Next-to-leading order perturbative QCD corrections to baryon correlators in matter

S. Groote,[1,2] J. G. Körner,[1] and A. A. Pivovarov[1,3]

[1]*Institut für Physik, Johannes-Gutenberg-Universität, Staudinger Weg 7, D-55099 Mainz, Germany*
[2]*Teoreetilise Füüsika Instituut, Tartu Ülikool, Tähe 4, 51010 Tartu, Estonia*
[3]*Institute for Nuclear Research of the Russian Academy of Sciences, Moscow 117312, Russia*
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We compute the next-to-leading order (NLO) perturbative QCD corrections to the correlators of nucleon interpolating currents in relativistic nuclear matter. The main new result is the calculation of the $\mathcal{O}(\alpha_s)$ perturbative corrections to the coefficient functions of the vector quark condensate in matter. This condensate appears in matter due to the violation of Lorentz invariance. The NLO perturbative QCD corrections turn out to be large which implies that the NLO corrections must be included in a sum rule analysis of the properties of both bound nucleons and relativistic nuclear matter.

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I. INTRODUCTION

The study of bound states in QCD is a difficult problem. With more than 30 years of research it is clear that the most promising approach to obtain quantitative information on the properties of hadrons are very likely lattice techniques, in particular, since both computer power and computational methods advanced dramatically since their first introduction in the early seventies. Lattice results are now available in many hadronic channels and further research is being actively pursued [1]. Nevertheless analytical nonlattice techniques can be used to verify at least the consistency of some models for hadron description and their predictions. The QCD sum rule analysis is based on the operator product expansion (OPE) and serves as a rigorous framework for many calculations in the theory of hadrons [2–5]. QCD sum rules are also useful for testing some model dependent approaches [6–8] such as the MIT quark-bag model [9]. A further important problem is the quantitative description of the properties of bound nucleons and relativistic nuclear matter within QCD.

The proton is the most abundant strongly interacting particle on Earth. It has played an important part in particle physics since long ago. In the theory of strong interactions the proton is a bound state of quarks and gluons with three valence quarks fixing its discrete quantum numbers. It has been intensively studied during the last 50 years. At present one cannot directly compute the properties of the proton analytically from QCD even for an isolated proton. The technique of the QCD sum rules provides a powerful tool for the phenomenological analysis of the proton. The properties of baryons have been successfully described within this approach when account is taken of the leading vacuum condensates [10–12]. The ever improving accuracy of experimental data requires improvements also in the theoretical description. Primarily this means that one has to account for the perturbative QCD corrections to the coefficient functions of the OPE for baryonic correlators. The next-to-leading order (NLO) perturbative QCD corrections have been calculated for the coefficient functions of the OPE for the unit operator and the scalar quark condensate in the massless quark limit in [13]. The perturbative corrections have been found to be large. The results for the unit operator were generalized to the massive quark case in [14] where again large perturbative corrections were found. The properties of the proton in vacuum are well studied in QCD although there is room for improvements in the numerical accuracy.

Protons are traditionally used as targets in accelerator experiments as e.g. in the electron scattering on iron at DESY. To analyze the data obtained in these experiments one needs to know the properties of the protons bound in nuclei, or more generally of the nuclear medium. Thus there is a considerable interest in computing the parameters of the proton medium. The most obvious reason is that protons are part of the nuclei which serve as targets in accelerator experiments. The scattering on nuclei is different from the scattering on the proton, and this is important for the interpretation of the data. One of the best known examples is the EMC effect (e.g. [15]). One of the possible theoretical approaches is to use an effective theory where the proton in the medium is treated as an effective particle [16–18]. Another approach is to analyze the properties of nucleon medium within the QCD sum rule approach in [19]. In this approach the problem of an accurate determination of the parton distributions in nuclei ultimately requires the calculation of perturbative corrections to the OPE in the framework of QCD sum rules.

