This implies that we have an observer naturalness problem.

The degree of unnaturalness P depends on a theory, since the relation between $\{m_u, m_d, m_e, \alpha\}$ and the fundamental parameters, as well as the form of the distribution function, depend on the ensemble we consider. However, we can still make a conservative estimate on P, based on the observation that the naturalness probability $\tilde{P}$ obtained by fixing all the fundamental parameters except for one generally satisfies $\tilde{P} \gtrsim P$. In the present context, we can fix y_u, y_d, y_e, α, and $\Lambda_{\rm QCD}$, as well as $\tan\beta$ for two Higgs doublet theories. The conditions of Eqs. (21), (26), and (28) then lead to

$$0.5 \lesssim \frac{v}{v_o} \lesssim 2, \qquad (29)$$

where we have used $a = 2.2$ MeV for illustrative purpose. The probability of v falling in this range $\tilde{P}_v$ then gives a conservative estimate of the naturalness probability P. Alternatively, we can fix v instead of $\Lambda_{\rm QCD}$. In this case, we have

$$0.5 \lesssim \frac{\Lambda_{\rm QCD}}{\Lambda_{{\rm QCD},o}} \lesssim 2. \qquad (30)$$

The probability of $\Lambda_{\rm QCD}$ falling in this range $\tilde{P}_{\Lambda_{\rm QCD}}$ can also give a conservative estimate of P, since $P \lesssim \min\{\tilde{P}_v, \tilde{P}_{\Lambda_{\rm QCD}}\}$. Which of $\tilde{P}_v$ and $\tilde{P}_{\Lambda_{\rm QCD}}$ we use is determined by which is easier to estimate and, in case both are easy to estimate, which gives a stronger bound on P.

Let us first consider an ensemble in which the distribution of v is flat in v^2, as in the standard model embedded into more fundamental theory at a high scale. In this case, we obtain an extremely small value of $\tilde{P}_v \approx v_o^2/M_*^2$, where $M_* \gg v_o$ is the cutoff scale. For $M_* \approx 10^{18}$ GeV, we obtain $\tilde{P}_v \approx 10^{-32}$. The origin of this small value, however, is precisely the existence of the gauge hierarchy problem. We thus see the existence of a severe observer naturalness problem in these theories, but it is hard to disentangle from the fine-tuning naturalness problem.

We therefore focus on theories in which the conventional gauge hierarchy problem is solved. In these theories, the distribution of v is expected to be flat in $\ln v$ or $1/\ln v$ within an ensemble. To estimate $\tilde{P}_v$, however, we need to know the range of v, which depends on how v is related to the fundamental parameters of the theory. To avoid model dependence coming from this, we here consider $\tilde{P}_{\Lambda_{\rm QCD}}$, instead of $\tilde{P}_v$. The value of the QCD scale, $\Lambda_{\rm QCD}$, is determined from the strong gauge coupling constant at the cutoff scale, $g_3(M_*)$, through renormalization group evolution. The probability $\tilde{P}_{\Lambda_{\rm QCD}}$ can then be estimated if (i) the theory between $\Lambda_{\rm QCD}$ and M_*, together with the value of M_*, is specified, and (ii) the distribution function for $g_3(M_*)$, including the range of $g_3(M_*)$, is given.[9] Specifying the theory between $\Lambda_{\rm QCD}$ and M_* is important because the existence of colored states whose masses are associated with v may increase the correlation between v and $\Lambda_{\rm QCD}$, enhancing $\tilde{P}_{\Lambda_{\rm QCD}}$. The distribution and the range of $g_3(M_*)$ are also important. For example, if the distribution of $g_3(M_*)$ were flat in $1/g_3^2(M_*)$ within the range $1/g_3^2(M_*) \gtrsim b_3/16\pi^2$, the QCD scale is distributed almost flat in $\ln\Lambda_{\rm QCD}$ for all $\Lambda_{\rm QCD} \lesssim M_*$. Here, b_3 is the one-loop beta function coefficient for g_3 evaluated at M_*. In this case, we would obtain $\tilde{P}_{\Lambda_{\rm QCD}}$ estimated using Eq. (30) to be infinitely small. To avoid this unphysical conclusion, we can introduce an arbitrary cutoff c on the distribution $1/g_3^2(M_*) \lesssim c$. Alternatively, we can consider that the distribution of $g_3(M_*)$ is flat in $g_3^2(M_*)$. In this case, restricting the range to be $g_3^2(M_*) \lesssim c'$ gives a finite answer to $\tilde{P}_{\Lambda_{\rm QCD}}$. The value of $\tilde{P}_{\Lambda_{\rm QCD}}$ depends on c', but we can make a reasonable conjecture on the value of c', e.g., $c' \approx 1$ or $\approx 16\pi^2/b_3$.

In Fig. 6, we plot the value of $g_3^2(M_*)$ as a function of $\Lambda_{\rm QCD}/\Lambda_{{\rm QCD},o}$ obtained using one-loop renormalization group equations for nonsupersymmetric theories (top) and supersymmetric theories (bottom). In nonsupersymmetric theories, M_* is taken to be 10^{14} GeV, which is the scale where the $SU(3)$ and $U(1)$ gauge couplings almost meet. (This is the scale where the three gauge couplings

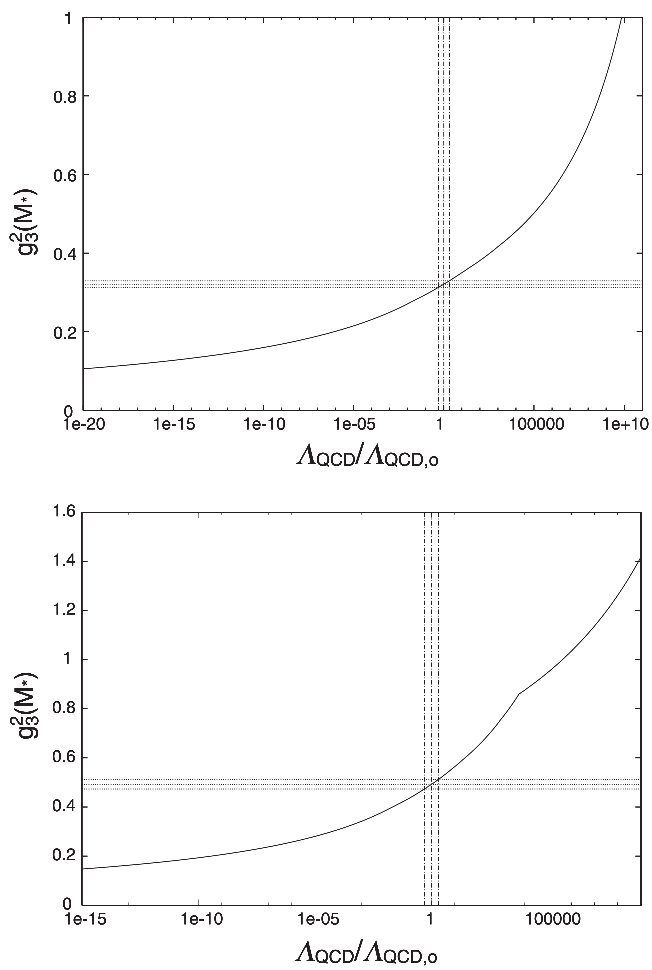

FIG. 6. The value of $g_3^2(M_*)$ as a function of $\Lambda_{\rm QCD}/\Lambda_{{\rm QCD},o}$ for nonsupersymmetric theories (top) and supersymmetric theories (bottom). The range of Eq. (30) is depicted by the vertical lines, while the corresponding range of $g_3^2(M_*)$ by the horizontal lines.

[9] Here we neglect higher order effects, which are expected to be small.

would almost meet if five Higgs doublet fields or a vector-like fermion with the Higgs quantum numbers were introduced at the weak scale.) In supersymmetric theories, we take M_* to be the unification scale 2×10^{16} GeV, and the scale of superparticle masses to be $m_{\rm SUSY} \simeq 1$ TeV. Dependence of the results on these parameters, however, is weak. From the figure, we find the probability of $\Lambda_{\rm QCD}$ falling in the range of Eq. (30) to be

$$\tilde{P}_{\Lambda_{\rm QCD}} \simeq 0.016\frac{1}{c'} \simeq 61^{-1}\frac{1}{c'}, \tag{31}$$

for nonsupersymmetric theories and

$$\tilde{P}_{\Lambda_{\rm QCD}} \simeq 0.038\frac{1}{c'} \simeq 26^{-1}\frac{1}{c'} \tag{32}$$

for supersymmetric theories. Here, we have taken the distribution function to be flat in $g_3^2(M_*)$. The value of c' is uncertain, but we expect $c' \gtrsim 1$. The numbers in Eqs. (31) and (32) depend on a in Eq. (25), through the dependence of the lower value of Eq. (30) on a. For $a =$ 1.3 MeV (5.5 MeV), 0.016 and 0.038 in these equations become 0.020 and 0.046 (0.012 and 0.029), respectively. Note that the analysis here provides the *most conservative* estimate for the level of an observer naturalness problem existing in theories beyond the standard model (in which the conventional gauge hierarchy problem is solved, so that $\tilde{P}_v$ can be larger than $\tilde{P}_{\Lambda_{\rm QCD}}$)

$$P \lesssim \tilde{P}_{\Lambda_{\rm QCD}}. \tag{33}$$

In theories where the gauge hierarchy problem is not solved, the degree of the observer naturalness problem is much more severe, $P \lesssim \tilde{P}_v \ll 1$. We thus find that the bounds on P derived here provide evidence for an observer naturalness problem in the standard model and beyond.

To obtain a conservative estimate on P, we have used here only $P \lesssim \min\{\tilde{P}_v, \tilde{P}_{\Lambda_{\rm QCD}}\}$. Physically, the case with $P \approx \min\{\tilde{P}_v, \tilde{P}_{\Lambda_{\rm QCD}}\}$ corresponds to the situation where we have the "best" model of flavor, i.e., the observed quark and lepton masses in units of the weak scale are reproduced essentially without any free parameter. This is, however, not the case for many theories of flavor, which will have $P < \min\{\tilde{P}_v, \tilde{P}_{\Lambda_{\rm QCD}}\}$, as discussed in the next section.

V. NATURALNESS PROBABILITIES AND THEORIES OF FLAVOR

The observed values of the first generation masses, $m_{u,d,e}$, are special: Fig. 5 shows that they are close to the boundaries of neutron, deuteron, and complex nuclei stability. To judge whether this is likely to be accidental we need to evaluate the naturalness probability, but this depends on the theory of flavor. We begin by considering ensembles of theories in which α and $v/\Lambda_{\rm QCD}$ are fixed, but the flavor observables x_F have some probability distribution $f(x_F)$.

In the standard model the relevant flavor observables are the Yukawa couplings: $x_F = y_{u,d,e}$. While they are all very small, of order 10^{-6}–10^{-5} at unified scales, they are all close to the maximal values allowed by the neutron, deuteron, and complex nuclei boundaries. Of particular importance, the observed values are especially close to the neutron (hydrogen) stability boundary. While y_e is about a factor of 3 from this boundary, in the y_u–y_d plane the distance from the observed point to this boundary corresponds to a variation in the coupling only of

$$\sqrt{\frac{(1-z)^2(1-A)^2}{1+z^2}} = 13^{+17}_{-3}\%, \tag{34}$$

where $z \equiv m_u/m_d$ and $A \equiv (\delta_{d-u} - m_n + m_p + m_e)/\delta_{d-u}$ evaluated in our universe. Here, the central value is obtained with $z = 0.56$ and $\delta_{d-u} = 2.26$ MeV. The electromagnetic interaction raises the proton mass above the neutron mass by about an MeV. Given that quarks can have masses up to 100 GeV or more, it is remarkable that the $m_d - m_u$ mass difference just overcompensates the electromagnetic shift to make the hydrogen mass only $\simeq 0.78$ MeV smaller than the neutron mass. The deuteron and complex nuclei stability boundaries are also close by, corresponding to changes in the Yukawa couplings by factors of $\approx$ (1.5–3) and $\approx$ 2, respectively.

It is important to numerically evaluate these accidents; for illustration, we consider two simple distribution functions. If $f(y_{u,d,e})$ are flat and nonzero in the range of $y_{u,d,e}$ from 0 to 1, then the corresponding naturalness probability for this ensemble, using Eq. (5), is $P_F \approx 10^{-16}$; while if $f(\log_{10} y_{u,d,e})$ are flat and nonzero in the range of $\log_{10} y_{u,d,e}$ from -6 to 0, then $P_F \approx 10^{-4}$. As expected from Fig. 5, the volume of parameter space closer to the special points on the observer boundary than the measured point is very small compared with the expected total volume of parameter space in the ensemble. From the viewpoint of neutron instability and deuteron and complex nuclei stability, the standard model description of flavor is highly unnatural.

Theories of flavor that go beyond the simple Yukawa coupling parameterization of the standard model typically involve further symmetries, such as flavor or unified gauge symmetries. While the set of flavor observables x_F in these theories can be smaller than that in the standard model, all known models do involve free parameters. If a theory could be found in which $y_{u,d,e}$ are precisely predicted, i.e., if variations in $y_{u,d,e}$ are less than the distances to the special points on the observer boundary when x_F vary, then such a theory would have $P_F \approx 1$. However, if $y_{u,d,e}$ vary significantly (and independently) as x_F vary in the ensemble, as in most theories of flavor, then the theory is likely to have P_F (much) smaller than of order unity. The best hope for a significant improvement in naturalness is then to obtain a successful prediction for the ratios of the first generation Yukawa couplings—symmetries that successfully predict the y_u, y_d, and y_e ratios could show that our universe lying so close to the observer boundary is just accidental. This is, however, not so easy, especially because of the extreme closeness of the observed point to the neutron stability boundary. Moreover, since the observer boundary involves the combinations $y_{u,d,e}v/\Lambda_{\rm QCD}$, the normalization of $y_{u,d,e}$ is coupled to the value of $v/\Lambda_{\rm QCD}$. Below, we do not take $v/\Lambda_{\rm QCD}$ as fixed, but consider it to vary in the ensemble as discussed in the previous section.

In certain theories of flavor, for example, with Abelian flavor symmetries, the smallness of y_f ($f = u, d, e$) follows from a single small parameter of the theory

$$y_f = c_f \epsilon^{p_f}, \tag{35}$$

where $\epsilon \ll 1$ while c_f are of order unity. The powers p_f are not free parameters of the theory, but result from a certain judicious choice of Abelian charges for all the quark and lepton fields [22]. The ratios of the Yukawa couplings can then be just numbers of order unity for a fixed value of ϵ (and $\tan\beta$ for two Higgs doublet theories). Let us consider, for example, that the coefficients c_f vary in the range $0 \leq c_f \leq 2c_{f,o}$ with a flat distribution on a linear scale, or in the range $\log_{10}c_{f,o} - 1/2 \leq \log_{10}c_f \leq \log_{10}c_{f,o} + 1/2$ with a flat distribution on a logarithmic scale. In this case, the extreme closeness of the neutron stability boundary gives $P_F \approx 1/18$ and $1/36$, respectively, for a fixed value of ϵ (and $\tan\beta$). In addition to this, there is the issue of the normalization of the masses $m_{u,d,e}$, arising from a variation of $v/\Lambda_{\rm QCD}$. Assuming that the distribution function is flat in $g_3^2(M_*)$, this leads to a further reduction of the probability at least by a factor of $\tilde{P} \lesssim 1/35$ and $1/15$ for nonsupersymmetric and supersymmetric theories, respectively. [These numbers are obtained by requiring that $v/\Lambda_{\rm QCD}$ should be within a factor of $\approx$ (2–3) from the observed value so that its variation can be absorbed into changes of c_f, without a large extra cost in P_F. The range of $g_3^2(M_*)$ here has been chosen to be $0 \leq g_3^2(M_*) \leq 1$.] Possible variations of ϵ (and $\tan\beta$) may or may not lead to further reductions of the probability. We thus conclude that theories with Abelian flavor symmetries considered here have

$$P \lesssim P_F\tilde{P} \lesssim (10^{-3}\text{–}10^{-2}), \tag{36}$$

from physics of flavor alone.

To increase P from flavor physics, it is necessary to predict ratios of $y_{u,d,e}$ to better than the factor of ≈ 2 we used in the above estimates. A more elaborate charge assignment for the quark and lepton fields under an Abelian flavor symmetry is not sufficient, since it will leave the relative coefficients of order unity undetermined. A non-Abelian flavor symmetry, or unified gauge symmetry, could improve P by successfully predicting y_u:y_d:y_e as Clebsch factors. For example, y_u:y_d:y_e = 1:2:1 at the unified scale agrees with inferred values of $m_{u,d,e}$, providing the eigenvalues are not affected much by mixing to heavier generations. A simple theory of flavor incorporating such a relation could lessen the significance of the observer naturalness problem of Eq. (36). However, uncertainties from QCD are too large for us to know if the relation really puts us sufficiently close to the neutron stability boundary, so that the problem is simply an accidental consequence of symmetry. Moreover, in conventional theories of flavor we expect simple Clebsch factors to apply to the heaviest generation not the lightest. This is because the lightest generation typically gets contributions from nontrivial matrix diagonalizations, so that the relation would have to involve more than one generation. For example, the inferred values of $m_{u,d,e}$ are consistent with unified boundary conditions $y_u = y_e$ and $y_d = 0$, provided the down quark mass arises from diagonalizing a symmetric 2×2 matrix that fully accounts for the Cabibbo angle. This requires, however, experimental inputs of the strange quark mass and the Cabibbo angle, so that it does not provide a real symmetry solution to the problem. We find it rather difficult to have a symmetry understanding for y_u:y_d:y_e, and even if we had, unnaturalness still arises from the coincidence between v and $\Lambda_{\rm QCD}$, given by Eqs. (31) and (32). We conclude that, in the absence of a convincing and significant progress in flavor physics, we have unnaturalness associated with flavor at the level given by Eq. (36).

VI. ENVIRONMENTAL SELECTION

We have seen that theories of particle physics and cosmology are likely to have observer naturalness problems, signaled by $P \ll 1$. The problems arise in an ensemble if the observed point is close to the observer boundary. The situations in which this happens can be classified into the following three cases: the observed point is (a) a typical point within a small observer region $\mathcal{O}$, (b) close to the boundary of a small observer region $\mathcal{O}$, and (c) close to the boundary of a large observer region $\mathcal{O}$, which may or may not be a closed region in the parameter space. These cases are depicted schematically in Fig. 7.

The heart of the observer naturalness problem is the coincidence: why is the observed point so close to the observer boundary, which is a very special region in the parameter space? To solve this problem, we must make the observer boundary *really* special—the existence and location of the observer boundary should somehow affect the process of selecting a member in the ensemble as the one describing our universe. This leads us to consider environmental selection, effects sensitive to the existence of an observer. In this section, we study how environmental selection on a multiverse can solve observer naturalness problems in general, and under what circumstances (and in what sense) we can identify evidence for it. We also discuss possible implications of this solution in identifying the correct theory describing our universe.

The nature of the observer naturalness problem is dramatically altered if the members in an ensemble are physi-

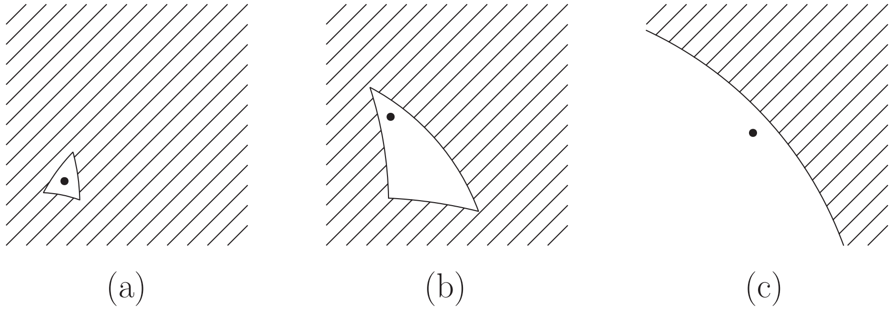

FIG. 7. The situations that lead to an observer naturalness problem. The observed point is denoted by the little dot, while the region outside $\mathcal{O}$ is shaded.

cally realized in a multiverse. Until now, the ensemble has been a useful fictitious mathematical device to study problems of naturalness; now we assume that each member of the ensemble is physically realized as a universe in spacetime. The fundamental theory of nature is assumed to have a huge number of vacua, the landscape of vacua, so that physics at low energy may be described by many possible effective theories. In fact, we are interested in a subset of these theories T that leads to physics described by the parameters x_i discussed in Sec. IV, with the parameters x_i taking different values in different universes. We assume that it is possible to define a distribution function of the multiverse $\tilde{f}_T(x_i)$ such that, averaging over the entire multiverse, we obtain $\langle x_i \rangle = \Sigma_T \int \tilde{f}_T(x_i) x_i dx_i$. We stress that this is the average value on the entire multiverse, independent of whether universes contain observers. If the population mechanism, including any relevant volume factors, were independent of x_i, then $\tilde{f}_T(x_i)$ would represent the distribution of vacua in the theory underlying the landscape. However, in general it includes both relevant population and volume factors.[10] We first focus on a single theory, so that we can drop the index T until later subsections.

Environmental selection on the multiverse involves two key points that give a central role to measurements made by observers

(i) Universes that have parameters outside the region $\mathcal{O}$ do not contain observers, hence values of x_i outside $\mathcal{O}$ cannot be measured. When discussing naturalness of the observed universe, we should not be asking questions about the entire multiverse but only about the observer region $\mathcal{O}$.

(ii) The number of observers in universes in $\mathcal{O}$ with x_i in the region x_i to $x_i + dx_i$ is given by $\tilde{f}(x_i)n(x_i)dx_i \equiv f(x_i)dx_i$. Each universe is to be weighted by an observer distribution $n(x_i)$, the factor associated with the number of observers that develop in a universe with parameters x_i. We make no attempt to define an "observer."

These two points are really two aspects of a single selection process, with $n(x_i)$ defined over the entire space of x_i. We, however, find it useful to consider that $n(x_i)$ rapidly drops to zero at certain observer boundaries, so that the region outside these boundaries does not affect the naturalness of a multiverse.

In the approximation that the observer boundary is sharp, the naturalness probability of Eq. (3) is then replaced by

$$P_{\mathcal{O}} = \left| \frac{\int_{\tilde{x}}^{x_o} f(x)dx}{\int_{\mathcal{O}} f(x)dx} \right|, \tag{37}$$

where the integral in the denominator implies that x is integrated only in the region $\mathcal{O}$. The expression for higher dimensional x space is also obtained similarly. We stress that here and throughout the rest of the paper

$$f(x_i) = \tilde{f}(x_i)n(x_i). \tag{38}$$

The entire region of parameter space of the multiverse is now irrelevant; the only question is whether we are typical observers in universes in $\mathcal{O}$. To calculate $\tilde{f}$ it is necessary to know both the landscape of vacua and the population mechanism. On the other hand, n is independent of the landscape and, with sufficient understanding of the physical environment for observers, could be calculated in principle from the low energy theory. We take the practical viewpoint that both $\tilde{f}$ and n are unknown, and hence give ourselves the freedom to assume any reasonable smooth distribution for f. It is then clear that distributions can be found that make $P_{\mathcal{O}} \approx 1$, solving the observer naturalness problems whether of the form of (a), (b), or (c) of Fig. 7. A more detailed description of each of these three types of solution is given in Sec. VI A. In Sec. VI C, we consider the relative probability of different theories T solving an observer naturalness problem.

It is quite clear that there are many origins for a significant x_i dependence of $f(x_i) = \tilde{f}(x_i)n(x_i)$ in the observer region $\mathcal{O}$. For $\tilde{f}$ these are the distribution of vacua in the

[10] For subtleties in defining the distribution function in the multiverse, see, e.g., [23] and references therein.

underlying theory as well as relevant volume and population factors, while for n a variation in x_i could change the density of galaxies, stars, nuclei, and so on that are relevant for observers. In Sec. VI B, we investigate a further effect: it could be that f or the shape of $\mathcal{O}$ depends on more variables, x_b, than the ones we focus on, x_a, and that when these extra variables are integrated out the resulting effective distribution acquires further x_a dependence. Finally, a crucial issue is how to evaluate evidence for environmental selection. In Sec. VI D, we argue that, for any observer naturalness problem, a low value for the naturalness probability P provides evidence for environmental selection. More precisely, for the known set of (simple) symmetry theories, T, the evidence is determined by the largest value of P_T

$$P_{\rm max} = \max_T \{P_T\}. \tag{39}$$

If $P_{\rm max} \approx 1$, there is no evidence, but as $P_{\rm max}$ decreases so the evidence becomes more substantial.

A. Three manifestations of environmental selection

Using a set of Lagrangian parameters x_i of some low energy theory, an observer naturalness problem might appear in the three ways illustrated in Fig. 7. In each case, environmental selection may be at work, but the description of the solution to the problem is different.

Suppose we find an observer naturalness problem of the form of (a) in Fig. 7. This suggests that the distribution function f is (approximately) constant in the Lagrangian basis x_i, since the replacement of $P \to P_{\mathcal{O}}$ then completely eliminates the observer naturalness problem. Assuming a constant f, the naturalness probability $P_{\mathcal{O}}$ of Eq. (37) becomes

$$P_{\mathcal{O}} = \left| \frac{x_o - \bar{x}}{\Delta x} \right|, \tag{40}$$

where Δx is the range of x in the observer region $\mathcal{O}$. Similarly, for multiple parameters the naturalness probability of Eq. (5) is replaced by

$$P_{\mathcal{O}} = \left| \frac{c_n \{\sum_{a=1}^n (x_{a,o} - \bar{x}_a)^2\}^{n/2}}{V_n(\mathcal{O})} \right| = \frac{v_n}{V_n(\mathcal{O})}, \tag{41}$$

where $V_n(\mathcal{O})$ is the volume of parameter space in the observer region $\mathcal{O}$. The elimination of observer naturalness problems by the replacement $P \to P_{\mathcal{O}}$ is illustrated in Fig. 8. The enormous increase in the probability here arises simply because all of the universes that lie outside the region $\mathcal{O}$ are cut out of the denominator.

The situation in Fig. 8 is the case of the overall picture for the cosmological constant Λ. For conventional naturalness, consider an ensemble with a distribution function f that is constant in Λ, which is the case in most theories. The probability that a member of this ensemble has a small value of Λ_o near the special value zero is $P(\Lambda) = \Lambda_o/M_*^4$,

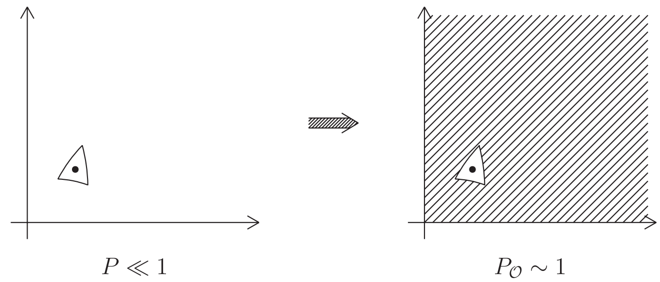

FIG. 8. Replacement of the probability $P \to P_{\mathcal{O}}$, illustrated for the case of (a) of Fig. 7.

where M_* is the fundamental scale. On the other hand, environmental selection, arising from a multiverse with $\tilde{f}$ and n assumed flat in Λ, replaces $\Lambda_{\rm max} \approx M_*^4$ by $\Delta\Lambda \approx Q^3 T_{\rm eq}^4$, giving

$$P_{\mathcal{O}}(\Lambda) \approx \frac{\Lambda_o}{Q^3 T_{\rm eq}^4}. \tag{42}$$

The vast majority of universes have a huge cosmological constant; but that is irrelevant because they contain only a dilute gas of elementary particles undergoing inflation. We should cut all these universes out of the measure, and consider only those where large scale structure forms, allowing the possibility of complex observers. This eliminates (or greatly ameliorates) the problem associated with the cosmological constant $P(\Lambda) \ll 1 \to P_{\mathcal{O}}(\Lambda) \sim 1$. A more refined analysis includes the effects of nontrivial $n(\Lambda)$ near the observer boundary [24].

Let us now consider observer naturalness problems in the forms of (b) or (c) of Fig. 7. In these cases, even after selection we are apparently left with some residual naturalness problem $P_{\mathcal{O}} \ll 1$ in some particular theory T. What does this imply? Does this mean that the observer naturalness problem in these forms cannot be solved by environmental selection?

The answer is clearly no. To understand what can be going on, it is important to realize that our knowledge can often be very incomplete. Imagine that we have assumed some distribution function f for an ensemble and found an observer naturalness problem in the form of (b) or (c) of Fig. 7 in the basis where f is constant. This seems to imply that the observer naturalness problem cannot be solved by a simple cutout procedure illustrated in Fig. 8. However, do we really know that we have identified all the relevant complexity boundaries, and thus identified the correct observer boundary? It could be that we have missed some relevant boundary and, after taking it into account, $P_{\mathcal{O}}$ may become order unity. This is illustrated in Fig. 9 for the case of (b) of Fig. 7.

Another possibility is that our initial assumption on the distribution function may not be correct. In practice, when we consider a theory, we start by assuming a "natural" distribution function $f(x_i)$, which is often taken to be

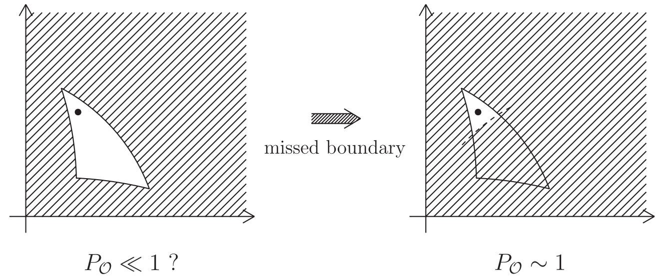

FIG. 9. Illustration of the situation in which a missed boundary (shown dashed) leads to a fictitious problem of $P_{\mathcal{O}} \ll 1$ after environmental selection.

constant in ''Lagrangian parameters'' x_i. However, there are *many* reasons why the distribution function in this parameter basis may not be constant even approximately. First, the Lagrangian parameters x_i of the low energy theory may be functions of the Lagrangian parameters y_j in the fundamental theory, $x_i = x_i(y_j)$, and the distribution function may be constant in y_j rather than x_i. Second, in the multiverse picture, a mismatch between the naive and true distribution functions may also result from the population mechanism depending on x_i, or may be induced by environmental selection via a nontrivial $n(x_i)$. Finally, environmental selection may depend not only on x_i but also on other variables, inducing an extra dependence of f on x_i, as we will discuss in the next subsection. Hence, the distribution function $f(x_i)$ may have a strong dependence on the Lagrangian parameters x_i of the low energy theory, and in this case, the situations of (b) or (c) in Fig. 7 actually correspond to typical observers with $P_{\mathcal{O}} \approx 1$, expected from environmental selection.

The situation in which naive and true distribution functions differ significantly is illustrated in Fig. 10 for the case of (c) of Fig. 7. (For the discussion here we assume that all relevant parts of the observer boundary have been correctly identified.) There are two (equivalent) ways to describe this situation. One is to go to the basis in which the true distribution function is constant. In this basis, which we may call the ''mathematical basis,'' the observed point is a typical point in $\mathcal{O}$, and the parameters chosen, y_i, can in general be nontrivial functions of the ''Lagrangian parameters'' x_i. In practice, the transformation from x_i to y_i should be inferred from the observed data and a calculated observer boundary. Another way to describe the situation is to keep using Lagrangian parameters, or parameters that have the most direct or intuitive physical meaning at low energies, x_i. In this basis, which we may call the ''physical basis,'' we have a nontrivial distribution function $f(x_i)$. The distribution function is peaked toward the region outside $\mathcal{O}$, so that a typical observer in $\mathcal{O}$ measures x_i close to the observer boundary.

In the physical basis, x_i, it is very convenient to introduce a ''probability force,'' which is simply the gradient of the probability distribution

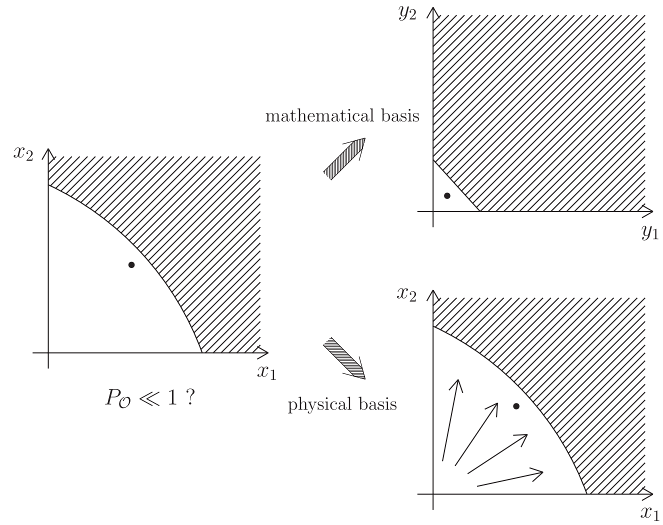

FIG. 10. Illustration of the situation in which naive and true distribution functions differ significantly. In the basis where the true distribution function is constant (the mathematical basis), the observed point is a typical point in $\mathcal{O}$, and the parameters chosen, y_i, are in general nontrivial functions of ''Lagrangian parameters'' x_i. In the basis spanned by x_i (the physical basis), the distribution function is nontrivial, giving a ''probability force'' field, defined in Eq. (43).

$$\mathbf{F} = \nabla f, \tag{43}$$

where $\nabla \equiv (\partial/\partial x_1, \partial/\partial x_2, \cdots)$. Within the region $\mathcal{O}$, this vector field gives an indication of what values of physical parameters are most likely to be measured in the multiverse. If the vector field indicates a flow toward an observer boundary, observers should be living close to the corresponding boundary. An example of the probability force field is depicted in the physical basis plot of Fig. 10.

B. Effective distributions from ''integrating out'' parameters

What are the possible origins of the probability force? One is a nontrivial prior distribution function $f_{\rm prior}$, which arises from a mismatch between the physical and mathematical bases, as discussed previously. In practice, however, this is not the only source of the probability force. The key is that when we discuss physical predictions in the multiverse picture, we typically choose a subset x_a of the entire parameter set x_i and study probabilistic predictions in the space of these chosen parameters x_a. An important point here is that when we focus on x_a, we cannot simply ignore the other variables x_b ($i = \{a, b\}$); they must be *integrated out*. This provides an effect on the distribution function *defined in the subspace* x_a, and thus modifies the probability force in x_a space.

To illustrate this idea, let us consider the simplest example of a constant $f_{\rm prior}$, i.e., the case in which the physical and mathematical bases coincide. In this case, the distribution of observers is given by

$$d\mathcal{N} = c dx_1 \cdots dx_N, \tag{44}$$

where $c = f_{\rm prior}$ is a constant, and the probability force in x_i space is zero, $\mathbf{F} = 0$. This, however, does not mean that the probability force is zero if we consider only a subset of the variables, e.g., $x_1, \cdots, x_{N-1}$. When we focus on the variables $x_1, \cdots, x_{N-1}$ (as we focused only on y_u, y_d, y_e, $v/\Lambda_{\rm QCD}$, and α out of the 13 parameters of Eqs. (9) and (10) in Secs. IV and V), we must integrate out the other variable x_N. Now, in the multiverse picture with environmental selection, the only relevant universes are those within the observer region $\mathcal{O}$, implying that integrals should be performed only over $\mathcal{O}$. The domain of x_N integration is then determined by the observer boundary, which is generically written as $x_N^{\rm min}(x_1, \cdots, x_{N-1}) \leq x_N \leq x_N^{\rm max}(x_1, \cdots, x_{N-1})$,[11] giving

$$d\mathcal{N} = f_{\rm eff}(x_1, \cdots, x_{N-1}) dx_1 \cdots dx_{N-1}, \tag{45}$$

where $f_{\rm eff}(x_1, \cdots, x_{N-1}) \equiv c\{x_N^{\rm max}(x_1, \cdots, x_{N-1}) - x_N^{\rm min}(x_1, \cdots, x_{N-1})\}$. The probability force *defined in* $x_1 \cdots x_{N-1}$ *space* is then obtained by Eq. (43) with f replaced by $f_{\rm eff}$: $\mathbf{F}_{\rm eff} = \nabla f_{\rm eff}$, where $\nabla = (\partial/\partial x_1, \cdots, \partial/\partial x_{N-1})$. This generically gives a nontrivial force $\mathbf{F}_{\rm eff} \neq \mathbf{0}$.

In general, when we consider physics of the landscape in the subspace of x_a ($a = 1, \cdots, n$), we must integrate out the other variables x_b ($b = n+1, \cdots, N$) to obtain the effective distribution function, and thus the probability force, defined in x_a space. The domain of integration is given by the observer region $\mathcal{O}$, leading to

$$f_{\rm eff}(x_1, \cdots, x_n) = \int \cdots \int_{\mathcal{O}} f_{\rm prior}(x_1, \cdots, x_N) dx_{n+1} \cdots dx_N, \tag{46}$$

where $f_{\rm prior} = \tilde{f}_{\rm prior} n$ is the prior distribution function defined in the entire parameter space x_i, whose x_b dependence could arise from both $\tilde{f}_{\rm prior}$ and n. The important point here is that the effective distribution function $f_{\rm eff}$ in x_a space is *not* obtained simply by "neglecting" the other variables x_b in $f_{\rm prior}$, i.e., by setting x_b to the observed values in $f_{\rm prior}$

$$f_{\rm eff}(x_1, \cdots, x_n) \neq f_{\rm prior}(x_1, \cdots, x_n, x_{n+1,o}, \cdots, x_{N,o}). \tag{47}$$

Since the observer region $\mathcal{O}$ can in general have a complicated shape in x_i space, the effective distribution function can have a quite different form than the expression in the right-hand-side of Eq. (47). The effective probability force defined in x_a space is then given by

$$\mathbf{F}_{\rm eff} = \nabla f_{\rm eff}, \tag{48}$$

where $\nabla = (\partial/\partial x_1, \cdots, \partial/\partial x_n)$.

The argument here suggests that it is a rather common phenomenon to have a nontrivial probability force when we focus only on a subset of the entire parameter set x_i, which is almost always the case. Then, if the resulting probability force is strong, we are likely to encounter an observer naturalness problem in the form of (b) or (c) in Fig. 7.

[11]Depending on the shape of $\mathcal{O}$, we may have to integrate x_N over multiple domains for some values of $x_1, \cdots, x_{N-1}$.

C. Cut factor and comparisons between different theories

In general, there are many theories T that lead to the low energy Lagrangian with parameters x and that are described by a multiverse distribution $f_T(x)$. As an example, x may be the Yukawa couplings of the up and down quarks and electron, $y_{u,d,e}$, and T may label the various theories of flavor. The number of observers in the multiverse who are governed by theory T is

$$n_T = \int_{\mathcal{O}} \tilde{f}_T(x) n(x) dx. \tag{49}$$

A typical observer will be governed by the theory T that has the maximum value of n_T (assuming that several T do not have comparable n_T). The absolute normalization of $n(x)$ is unimportant, since we are only interested in relative numbers of observers governed by different theories

$$\frac{n_T}{n_{T'}} = \frac{\int_{\mathcal{O}} \tilde{f}_T(x) n(x) dx}{\int_{\mathcal{O}} \tilde{f}_{T'}(x) n(x) dx} = \frac{\mathcal{N}_T}{\mathcal{N}_{T'}} \frac{\tilde{C}_T^{-1}}{\tilde{C}_{T'}^{-1}}, \tag{50}$$

where $\mathcal{N}_T = \int \tilde{f}_T(x) dx$ is the total "number" of universes (including volume factor weights) described by theory T and

$$\tilde{C}_T = \frac{\int \tilde{f}_T(x) dx}{\int_{\mathcal{O}} \tilde{f}_T(x) n(x) dx}. \tag{51}$$

Suppose now that $n(x)$ is relatively flat over $\mathcal{O}$ so that it is a good approximation to take it constant. Multiplying this constant by $\tilde{C}_T$ then gives

$$C_T = \frac{\int \tilde{f}_T(x) dx}{\int_{\mathcal{O}} \tilde{f}_T(x) dx}. \tag{52}$$

In this case, there are a factor C_T more universes described by T in the multiverse than in $\mathcal{O}$. Environmental selection solves observer naturalness problems at a cost of removing a factor C_T of these universes—we call C_T the cut factor for theory T.

Consider, for example, the following two theories—the standard model where only the Higgs mass-squared parameter scans with a distribution function constant in $|m_h^2|$ and a theory beyond the standard model where only the weak scale scans with a distribution function constant in $1/\ln v$. Considering selection by Eq. (29), the cut factors in these theories are $O(10^{32})$ and $O(100)$, respectively. If we take this naively, it seems to suggest that the latter theory is preferred over the former, and we might say that the standard model is less likely since it involves the cost of a much larger cut factor.[12] (This result is similar to that of the conventional naturalness argument, although here observable universes always have $v \ll M_{\rm Pl}$ due to selection effects.) Of course this is not rigorous—our ignorance of $\mathcal{N}_T$, as well as other possible selection effects, makes the argument based only on the cut factor unreliable; without knowledge of $\mathcal{N}_T$ there is no real reason to prefer a theory beyond the standard model over the standard model (other than considerations based on other physics such as gauge coupling unification and dark matter). Nevertheless, the relative cut factors for two theories do contribute to the relative number of observers described by each theory, as in Eq. (50), hence we may describe a large cut factor for some theory as a cost in its solution to an observer naturalness problem.