In the present paper we compute NLO perturbative QCD corrections to the correlators of baryon interpolating currents in matter. We focus on the calculation of the $\mathcal{O}(\alpha_s)$ perturbative corrections to the coefficient functions of the bilinear quark operators that may lead to the emergence of nonvanishing condensates upon averaging over the appropriate physical states. We present new results for the coefficient functions of the quark operators in the vector representation $(1/2,\ 1/2)$ of the Poincaré group: NLO accuracy is achieved in the expansion in the coupling constant of QCD. When averaged over the ground state of matter one obtains nonvanishing values of the quark operators in the vector representation (1/2, 1/2) when studying the properties of the nucleon medium within the QCD sum rules approach. The presence of matter violates Lorentz invariance and thus allows for the appearance of a vector condensate averaged over the matter states. The NLO perturbative QCD corrections to the coefficient functions turn out to be large in the $\overline{\rm MS}$ scheme. This means that one must account for the perturbative corrections in applications of sum rules analysis of baryon properties in matter.

II. BASIC EXPRESSIONS FOR THE ANALYSIS

The formulation of the OPE analysis is standard by now. In accordance with the QCD sum rule approach, we shall calculate the operator product expansion of two interpolating currents $J(x)$ which have a nonvanishing overlap with the state of interest. The OPE for the quantity

$$T(q) = i \int d^4x e^{iqx} T\{J(x)\bar{J}(0)\} \tag{1}$$

is performed by means of Wilson's operator product expansion. For the analysis of the properties of isolated hadrons in the vacuum one then averages the operator product over the ground state of QCD or the physical vacuum to obtain the correlation function of two interpolating currents $J(x)$

$$\Pi(q) = i \int d^4x e^{iqx} \langle 0|T\{J(x)\bar{J}(0)\}|0\rangle \tag{2}$$

in vacuum. The assumption of the QCD sum approach is that the vacuum expectation values of the local operators that appear in the OPE (the so-called condensates) are nonzero. The calculations are done in perturbative QCD and therefore must be performed in the region $-q^2 \geq 1\ \text{GeV}^2$ where perturbative QCD is valid. However, in this region, the effective strong interaction constant α_s is not very small numerically [20]. This forces one to calculate the coefficient functions of the operator product expansion in perturbation theory at least up to NLO in order to have sensible results. One more reason is of course the general property of perturbation theory that only at this order of the perturbative expansion can one reliably fix the renormalization group scale μ which determines the numerical values of the coupling constant and condensates within the OPE. It turns out that the NLO corrections are rather large in many hadronic channels. Even in the case of the correlators of the classical quark-antiquark currents the perturbative QCD corrections in the standard $\overline{\rm MS}$ scheme are not small [21]. Thus, the calculations of $\Pi(q)$ should be done at least at NLO in α_s to have the precision required by modern applications.

In the original paper of Ioffe the current

$$\eta = \epsilon_{abc}(u_a^T C\gamma_\mu u_b)\gamma_5\gamma^\mu d_c \tag{3}$$

was used to analyze the properties of the proton [10]. u and d are light quark fields and C is the charge conjugation matrix with the properties $C\gamma_\mu^T C = \gamma_\mu$ and $C = -C^{-1} = -C^T = -C^\dagger$. By using a Fierz transformation the Ioffe current $\eta(x)$ can also be rewritten as a linear combination of the two current operators O_1 and O_2,

$$\eta(x) = 2(O_1 - O_2) \tag{4}$$

where

$$O_1 = \epsilon_{abc}(u_a^T C d_b)\gamma_5 u_c, \qquad O_2 = \epsilon_{abc}(u_a^T C\gamma_5 d_b)u_c. \tag{5}$$

In fact, the operators O_1 and O_2 form a complete basis for the lowest dimension interpolating currents of the proton with no derivative couplings. We generalize Ioffe's current by writing a linear combination of the operators O_1, O_2 of the form

$$J(x) = O_1 + tO_2 \tag{6}$$

where t is a mixing parameter.

The topology needed in the calculation of the LO correlator [see Fig. 1(a)] falls into the category of the well-known sunset diagrams. These only contain lines that connect two vertices [22,23]. Such a topology also appears in the effective gluon low energy correlator for heavy quarks below the production threshold that leads to the decay of heavy quarkonia into gluon/photons [24,25]. Sunset diagrams can be calculated very efficiently in configuration space. The NLO perturbative QCD corrections are of two types. The first correction is a propagator-type correction which is not difficult to compute [cf. Figure 1 (b)]. The second correction [Fig. 1(c)] comes from the diagrams of the fish type and involves the calculation of an irreducible two-loop subdiagram.