D. Evidence for environmental selection

We have seen that observer naturalness problems, which appear in the form of (a), (b), or (c) of Fig. 7, can be solved in general by environmental selection. Turning this argument around, we find that effects of environmental selection can show up in one or both of the following forms:

(i) *Amazing coincidences:* we are living in a region of parameter space that admits complex observers, which, however, is only a (very) small portion of the entire parameter space.

(ii) *Living on the edge:* we are living (very) near an ‘‘edge’’ of the parameter region $\mathcal{O}$ that admits complex observers, i.e., physical parameters take values that correspond to a point very close to the observer boundary, compared with the extent of the region $\mathcal{O}$.

In fact, how environmental selection manifests itself depends on the basis—we have seen that a phenomenon that appears as (ii) in one basis can appear as (i) in another basis, or vice versa. However, since we are often presented with a natural basis in which the physical meaning of parameters is most direct and/or intuitive, it is meaningful to consider the manifestation of environmental selection in that particular basis (physical basis). As we have seen, the manifestation then takes the form of (i) if the effective distribution function $f_{\rm eff}(x)$ is (nearly) constant over the observer region $\mathcal{O}$, while it takes the form of (ii) or a combination of (i) and (ii) if $f_{\rm eff}(x)$ is peaked toward a boundary of $\mathcal{O}$. There are many possible origins for a nontrivial form of $f_{\rm eff}(x)$: an x dependence of the landscape vacua, the dynamics of the population mechanism, the observer factor $n(x)$, and environmental selection acting on variables other than x.

Can the observation of one or both of the above two phenomena, (i) and (ii), be viewed as evidence for environmental selection? In a given theory T, the answer is yes, since otherwise it is very difficult to explain these features. Even though there is an observer naturalness problem associated with a very small value for P_T, after environmental selection a typical point within $\mathcal{O}$ results, i.e.,

$$P_T \to P_{\mathcal{O},T} \approx 1. \tag{53}$$

The smaller the original P_T, the more amazing the coincidence, or the closer to the edge we are living. The significance for the evidence for environmental selection is then quantified by the size of the naturalness probability P_T, with a smaller value of P_T corresponding to stronger evidence.

In practice, however, we do not know beforehand the correct theory describing our universe. Suppose we encounter a situation described by (i) or (ii) above. Do we conclude that we have found evidence for environmental selection, or that the theory we are considering is simply wrong? Whether an observation of the phenomenon described in (i) or (ii)—an observer naturalness problem—can be viewed as evidence for environmental selection depends on whether one can find an alternative theory in which the problem does not arise. A simple alternative theory without the naturalness problem may provide a better description of our universe. On the other hand, it is possible that we cannot find such a theory, or can find only theories that are significantly more complicated. Then we may conclude that environmental selection provides the best explanation of the phenomenon, so that we provisionally accept the multiverse theory.

In principle, with competing theories to describe nature, the evidence for environmental selection at a particular observer boundary is given by the largest value of P_T associated with that boundary. However, it may be that a theory with large P_T is very complicated or *ad hoc*, so that it does not appear to be an adequate solution to the observer naturalness problem. In this case, it may be judged that the evidence for environmental selection is better represented by a smaller value of P_T associated with some simpler theory.

The discussion given above illustrates why the cosmological constant is a powerful argument for environmental selection: in all known quantum field theories the naturalness probability is extremely small ($P_T \approx 10^{-120}$–10^{-60} depending on the presence of weak scale supersymmetry

[12]Here we do not consider the cut factor arising from selection of the cosmological constant $\Lambda \lesssim Q^3 T_{\rm eq}^4$. In fact, the size of the cut factor associated with the cosmological constant does not depend, e.g., on the scale of supersymmetry breaking if the constant term in the superpotential W_0 scans up to the fundamental scale M_* with the distribution flat in $|W_0|^2$, as suggest by string theory [25]. This part of the cut factor is then of order $M_*^4/\Lambda \approx 10^{120}$ regardless of the theory. (An exception for this is given by supersymmetric theories in which both supersymmetry and R symmetry are broken at low energies.)

and the nature of its breaking), implying that the largest value of P_T is of order 10^{-60} or smaller. The absence of a simple theory with $P_T = O(1)$ is crucial in this argument. A similar argument may also be made for the quark and lepton masses discussed in Secs. IV and V. We have argued that all known theories of flavor are quite inadequate to explain the relevant observer naturalness problems, leading to $P_T \lesssim (10^{-3}\text{–}10^{-2})$. Although this is numerically less impressive than the case of the cosmological constant, it is nevertheless very important. A single piece of evidence for environmental selection, no matter how significant, could be completely erased by the discovery of a single new theory. The more independent pieces of evidence for environmental selection, the more convincing the overall picture becomes.

One might think it always difficult to "confirm" the absence of alternative theories that do not have the problem. Indeed, the absence of such theories can in general be inferred only from negative results of theoretical search. However, in the case that the observer naturalness problem is related to a fine-tuning naturalness problem, it is possible that we can be convinced rather firmly that the problem does in fact exist. This will be the case, for example, if we do not see any deviation of gravity from Newton's law down to a scale (much) smaller than $O(100\ \mu\text{m})$, since it will tell us the absence of a physical threshold that can control the observed value of the cosmological constant. In Secs. VIII and IX, we will also argue that the observation (or nonobservation) of new physics at the TeV scale may also be viewed, depending on what we will see, as evidence for the existence of an observer naturalness problem, and hence environmental selection.

We stress that environmental selection can provide numerical predictions that are difficult to obtain in other ways. Specifically, this occurs if the effective distribution function $f_{\rm eff}$ has a nontrivial form in the physical basis. In this case, the physical parameters take values corresponding to a point close to the observer boundary, giving nontrivial predictions.

An example of predictions made possible by environmental selection is given by the stability boundary of the desired electroweak phase of the standard model [16]. Suppose that the standard model is valid up to some high scale M_* near the Planck scale and that the weak scale v results from environmental selection. Suppose further that the Higgs-quartic coupling at M_*, $\lambda_{h,*}$, and the top quark Yukawa coupling at M_*, $y_{t,*}$, vary from one universe to another. There is then an observer boundary $O(\lambda_{h,*}, y_{t,*}) = 1$ corresponding to sufficient stability of the desired electroweak phase. Now, if the distribution function $f_{\rm eff}(\lambda_{h,*}, y_{t,*})$ is strongly peaked toward the phase boundary then our universe is expected to be close to this edge. In particular, if the peaking is stronger in $\lambda_{h,*}$ than in $y_{t,*}$ then the most probable point on the phase boundary has $M_{\rm Higgs} \simeq 107$ GeV and $m_t \simeq 175$ GeV. Discovery of the Higgs boson near this mass, together with the absence of any new electroweak physics, would then provide evidence that our universe is near the edge of this observer phase boundary, and hence of environmental selection. In general, if we find ourselves living close to an observer boundary and if we do not have a simple (alternative) theory explaining that fact, then we may regard it as evidence for environmental selection.

VII. PREDICTIONS FOR m_u, m_d AND m_e FROM ENVIRONMENTAL SELECTION

The observer naturalness problem associated with the stability of neutrons, deuterons, and complex nuclei was introduced in Sec. IV, and is illustrated in Fig. 5. We showed that no matter what the theory of flavor, there is always a factor in the naturalness probability $\tilde{P}$ from the dependence on $v/\Lambda_{\rm QCD}$. In Sec. V, we argued that there is a factor P_F in the naturalness probability that is highly dependent on the theory of flavor. In all known theories $P = P_F\tilde{P} \lesssim (10^{-3}\text{–}10^{-2})$. In this section, we argue that this naturalness problem can be solved by environmental selection; furthermore, there are several possible solutions with different consequences.

We begin by restating the naturalness problem in terms of standard model parameters. For the neutron and complex nuclei stability boundaries a crucial quantity is

$$m_n - m_p - m_e = C_I(y_d - y_u)v - y_e v - C_\alpha \alpha \Lambda_{\rm QCD}, \tag{54}$$

where $C_I \approx (0.5\text{–}2)$ and $C_\alpha \approx (0.5\text{–}2)$ are strong interaction coefficients, and we choose a definition of the QCD scale such that $\Lambda_{\rm QCD} = 100$ MeV. Throughout we use approximate ranges for $m_{u,o}$ and $m_{d,o}$ from Ref. [21]. For complex nuclei the binding energy per nucleon is $E_{\rm bin} = C_B\Lambda_{\rm QCD}$, with $C_B \approx 0.08$ another strong interaction coefficient. The neutron and complex nuclei boundaries are then described by the inequalities

$$0 < C_I(y_d - y_u)\frac{v}{\Lambda_{\rm QCD}} - y_e\frac{v}{\Lambda_{\rm QCD}} - C_\alpha\alpha < C_B. \tag{55}$$

In the approximation that the deuteron binding energy B_D is linear in $m_u + m_d$ in the region of interest, the stability boundary for the deuteron can be written as

$$\frac{B_D}{\Lambda_{\rm QCD}} = C_1 - C_2(y_d + y_u)\frac{v}{\Lambda_{\rm QCD}} > 0, \tag{56}$$

where $C_{1,2}$ are two further strong interaction coefficients. These three boundaries involve just four independent combinations of standard model parameters, $y_{u,d,e}v/\Lambda_{\rm QCD}$ and α.

Much of the observer naturalness problem arises because the three terms that contribute to $m_n - m_p - m_e$ in Eq. (54) are comparable, as shown in Fig. 11, even though Yukawa couplings and the ratio of mass scales $v/\Lambda_{\rm QCD}$ are

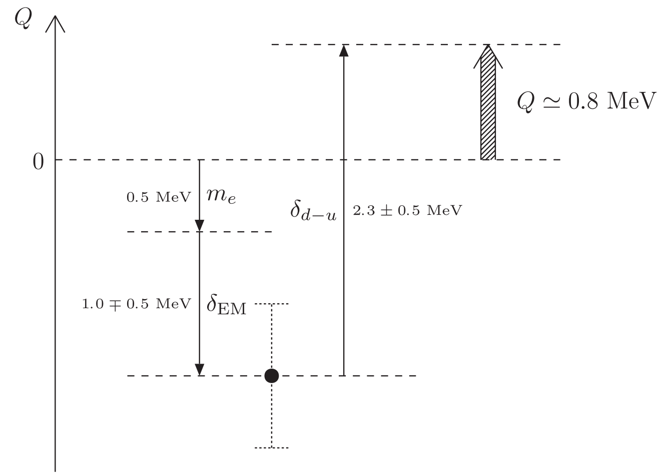

FIG. 11. Q value for the reaction $n \to p e^- \bar{\nu}$.

expected to range over many orders of magnitude. The electron mass, $y_e v \simeq 0.5$ MeV and the electromagnetic mass splitting $\delta_{\rm EM} \equiv C_\alpha \alpha \Lambda_{\rm QCD} \simeq 1.0 \mp 0.5$ MeV differ by only about a factor of 2. They both stabilize the neutron, and hence an unstable neutron requires $y_d > y_u$. Not only is this the case, but the isospin breaking term $\delta_{d-u} \equiv C_I(m_d - m_u) = 2.3 \pm 0.5$ MeV only just overcompensates the negative terms to give a net available energy for neutron beta decay of just 0.8 MeV, an amount that is also very close to the electron mass and electromagnetic terms. In any theory of flavor where $y_{u,d,e}$ are determined by symmetries, these numerical coincidences, allowing neutron instability and the existence of complex nuclei, must simply be accidents. Furthermore, these accidents also involve $v/\Lambda_{\rm QCD}$ and α. The other standard model combination, $(y_d + y_u)v/\Lambda_{\rm QCD}$, which could also range over many orders of magnitude, is numerically within about a factor of 2 of the deuteron stability boundary. Finally, as is apparent from Fig. 5, the observed parameters are actually much closer to the neutron stability boundary than a factor of 2. The perpendicular distance in coupling parameter space is (10–30)%, depending on the values of C_I and $y_{u,o}/y_{d,o}$. As discussed in previous sections, some aspects of these accidents could arise from a theory of flavor that determines $y_{u,d,e}$ to be small and in the approximate ratio 1:2:1 at a unified scale; but the accidents associated with the value of $v/\Lambda_{\rm QCD}$ and the closeness to the neutron stability boundary remain.

Environmental selection cuts out a large region of parameter space where neutrons are stable, or deuterons or complex nuclei are unstable, greatly reducing the observer naturalness problem. The only remaining question is whether our observed universe is a typical universe within the observer region $\mathcal{O}$ shown in Fig. 5. This depends on the probability distribution for $y_{u,d,e} v/\Lambda_{\rm QCD}$ and α, and on whether $\mathcal{O}$ has been correctly identified, allowing several solutions of the problem with different implications. We begin by taking α fixed to its observed value while allowing $y_{u,d,e}$, v, and $\Lambda_{\rm QCD}$ to scan. Since there are only two scales v and $\Lambda_{\rm QCD}$ appearing in the nuclear stability observer boundaries, we can take $\Lambda_{\rm QCD} = 100$ MeV without loss of generality, setting the unit of mass. Furthermore, since v always multiplies a Yukawa coupling, environmental selection for the light fermion masses can be discussed in terms of a distribution function $f(m_{u,d,e})$ without loss of generality.[13] For a given f we can compute $\langle m_u \rangle$, $\langle m_d \rangle$, and $\langle m_e \rangle$ and compare them with the values observed in our universe, $m_{u,o}$, $m_{d,o}$, and $m_{e,o}$. We can also compute how close a typical universe in $\mathcal{O}$ is to the neutron stability boundary.

Predictions for $\langle m_{u,d,e} \rangle$ involve the mass scales that arise in the observer boundaries, namely, the maximum value of $m_+ = m_d + m_u$ allowed by deuteron stability

$$m_{+\,\rm max} = \frac{C_1}{C_2} \Lambda_{\rm QCD} \simeq (1.4\text{–}2.7) m_{+,o} \simeq (1.4\text{–}2.7) \times (5\text{–}12)\ \text{MeV}, \tag{57}$$

for $a = (1.3\text{–}5.5)$ MeV, the binding energy per nucleon in stable complex nuclei

$$E_{\rm bin} = C_B \Lambda_{\rm QCD} \approx 8\ \text{MeV}, \tag{58}$$

and the electromagnetic contribution to the proton mass

$$\delta_{\rm EM} = C_\alpha \alpha \Lambda_{\rm QCD} \simeq (0.5\text{–}1.5)\ \text{MeV}. \tag{59}$$

While all three mass scales are proportional to $\Lambda_{\rm QCD}$, $\delta_{\rm EM}$ is significantly smaller than $m_{+\,\rm max}$ and $E_{\rm bin}$. Hence, it is important to see which of these mass scales enter the predictions for $\langle m_{u,d,e} \rangle$.

We consider three situations that solve the observer naturalness problem:

(I) An important part of the observer boundary is missing.

(II) The probability distribution is flat in mass space, $d\mathcal{N} = A dm_u dm_d dm_e$. In this case, the closeness to the neutron stability boundary discussed above is accidental.

(III) The probability distribution $d\mathcal{N} = f(m_u, m_d, m_e) dm_u dm_d dm_e$ yields a nontrivial probability force toward the neutron stability boundary.

[13]The distribution function $f(m_{u,d,e})$ here and below really means $f(m_{u,d,e}/\Lambda_{\rm QCD})$, but we omit $\Lambda_{\rm QCD}$ for notational simplicity. In the language of Sec. VI B, this is the effective distribution for $m_{u,d,e}/\Lambda_{\rm QCD}$ after integrating out the other parameters. Specifically, $f(m_{u,d,e}/\Lambda_{\rm QCD}) = \int_{\mathcal{O}} \delta(m_u/\Lambda_{\rm QCD} - y_u v/\Lambda_{\rm QCD}) \delta(m_d/\Lambda_{\rm QCD} - y_d v/\Lambda_{\rm QCD}) \delta(m_e/\Lambda_{\rm QCD} - y_e v/\Lambda_{\rm QCD}) \times f(y_u, y_d, y_e, v, \Lambda_{\rm QCD}) dy_u dy_d dy_e dv d\Lambda_{\rm QCD}$. Using $f(m_{u,d,e})$, we are able to discuss environmental selection at nuclear boundaries without answering how the degeneracy inside $m_{u,d,e}$ (scaling $v/\Lambda_{\rm QCD}$ and $y_{u,d,e}$ oppositely keeping $m_{u,d,e}$ fixed) is determined.

For case I, consider a multiverse with m_u, m_d, and m_e uniformly distributed on logarithmic scales so that the relevant diagrams are shown in the left panels of Fig. 5. Although the original naturalness problem $P \ll 1$ has been ameliorated by a large cut factor, the naturalness problem is not entirely removed by environmental selection for neutron instability and deuteron and complex nuclei stability, since $P_{\mathcal{O}} \ll 1$. For example, there are large regions of $\mathcal{O}$ at small $m_{u,e}$ that are distant from the observer boundary. A complete solution follows if there are additional relevant boundaries that we have failed to identify, that reduce $\mathcal{O}$ to the point where our universe becomes typical within $\mathcal{O}$. This would certainly require new physical constraints to remove the large regions with low values of $m_u/m_{u,o}$ and $m_e/m_{e,o}$. A complete solution may need further cuts to remove large values of $m_e/m_{e,o}$ not already excluded, for example, using the threshold for the pp reaction. Furthermore, to understand our closeness to the neutron stability boundary, it would be necessary for other cuts to approach our universe with a similar closeness on a logarithmic scale. While we can certainly identify physical processes that would introduce extra boundaries, we are unable to argue that they induce catastrophic changes rather than just the substantial changes discussed in Sec. IV.

For the second case, II, suppose that the distributions of m_u, m_d, and m_e are flat on linear scales, so that the effects of environmental selection can be understood from the cuts of the observer boundaries drawn in the right panels of Fig. 5. The observed masses are relatively typical within $\mathcal{O}$, so that $P_{\mathcal{O}}$ is not much smaller than unity, and the naturalness problem is largely solved. In fact, the observed masses are still close to the neutron stability boundary, even in the right panels of Fig. 5, which in this example is accidental. Having assumed a simple form for the multiverse distribution, i.e., that $f(m_u, m_d, m_e)$ is constant, we are able to use the precise form of the observer boundary to compute the average observed values of the electron, up quark and down quark masses by integrating over $\mathcal{O}$

$$\langle m_e \rangle = \frac{1}{4} C_I m_{+\,\mathrm{max}} \approx C_I \left(\frac{m_{+,o}}{5\ \mathrm{MeV}} \right) (2\text{–}3)\ \mathrm{MeV}, \qquad (60)$$

$$\langle m_+ \rangle = \tfrac{3}{4} m_{+\,\mathrm{max}} \approx (1\text{–}2) m_{+,o}, \qquad (61)$$

$$\langle m_- \rangle = \tfrac{1}{2} m_{+\,\mathrm{max}} \approx (0.7\text{–}1.4) m_{+,o}, \qquad (62)$$

where $m_\pm = m_d \pm m_u$. In these equations the analytic expressions are obtained without including the effect of the complex nuclei boundary, and the numerical range corresponds to $a = (1.3\text{–}5.5)$ MeV. (Including the effect of the complex nuclei boundary changes the numerical values only up to about 30%.) These results demonstrate that environmental selection yields predictions for $m_{u,d,e}$ once a simple form for the probability distribution has been assumed. The predictions for m_u and m_d are good. The prediction for m_e is quite uncertain. For many values of the strong interaction parameters it is somewhat large; for example, for central values of C_I, a, and $m_{+,o}$, $\langle m_e \rangle \approx 6 m_{e,o}$, so that low values of C_I and $m_{+,o}$ as well as a high value of a are preferred. Nevertheless, we stress that *a major part of the observer naturalness problem is solved if $f(m_u, m_d, m_e)$ is relatively flat within $\mathcal{O}$*. On linear scales for $m_{u,d,e}$, our universe is quite typical of $\mathcal{O}$. Of course, case II does imply that the closeness to the neutron boundary is accidental, and the rest of this section is devoted to understanding this closeness. A peaked distribution function can also give $\langle m_e \rangle \propto \delta_{\mathrm{EM}}$, rather than $\propto m_{+\,\mathrm{max}}$, leading immediately to an understanding of the lightness of the electron.

For case III, inside $\mathcal{O}$ the distribution f is peaked toward the neutron stability boundary, allowing us to explore the consequences of an environmental explanation for why $m_{u,d,e}$ are so close to this boundary. We call this stability boundary the n surface—it is a 2-dimensional plane in the 3-dimensional space of masses $m_{u,d,e}$. Within $\mathcal{O}$, the probability force $\mathbf{F} = \nabla f$ can be resolved into components parallel $F_\parallel$ and perpendicular $F_\perp$ to the n surface. We assume that $F_\perp$ points toward the n surface. The $F_\parallel$ field will determine the most probable location on the n surface.

The n surface has a triangular shape, as shown in Fig. 12. Two of the edges of the triangle correspond to edges of physical space $m_{u,e} = 0$, while the other edge corresponds to the intersection of the n surface with the D surface, the boundary for deuteron stability. We call these three edges the u, e, and D edges, and label the three vertices of the triangle as ue, uD and eD, as shown in Fig. 12. The $F_\parallel$ field will determine where on the n surface triangle the distribution f is maximized; there are three possibilities:

(1) In the interior, not close to an edge.

(2) On an edge, not close to a vertex.

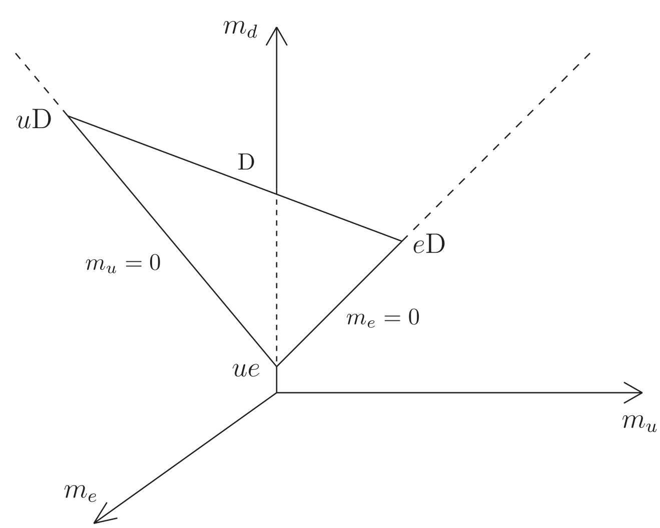

FIG. 12. 2-dimensional n surface in the 3-dimensional space of masses $m_{u,d,e}$.

(3) On a vertex.

On a linear scale, the n surface is small compared with the expected available parameter space, so that it is unlikely that $\tilde{f}$ would have a sharp peak in the interior of the triangle. However, the observer factor n may vary significantly across the triangle inducing a maximum in the interior. For case III-2, as we move along an edge, f rises to reach a maximum and then falls. As an example consider the distribution in $\mathcal{O}$ to be

$$f(m_u, m_d, m_e) = A(m_d - m_u)^{-p}, \tag{63}$$

where A is a normalization constant, and p is positive. This situation would arise if the populated landscape has a low probability to yield a universe with large breaking of isospin, or if there are more observers in a universe as isospin is restored. In a slice through parameter space at constant m_e, the resulting probability force $\mathbf{F}_- \equiv \nabla f$ is toward and perpendicular to the n surface. However, in the full 3-dimensional space $\mathbf{F}_-$ is not perpendicular to the n surface. This is apparent in Fig. 13, which shows the n surface in the 3-dimensional space of m_+, m_-, and m_e. Clearly the force will lead to a preference for low values of m_e, so that if p is large enough the most probable universes will have m_e close to zero, i.e., close to the e edge of the n surface triangle of Fig. 12. From Fig. 13, all points on the e edge are equally probable, so that there is no expectation of being close to either the ue or eD vertices. We can compute the expectation values of $m_{u,d,e}$ by integrating over the 3-dimensional region $\mathcal{O}$. For $p > 3$ the regions in Fig. 13 at low m_- and low m_e dominate the integrals, and one sees that $\delta_{\rm EM}$ sets the scale for both m_e and m_-, giving

$$\langle m_e \rangle = \frac{1}{p-3}\delta_{\rm EM}, \tag{64}$$

and

$$C_I \langle m_- \rangle = \frac{p-1}{p-3}\delta_{\rm EM}. \tag{65}$$

This explains why the three contributions to $m_n - m_p - m_e$ shown in Eq. (54) are comparable. On the other hand, Fig. 13 shows that m_+ is uniformly distributed along the n surface from small values to $m_{+\,\rm max}$, and hence

$$\langle m_+ \rangle = \tfrac{1}{2} m_{+\,\rm max}. \tag{66}$$

Notice that of the three mass scales that enter the observer boundaries, Eqs. (57)–(59), $E_{\rm bin}$ does not appear. This is because the probability distribution is peaked away from the complex nuclei boundary, which therefore becomes irrelevant in determining the averages.[14]

[14]The appearance of the mass scales in Eqs. (64)–(66) depends only on a probability force toward low m_-, and not on the particular choice of the power law f in Eq. (63).

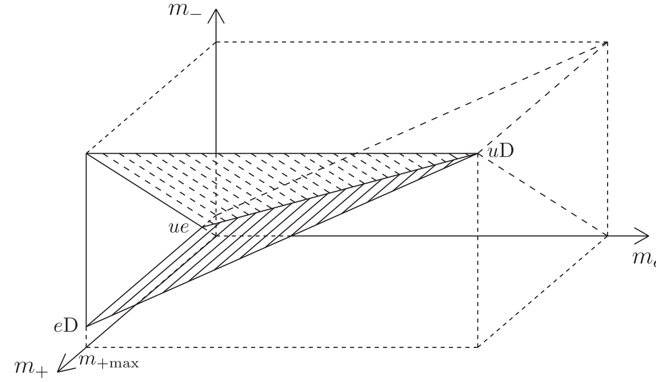

FIG. 13. The n surface in the 3-dimensional space of m_+, m_-, and m_e (shaded by solid lines). The m_e axis is stretched relative to the $m_\pm$ axes to make the figure more visible. The observer region $\mathcal{O}$ is the region surrounded by the n surface, the $m_u = 0$ surface (shaded by dashed lines), the $m_e = 0$ surface, and the deuteron stability surface at $m_+ = m_{+\,\rm max}$. The complex nuclear stability boundary is not shown.

For case III-3 there are three possible vertices of the n surface triangle to consider. Our universe is certainly not close to the uD vertex, since this gives m_e too large, hence, we study the remaining two vertices. There are many f that lead to these vertices; we begin by identifying and studying the simplest cases. The ue vertex can be reached by a preference for low values of $m_{u,d,e}$. However, low values of m_u or m_e are not sufficient as they lead to the u or e edges, hence, as a simple example we study the distribution

$$f(m_u, m_d, m_e) = A m_d^{-q}, \tag{67}$$

with q positive. At the ue vertex the value of m_d is determined by $\delta_{\rm EM}$. For sufficiently large q this is the only relevant scale so that the average values of $m_{u,d,e}$ over $\mathcal{O}$ will all be determined by $\delta_{\rm EM}$

$$\langle m_e \rangle = \frac{1}{q-4}\delta_{\rm EM}, \tag{68}$$

$$C_I \langle m_+ \rangle = \frac{q}{q-4}\delta_{\rm EM}, \tag{69}$$

$$C_I \langle m_- \rangle = \frac{q-2}{q-4}\delta_{\rm EM}. \tag{70}$$

While choices of q and C_I can lead to a hierarchy between these averages (for example $\langle m_e \rangle \simeq \delta_{\rm EM}$, $\langle m_+ \rangle \simeq 10\delta_{\rm EM}$ and $\langle m_- \rangle \simeq 6\delta_{\rm EM}$ for $q = 5$ and $C_I = 0.5$), the hierarchy is more readily generated when $\langle m_e \rangle \propto \delta_{\rm EM}$ and $\langle m_+ \rangle \propto m_{+\,\rm max}$, as in Eqs. (64) and (66).

The eD vertex is favored by large m_u and small m_e; however, large or small m_d both lead away from the eD vertex. A simple distribution peaking toward the eD vertex is

$$f(m_u, m_d, m_e) = A m_u^r, \tag{71}$$

with $r \gtrsim m_{+\,\mathrm{max}}/\delta_{\mathrm{EM}}$. Any distribution peaking near this vertex will lead to $\langle m_- \rangle \propto \delta_{\mathrm{EM}}$ and $\langle m_+ \rangle \simeq m_{+\,\mathrm{max}}$, which agree with observations. The prediction for the electron mass depends on the strength and direction of the force. In the example of Eq. (71), $\langle m_e \rangle \approx m_{+\,\mathrm{max}}/r$.

In two Higgs doublet theories a probability distribution for the ratio of vacuum expectation values $\tan\beta$ contributes to the distributions for $m_{u,d,e}$. Suppose that this distribution favors large $\tan\beta$ and that $\tan\beta_o \gg 1$. In this case, low m_d and m_e are preferred, so that within the observer region the probability distribution is peaked toward the ue vertex of the n surface, leading to $\langle m_e \rangle$, $\langle m_+ \rangle$, $\langle m_- \rangle \propto \delta_{\mathrm{EM}}$. The probability distribution in the electroweak symmetry breaking sector may be the origin of the force determining $\langle m_{u,d,e} \rangle$.

Finally, we consider the possibility that α also scans. In this case, the size of the electromagnetic mass difference δ_{EM} scans relative to the purely QCD scales of E_{bin} and $m_{+\,\mathrm{max}}$. This means that there is a shift in the position of the allowed window for $(C_I m_- - m_e)$ from neutron and complex nuclei stability. If α increases too much this window shifts to a nonphysical region where $m_- > m_+$, leading to an upper limit on α

$$\alpha < \frac{C_I C_1}{C_\alpha C_2}, \tag{72}$$

which is numerically about 0.2, with large uncertainties. Thus, α is about an order of magnitude away from the maximum value that it may take anywhere in $\mathcal{O}$.

If α is very small then the range for $(C_I m_- - m_e)$ increases. The amount of increase is negligible on a linear scale, but is sizable on a logarithmic scale. For example, with $\alpha = 10^{-4}$, $(C_I m_- - m_e)$ can range from 10^{-2} MeV to 8 MeV. However, if gauge couplings unify then α and Λ_{QCD} become related. As α is decreased, so Λ_{QCD} becomes exponentially smaller. It could be that selection effects on the size of Λ_{QCD} compared with the unified scale and/or the electroweak scale dominate over selection effects of α in nuclear physics.

In many circumstances, precise predictions for environmental selection follow from assuming sharply varying distribution functions f. However, in the present example of nuclear physics in the parameter space $m_{u,d,e}$ and α, it is possible that $\tilde{f}$ is sufficiently slowly varying over $\mathcal{O}$ that physical arguments based on the observer factor n could lead to predictions without any assumptions of sharply varying f. For example, consider variations in the parameters within $\mathcal{O}$ that lead from the neutron surface to the observer boundary with no complex stable nuclei. Moving toward the boundary of no complex stable nuclei, the observer factor may be reduced by successive nuclei becoming unstable. On the other hand, moving closer to the neutron surface than our universe will lead to a longer neutron lifetime and therefore to more primordial helium production; the reduction in primordial hydrogen will result in fewer hydrogen burning stars. The competition of these (and other) effects may determine the location of our universe within $\mathcal{O}$. Of course, even in this case, some assumption about $\tilde{f}$ is still needed.

VIII. ELECTROWEAK SYMMETRY BREAKING SELECTED BY NUCLEAR STABILITY

The origin of electroweak symmetry breaking is one of the largest mysteries remaining in the standard model. The quadratic divergence of the Higgs mass-squared parameter in the standard model implies that if the scale of new physics M is (much) larger than the weak scale v, the theory requires fine-tuning. This hierarchy problem was a key motivation for much of the model building in the last 30 years. What do we know about the scale M? In many (nonsupersymmetric) theories beyond the standard model, precision electroweak data indicates a "little hierarchy problem": v/M is uncomfortably small, typically of order $((10^{-2}$–$10^{-1}))$ or smaller (see, e.g., [26]). There is also a similar fine-tuning problem in supersymmetric theories, although its origin is different. A sufficiently heavy Higgs boson typically requires superparticles to be somewhat heavier than the weak scale, leading to some amount of fine-tuning (see, e.g., [27]). In either case, we find that some amount of unnaturalness is present, at least for the simplest theories, suggesting that environmental selection may be playing a role. In this section, we investigate whether a hierarchy between v and M is to be expected from environmental selection, the size of any such hierarchy, and how the hierarchy depends on which parameters are assumed to scan.

To address the question of the environmental selection of v/M, several issues must be addressed: what is the theory under discussion, which parameters of that theory scan, and what observer boundaries implement the selection? Below, we formulate a fairly general class of theories that describes electroweak symmetry breaking, and the nuclear stability boundaries of Sec. IV are used to implement selection. In Sec. IX, we consider an alternative possibility that selection occurs at the electroweak phase boundary.

The mass scale of the new physics that generates electroweak symmetry breaking is defined to be M, and we assume that the effective theory below M is the standard model with the Higgs potential

$$V = m_h^2 h^\dagger h + \frac{\lambda_h}{2}(h^\dagger h)^2. \tag{73}$$

Integrating out the physics of the electroweak symmetry breaking sector at scale M in general leads to several contributions to m_h^2, some positive and some negative, which we write as

$$m_h^2 = (g_1(x_i) - g_2(x_i))M^2, \tag{74}$$

where the functions $g_{1,2}$ are both positive. The dimensionless parameters x_i are the set of parameters of the theory above M that substantially affect electroweak symmetry breaking. These parameters are evaluated at the scale M, so that $g_{1,2}$ are also functions of M through the logarithmic sensitivity of x_i on M: $g_{1,2}(x_i(M))$. Note that m_h^2 here includes the quadratically divergent radiative corrections in the standard model that are regulated by the theory above M. The parameters x_i thus include the standard model gauge and Yukawa couplings.

The precise nature of the couplings x_i and the functional form of $g_{1,2}$ are unimportant for our discussion, so that we write $g_1(x_i) = Ax$ and $g_2(x_i) = Ay$, giving

$$m_h^2 = (x - y)AM^2, \tag{75}$$

where $x, y, A > 0$. The numerical constant A is chosen such that typical values for y in the observer region are of order unity. The parameters x and y depend logarithmically on M through renormalization group evolution. (A is a one-loop factor in many theories beyond the standard model. Since m_h^2 contains quadratically divergent contributions in the standard model, and thus x and y contain pieces proportional to the $SU(2)$ gauge and top Yukawa couplings squared, respectively, the value of A should not be much smaller than the one-loop factor.) In the context of conventional naturalness criteria, electroweak symmetry breaking is unnatural if x is typically larger than y in an ensemble (only a small fraction of members in the ensemble leads to electroweak symmetry breaking), whereas if x is typically of order y or less then the natural value of the weak scale is $\sqrt{A}M$ for a quartic coupling of order unity.

A key observation is that as experimental limits on physics beyond the standard model get stronger so the mass scale M is constrained to be larger. In some (nonsupersymmetric) theories of electroweak symmetry breaking, this typically arises from contributions of particles of mass M to the precision electroweak observables. In many supersymmetric models, the increased lower limits on the Higgs boson and superparticle masses have pushed up the mass scale of some superparticles to about a TeV or larger. These little hierarchies between v and M imply that the parameters x_i are constrained toward the phase boundary of electroweak symmetry breaking. As experiments push up M, so g_1 and g_2 of Eq. (74) cancel to give $v/\sqrt{A}M$ (much) smaller than unity. This is illustrated in Fig. 14 for a 2-dimensional slice through the parameter space. The unusual closeness of the parameters to this boundary can be viewed as an observer naturalness problem.

In this section, we investigate the various possible ways in which environmental selection on a multiverse can play a role in electroweak symmetry breaking. We use the class of theories described above, which contains most nonsupersymmetric and supersymmetric theories beyond the standard model. The relevant observer boundaries are

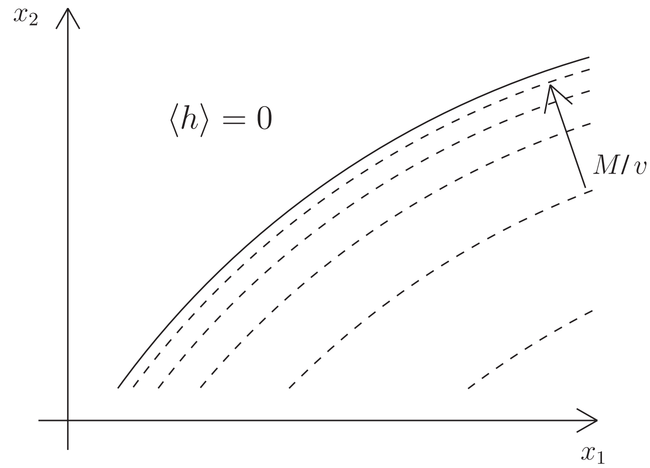

FIG. 14. The (supersymmetric) little hierarchy problem as an observer naturalness problem. Contours of M/v are drawn in a 2-dimensional slice of parameter space of a generic electroweak symmetry breaking sector. As the experimental limit on M increases, so the allowed region of parameter space shrinks to values of larger M/v close to the phase boundary.

those of nuclear stability, shown in Fig. 15, so that in general one must consider scanning parameters that affect nuclear physics as well as the parameters of the electroweak symmetry breaking sector. We start by considering only a few parameters scanning, and then progress to more general situations.

First of all, it is natural to expect that some parameters of the theory at M scan in the multiverse, so that there are universes with different values of the weak scale v. Universes with $x > y$ have $v = 0$, while those with $y > x$ have

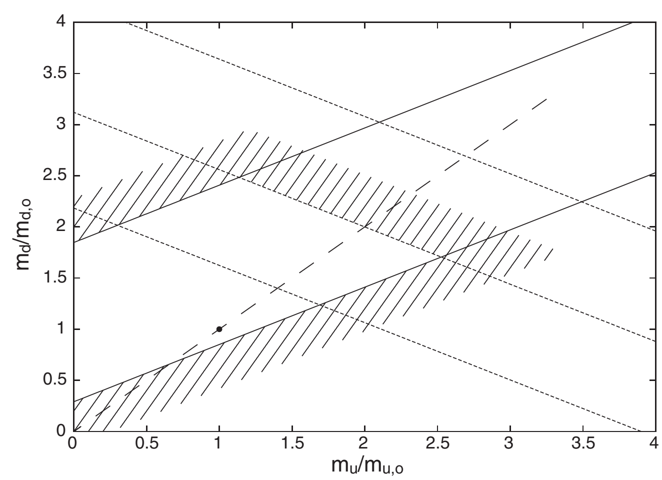

FIG. 15. The observer region in m_u–m_d space (inside the shading). The dashed line drawn from the lower left to the upper right represents the trajectory followed when only the electroweak vacuum expectation value v is varied (neglecting the effects of a variation of m_e).

$$v^2 = (y - x)\frac{AM^2}{\lambda_h(v)}. \tag{76}$$

Scanning of parameters in the electroweak symmetry breaking sector at M may also lead to a nontrivial effective distribution for $\lambda_h(v)$. Since a light Higgs boson has not been discovered, however, this distribution should lead to $\lambda_h(v)$ typically having an order unity value in the observer region. This implies

$$\frac{v^2}{AM^2} \approx y - x \tag{77}$$

in our universe, where y is of order unity due to definition of A. The crucial question is if some cancellation between y and x is expected, which cannot be understood in the conventional symmetry approach. If not, then $v \approx \sqrt{A}M$ as in the conventional case. If so, however, we obtain an extra hierarchy between v and M that cannot be explained by a symmetry, and the question becomes the following: is $y - x$ typically of order 10^{-2}–10^{-1}, with the multiverse generating a little hierarchy, or is it extremely small, for example 10^{-30}, giving a large hierarchy?

In addition to v, the nuclear stability observer boundaries depend on parameters $y_{u,d,e}$, α, and $\Lambda_{\rm QCD}$. In Secs. VIII A and VIII B we assume that these additional parameters do not scan, so that the only scanning in the standard model at low energies relevant for electroweak symmetry breaking is that of v. In this case, environmental selection for neutron instability and deuteron stability defines a fixed observer window for v

$$v_- < v < v_+, \tag{78}$$

where, from Eq. (29), $v_- \simeq 0.5 v_o$ and $v_+ \simeq 2 v_o$. (The value of v_+ depends on the parameter a describing the strength of the deuteron binding; here we take $a = 2.2$ MeV.) This is illustrated in the m_u–m_d plane in Fig. 15. As v is varied about v_o so m_u and m_d vary, but with a fixed ratio, as shown by the dashed line. (The corresponding variation of m_e gives only small effects.) Since v_o lies centrally in the observer window, it is consistent with a distribution $f_v(v)$ that is slowly varying. Can environmental selection for v generate a little or large hierarchy, and if so is the effective distribution for v mildly varying?