For completeness, and for the convenience of the reader, we present general expressions for the corrections in configuration space which can be used for a variety of interpolating currents. The efficiency of the configuration space approach has already been proven in computing NLO corrections to pentaquark correlators [26].

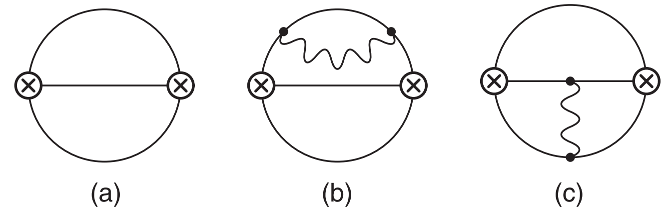

FIG. 1. LO (a) and NLO propagator-type (b) as well as fish-type (c) corrections of the baryonic two-point correlator.

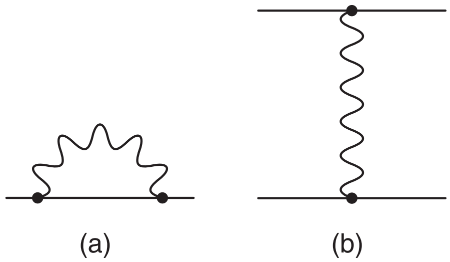

FIG. 2. Propagator (a) and dipropagator (b) correction with open Dirac indices.

First we list the NLO correction to the fermion propagator [see Fig. 2(a)] in configuration space. One has

$$S(x)|_{\rm NLO} = S(x)|_{\rm LO}\left\{1 - C_F\frac{\alpha_s}{4\pi}\frac{1}{\varepsilon}(\mu_X^2x^2)^\varepsilon\right\}, \qquad (7)$$

where $C_F = (N_c^2 - 1)/2N_c$ is the Casimir operator of the color group $SU(N_c)$ ($N_c = 3$ for QCD) and $S(x)|_{\rm LO} = \gamma^\mu x_\mu F_0(x^2)$ is the LO fermion propagator where $F_0(x^2)$ is defined in the Euclidean domain and is given by

$$F_0(x^2) = \frac{-i\Gamma(2-\varepsilon)}{2\pi^{2-\varepsilon}(x^2)^{2-\varepsilon}}. \qquad (8)$$

$\Gamma(z)$ is Euler's gamma-function. The space-time dimension is parametrized by $D = 4 - 2\varepsilon$ throughout. Written in terms of $F_0(x^2)$ one has

$$S(x)|_{\rm NLO} = F_0(x^2)\gamma^\mu x_\mu\left\{1 - C_F\frac{\alpha_s}{4\pi}\frac{1}{\varepsilon}(\mu_X^2x^2)^\varepsilon\right\}. \qquad (9)$$

The renormalization scale μ_X is the appropriate one for calculations in configuration space. This choice avoids the appearance of $\ln(4\pi)$ and γ_E (Euler constant) factors in configuration space calculations. The relation of μ_X and the usual renormalization scale μ of the $\overline{\rm MS}$-scheme is given by $\mu_X = \mu e^{\gamma_E}/2$. Note that the NLO fermion propagator is gauge dependent even if the complete calculation is gauge invariant. In our calculation we have used diagonal or Feynman gauge.

The next quantity needed in our calculation is the NLO correction to the propagator of a pair of fermions. We call this diquark propagator a dipropagator for short and denote it by $S_2(x)$. The dipropagator is given in terms of a two-loop amplitude with open Dirac indices [see Fig. 2(b)] which requires a genuine two-loop calculation. The result for the dipropagator up to NLO reads [26]

$$\begin{aligned} S_2(x)|_{\rm NLO} = F_0(x^2)^2\Big\{&\gamma^\mu x_\mu \otimes \gamma^\nu x_\nu + t^a\otimes t^a\frac{\alpha_s}{4\pi}\frac{1}{\varepsilon} \\ &\times(\mu_X^2x^2)^\varepsilon(\gamma^\mu\otimes\gamma^\nu(a_1x_\mu x_\nu + b_1x^2g_{\mu\nu}) \\ &+ a_3\Gamma_3^{\alpha\beta\mu}\otimes\Gamma_3{}^\nu{}_{\alpha\beta}x_\mu x_\nu)\Big\} \end{aligned} \qquad (10)$$

where the coefficients a_1, b_1 and a_3 are given by

$$a_1 = -1 - \tfrac{11}{2}\varepsilon, \qquad b_1 = -1 - \tfrac{1}{2}\varepsilon, \qquad a_3 = -\tfrac{1}{2} - \tfrac{1}{4}\varepsilon,$$

and where

$$\Gamma_3^{\mu\alpha\nu} = \tfrac{1}{2}(\gamma^\mu\gamma^\alpha\gamma^\nu - \gamma^\nu\gamma^\alpha\gamma^\mu). \qquad (11)$$