Another possibility, studied in Sec. VIII C, is that the entire set $y_{u,d,e}$, α, $\Lambda_{\rm QCD}$, and v scans, so that there is a nonzero probability distribution throughout the nuclear observer region. A 2-dimensional slice through the observer region is shown in Fig. 15; the scanning is no longer restricted to the dashed line. An interesting question then is whether the closeness to the neutron stability boundary is a statistical accident, or whether it results with high probability due to a strongly varying distribution function. As stressed in Sec. IV, the nuclear observer region depends on only four combinations of these quantities, $m_{u,d,e}/\Lambda_{\rm QCD}$ and α. For example, a common scanning of v and $\Lambda_{\rm QCD}$ does not affect nuclear physics. This implies that a numerical value for v is no longer determined by the nuclear observer region alone. Can environmental selection from the nuclear observer boundaries, shown in Fig. 15, lead to a little or large hierarchy?

A. Scanning the mass scale M

The fundamental field theory at the cutoff scale M_* ($\gg M$) will have a certain set of parameters. We assume that the scanning at M_* is limited to those parameters that affect the scale M (which may be only M itself), leading to a distribution function $f(M)$. In particular, the dimensionless parameters $x_i(M_*)$ do not scan. Assuming that effects on x_i from the parameters controlling M are small, this implies that the scanning of the parameters x and y in Eq. (75) comes only through a calculable logarithmic dependence on M, which arises from nontrivial scaling of these parameters under renormalization group evolution.

Suppose now that $y - x$ is positive and of order unity throughout the multiverse (which will be the case if $y - x$ is positive at M_* and becomes larger as the renormalization scale is lowered). Environmental selection requires that v lies in the fixed range $v_- < v < v_+$. This selects M to be in the range

$$v_- \sqrt{\frac{\lambda_h}{(y-x)A}} < M < v_+ \sqrt{\frac{\lambda_h}{(y-x)A}}. \tag{79}$$

The ratio between v and M is then the same as if there were no selection, $v/M \approx \sqrt{A}$.

There is, however, another possibility. It could be that as M varies so $y - x$ passes through zero. In this case, there is a critical value $M = M_c$ corresponding to the electroweak phase boundary, $y - x = 0$. By assumption, neither the critical value M_c, nor the nuclear window parameters $v_\pm$ are scanning. It would be an accident for M_c to be close to $v_\pm$, so we assume that they are distant. A physically interesting new possibility arises if $y - x$ is small in our universe, which is possible if $M_c \gg v_\pm$. This new possibility corresponds to the environmental selection of a large hierarchy.

Such a large hierarchy can occur in two ways, depending on the sign of the beta function β for $y - x$, as illustrated in the two panels in Fig. 16. (The definition of β here is given by $d(y - x)/d\ln\mu = \beta$.) For $\beta > 0$ (the top panel), electroweak symmetry breaking is only possible for universes with $M > M_c$ and, since $M_c \gg v_\pm$, the observer condition Eq. (78) selects M to be just above M_c in the range $M_c(1 + \delta_-) < M < M_c(1 + \delta_+)$, where $\delta_\pm = \lambda_h v_\pm^2/|\beta| A M_c^2$. The value of $y - x$ is of order $v^2/AM_c^2 \ll 1$, i.e., the cancellation of order v^2/AM_c^2 is forced by environmental selection, and the hierarchy between the weak scale and the scale of new physics is very large $M_c/v \gg 1$. (Note that x

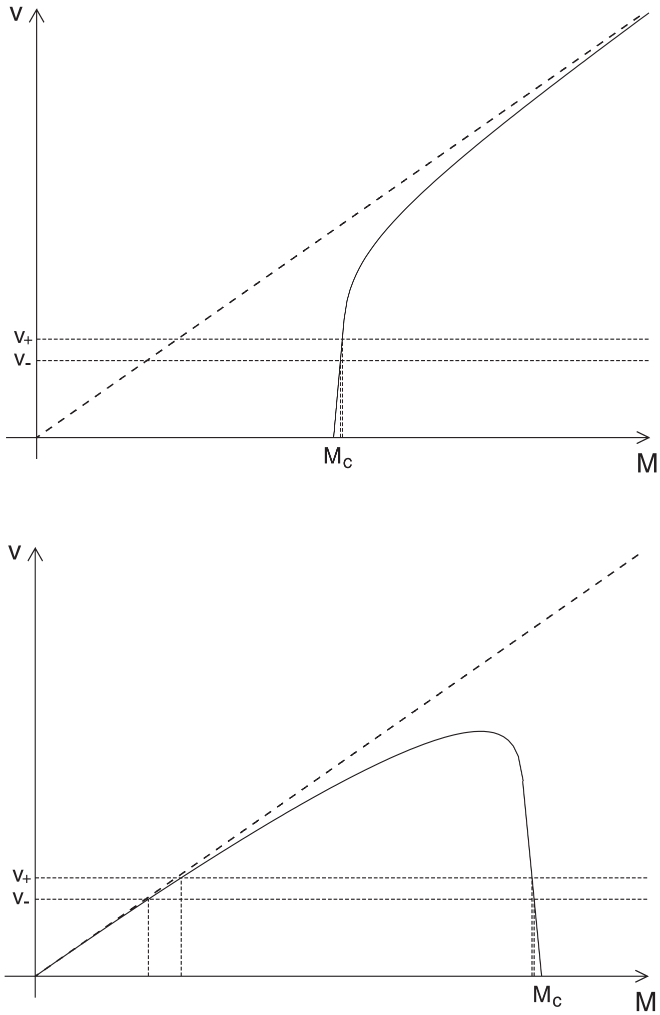

FIG. 16. Illustration of the ranges for M selected by the observer condition $v_- < v < v_+$. The solid curve gives $v(M)$ in regions of M where electroweak symmetry is broken.

and y themselves are of order unity.) On the other hand, in the case that $\beta < 0$ (the bottom panel) the broken phase has $M < M_c$, and environmental selection gives $M_c(1 - \delta_+) < M < M_c(1 - \delta_-)$; the value of $y - x$ in this case is, again, of order $v^2/AM_c^2 \ll 1$. For $\beta > 0$ only a large hierarchy is possible. For $\beta < 0$, however, universes with $v_- < v < v_+$ are also possible with $y - x \approx 1$ and $M \approx v/\sqrt{A}$, the case corresponding to conventional natural theories. The case of a large hierarchy is more probable if the distribution function for M, $f(M)$ is sufficiently weighted toward large M.

One possibility is that M is the overall scale of superparticle masses, so that the theory above M is supersymmetric. If the hierarchy is large, the situation described above then corresponds to the split supersymmetry scenario discussed in Refs. [8,28]. However, the theory above M does not have to be supersymmetric. It may, for example, be a strongly interacting theory leading to a composite Higgs boson below M, possibly as a pseudo Nambu-Goldstone boson [29–31]. In this case, we obtain a prediction on the Higgs boson mass as a function of M, by setting the Higgs-quartic coupling to be either very large or small at the scale M (compared with the logarithmically enhanced contribution between M and v). This could be interesting because for $M \ll M_{\rm Pl}$ predicted values of the Higgs boson mass are outside the range of $\approx$ (130–180) GeV, expected if the theory above v is the standard model up to a high scale of order the Planck scale with a (absolutely) stable electroweak symmetry breaking vacuum [32,33].

We stress that the situation described here is very special; for example, if $\Lambda_{\rm QCD}$ scans in the multiverse, it is possible that the resulting hierarchy is "little," e.g., $v/M \approx (10^{-2}$–$10^{-1})$, since the value of M may be set close to the observed weak scale by environmental selection, as we show below.

B. Full scan of the electroweak symmetry breaking sector

Next we consider a general scanning of the electroweak symmetry breaking sector, so that all of x, y, λ_h, and M vary independently, but we keep $y_{u,d,e}$, α, and $\Lambda_{\rm QCD}$ fixed. In this case, it is useful to consider a 2-dimensional observer region in the M–z plane, as shown in Fig. 17, where $z \equiv y - x$. As before, we assume that $\lambda_h(v)$ has a typical value of order unity, e.g., with its distribution function being strongly peaked at ~ 1, and hereafter we neglect the effect of its scanning.[15] Electroweak symmetry breaking occurs in universes with $z > 0$, and the observer boundaries $v = v_\pm$ are shown in Fig. 17. Other parts of the observer boundary correspond to $M = M_*$, the maximum value of M (the cutoff scale), and $z = z_{\rm max}$, the maximum value of z determined by the ranges for x and y in the landscape, which we take to be of order unity.

The effective probability distribution in this 2-dimensional observer region $\mathcal{O}$ is obtained from the distribution function for x, y, M as

$$f_{\rm eff}(z, M) = \int_{\mathcal{O}} dx dy f(x, y, M) \delta(z - (y - x)). \qquad (80)$$

With x and y scanning over the multiverse, the logarithmic dependence of these parameters on M frequently does not give a major effect, and hence we neglect it here. When these effects are important they lead to very interesting results, and we defer a discussion of this until Sec. VIII E. If $f_{\rm eff}(z, M)$ were constant over $\mathcal{O}$, it is clear that our universe would be expected to have $z \approx 1$ and $v \approx \sqrt{A} M$, since this corresponds to most of the area of $\mathcal{O}$.

[15]Our conclusions are not affected by the scanning of $\lambda_h(v)$. (The distribution of $\lambda_h(v)$ should, of course, be consistent with the bound on the Higgs boson mass.) In some cases, for example, in the minimal supersymmetric standard model, $\lambda_h(M)$ is determined by the theory at M and has very little ability to scan.

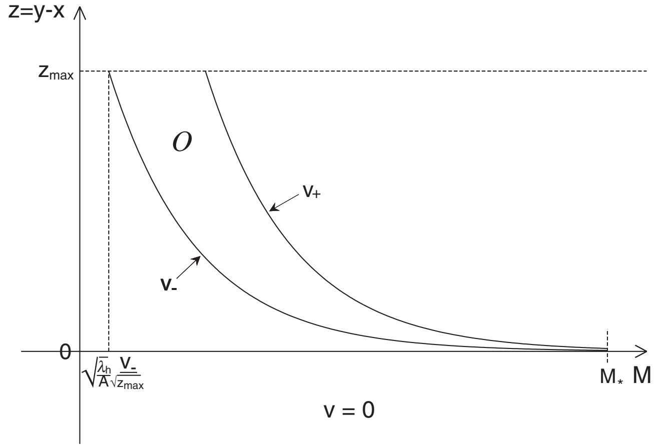

FIG. 17. Sketch of the 2-dimensional observer region in the M–z plane.

However, it is also clear that as $f_{\rm eff}(z, M)$ becomes progressively more peaked toward low z and/or high M, so the typical hierarchy will first grow to a little hierarchy $z \approx O(10^{-2}$–$10^{-1})$ and finally to a large hierarchy $z \ll O(10^{-2})$. A power distribution is easily able to overcome the narrowing of the observer region at large M. The probability distribution for the hierarchy, z, is given by

$$f_z(z) = \int_{\mathcal{O}} dM f_{\rm eff}(z, M), \tag{81}$$

and a general scanning of the electroweak symmetry breaking sector of the theory could lead to a large probability for either a little hierarchy or a large hierarchy. Indeed, since a strong dependence of $f_{\rm eff}(z, M)$ on z and M could result from many sources—the landscape of vacua, the population mechanism, integrating out other parameters, and the observer distribution n—it would certainly not be surprising if either a little or large hierarchy resulted. *Environmental selection of the weak scale can provide a simple generic explanation for our present difficulties in constructing natural theories of electroweak symmetry breaking.*

Note that this mechanism works both in the context of nonsupersymmetric and supersymmetric theories,[16] and that the small value of z implies a cancellation between the positive and negative contributions in Eq. (74) (x and y) that cannot be explained in the conventional symmetry viewpoint. In Sec. VIII D, we study in detail the simplified case that y does not scan, and x and M have power-law or logarithmic distributions. We compute $f_z(z)$, find conditions for large and little hierarchies, and obtain an analytic result for the size of the hierarchy when it is little. We also compute the distribution for v^2, $f_v(v^2)$ in the observer window.

Before closing this subsection, let us consider the case that only the dimensionless variables in the electroweak symmetry breaking sector x_i scan, with the value of M fixed. In this case, experiments constrain the value of M to be larger than about a TeV. What value should we expect for M? Without any special reason, we expect M to take some "random" value between TeV and fundamental scales; it is unlikely that M is close to the observed weak scale, since it requires an accident of order $O(0.01$–$0.1)$, as discussed in Sec. IV D. Hence, we typically expect a large hierarchy with $M \gg v$. The values of x_i are environmentally selected by Eq. (78) to a very small range

$$\frac{\lambda_h v_-^2}{AM^2} < y - x < \frac{\lambda_h v_+^2}{AM^2}, \tag{82}$$

in which there is a large cancellation between x and y.

C. Scanning over the entire nuclear stability observer region

We continue to study theories of electroweak symmetry breaking that lead to the standard model as an effective theory below M, with $m_h^2 = (x - y)AM^2$. However, as well as having v scan, via the scanning of x, y, and M, we now allow the other parameters of the nuclear stability observer boundaries to also scan. This means that instead of exploring a distribution through the observer region corresponding to only varying v, as shown by the dashed line of Fig. 15, we are now exploring a distribution over the entire observer region discussed in Sec. IV. With $y_{u,d,e}$, α, and $\Lambda_{\rm QCD}$ all scanning, the problem is apparently very complex. However, here we are only interested in two aspects of the problem: the probability distribution for $z = y - x$ that determines the size of the hierarchy between v and M, and the probability force perpendicular to the neutron surface that determines how close typical observers are to the neutron stability boundary. These questions can be addressed by studying an effective distribution over a reduced 2-dimensional projection of the parameter space.

The neutron and complex nuclei stability boundaries of Eq. (55) can be written in the form

$$\xi_- < \xi < \xi_+, \tag{83}$$

where

$$\xi^2 = z\frac{A\tilde{M}^2}{\lambda_h}, \tag{84}$$

with

$$\tilde{M} = \frac{C_I(y_d - y_u) - y_e}{\alpha\Lambda_{\rm QCD}}M, \tag{85}$$

and $\xi_- = C_\alpha$ and $\xi_+ = C_\alpha + C_B/\alpha$. Note the similarity in the form of the equations for ξ, Eqs. (83) and (84), to

[16]In supersymmetric theories, there are at least two Higgs doublets, but our analysis can be applied to these cases by identifying h as the linear combination causing electroweak symmetry breaking.

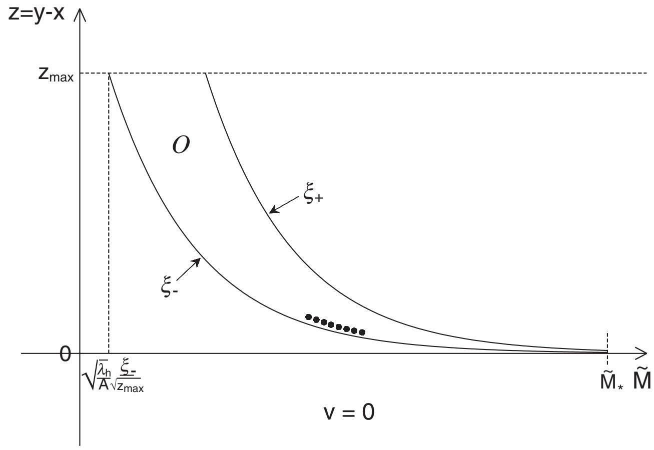

FIG. 18. Sketch of the 2-dimensional observer region in the $\tilde{M}$–z plane.

those for v, Eqs. (76) and (78). This means that the shape of the observer region in the 2-dimensional projection of the parameter space on the $\tilde{M}$–z plane, shown in Fig. 18, is the same as in the last subsection on the M–z plane. The curved observer boundaries at $\xi = \xi_-$ and $\xi = \xi_+$, however, now correspond to neutron and complex nuclei stability, respectively. Our universe thus lies very close to $\xi = \xi_-$. How probable this is will be determined by the effective distribution for ξ^2, $f_\xi(\xi^2)$. Similarly, the expected size for the hierarchy will be determined by the distribution for z, $f_z(z)$.

In general, the probability distribution in our whole parameter space is given by the multidimensional distribution function $f(x, y, M, \lambda_h, y_u, y_d, y_e, \alpha, \Lambda_{\rm QCD})$, which is not easy to deal with. A crucial point, however, is that *as far as questions of the size of the hierarchy z and the closeness to the neutron stability boundary are concerned, we only need to study the effective distribution function $f_{\rm eff}(z, \tilde{M})$ obtained after integrating out all the other variables*. In particular, we can parameterize our ignorance of the (potentially) complicated distribution function f and the shape of the observer region in this multidimensional space by the effective distribution function $f_{\rm eff}$ in the 2-dimensional $\tilde{M}$–z space. Note that as long as the form of $f_{\rm eff}$ is kept arbitrary, there is no loss of generality.

The effective distribution function $f_{\rm eff}(z, \tilde{M})$ is formally given by

$$f_{\rm eff}(z, \tilde{M}) = \int_{\mathcal{O}} dx dy dM d\lambda_h dy_u dy_d dy_e d\alpha d\Lambda_{\rm QCD} \times f(x, y, M, \lambda_h, y_u, y_d, y_e, \alpha, \Lambda_{\rm QCD}) \times \delta(z - (y - x)) \times \delta\left(\tilde{M} - \frac{C_I(y_d - y_u) - y_e}{\alpha \Lambda_{\rm QCD}} M\right). \qquad (86)$$

The problem of studying selection in the $\tilde{M}$–z plane then becomes identical to that in the M–z plane discussed in the previous subsection with the replacement $M \to \tilde{M}$ and $v \to \xi$, except that the upper boundary ξ_+ now depends on a scanning parameter α. The effect of the α scanning on the analysis, however, is small as long as the scanning is mild, e.g., the range of the α scanning does not span many orders of magnitudes. We assume this to be the case, and hereafter we neglect the effect from this scanning. The effective distributions for z and ξ^2 are given in terms of $f_{\rm eff}(z, \tilde{M})$ by

$$f_z(z) = \int_{\mathcal{O}} d\tilde{M} f_{\rm eff}(z, \tilde{M}), \qquad (87)$$

and

$$f_\xi(\xi^2) = \int_{\mathcal{O}} dz d\tilde{M} f_{\rm eff}(z, \tilde{M}) \delta\left(\xi^2 - \frac{zA\tilde{M}^2}{\lambda_h}\right), \qquad (88)$$

respectively.

We can identify three very different situations. The first is that $f_{\rm eff}(z, \tilde{M})$ is very mildly varying. In this case, from Fig. 18 we see that a typical universe will lie in the middle of the observer region and hence environmental selection will lead to neither a hierarchy nor a closeness to the neutron boundary. A second situation has a strongly varying $f_{\rm eff}(z, \tilde{M})$, but with the $\tilde{M}$ component of the probability force field unable to overcome the narrowing of the observer region at large $\tilde{M}$, shown in Fig. 18. In this case, if there is a strong force to low values of z, the multiverse yields a little hierarchy by making low values of $z \approx O(10^{-2}$–$10^{-1})$ typical. If in addition there is a significant probability force to low $\tilde{M}$, *the combined effects of the z and $\tilde{M}$ distributions lead to a closeness to the neutron stability boundary as well as to a little hierarchy*, as shown by the dots in Fig. 18. Finally, the probability force toward large $\tilde{M}$ may be strong enough to give a large hierarchy with extremely small z. In this case, the observer boundaries at ξ_- and ξ_+ are very close to each other. At such low values of z, is it reasonable to have $f_{\rm eff}$ sufficiently different on these boundaries to favor the boundary at ξ_-? Here we must recall that f is a product of a populated landscape distribution $\tilde{f}$ and an observer distribution n. It is certainly unreasonable for $\tilde{f}$ to have such a large variation over such a small region of parameter space. However, as we move from the ξ_- boundary to the ξ_+ boundary, nuclear physics changes very significantly, which is in a way independent of how small z is. Hence, a closeness to the neutron boundary may be typical with a large hierarchy, *but only if it is induced by n*, through such arguments as appeared in the last paragraph of Sec. VII.

The closeness to the neutron boundary may be a hint that $f_{\rm eff}(z, \tilde{M})$ is not flat. As argued previously, there are many origins for strong probability forces, so that a little or large hierarchy is quite natural in the multiverse. In the next subsection, we explore in some detail a subclass of the theories specified by Eq. (74), allowing explicit formulae for the effective distributions and the size of the hierarchy.

D. An explicit example with power-law distributions

In the previous two subsections, we assumed that the positive and negative terms in m_h^2/M^2 scan independently. To provide a simple explicit illustration of our results, in this subsection we choose to scan only the positive term, i.e., we fix $y = 1$ so that

$$m_h^2 = (x - 1)AM^2, \tag{89}$$

and $z = 1 - x$. The analysis with x fixed and y scanning is very similar. For ease of calculation we assume a polynomial distribution function

$$f(x, M)dxdM \propto x^n dx M^q d\ln M, \tag{90}$$

and the range of x is given by $0 \leq x \leq x_{\max}$, with $x_{\max}$ some number larger than 1, while M has some very large maximum value M_*.

We begin by assuming that the only parameter appearing in the nuclear stability boundary that scans is v, so that the condition of Eq. (78) gives the observer region

$$v_-^2 < v^2 = (1 - x)\frac{AM^2}{\lambda_h} < v_+^2. \tag{91}$$

The observer region $\mathcal{O}$ is sketched in the M–z plane in Fig. 19, which is obtained simply by setting $y = 1$ in Fig. 17. The region is separated by the nuclear physics observer boundaries from regions with $v < v_-$ and $v > v_+$, and approaches very close to $x = 1$, i.e., $z = 0$, as M approaches $M_* \gg v_o$. The narrowing of the observer region at large M is as we have seen in Fig. 17.

What is the typical value of the scale M? For a logarithmic distribution $q = 0$, one might guess that all decades of M are equally probable; but this is not the case, because at large M the observer region narrows. It is useful to compute an effective probability distribution for the hierarchy $z = 1 - x = |m_h^2|/AM^2$ over the observer region

$$f_z(z) = \int_{\mathcal{O}} dxdM f(x, M)\delta(z - (1 - x)) \sim (1 - z)^n \frac{1}{z^{q/2}} \tag{92}$$

for sufficiently large M_*. From the viewpoint of the hierarchy, this shows that the critical value of q is 2 not zero: when $q = 2$ the distribution for the hierarchy z is flat on logarithmic scales, at least for large M where z is small. Thus, $q \geq 2$ gives a large hierarchy, shown by the narrow wedge at large M in Fig. 19, and $q < 2$ gives a little (or no)

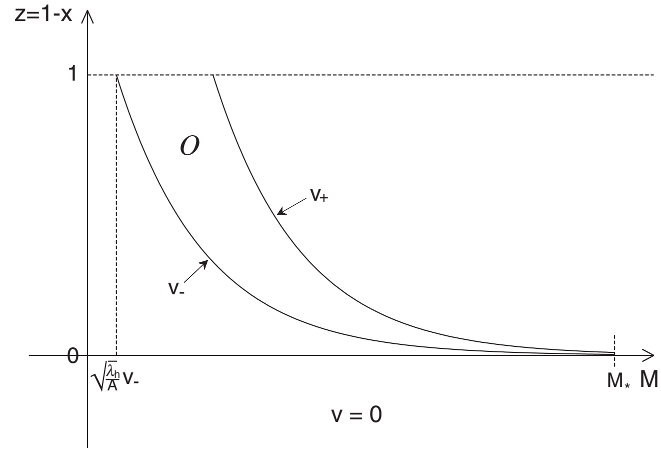

FIG. 19. Sketch of the 2-dimensional observer region in the M–z plane, where $z = 1 - x$.

hierarchy, corresponding to the bulk of the observer region in Fig. 19. For example, if the *a priori* distribution for M is logarithmic, as might happen if the scale is triggered by dimensional transmutation associated with some gauge dynamics, the most probable observed value of M is small because of the weighting from the narrowing of the observer region $\mathcal{O}$.[17] On the other hand, if $d\mathcal{N} \sim dM^2$, as expected in a typical nonsupersymmetric perturbative theory with mass scale M, we find $q = 2$, so that all scales are equally probable, implying a large hierarchy, but not one with M near M_*.

If $q < 2$, how large is the little hierarchy? For $n > 0$, the first factor in Eq. (92) suppresses the probability of having $z \sim 1$, i.e., the probability of having the lowest values of M gets suppressed, so that some amount of hierarchy between v and M arises from the scanning in the multiverse. For $q < 2$ the distribution of Eq. (92) leads to the average value of z

$$\langle z \rangle = \left\langle \left| \frac{m_h^2}{AM^2} \right| \right\rangle = \frac{2 - q}{2n + 4 - q}, \tag{93}$$

so that for $2n \gg 2 - q$ we obtain an extra hierarchy of a factor of $\simeq (2 - q)/2n$ from the multiverse, which cannot be explained in the conventional symmetry approach. Since $|q|$ is expected to be small, this is an appreciable effect for large n. In Fig. 20, we plot $f_z(z)$ of Eq. (92) for $(n, q) = (3, 0)$, $(10, 0)$, $(3, 1)$. We clearly see that the

[17] In the case that M arises as a dimensional transmutation, the expected distribution of M below M_* is not exactly $\propto d\ln M$, since it would lead to an unphysical conclusion that the probabilities of having M between $10^n M_*$ and $10^{n+1} M_*$ ($n \leq -1$) are equal for all n down to $n \to -\infty$. (The distribution should, for example, be cut off at some value $M_{\min}$ or in fact be $\propto d(1/\ln M)$; see discussions at the end of Sec. II B and in Sec. IV D.) The assumption/approximation of the exact logarithmic distribution, however, is sufficient for our purposes here.

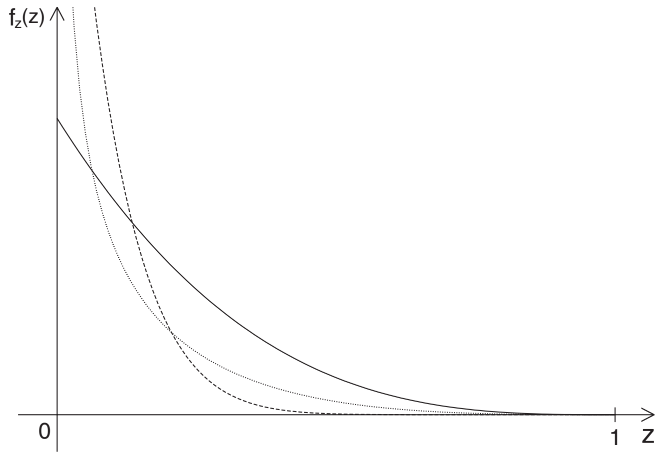

FIG. 20. The distribution function $f_z(z)$ for $(n, q) = (3, 0)$ (solid), (10, 0) (dashed), and (3, 1) (dotted). Each function is normalized such that $\int_0^1 f_z(z)dz = 1$.

distribution is peaked toward $z \ll 1$. The probability of us observing fine-tuning of Δ^{-1} or more that cannot be understood by the symmetry approach is given by

$$\mathcal{P}_\Delta = \frac{\int_0^{\Delta^{-1}} f_z(z)dz}{\int_0^1 f_z(z)dz}, \tag{94}$$

which for $\Delta \gg n \gg 1$ reduces to

$$\mathcal{P}_\Delta \approx \frac{n}{\Delta}, \tag{95}$$

for $q = 0$ (the case with dimensional transmutation) and

$$\mathcal{P}_\Delta \approx \sqrt{\frac{4n}{\pi\Delta}} \tag{96}$$

for $q = 1$. For example, for $n = 10$ and $q = 0$ ($n = 10$ and $q = 1$) we have a $\simeq 20\%$ ($\simeq 50\%$) probability of observing fine-tuning $\Delta^{-1} < 2\%$, which cannot be explained in the conventional symmetry picture.

The observer boundaries of Fig. 19 are the contours for $v = v_\pm$ so that, by construction, v can only be observed between v_- and v_+. What are the most probable values of v to be observed? The effective distribution for v^2 is defined by

$$f_v(v^2) = \int_{\mathcal{O}} dxdMf(x, M)\delta\left(v^2 - (1 - x)\frac{AM^2}{\lambda_h}\right). \tag{97}$$

For a large hierarchy with $q \geq 2$, we find a flat distribution in v^2

$$f_v(v^2) \sim \text{const.} \tag{98}$$

When $y_{u,d,e}$, α, and $\Lambda_{\rm QCD}$ do not scan, the observed value of the weak scale v_o is roughly midway between v_- and v_+, as shown in Eq. (29), which is consistent with Eq. (98).[18] For a little (or no) hierarchy, with $q < 2$, we find a distribution

$$f_v(v^2) \sim v^{q-2}. \tag{99}$$

This is again consistent with v_o centrally located in its observer window, provided that q is not too negative.

Next, we allow the other parameters that appear in the nuclear stability observer boundaries to also scan. As described in Sec. VIII C, the issues of the size of the hierarchy and the closeness to the neutron surface can be addressed by studying a 2-dimensional projection of parameter space. Since y does not scan, $z = 1 - x$ and so here we discuss the $\tilde{M}$–x plane. We assume the effective probability distribution in this plane has a power-law behavior

$$f_{\rm eff}(x, \tilde{M})dxd\tilde{M} \propto x^n dx\tilde{M}^{\tilde{q}} d\ln\tilde{M}. \tag{100}$$

The neutron and complex nuclei boundaries are again given by Eqs. (83)–(85), but with $z = 1 - x$, and are represented in Fig. 18 in the $\tilde{M}$–x plane (with y now set to unity). The observer region as $x \to 1$ again becomes a narrow wedge where the hierarchy is large, $M \gg v$.

The equations describing this setup are very similar to those in the case with only v scanning, except that the observer boundaries are now at $\xi_\pm$ rather than $v_\pm$, and $\tilde{M}$ is not the scale of the new physics, see Eqs. (83)–(85). The size of the hierarchy is governed by the effective distribution for z and is given by the analogue of Eq. (92)

$$f_z(z) \sim (1 - z)^n \frac{1}{z^{\tilde{q}/2}}. \tag{101}$$

Thus, $\tilde{q} \geq 2$ gives a large hierarchy, and $\tilde{q} < 2$ gives a little (or no) hierarchy. The size of the little hierarchy depends on n, and using the distribution Eq. (100) we find

$$\left\langle \left| \frac{m_h^2}{AM^2} \right| \right\rangle = \frac{2 - \tilde{q}}{2n + 4 - \tilde{q}}. \tag{102}$$

The little hierarchy increases with n, as shown by the sequence of dots in Fig. 18 near the neutron stability boundary.

What makes the proximity to the neutron boundary typical? The effective distribution for ξ, which determines the probability force perpendicular to the neutron surface, follows immediately from a calculation analogous to that which led to Eqs. (98) and (99) giving

$$f_\xi(\xi^2) \sim \text{const.} \quad \text{for} \quad \tilde{q} \geq 2, \tag{103}$$

[18]Caution, however, is needed in interpreting Eq. (98). This result arises because at large M the contours of v_- and v_+ are extremely close in M–x space, so that the assumed form for f, Eq. (90), implies little variation between the contours. While this is expected for the multiverse distribution $\tilde{f}$, it may not be true for the observer distribution n. In this case, Eq. (90) needs to be modified to incorporate the effect represented by n.

and

$$f_\xi(\xi^2) \sim \xi^{\tilde{q}-2} \quad \text{for} \quad \tilde{q} < 2. \tag{104}$$

As with Eq. (98), caution is necessary in interpreting Eq. (103). At large $\tilde{M}$ our assumed distribution, Eq. (100), may not adequately account for variations in the observer factor $n(\xi)$ between ξ_- and ξ_+. In particular, we cannot conclude from Eq. (103) that a large hierarchy is incompatible with a multiverse explanation for the closeness to the neutron stability boundary. With a little hierarchy, sufficient closeness to the neutron stability boundary results with the distribution of Eq. (100) for $-10 \lesssim \tilde{q} \lesssim -2$.

E. Landscapes with a critical value for M

In the previous subsections we have ignored the logarithmic evolution of x and y. For many landscapes this is permissible, but for some this evolution plays a critical role. Consider a landscape such that $x_* - y_*$ scans only in a restricted region with $x_* - y_* > 0$, where $x_* \equiv x(M_*)$ and $y_* \equiv y(M_*)$, which are the fundamental scanning parameters. In this case, electroweak symmetry breaking is only possible because of the evolution of $x - y$ to lower energies according to the beta function $\beta = d(x-y)/d\ln\mu > 0$. For each value of (x_*, y_*), the quantity $x(M) - y(M)$ passes through zero at some $M_c(x_*, y_*)$, so that in these universes electroweak symmetry breaking is only possible if $M < M_c(x_*, y_*)$. As (x_*, y_*) vary over the entire landscape, suppose that the largest value of $M_c(x_*, y_*)$ is $M_{c,\max}$, which is much less than M_*. This maximum critical mass is clearly a property of the particular landscape under consideration; it defines a maximum possible value for the electroweak scale anywhere in the multiverse, and has an important effect on the observer region of the electroweak symmetry breaking sector. For the critical universes with $M_c(x_*, y_*) = M_{c,\max}$ the evolution equation for $z = y - x$ can be solved as

$$z_{\max}(M) = \beta \ln \frac{M_{c,\max}}{M}, \tag{105}$$

where we have approximated that β is constant. The trajectory of Eq. (105) is sketched in Fig. 21 for the case that $M_{c,\max} \gg v_\pm$, which provides a new boundary to the observer region, since $z_{\max}$ represents the maximal value of z in the multiverse for a given M. In this subsection, we assume that the distribution for M and z leads to most universes in the observer region having large M, close to $M_{c,\max}$, so that this new part of the observer boundary plays an important role.[19]

[19]For many landscapes the evolution of $x - y$ from M_* to M can be ignored. It induces a shift in the origin of $z = y - x$ by an amount $\beta\ln(M_*/M)$, which is less than unity since $\beta\ln(M_*/M)$ is expected to be of $O(1)$ or smaller. In many cases the local property of $f_{\rm eff}(z, M)$ is not significantly changed by this shift.

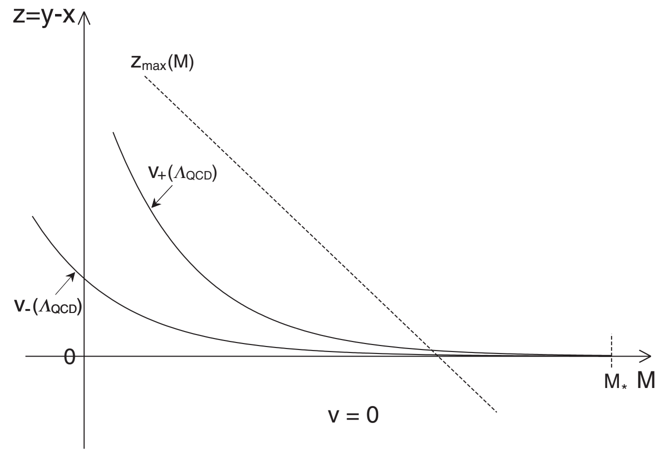

FIG. 21. Sketch of the 2-dimensional observer region in the M–z plane. The maximum value of z, $z_{\max}(M)$ is depicted by the dashed line.

If $y_{u,d,e}$, α, and $\Lambda_{\rm QCD}$ do not scan then $v_\pm$ are fixed numbers that arise from nuclear selection. Since there is no reason for the critical mass $M_{c,\max}$ of the landscape to be close to $v_\pm$, a large hierarchy is expected. Here, we are more interested in the case that $v_\pm$ varies in the landscape, and for simplicity we accomplish this via a scanning of $\Lambda_{\rm QCD}$, which we take to have a mild distribution, for example flat on a logarithmic scale. In this case, $v_\pm \propto \Lambda_{\rm QCD}$, so that the observer boundaries in Fig. 21 labeled by $v_\pm$ have $z_\pm \propto \Lambda^2_{\rm QCD}$, and hence move up and down as $\Lambda_{\rm QCD}$ scans. For a large hierarchy, M is so much larger than v that the interval of z that can contribute to universes in the observer region is very small $\Delta z \approx v^2/M^2$. Since the distribution for $\Lambda_{\rm QCD}$ is mild, a gain in probability is accomplished by having the $v_\pm$ boundary curves of Fig. 21 move upwards, allowing a larger interval of z to contribute. As the $v_\pm$ curves move up so the values of M that are on the $z_{\max}$ observer boundary decrease. At some point this will imply a significant loss of probability via the distribution for M. For example, for $f(M)dM \propto M^q d\ln M$ with large q, this loss of probability begins to set in once $M \sim (1 - 1/q)M_{c,\max}$ at which point $z_{\max} \sim \beta/q$. The mild distribution for $\Lambda_{\rm QCD}$ allows the $v_\pm$ curves of Fig. 21 to rise so that, at this value of M, $z_\pm$ will be close to $z_{\max}$. A further drop in M causes a significant drop in probability from $f(M)$, which is not offset by a sufficient growth in Δz, since $z_c(M)$ grows logarithmically. Hence, we predict that a little hierarchy develops. Specifically, for a distribution function

$$f(z, M, \Lambda_{\rm QCD})dzdMd\Lambda_{\rm QCD} \propto f(z)M^q dzd\ln Md\ln\Lambda_{\rm QCD}, \tag{106}$$

integrating over the observer region we obtain a typical size of the hierarchy

$$\langle z\rangle = \frac{\int z f(z) e^{-(qz/\beta)} dz}{\int f(z) e^{-(qz/\beta)} dz} \approx \frac{\beta}{q}, \tag{107}$$

where the range of the z integration is from 0 to the maximum value that z takes in the energy interval between $\approx v$ and M_*, and we have taken $f(z)$ to be constant in the last expression for illustration, although the result is not very sensitive to the form of $f(z)$. Note that this result applies to an arbitrary positive value of $q \gtrsim O(\beta)$.

Integrating over $\Lambda_{\rm QCD}$ effectively "removes" the $v_\pm$ boundaries in the 2-dimensional parameter space of $\{M, z\}$, and the resulting effective distribution in the M–z plane is unaltered for a prior distribution flat in $\ln\Lambda_{\rm QCD}$. Allowing any mild prior distribution for $\Lambda_{\rm QCD}$, or allowing other standard model parameters such as the Yukawa couplings $y_{u,d,e}$ to scan with mild distributions, would lead to an effective distribution $f_{\rm eff}(z, M)$ that would lead to the basic result of Eq. (107) being unaffected. The physics behind this result is both simple and general: the $1/q$ factor results from a probability force in the M direction making the most probable universes in the observer region those close to $M_{c,\max}$, while the factor of β arises from the shape of the observer boundary near $M_{c,\max}$, as given in Eq. (105).

We conclude that landscapes having a critical value of M, above which electroweak symmetry breaking is impossible, very easily lead to a little hierarchy with z suppressed by both a loop factor β and a probability force factor, for example $1/q$. Unlike the case of Secs. VIII B and VIII C large q does not lead to a large hierarchy. Instead, it acts to make M close to $M_{c,\max}$ and, since $z_{\max}(M_{c,\max}) = 0$, this leads to a little hierarchy. Note that the loop factor can provide an extra hierarchy corresponding to 1–2 orders of magnitude of fine-tuning, on top of the $1/q$ factor. Hence, this mechanism typically yields M in the TeV region or larger.

If the physics at M is supersymmetric, this mechanism is the one considered in Ref. [9]. If the only scanning parameters are a uniform scaling of the overall scale of supersymmetry breaking and of the supersymmetric Higgs mass parameter μ, the scale of M_c corresponds to the scale where the determinant of the Higgs boson mass matrix $\det(\mathcal{M}_H^2)$ passes through zero. To avoid a large hierarchy, some other parameter, such as $\Lambda_{\rm QCD}$ should scan. One again needs to assume that, for the landscape as a whole, there is some maximum value of the scale where $\det(\mathcal{M}_H^2)$ passes through zero, and this is taken to be $M_{c,\max}$. In this subsection, we have stressed the generality of the idea, and that the loop suppression factor appearing in Eq. (107) does not depend on whether the physics at M is supersymmetric or not. In the supersymmetric case, β is determined by the specific form of the renormalization group equations for the Higgs boson mass parameters, and one finds that $\beta \approx O(0.1)$. Nevertheless, this is an important accomplishment, since without the landscape the ratio between v and M involves a large logarithm $\ln(M_*/v)$ (in the case of high scale supersymmetry breaking) so that $A \approx \beta \ln(M_*/v) \approx O(1)$, giving $M \approx \sqrt{\lambda_h} v \approx M_Z/\sqrt{2}$, where M_Z is the Z boson mass. The factor of β, together with $1/q$, can easily make M at the TeV scale or larger.

F. Summary and discussion

The conventional naturalness argument implies that the mass scale M of new physics beyond the standard model that generates electroweak symmetry breaking should not be much larger than the Higgs vacuum expectation value v. In contrast, we have shown that environmental selection for nuclear stability can lead to M substantially larger than v. We have considered a very general framework that is independent of the model of the new physics, assuming only that it generates both positive and negative contributions to the Higgs mass-squared parameter $m_h^2 = (x - y)AM^2$. If the landscape does not possess a maximum critical mass $M_{c,\max}$ significantly less than M_*, the relevant observer boundaries are shown in Fig. 17 when only x, y, and M scan, and in Fig. 18 when $y_{u,d,e}$, α, and $\Lambda_{\rm QCD}$ also scan. In both cases we find

(i) A large hierarchy is generated by a distribution for M ($\tilde{M}$) that grows sufficient with M ($\tilde{M}$) to overcome the narrowing observer region of Fig. 17 (Fig. 18). While this does not require a very strong peaking, the distribution must continue growing over many orders of magnitude in M ($\tilde{M}$).