We use the standard notation $\otimes$ for the direct product of two Dirac or color matrices. The generators of the color group algebra t^a appearing in the above expression are normalized by the condition $\mathrm{tr}(t^at^b) = \delta^{ab}/2$. Equations (7) and (10) allow one to calculate the NLO corrections to n-quark(antiquark) current correlators of any composition using purely algebraic algorithms without having to compute any integrals. For example, the form (10) has been used in [26] to compute the radiative corrections to the pentaquark current correlator.

The above results need to be renormalized. The renormalization can be done in configuration space. To renormalize the single propagator one can use multiplicative renormalization. The only ingredient needed is the wave function renormalization constant of the fermion. The diagrams involving dipropagators result in mixing of the operators under renormalization. Mixing is taken into account through a subtraction of the corresponding vertex divergences generated by the operator that can admix to the initial current. The general formula reads

$$\begin{aligned} \psi_i\otimes\psi_j|_R^{\rm IR} &= \psi_i\otimes\psi_j \\ &\quad - \frac{\alpha_s}{4\pi\varepsilon}\left(1_{ii'}\otimes 1_{jj'} + \frac{1}{4}\sigma_{ii'}^{\alpha\beta}\otimes\sigma_{jj'}^{\alpha\beta}\right)\psi_{i'}\psi_{j'} \\ &= \psi_i\otimes\psi_j \\ &\quad - \frac{\alpha_s}{4\pi\varepsilon}\left(\psi_i\otimes\psi_j + \frac{1}{4}\sigma_{ii'}^{\alpha\beta}\psi_{i'}\otimes\sigma_{jj'}^{\alpha\beta}\psi_{j'}\right), \end{aligned} \qquad (12)$$

where i and j are color indices and ψ stands for either the up or down quark fields. For definiteness we define our $\sigma^{\alpha\beta} = i/2[\gamma^\alpha, \gamma^\beta]$. The results are again given in diagonal or Feynman gauge where we emphasize again that the complete result is gauge independent. Note that the part proportional to the product of σ-matrices is gauge independent.

Before presenting the results of our calculation we want to remark on the renormalization group properties of the operators $O_{1,2}$ defined in Eq. (5). They represent a complete basis of the operators mixing under renormalization and suffice to perform the calculation of the baryonic correlators. As mentioned before the operators $O_{1,2}$ form a basis of operators of lowest dimension for the interpolating currents of the nucleon. Since their anomalous dimensions are identical at this order they satisfy the same renormalization group evolution. One has

$$\mu^2\frac{d}{d\mu^2}O_{1,2}(\mu) = \frac{\alpha_s}{2\pi}O_{1,2}(\mu).$$

Note that the numerical value of the anomalous dimension is such that the product $\sqrt{m(\mu)}O_{1,2}(\mu)$ with $m(\mu)$ is renormalization group invariant at this order of QCD. At NLO the operators mix. The two-loop anomalous dimensions have been computed in Ref. [27]. These ingredients allow one to compute the necessary correlator functions.

III. CORRELATOR INCLUDING THE SCALAR CONDENSATE

In the OPE one computes the contributions of local operators to the correlator function. In case of the vacuum correlators only Lorentz scalars contribute which means that the only nonvanishing condensate is of the form $\langle\bar{q}q\rangle$. The standard vacuum condensate contributions have been calculated before in Ref. [13] including the $\mathcal{O}(\alpha_s)$ corrections. They read

$$i\int dx e^{iqx}\langle T\{J(x)\bar{J}(0)\}\rangle = \not{q}\Pi_q(q^2) + \Pi_m(q^2) \tag{13}$$

with

$$\Pi_q(q^2) = -\frac{1}{8(4\pi)^4}(5t^2+2t+5)Q^4\ln\left(\frac{Q^2}{\mu^2}\right)\times\left\{1+\frac{\alpha_s}{\pi}\left(\frac{71}{12}-\frac{1}{2}\ln\left(\frac{Q^2}{\mu^2}\right)\right)\right\}$$

and

$$\Pi_m(q^2) = \frac{\langle\bar{\psi}\psi\rangle}{4(4\pi)^2}Q^2\ln\left(\frac{Q^2}{\mu^2}\right)(1-t)\times\left(5+7t+(3+5t)\frac{3\alpha_s}{2\pi}\right). \tag{14}$$

Here $Q^2=-q^2$, $\langle\bar{\psi}\psi\rangle = \langle\bar{u}u\rangle = \langle\bar{d}d\rangle$.

In matter a new type of condensate $\langle M|\bar{q}\gamma_\mu q|M\rangle \neq 0$ appears where $|M\rangle$ is the ground state of the matter. The quantity $\langle M|\bar{q}\gamma_\mu q|M\rangle$ violates Lorentz invariance. This is expected since the matter itself fixes a special frame. Therefore one needs to account for a new operator in the OPE for the baryonic currents. Including the new vector operator one now has

$$T\{J(x)J(0)\} = C_{\rm I}(x^2) + C_{\bar{q}q}(x^2)\{\bar{q}q\} + C^{\mu}_{\bar{q}\gamma_\mu q}(x)\{\bar{q}\gamma_\mu q\} \tag{15}$$

where the coefficient function $C^{\mu}_{\bar{q}\gamma_\mu q}(x)$ of the vector operator $\{\bar{q}\gamma_\mu q\}$ is a four-vector. We calculate the coefficient function $C^{\mu}_{\bar{q}\gamma_\mu q}(x)$ at NLO accuracy by using again configuration space techniques which were developed in a different setting, namely, the NLO analysis of pentaquark sum rules [26].

First we check on known results using configuration space techniques. We split the result for the noncondensate contribution into two parts which reflect the two ways of how the Dirac indices have been contracted. One has

$$\begin{aligned}\Pi^{\rm o}(x) &= -4N_c!(F_0(x^2))^3(1+t^2)\\ &\quad\times\left\{1+\frac{\alpha_s}{\pi}\left((\mu_x^2x^2)^\varepsilon\left(\frac{1}{\varepsilon}+\frac{7}{3}\right)-\frac{1}{\varepsilon}\right)\right\}x^2\not{x}\\ &= -4N_c!(F_0(x^2))^3(1+t^2)\\ &\quad\times\left\{1+\frac{\alpha_s}{\pi}\left(\frac{7}{3}+\ln(\mu_x^2x^2)\right)\right\}x^2\not{x},\end{aligned} \tag{16}$$

and

$$\begin{aligned}\Pi^{\rm x}(x) &= -N_c!(F_0(x^2))^3(1+t)^2\\ &\quad\times\left\{1+\frac{\alpha_s}{\pi}\left((\mu_x^2x^2)^\varepsilon\left(\frac{1}{\varepsilon}+\frac{7}{3}\right)-\left(\frac{1}{\varepsilon}+\frac{7}{6}\right)\right)\right\}x^2\not{x}\\ &= -N_c!(F_0(x^2))^3(1+t)^2\\ &\quad\times\left\{1+\frac{\alpha_s}{\pi}\left(\frac{7}{6}+\ln(\mu_x^2x^2)\right)\right\}x^2\not{x}\end{aligned} \tag{17}$$

for the direct and crossover part of the Wick contraction, respectively, as shown symbolically in Fig. 3. The singular parts $\propto 1/\varepsilon$ in each of the contributions cancel against counter terms in the course of renormalization performed in Eqs. (16) and (17). Note the different dependence on the mixing parameter t in the two parts of $\Pi(x)=\Pi^{\rm o}(x)+\Pi^{\rm x}(x)$. Note also that our techniques allow for the calculation of all condensate corrections but require new modules as indicated in Fig. 4. These modules are relevant for the calculation of the coefficient functions of the scalar operator $\{\bar{\psi}\psi\}$ and the vector operator $\{\bar{\psi}\gamma^\mu\psi\}$.