(ii) A little hierarchy is generated when the distribution for M ($\tilde{M}$) does not grow sufficiently to generate a large hierarchy, and the distribution for z is strongly peaked to low values of z. This strong peaking need only persist for 1 or 2 orders of magnitude in z.

In both cases, there is necessarily a large cut factor, either from a mild growth in the distribution for M ($\tilde{M}$) over many decades for a large hierarchy, or from a strong distribution for z over a much more limited range for a little hierarchy. In general, a significant variation of a distribution function can arise from many sources—the distribution of vacua in the landscape, the population of these vacua, integrating out parameters, and the number density of observers—so that in the multiverse the existence of a hierarchy, either little or large, is not surprising.

For landscapes with a maximum critical mass $M_{c,\max}$

(i) A little hierarchy is generated by a distribution that favors large M near $M_{c,\max}$. The size of the little hierarchy depends on both the strength of this distribution, and also on a loop factor that arises from the beta function for $x - y$. The combination of these factors makes it probable that M is as large as several TeV.

In Sec. X, we argue that, in the presence of WIMP dark matter, a strong distribution for M can arise from the probability distribution for the cosmological constant.

Until now, we have not specified the physics behind x and y; here, we consider a few simple schemes, stressing how a cancellation between x and y could lead to a strong distribution for z and hence a little hierarchy. The quadratic divergences of the standard model contribute to both x (e.g., the $SU(2)$ gauge contribution) and y (e.g., the top quark contribution). At mass scale M suppose that these quadratic divergences are cut off by particles of mass $M_{W'}$ and $M_{t'}$, respectively. If the theory at M is supersymmetric, these are the w-ino and top squark masses, while if the theory at M is nonsupersymmetric (for example, with composite Higgs dynamics) they are some states of the model. In all of these schemes, integrating out the physics at M gives a low energy theory with

$$m_h^2 \sim \frac{g^2}{16\pi^2} M_{W'}^2 - \frac{3y_t^2}{16\pi^2} M_{t'}^2, \tag{108}$$

where g is the standard model $SU(2)$ gauge coupling. The numerical coefficients are model dependent and extra contributions to m_h^2 are expected, for example, those that regulate the hypercharge and Higgs-quartic divergences; but neither of these affects the arguments below.

Whether the theory at M is supersymmetric or not, it may well be that M arises as a dimensional transmutation, in which case it is reasonable that the distribution for $\ln M$ is sufficiently flat that a large hierarchy does not develop. In this case, what distribution for the parameters in Eq. (108) would lead to a little hierarchy?

We need a distribution $f_z(z)$ peaked at low values, i.e., in Fig. 17 or Fig. 18 the probability force must have a large component downwards. Since $z = 0$ is not a special point from the fundamental theory point of view (see the discussion in Sec. II B), this implies that most universes will be in the phase with $z < 0$, i.e., $m_h^2 > 0$. In the multiverse, the positive term in Eq. (108) typically dominates over the negative term. Such a distribution could arise in several ways. For example, suppose that $M_{W'}/M_{t'}$ does not scan, but g and y_t do. If g has a distribution peaked at a (much) larger value than y_t, then most universes will have negative z, and near the observer boundary the probability force will be toward smaller values of z. The shapes of the distributions for g and y_t need not be power law, as assumed for simplicity in Sec. VIII D. For example, they could be Gaussians with peaks at $\bar{g}$ and $\bar{y}_t$, with $\bar{g}$ sufficiently larger than $\bar{y}_t$, so that most universes have $m_h^2 > 0$. The few universes that have m_h^2 negative will typically have low z and therefore a little hierarchy. For narrow Gaussians for g^2 and y_t^2 with the standard deviations δ_g and δ_y, respectively, we find

$$\langle z\rangle = \left\langle \left| \frac{m_h^2}{A_y M^2} \right| \right\rangle \approx \frac{\int_0^\infty z e^{-((z+z_c)^2/2\delta^2)} dz}{\int_0^\infty e^{-((z+z_c)^2/2\delta^2)} dz} \sim \frac{\delta^2}{z_c}, \tag{109}$$

where $z_c \equiv (A_g/A_y)\bar{g}^2 - \bar{y}_t^2 > 0$ and $\delta^2 \equiv (A_g/A_y)^2 \delta_g^2 + \delta_y^2$, with $A_{g,y}$ positive coefficients defined by $m_h^2 = (A_g g^2 - A_y y_t^2) M^2$. Alternatively, it could be that the scanning of the masses $M_{W'}$ and $M_{t'}$ is more important than that of the couplings, and that the little hierarchy results because the distributions typically give $M_{W'}$ (much) larger than $M_{t'}$.

In particular models, it is possible to see other situations that lead to a little hierarchy. For example, in the minimal supersymmetric standard model it could be that the distributions for the supersymmetric Higgs mass parameter μ and the scale of the soft supersymmetry breaking mass parameters $\tilde{m}$ differ. If μ is typically (much) larger than $\tilde{m}$, then most universes do not have electroweak symmetry broken by $\langle h \rangle \neq 0$. This generically leads to a strong distribution preferring low z, and universes in the observer region will have a little hierarchy. More generally, in the minimal supersymmetric standard model the electroweak phase boundary takes the form

$$(|\mu|^2 + m_{H_1}^2)(|\mu|^2 + m_{H_2}^2) = |\mu B|^2, \tag{110}$$

with the soft Higgs mass-squared parameters $m_{H_1}^2$ and $m_{H_2}^2$ depending on other parameters of the theory via renormalization group scaling. Any multiverse distribution for the parameters of the model that typically makes the left-hand-side of Eq. (110) larger than the right-hand-side will generically lead to a little hierarchy. The scale of the superparticle masses are then raised significantly above v, and the measured values of the parameters should be close to satisfying the critical condition, Eq. (110).

IX. ELECTROWEAK SYMMETRY BREAKING AS OBSERVER BOUNDARY

In the last section, we assumed that the relevant observer boundaries for selecting the electroweak vacuum expectation value $\langle h \rangle = v$ were those of neutron, deuteron, and complex nuclei stability. Of these three boundaries, the requirement that some complex nuclei are stable seems clearly to be the most robust requirement for observers. For example, if the neutron is stable, nuclear energy is produced in diffuse protogalaxies rather than in stars. While this is a drastic change from our universe, some form of observers might still be possible. On the other hand, it is harder to imagine that some complex observers develop in the world in which the only stable nucleus is p or Δ^{++}. In this section, we retain only the complex stable nuclei boundary, dropping the neutron and deuteron (in)stability requirements from the observer boundary. This allows much smaller values for v. How small can v become while remaining in the observer region? If electroweak symmetry is broken dominantly by the QCD condensate, there is a strong washout of the baryon asymmetry of the universe due to sphaleron effects. This implies that complex structures involving baryons do not arise if $v \lesssim \Lambda_{\rm QCD}$, so that there is a complexity boundary for v near $\Lambda_{\rm QCD}$, which is therefore a candidate for being part of the observer boundary. Since $\Lambda_{\rm QCD}$ is much smaller than the scale of the electroweak symmetry breaking sector M, we can speak of this boundary as the phase boundary between $\langle h \rangle = 0$ and $\langle h \rangle \neq 0$ phases. In this section, we use only two boundaries: complex nuclear stability and the Higgs breaking of electroweak symmetry, which were both classified as catastrophic boundaries in Sec. IV.

We consider the generic electroweak symmetry breaking sector discussed in the previous section, with relevant scanning parameters x, y, and M and a hierarchy

$$z \equiv y - x = \frac{|m_h^2|}{AM^2}. \qquad (111)$$

With $y_{u,d,e}$, α, and $\Lambda_{\rm QCD}$ fixed, the relevant observer region is now as shown in Fig. 22. Compared with Fig. 17, the observer boundary at v_+ remains while that at v_- has been replace by one at $v \approx \Lambda_{\rm QCD}$. This greatly enlarges the observer region, with v varying over about 4 orders of magnitude compared with a factor 4 in Sec. VIII. Despite this, we find that there is very little change in the physical picture of environmental selection.

For example, in the case that y is fixed and x and M have polynomial distributions as in Eq. (90), the effective distributions for z and v are not changed and given as before [see Eqs. (92), (98), and (99)] by

$$d\mathcal{N} = f_z(z)dz \sim (1-z)^n \frac{1}{z^{q/2}} dz, \qquad (112)$$

and

$$d\mathcal{N} = f_v(v^2)dv^2 \sim \begin{cases} v dv & \text{for} \quad q \geq 2, \\ v^{q-1} dv & \text{for} \quad q < 2. \end{cases} \qquad (113)$$

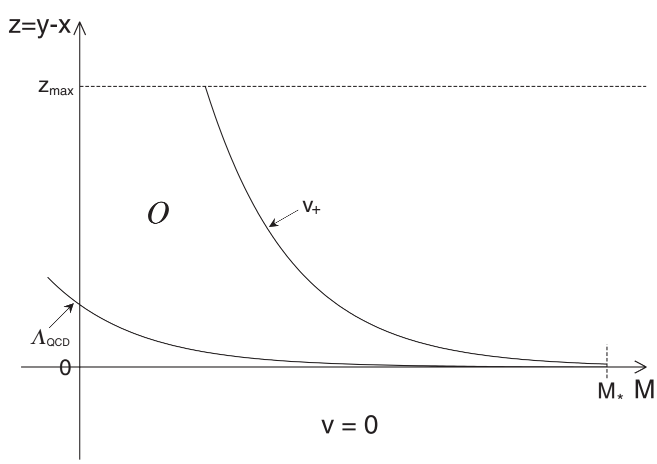

FIG. 22. Sketch of the 2-dimensional observer region in the M–z plane. The two boundaries of the observer region depicted are $v = v_+$ and $v \approx \Lambda_{\rm QCD}$.

Once again, $q \geq 2$ gives a large hierarchy and $q < 2$ a little (or no) hierarchy. Furthermore, the size of the little hierarchy is largely governed by n, as discussed in Sec. VIII D. One difference is that the range of v in the observer region is now much larger; how reasonable is it that our universe is about a factor of 5 from v_+ and 3 orders of magnitude from $\Lambda_{\rm QCD}$? For $0 \lesssim q \lesssim 1$ the observed value of v is quite typical, but for larger q, v becomes progressively more peaked near v_+. However, this cannot be viewed as evidence against a large hierarchy. For large q it is unlikely that Eq. (113) is the correct distribution near v_+: as v approaches this observer boundary so successively more nuclei become unstable, so that the observer factor n is likely to become smaller.

Given the observer region of Fig. 22, we can understand the origin of large or little hierarchies from the qualitative features of the distribution $f(z, M)$, without recourse to any particular functional form. The observer boundary at v_+ causes a narrowing of the observer region at large M, so that a large hierarchy results only if the distribution gives a sufficient preference to large M to overcome this narrowing. If the probability force in the M direction is insufficient, then the size of the little hierarchy depends on the strength of the probability force in the negative z direction. At first sight a force in the negative z direction apparently pushes the electroweak vacuum expectation value close to the value $\Lambda_{\rm QCD}$ at the baryon washout boundary. This is incorrect: such a force makes the most probable region that with low z, but the value of v that this corresponds to changes with M. This is why Eq. (113) involves q and not n.

We again conclude that the little hierarchy problem (supersymmetric or not) is very easily solved by environmental selection. Strongly varying distribution functions of the electroweak symmetry breaking sector are able to generate either a large or little hierarchy; furthermore, this is not sensitive to which observer boundary is used to limit the lower value of v.

X. CONNECTIONS TO THE COSMOLOGICAL CONSTANT

In this paper, we argue that evidence for the multiverse can be found in unnaturalness in the cosmological constant, nuclear physics, and electroweak symmetry breaking. This implies that scanning of parameters is occurring in all three different arenas. Is it correct to consider each scanning problem separately, or should there be a combined treatment? In Sec. VIII C, we provided an analysis of the combined scanning in the standard model and electroweak symmetry breaking sectors. In this section, we study connections with the scanning of the cosmological constant.

In Sec. VI B, we argued that "integrating out" a set of parameters x_b modifies the distribution function for a set x_a

$$f_{\rm eff}(x_a) = \int_{\mathcal{O}(x_a)} f_{\rm prior}(x_a, x_b) dx_b, \qquad (114)$$

if the observer region $\mathcal{O}$ for x_b depends on x_a. In the present case, we consider $x_b = \Lambda$ and x_a to be the set of scanning parameters of the standard model and the electroweak symmetry breaking sector. Assuming the multiverse probability distribution for Λ to be flat on a linear scale

$$f_{\rm eff}(x_a) \propto f_{\rm prior}(x_a) \int^{\rho_{\rm NL}(x_a)} d\Lambda = f_{\rm prior}(x_a)\rho_{\rm NL}(x_a), \qquad (115)$$

where $\rho_{\rm NL}$ is the energy density of the universe when it goes nonlinear. Hence, the distribution function for the parameters of the standard model and the electroweak symmetry breaking sector is modified by the scanning of the cosmological constant if $\rho_{\rm NL}$ depends on the parameters x_a. In our previous analyses, the assumed form of the distribution should apply to $f_{\rm eff}$, rather than to $f_{\rm prior}$. This implies that the scanning of the cosmological constant can affect the form of the distribution functions that appeared in the analyses in previous sections.

In this section, we explore some consequences of a nontrivial dependence of $\rho_{\rm NL}$ on x_a. Perhaps the simplest origin for such a dependence is the case of WIMP dark matter. The temperature of matter-radiation equality $T_{\rm eq}$ depends on the WIMP mass via its annihilation cross section, $T_{\rm eq} \propto 1/\sigma_A \sim m_{\rm WIMP}^2$. Hence, $\rho_{\rm NL} \sim Q^3 T_{\rm eq}^4 \sim Q^3 m_{\rm WIMP}^8$, where Q is the primordial density perturbation. Working with the generic electroweak symmetry breaking sector introduced in Sec. VIII, with mass scale M and dimensionless parameters x_i, the WIMP mass is proportional to M so that

$$T_{\rm eq} = g(x_i) M^2, \qquad (116)$$

where $g(x_i) > 0$ is a model dependent function, depending on the mass and interactions of the WIMP. The effective distribution function for the electroweak symmetry breaking sector now becomes

$$f_{\rm eff}(x_i, M) \propto f_{\rm prior}(x_i, M) \int^{\rho_{\rm NL}(x_i, M)} d\Lambda \sim f_{\rm prior}(x_i, M) g(x_i)^4 M^8 Q^3. \qquad (117)$$

Note that here we have assumed that the parameter Q does not scan. If Q also scans, the result of integrating out cosmological parameters is altered from Eq. (117), as we will see below.

If Q does not scan, Eq. (117) shows that the effective distribution for the parameters of the electroweak symmetry breaking sector $\{x_i, M\}$ can acquire an important component from integrating out the cosmological constant. In particular, the cosmological constant may provide a strong probability force to larger values of M through the M^8 factor. At first sight it appears that this factor drives a large hierarchy $M \gg v$, but this is not the case. A crucial issue is: what stops the runaway to large M? It is important to separate two cases.

In the first case, the factor $T_{\rm eq}^4 \sim \{g(x_i)M^2\}^4$ in $f_{\rm eff}$ pushes the amount of dark matter up to the maximum allowed by some astrophysical observer boundary, for example, that of stellar collisions in galaxies [34], so that $T_{\rm eq}$ is essentially fixed to the boundary value $T_{\rm eq} = T_{\rm eq,*}$. The distribution function for the parameters relevant for electroweak symmetry breaking, $\{x, y, \lambda_h, M\}$ where x, y, $\lambda_h \subset x_i$, is obtained by integrating out parameters that appear in $g(x_i)$ but not in m_h^2 or λ_h, within the observer region $T_{\rm eq} < T_{\rm eq,*}$. This in general leads to a complicated dependence of $f_{\rm eff}$ on $\{x, y, \lambda_h, M\}$, not in the simple form of Eq. (117). Note that as M grows beyond the TeV scale, since $T_{\rm eq}$ stays to be $T_{\rm eq,*}$, cancellations must occur in $g(x_i)$ so that $m_{\rm WIMP} \ll M$. This implies that a large hierarchy can develop only if $f_{\rm prior}$ has a strong preference toward larger values of M.

In the second case, the most probable region of scanning parameter space does not lead to the dark matter density being on the edge of its maximal value determined by astrophysics. The values of the parameters $\{x_i, M\}$ are determined by other physics, including electroweak symmetry breaking. The distribution of the parameters relevant for electroweak symmetry breaking may take a form $f_{\rm eff} \sim f T_{\rm eq}^4$, so that the factor of $g(x_i)^4 M^8$ can play an important role in electroweak symmetry breaking. Some examples of this are given in the following subsection.

A completely different situation arises if the space of scanning parameters is increased to include Q. As illustrated in Fig. 23, the probability force from the cosmological constant no longer acts in the direction of increasing $m_{\rm WIMP}$, but in the direction of increasing $\rho_{\rm NL}$, so that now

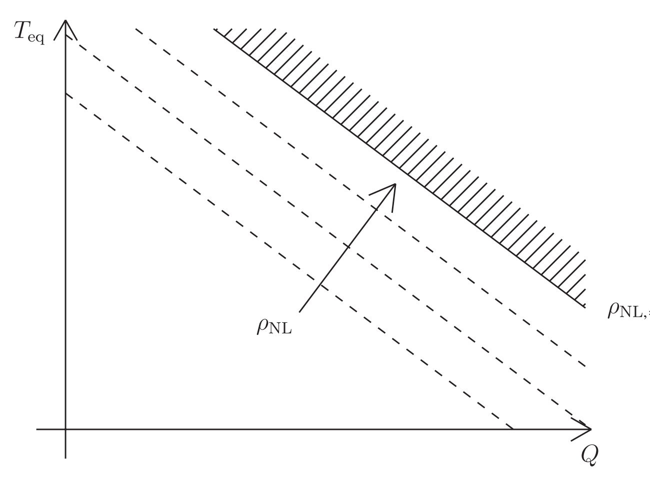

FIG. 23. The probability force from the cosmological constant in the Q–$T_{\rm eq}$ plane. The contours of $\rho_{\rm NL}$ are drawn by dashed lines, while the observer boundary from astrophysics, $\rho_{\rm NL} = \rho_{\rm NL,*}$ is depicted in the solid line.

the issue becomes: what stops the runaway in the ρ_{NL} direction? It is again an astrophysical limit on the amount of dark matter, but this is now a limit on ρ_{NL} rather than on T_{eq}: $\rho_{NL} < \rho_{NL,*}$. The probability distribution along this astrophysical boundary now depends on the distribution $f_Q(Q)$. Integrating first Λ and then Q over the observer region, we obtain

$$f_{\rm eff}(x_i, M) \propto f_{\rm prior}(x_i, M) T_{\rm eq}^4 \int^{(\rho_{\rm NL,*}/T_{\rm eq}^4)^{1/3}} Q^3 f_Q(Q) dQ, \tag{118}$$

where T_{eq} is given by Eq. (116). The induced contribution to the effective distribution now depends on the form of the original distribution for the density perturbation Q, since the observer boundary for Q depends on $T_{eq} = T_{eq}(x_i, M)$. However, in the simple case that $f_Q \propto 1/Q$, so that the Q distribution is flat on a logarithmic scale, $f_{\rm eff}(x_i, M) \sim f_{\rm prior}(x_i, M)$, and the T_{eq}^4 contribution is removed. Hence, a nontrivial contribution to $f_{\rm eff}$ is generated by a nontrivial distribution for $\ln Q$, not by the probability force from the cosmological constant.

Examples of the cosmological constant affecting electroweak symmetry breaking

We have seen that integrating out cosmological parameters with WIMP dark matter can generate a nontrivial component in the distribution of parameters relevant for electroweak symmetry breaking. In general, the induced distribution takes a complicated form depending on what parameters scan and how they are determined. In the case that Q does not scan and the dark matter density is not determined by an astrophysical bound, the effective distribution function receives a factor of $g(x_i)^4 M^8$, as shown in Eq. (117). Assuming that parameters appearing in $g(x_i)$ but not in m_h^2 or λ_h are determined independently of M, the distribution for the parameters x, y, and M, appearing in $m_h^2 = (x - y)AM$, takes the form

$$f_{\rm eff}(x, y, M) \sim f_{\rm prior}(x, y, M) \tilde{g}(x, y)^4 M^8, \tag{119}$$

where $\tilde{g}(x, y)$ is related to $g(x_i)$ in Eq. (116). In this subsection, we illustrate how the scanning of cosmological parameters can affect electroweak symmetry breaking, using the example of Eq. (119).