For the scalar condensate we obtain the results

$$\begin{aligned}\Pi_S^{\rm o}(x) &= 4N_c!(F_0(x^2))^2(1-t)(1+t)\\ &\quad\times\left\{1+\frac{\alpha_s}{\pi}(\mu_x^2x^2)^\varepsilon\right\}x^2\langle\bar{\psi}\psi\rangle/12,\\ \Pi_S^{\rm x}(x) &= N_c!(F_0(x^2))^2(1-t)\\ &\quad\times\left\{1+3t+\frac{\alpha_s}{2\pi}(\mu_x^2x^2)^\varepsilon(1+7t)\right\}x^2\langle\bar{\psi}\psi\rangle/12,\end{aligned}$$

where we have assumed $\langle\bar{u}u\rangle = \langle\bar{d}d\rangle = \langle\bar{\psi}\psi\rangle$. Adding up the two contributions gives

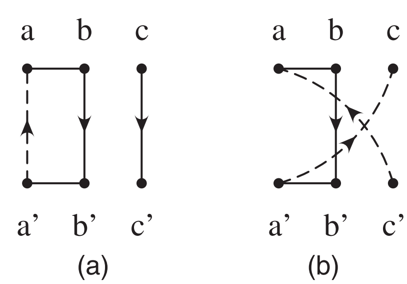

FIG. 3. Ordinary (a) and crossover part (b) of the Wick contraction for the operators $O_{1,2}$ in a symbolic representation. The arrows indicate the direction for the quark current within the correlator, the dashed lines are inverted by using transposition and charge conjugation.

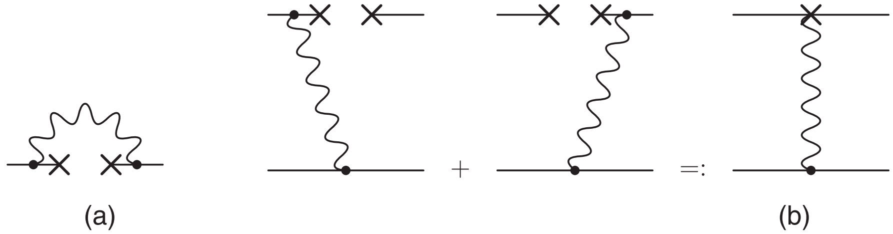

FIG. 4. New modules for the correlator corrections including condensates: condensate correction (a) and condensate-propagator correction consisting of two diagrams (b).

$$\Pi_S(x) = N_c!(F_0(x^2))^2(1-t) \times \left\{5 + 7t + \frac{3\alpha_s}{2\pi}(\mu_x^2 x^2)^\epsilon(3+5t)\right\} x^2\langle\bar\psi\psi\rangle/12. \tag{18}$$

These results are finite and need not be renormalized. After a Fourier transformation the results are in agreement with the results for $\Pi_m(q^2)$ in Eq. (14) obtained by direct integration in momentum space. The corresponding spectral density that appears in the integrand of the dispersion representation reads

$$\rho_S(s) = \frac{N_c!s}{2(4\pi)^2}(1-t)\left\{5 + 7t + \frac{3\alpha_s}{2\pi}(3+5t)\right\}\langle\bar\psi\psi\rangle/12 \tag{19}$$

with $s = q^2 > 0$. The result is proportional to $(1-t)$ and thus vanishes for $t = 1$. The vanishing of the spectral density at $t = 1$ is a general property of the correlator function related to the chiral structure of the current. Indeed, in the massless limit where the chiral symmetry is exact the contribution of the scalar quark condensate vanishes in all orders of perturbation theory.

IV. CORRELATOR INCLUDING THE VECTOR CONDENSATE

We now present our new results for the vector condensate. As mentioned before the vector condensate violates Lorentz invariance as a manifestation of the presence of matter. It can be calculated by using the same set of diagrams as before (see Fig. 5). For the part proportional to the vector quark operator in the OPE we write

$$\Pi_V(x) = \frac{\{\bar\psi\gamma^\mu\psi\}}{12}(\not{x}x_\mu A_V(x^2) + x^2\gamma_\mu B_V(x^2)) \tag{20}$$

where $A_V(x^2) = A_V^{\rm o}(x^2) + A_V^{\rm x}(x^2)$, $B_V(x^2) = B_V^{\rm o}(x^2) + B_V^{\rm x}(x^2)$ and find the renormalized coefficient functions

$$\begin{aligned} A_V^{\rm o}(x^2) &= -8N_c!(F_0(x^2))^2(1+t^2)\left\{1 + \frac{\alpha_s}{\pi}\left(\ln(\mu_x^2x^2) + \frac{5}{3}\right)\right\},\\ B_V^{\rm o}(x^2) &= -4N_c!(F_0(x^2))^2(1+t^2)\left\{1 + \frac{\alpha_s}{\pi}(\ln(\mu_x^2x^2) + 1)\right\},\\ A_V^{\rm x}(x^2) &= -2N_c!(F_0(x^2))^2(1+t)^2\left\{1 + \frac{\alpha_s}{\pi}\left(\ln(\mu_x^2x^2) + \frac{1}{2}\right)\right\},\\ B_V^{\rm x}(x^2) &= -N_c!(F_0(x^2))^2(1+t)^2\left\{1 + \frac{\alpha_s}{\pi}\left(\ln(\mu_x^2x^2) - \frac{1}{6}\right)\right\}. \end{aligned} \tag{21}$$

Note that the curly bracket notation $\{\bar\psi\gamma^\mu\psi\}$ refers to an operator before averaging. We emphasize again that the vacuum expectation value of the vector operator vanishes, i.e. $\langle\bar\psi\gamma^\mu\psi\rangle = 0$, while in matter one has $\langle M|\bar\psi\gamma^\mu\psi|M\rangle \neq 0$.

In momentum space the correlator function is expanded as

$$\Pi_V(q) = \frac{\{\bar\psi\gamma^\mu\psi\}}{12}(\not{q}q_\mu A_V(q^2) + q^2\gamma_\mu B_V(q^2)). \tag{22}$$

For the spectral density one obtains $\rho_{A_V}(s) = \rho_{A_V^{\rm o}}(s) + \rho_{A_V^{\rm x}}(s)$ and $\rho_{B_V}(s) = \rho_{B_V^{\rm o}}(s) + \rho_{B_V^{\rm x}}(s)$ where

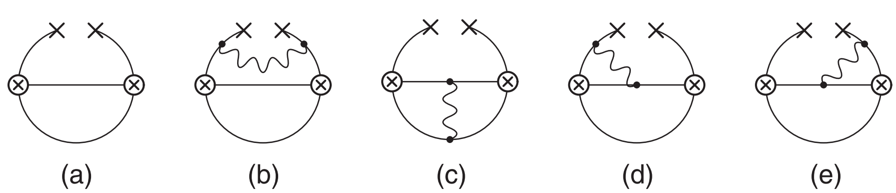

FIG. 5. LO contribution (a) and NLO contributions (b–e) to the correlation function including a (scalar or vector) condensate.

$$\rho_{A_V^o}(s) = \frac{4N_c!}{3(4\pi)^2}(1+t^2)\left\{1+\frac{\alpha_s}{\pi}\left(\frac{7}{2}+\ell\right)\right\},$$
$$\rho_{B_V^o}(s) = -\frac{8N_c!}{3(4\pi)^2}(1+t^2)\left\{1+\frac{\alpha_s}{\pi}\left(\frac{15}{4}+\ell\right)\right\},$$
$$\rho_{A_V^x}(s) = \frac{N_c!}{3(4\pi)^2}(1+t)^2\left\{1+\frac{\alpha_s}{\pi}\left(\frac{7}{3}+\ell\right)\right\},$$
$$\rho_{B_V^x}(s) = -\frac{2N_c!}{3(4\pi)^2}(1+t)^2\left\{1+\frac{\alpha_s}{\pi}\left(\frac{31}{12}+\ell\right)\right\} \tag{23}$$

and $\ell = \ln(\mu^2/s)$. As an example we take Ioffe's current Eqs. (3) and (4) which is obtained from our general current (6) by setting $t = -1$ and multiplying by a factor of 2. Including the noncondensate contribution and the scalar and vector condensate contributions the spectral density is now given by

$$\rho_\eta(s) = \frac{4}{(4\pi)^4}\not{q}s^2\left\{1+\frac{\alpha_s}{\pi}\left(\frac{71}{12}+\ell\right)\right\}$$
$$-\frac{4}{(4\pi)^2}\{\bar\psi\psi\}s\left\{1+\frac{3\alpha_s}{2\pi}\right\}$$
$$-\frac{1}{3\pi^2}\{\bar\psi\gamma^\mu\psi\}\left[\not{q}q_\mu\left\{1+\frac{\alpha_s}{\pi}\left(\frac{7}{2}+\ell\right)\right\}\right.$$
$$\left.+2s\gamma_\mu\left\{1+\frac{\alpha_s}{\pi}\left(\frac{15}{4}+\ell\right)\right\}\right]. \tag{24}$$