Suppose that $f_{\rm prior}$ is a rather mild function of x, y, and M. The effective distribution $f_{\rm eff}$ is then determined essentially by the factor $\tilde{g}(x, y)^4 M^8$. The fact that the dark matter density does not saturate the astrophysical bound implies that the runaway to larger M due to the M^8 factor should be stopped not by the condition $T_{eq} < T_{eq,*}$ but by some other physics. The simplest possibility is that M does not scan in the multiverse. At the end of Sec. VIII B, we argued that if M is fixed then a large hierarchy is to be expected, as in Eq. (82). However, this conclusion is reversed if Λ_{QCD} scans. Since the nuclear physics boundaries depend on M only through the ratio M/Λ_{QCD}, it makes no difference to the argument on the size of the hierarchy whether M scans or Λ_{QCD} scans. Allowing x, y, and Λ_{QCD} to scan, the observer region is that of Fig. 17, with M replaced by $1/\Lambda_{QCD}$. Taking a mild distribution for Λ_{QCD} avoids a large hierarchy.[20] The amount of the little hierarchy is then determined by the factor $\tilde{g}(x, y)^4$. For example, for $\tilde{g} \sim x^m$ and a logarithmic distribution for Λ_{QCD}, $\langle z \rangle \approx 1/(4m + 2)$, so that a little hierarchy of order 10^{-2}–10^{-1} can be easily obtained for m a factor of a few. In general, with $\tilde{g}^4$ strongly preferring $x > y$, we obtain a little hierarchy. The crucial physics here is that the large scale structure observer boundary, $\Lambda \approx \rho_{NL}$, depends sensitively on the electroweak symmetry breaking parameters x and y through WIMP dark matter. The probability distribution for the cosmological constant then generates an effective distribution for x and y, which can be sharply varying.

Another possibility of preventing the runaway to large M arises if the range for the scanning of x and y are such that there is a maximum energy $M_{c,\rm max}$ above which the m_h^2 parameter is always positive. As shown in detail in Sec. VIII E, a strong probability force to large M leads to a little hierarchy, providing the nuclear observer boundaries of $v_\pm$ are able to scan, for example, via a scanning of Λ_{QCD}. Here, we simply point out that the strong force in the M direction can arise from integrating out the cosmological constant, i.e., from the M^8 factor in Eq. (119). This leads to a little hierarchy without the need for any cut factor beyond that for the cosmological constant. In the case of supersymmetry, with $d\mathcal{N} \sim M^8 dM$ ($q \approx O(10)$), the multiverse allows an improvement in the naturalness by a factor of $\approx q/\beta \approx O(100)$.

XI. CONCLUSIONS

Environmental selection on a multiverse is a radical departure from conventional methods of fundamental physics for explaining physical phenomena. Nevertheless, in light of the cosmological constant problem and the discovery of dark energy, it warrants further exploration. Will sufficient evidence emerge to convince us that the multiverse exists?

Since we cannot directly explore other universes, it may be questioned whether evidence for the multiverse can be found at all. However, theories in physics and cosmology that cannot be directly tested in the laboratory are far from new—one only has to think of unified theories and inflation. The inability to make direct laboratory tests of the new particles and interactions does not put these theories beyond the realm of science; rather, it leads to careful

[20] In fact, consistency of the setup requires that the hierarchy is not very large, since a value of M much larger than the weak scale makes the natural size of $T_{eq} \sim 8\pi M^2/M_{Pl}$ much larger than $T_{eq,*}$, implying that the astrophysical bound is saturated.

investigations of whether they make successful indirect numerical predictions for data that cannot be satisfactorily explained by other means. Thus, for any new theory or framework that cannot be directly probed, two questions are important

(a) In the absence of the new theory, is there numerical data that cannot be adequately explained using other known theories? Indeed, does the data represent a problem for existing theories?
(b) Does the new theory provide a numerical understanding of the data, thus solving the previous problem?

For inflation, the data of (a) includes the flatness and isotropy of the Universe, and the spectrum of density perturbations. So far, these highly significant cosmological problems have only been solved by inflation. Similarly, unified theories provide an understanding of gauge coupling constant unification. If the numerical understanding of the data is sufficiently precise, and thought to be better than competing theories, then the new theory may become the provisional standard view. As long as the numerical significance of its predictions is not overwhelming, one must stress the provisional nature of the understanding and the importance of seeking both further arenas in which it can be tested and new competing theories. For example, proton decay would provide further evidence for unified theories, but despite intensive searches, such evidence is still lacking.

Seeking evidence for the multiverse is no different in principle than seeking evidence for other theories that cannot be directly probed in the laboratory. In this paper, we have argued that evidence for the multiverse can be found in three different arenas: the cosmological constant, nuclear physics, and electroweak symmetry breaking. In all three cases, the conventional approach based on symmetries has not provided a numerical understanding of the data, rather in each case it leads to naturalness problems. The observed values of parameters are very close to special values that are critical for the formation of some complex structure, yet this closeness is not adequately explained by any symmetry. An observer region in the parameter space of some theory is defined by requiring the existence of certain complex structures necessary for observers. Unnaturalness results if the observer region is very small compared with the entire parameter space, or if we observe values of the parameters very close to the boundary of the observer region. We have introduced a naturalness probability P that allows a numerical evaluation of these problems. The value of P can be highly dependent on the theory T under consideration. The evidence for unnaturalness, (a), is governed by the maximal value of P_T that can be obtained in *simple* theories: the lower the maximal P_T, the more severe the naturalness problem.

In each arena the multiverse easily and generically solves the naturalness problem. Each arena is somewhat different, and we summarize our results for each below; but there are also some common features. In all three cases, environmental selection elegantly explains why our universe is not to be found in the largest region of parameter space. Furthermore, the multiverse distribution is likely to have a strong dependence on the parameters of the low energy effective theory, through the landscape of vacua of the fundamental theory, the population mechanism, integrating out parameters, and from the physics that determines the density of observers. With a strongly varying distribution, it is most probable to observe a universe close to the observer boundary, solving naturalness problems and leading to predictions. Given the current theoretical status, the multiverse appears to us to provide the most elegant and plausible prediction for several parameters, including Λ and $m_{u,d,e}$.

The cosmological constant problem is the most severe naturalness problem, with a naturalness probability in the range

$$P_\Lambda \approx (10^{-120}\text{–}10^{-60}). \tag{120}$$

The robustness of the observer boundary is particularly convincing—how could observers form in a dilute gas of inflating elementary particles? The interesting open questions are whether our universe is sufficiently close to the observer boundary, and how runaway behavior can be prevented if $T_{\rm eq}$ and Q scan. These questions, however, are secondary: a notoriously intractable problem has an elegant solution that predicts dark energy.

Astrophysicists have often remarked on the special values of parameters required for a variety of phenomena in nuclear physics. We have obtained the observer region in the 4-dimensional parameter space $m_{u,d,e}/\Lambda_{\rm QCD}$, α resulting from the stability boundaries for neutrons, deuterons, and complex nuclei, as shown in Fig. 5. We find this observer region to be small, with our universe within (10–30)% of the neutron stability boundary, so that in the standard model the naturalness probability is $P_{\rm nuc,SM} \approx (10^{-16}\text{–}10^{-4})$. Even if we could construct a theory of flavor with successful, precise predictions for the Yukawa couplings $y_{u,d,e}$ and at the same time find a theory that correctly predicted the weak scale v, there would still be a naturalness probability associated with the value of $\Lambda_{\rm QCD}$ that we estimate to be $\approx$ (0.01–0.04). Despite decades of experiments on flavor physics, with recent increasing levels of accuracy in B meson and neutrino physics, progress on a theory of flavor has been limited. The most promising theories appear to be based on flavor symmetries, with a sequential pattern of transferring symmetry breaking to successive generations of quarks and charged leptons. There are a great number of candidate theories, but none is sufficiently promising to be widely recognized as the standard. We have estimated that such a lack of progress in flavor physics, especially in the first generation masses, decreases the naturalness probability of the nuclear observer region by about an order of magnitude, leading to

$$P_{\mathrm{nuc}} \lesssim (10^{-3}\text{–}10^{-2}). \tag{121}$$

We think that this estimate is conservative. For example, in theories with Abelian flavor symmetries this estimate does not take into account that there are many simple ways of assigning charges to the three generations. Given the state of theories of flavor, we suspect that the naturalness probability of the nuclear observer boundary is less than this conservative estimate. Suppose that $m_{u,d,e}$ had each been significantly different, for example, by an order of magnitude. It would still be possible to accommodate this in theories with Abelian flavor symmetries, with about the same level of success as for the actual observed values, by changing the charges of the first generation.

The nuclear naturalness problem certainly requires more than 1% fine-tuning, and has not received sufficient recognition. If symmetries rule flavor physics, it must be viewed as purely accidental; while in the multiverse it can be viewed as a prediction. After decades of studying theories of flavor, the current status is that we are far from a convincing explanation for the masses of the electron, the up quark, and the down quark. The multiverse allows simple arguments that relate these masses to the QCD scale. For example, if the multiverse distribution strongly favors isospin restoration, then m_e and $m_d - m_u$ are expected to be close to $\delta_{\mathrm{EM}} \simeq 1.0 \pm 0.5$ MeV, the electromagnetic mass difference of the proton and neutron. It is true that such multiverse predictions require an assumption on the form of the distribution function; but such assumptions may be much simpler than the choice of flavor group, representations, and sequential symmetry breaking of the standard approach.

The multiverse predictions for $m_{u,d,e}/\Lambda_{\mathrm{QCD}}$ can be retained even if unified and/or flavor symmetries describe the overall pattern of quark and lepton masses and mixings. This allows us to preserve particular successful relations, such as $\theta_C \sim \sqrt{m_d/m_s}$ or $m_b = m_\tau$ at the unified scale. The only requirement is that the theory contain three independent scanning parameters that allow $m_{u,d,e}/\Lambda_{\mathrm{QCD}}$ to be environmentally selected.

A simple, natural theory of electroweak symmetry breaking is lacking. The natural regions of simple technicolor, supersymmetric, and composite Higgs models have been excluded. Precision measurements of electroweak observables, together with direct searches for the Higgs boson, have led to successive increases in unnaturalness, leading to

$$P_{\mathrm{EWSB}} \lesssim (10^{-2}\text{–}10^{-1}), \tag{122}$$

in simple models. This problem in the data is of type (a) and is widely appreciated, so that much recent research has focused on building models that alleviate the problem. We have shown that, no matter what the ultimate physics at mass scale M behind electroweak symmetry breaking, environmental selection on a multiverse leads very easily to $M \gg v$, with either a little or large hierarchy. This is a very robust result requiring only that some parameters of electroweak symmetry breaking scan, and that the multiverse distribution is strongly varying. A distribution $f(M) \sim M^q$ gives a large hierarchy if $q \geq 2$ and a little hierarchy (or no hierarchy) for $q < 2$, which includes the important case of M being induced by a dimensional transmutation ($q = 0$) For $q < 2$ the little hierarchy v^2/M^2 gains a factor of $\approx 1/n$ from the multiverse, where n describes the peaking of a distribution in some other parameter of the electroweak symmetry breaking sector. In multiverses where electroweak symmetry breaking is only possible if M is below some critical value $M_{c,\max}$, a little hierarchy develops from a distribution favoring large M. The size of the hierarchy v^2/M^2 is enhanced by a loop factor β, gaining a factor of $\approx \beta/q$, so that M is typically in the TeV region or larger. These results are independent of what other standard model parameters are scanning, including Yukawa couplings and Λ_{QCD}, and do not even depend on whether selection is happening at the nuclear stability boundaries, or at the phase boundary for electroweak symmetry breaking itself. Hence, we stress that the multiverse provides a very general solution to the hierarchy problem, whether little or large.

All three naturalness problems have the common feature of being solved by a multiverse distribution that makes observers typically close to an observer boundary. There may be connections between the three problems, arising from integrating out certain parameters in a more fundamental theory. For example, the probability force driving a little hierarchy may originates from the probability distribution for the cosmological constant, with WIMP dark matter acting as a mediator.

The current status of naturalness in electroweak symmetry breaking, Eq. (122), indicates that either we have not yet arrived at the right theory or that environmental selection is playing an important role. The LHC will determine the correct interpretation of Eq. (122), leading us either to a new natural theory, or to a third arena for multiverse evidence. Even if the naturalness probability is much larger than for the cosmological constant, the pervasive pattern of a finely tuned universe will make the multiverse much harder to dismiss. The importance of the LHC in this regard cannot be overemphasized: for the cosmological constant and nuclear naturalness problems we are stuck—we may not be able to experimentally determine the relevant theory beyond what we already know, in which case increasing the naturalness probability is a theoretical enterprise. However, the LHC will teach us a great deal about the theory of electroweak symmetry breaking and hence will lead to a better determination of P_{EWSB}. One possibility is that the LHC will reveal a completely natural theory that we have not been able to invent. Below, we mention a few examples where LHC data could determine a small value for P_{EWSB}.

If the LHC discovers a light Higgs boson of mass $m_{\rm Higgs}$ and sets a limit of $M_{\rm col}$ on new colored particles, then

$$P_{\rm EWSB} \lesssim 0.08\left(\frac{m_{\rm Higgs}}{150\ {\rm GeV}}\right)^2\left(\frac{2\ {\rm TeV}}{M_{\rm col}}\right)^2. \qquad (123)$$

This result is true in the vast majority of theories, although some counterexamples are known. Another possibility is that only a light Higgs boson is discovered, with a mass very close to the vacuum instability limit of the standard model. While Eq. (123) still applies as a direct consequence, there is the additional implication that the hierarchy is large with a very much smaller $P_{\rm EWSB}$. Evidence for a large hierarchy could also emerge from the discovery of split supersymmetry. Alternatively, the discovery of weak scale supersymmetry, with a light Higgs boson and a top squark heavier than ≈ 1 TeV would indicate a little hierarchy with a naturalness probability

$$P_{\rm EWSB} \lesssim 0.05\left(\frac{m_{\rm Higgs}}{130\ {\rm GeV}}\right)^2\left(\frac{1\ {\rm TeV}}{m_{\tilde{t}}}\right)^2\left(\ln\frac{M_{\rm mess}/m_{\tilde{t}}}{10}\right)^{-1}, \qquad (124)$$

where $M_{\rm mess}$ is the messenger scale of supersymmetry breaking.

The discovery of dark energy has verified a remarkable prediction of the multiverse; but this could be undermined by the discovery of an alternative solution to the cosmological constant problem. The nuclear stability boundaries imply at least 1% fine-tuning in any known theory, and the multiverse allows a striking understanding of $m_{u,d,e}$. Our current theories of electroweak symmetry breaking are unnatural; a confirmation by the LHC would solidify evidence for the multiverse in a third arena. Even with $P_{\rm nuc}$ and $P_{\rm EWSB}$ much larger than P_Λ, the three arenas together would provide significant, robust evidence for a multiverse.

ACKNOWLEDGMENTS

We thank Jesse Thaler for useful conversations. This work was supported in part by the Director, Office of Science, Office of High Energy and Nuclear Physics, of the U.S. Department of Energy under Contract No. DE-AC02-05CH11231, and in part by the National Science Foundation under Grant No. PHY-0457315. The work of Y. N. was also supported by the National Science Foundation under Grant No. PHY-0555661, by a DOE OJI, and by the Alfred P. Sloan Foundation.

[1] J. D. Barrow and F. J. Tipler, *The Anthropic Cosmological Principle* (Oxford University Press, Oxford, United Kingdom, 1986); C. J. Hogan, Rev. Mod. Phys. **72**, 1149 (2000).
[2] S. Weinberg, Phys. Rev. Lett. **59**, 2607 (1987).
[3] R. Bousso and J. Polchinski, J. High Energy Phys. 06 (2000) 006; S. Kachru, R. Kallosh, A. Linde, and S. P. Trivedi, Phys. Rev. D **68**, 046005 (2003); L. Susskind, arXiv:hep-th/0302219; M. R. Douglas, J. High Energy Phys. 05 (2003) 046.
[4] A. Vilenkin, Phys. Rev. D **27**, 2848 (1983); A. D. Linde, Mod. Phys. Lett. A **1**, 81 (1986); Phys. Lett. B **175**, 395 (1986); A. H. Guth and E. J. Weinberg, Nucl. Phys. **B212**, 321 (1983).
[5] A. Vilenkin, Phys. Rev. Lett. **74**, 846 (1995).
[6] V. Agrawal, S. M. Barr, J. F. Donoghue, and D. Seckel, Phys. Rev. D **57**, 5480 (1998).
[7] S. M. Barr and A. Khan, Phys. Rev. D **76**, 045002 (2007).
[8] N. Arkani-Hamed and S. Dimopoulos, J. High Energy Phys. 06 (2005) 073.
[9] G. F. Giudice and R. Rattazzi, Nucl. Phys. **B757**, 19 (2006).
[10] T. Banks, M. Dine, and E. Gorbatov, J. High Energy Phys. 08 (2004) 058; L. Susskind, arXiv:hep-th/0405189; M. R. Douglas, arXiv:hep-th/0405279; M. Dine, E. Gorbatov, and S. Thomas, arXiv:hep-th/0407043.
[11] R. Barbieri and G. F. Giudice, Nucl. Phys. **B306**, 63 (1988); J. R. Ellis, K. Enqvist, D. V. Nanopoulos, and F. Zwirner, Mod. Phys. Lett. A **1**, 57 (1986).
[12] S. Weinberg, Phys. Rev. D **13**, 974 (1976); **19**, 1277 (1979); L. Susskind, Phys. Rev. D **20**, 2619 (1979).
[13] E. Witten, Nucl. Phys. **B188**, 513 (1981).
[14] H. Georgi and S. L. Glashow, Phys. Rev. Lett. **32**, 438 (1974); H. Georgi, H. R. Quinn, and S. Weinberg, Phys. Rev. Lett. **33**, 451 (1974).
[15] S. Dimopoulos and H. Georgi, Nucl. Phys. **B193**, 150 (1981); N. Sakai, Z. Phys. C **11**, 153 (1981); S. Dimopoulos, S. Raby, and F. Wilczek, Phys. Rev. D **24**, 1681 (1981).
[16] B. Feldstein, L. J. Hall, and T. Watari, Phys. Rev. D **74**, 095011 (2006).
[17] M. Fukugita and T. Yanagida, Phys. Lett. B **174**, 45 (1986).
[18] N. Arkani-Hamed, S. Dimopoulos, and S. Kachru, arXiv: hep-th/0501082.
[19] C. J. Hogan, Phys. Rev. D **74**, 123514 (2006).
[20] S. R. Beane, K. Orginos, and M. J. Savage, Nucl. Phys. **B768**, 38 (2007); see also S. Weinberg, *The Quantum Theory of Fields Volume II* (Cambridge University Press, Cambridge, United Kingdom, 1996).
[21] W. M. Yao *et al.* (Particle Data Group), J. Phys. G **33**, 1 (2006).
[22] C. D. Froggatt and H. B. Nielsen, Nucl. Phys. **B147**, 277 (1979).
[23] A. Linde, J. Cosmol. Astropart. Phys. 06 (2007) 017; J. Garriga, D. Schwartz-Perlov, A. Vilenkin, and S. Winitzki, J. Cosmol. Astropart. Phys. 01 (2006) 017; R. Bousso, Phys. Rev. Lett. **97**, 191302 (2006).

[24] H. Martel, P. R. Shapiro, and S. Weinberg, Astrophys. J. **492**, 29 (1998); R. Bousso, R. Harnik, G. D. Kribs, and G. Perez, Phys. Rev. D **76**, 043513 (2007).
[25] F. Denef and M. R. Douglas, J. High Energy Phys. 05 (2004) 072.
[26] R. Barbieri and A. Strumia, arXiv:hep-ph/0007265.
[27] R. Kitano and Y. Nomura, Phys. Rev. D **73**, 095004 (2006).
[28] G. F. Giudice and A. Romanino, Nucl. Phys. **B699**, 65 (2004); **B706**, 487(E) (2005).
[29] D. B. Kaplan and H. Georgi, Phys. Lett. **136B**, 183 (1984); D. B. Kaplan, H. Georgi, and S. Dimopoulos, Phys. Lett. **136B**, 187 (1984).
[30] N. Arkani-Hamed, A. G. Cohen, and H. Georgi, Phys. Lett. B **513**, 232 (2001).
[31] R. Contino, Y. Nomura, and A. Pomarol, Nucl. Phys. **B671**, 148 (2003).
[32] N. Cabibbo, L. Maiani, G. Parisi, and R. Petronzio, Nucl. Phys. **B158**, 295 (1979); T. Hambye and K. Riesselmann, Phys. Rev. D **55**, 7255 (1997).
[33] G. Altarelli and G. Isidori, Phys. Lett. B **337**, 141 (1994); J. A. Casas, J. R. Espinosa, and M. Quiros, Phys. Lett. B **342**, 171 (1995).
[34] M. Tegmark, A. Aguirre, M. Rees, and F. Wilczek, Phys. Rev. D **73**, 023505 (2006).