Our results for the vector condensate confirm the LO results given in [28,29]. The NLO corrections to the vector condensate are new. One can see that they are numerically large in the $\overline{\rm MS}$ renormalization scheme at the standard value $\mu = \sqrt{s}$ for the renormalization scale. The numerical values of the condensates $\langle M|\bar q q|M\rangle$ and $\langle M|\bar q\gamma_\mu q|M\rangle$ are nonperturbative parameters of QCD that are built into the sum rule analysis. Following [28,29] we take $\langle M|\bar q\gamma_\mu q|M\rangle = u_\mu\frac{3}{2}\rho_N$ where u_μ is the four-velocity of relativistic nuclear matter and ρ_N is its density. For the contribution of the vector condensate we obtain

$$\rho_\eta^V(s) = -\frac{1}{2\pi^2}\rho_N\left[\not{q}(qu)\left\{1+\frac{\alpha_s}{\pi}\left(\frac{7}{2}+\ell\right)\right\}\right.$$
$$\left.+2s\not{u}\left\{1+\frac{\alpha_s}{\pi}\left(\frac{15}{4}+\ell\right)\right\}\right]. \tag{25}$$

Canonically QCD sum rules are analyzed at a low scale of the order of 1 GeV. The running of the coupling $\alpha_s(M_Z) = 0.1176 \pm 0.002$ [30] to this low scale $\mu = 1$ GeV results in $\alpha_s(1\ \text{GeV})/\pi = 0.15 \pm 0.1$. With this value of the coupling constant the NLO correction amounts up to 60% of the leading order result.

The inclusion of terms proportional to the light quark masses $m_{u,d}$ do not substantially change the quantitative results as the masses of light quarks are small [31]. Even for exotic strange matter the results are still valid since the s-quark mass is still reasonably small. Two recent $O(\alpha_s^4)$ QCD sum rule determinations give $m_s(2\ \text{GeV}) = 105 \pm 6 \pm 7$ MeV [32] and $m_s(2\ \text{GeV}) = 92 \pm 9$ MeV [33]. These numbers are rather close though somewhat smaller than previous results based on τ decay data [34]. Even if there is not much hope to detect strange matter on Earth, strange matter can appear as an intermediate state in the high energy collisions of heavy ions. In view of such possible applications the inclusion of strange quark mass corrections is rather topical. We mention that the contribution of four-quark operators are also important [35]. Their contribution can be accounted for in the factorization approximation. The result reads

$$\Pi_{4q}(q^2) = -\frac{\langle\bar\psi\psi\rangle^2}{24Q^2}\left(5\left\{1+\frac{\alpha_s}{\pi}\left(\frac{61}{15}L_Q-\frac{511}{90}\right)\right\}\right.$$
$$+2t\left(1+\frac{\alpha_s}{\pi}\left(\frac{5}{3}L_Q-\frac{224}{9}\right)\right\}$$
$$\left.-7t^2\left\{1+\frac{\alpha_s}{\pi}\left(\frac{47}{21}L_Q+\frac{325}{126}\right)\right\}\right) \tag{26}$$

where $L_Q = \ln(\frac{Q^2}{\mu^2})$. The accuracy of the factorization approximation for four-quark operators has been checked in [36] where the configuration space technique was heavily used (see also [37,38]).

V. CONCLUSIONS

To summarize, we found an important correction to the correlator of baryon currents in media which is needed in the analysis of the properties of relativistic nuclear matter and bound nucleons within the QCD sum rule approach. The correction is given by the NLO contribution of QCD perturbation theory expansion to the coefficient function of the vector condensate in the OPE of the baryon currents and is not small. It amounts to 60% of the leading order term at a low energy scale relevant to the analysis of nuclear matter and therefore should be taken into account in phenomenological applications.

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