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Possible complex annihilation and $B \to K\pi$ direct CP asymmetry

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We point out that a strong phase from the penguin annihilation channel in B meson decays is sensitive to the treatment of subleading terms with physical b quark mass m_b. In the limit of infinite heavy-quark mass, both the soft-collinear effective theory and the perturbative QCD approach based on the k_T factorization theorem agree that the strong phase is suppressed. For finite heavy-quark mass, the two approaches predict different results, which are related to the treatment of subleading terms. This is illustrated by taking a toy model for soft-collinear effective theory, in which a small quantity, suppressed by $O(\Lambda/m_b)$, with Λ being a hadronic scale, is kept in the denominators of internal particle propagators. This model can generate a sizable strong phase to accommodate the data of the $B^0 \to K^{\mp}\pi^{\pm}$ direct CP asymmetry.

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The effect of scalar penguin annihilation on charmless nonleptonic B meson decays has attracted intensive attention. This power-suppressed contribution is chirally enhanced, i.e., proportional to μ_P/m_b in $B \to PP$ decays, where μ_P is the chiral scale associated with the pseudoscalar meson P and m_b the b quark mass. Since it involves end-point singularities, it was described by a free parameter $X_A = \ln(m_b/\Lambda)[1 + \rho_A \exp(i\phi_A)]$ in QCD-improved factorization (QCDF) [1], with Λ being a hadronic scale and ρ_A and ϕ_A varied arbitrarily within some artificially specified ranges. In order to fit data such as the $B^0 \to K^{\mp}\pi^{\pm}$ direct CP asymmetry $A_{CP}(B^0 \to K^{\mp}\pi^{\pm})$, ϕ_A must take a sizable value. On the other hand, the contribution from scalar penguin annihilation has been found to be almost imaginary in the perturbative QCD (PQCD) approach based on the k_T factorization theorem [2,3], and the resultant strong phase leads to a prediction consistent with the measured $A_{CP}(B^0 \to K^{\mp}\pi^{\pm})$. In soft-collinear effective theory (SCET) at leading power [4–6], a nonperturbative complex charming penguin was introduced to accommodate the data of $A_{CP}(B^0 \to K^{\pm}\pi^{\mp})$ without considering the annihilation amplitude. In the recent SCET formalism with the zero-bin subtraction [7], the annihilation contribution becomes factorizable and has been concluded to be almost real [8].

The motivation of this paper is to explain the opposite theoretical observations on the almost imaginary or almost real penguin annihilation derived in PQCD and in SCET. We shall first point out that the comparison of the measured $A_{CP}(B^{\pm} \to K^{\pm}\pi^0)$ and $A_{CP}(B^{\pm} \to K^{\pm}\rho^0)$ indicates an imaginary penguin annihilation amplitude [9,10]: The decays $B^{\pm} \to K^{\pm}\pi^0$ ($B^{\pm} \to K^{\pm}\rho^0$) involve a $B \to P$ ($B \to V$) transition, so the penguin emission amplitude is proportional to the constructive (destructive) combination of the Wilson coefficients $a_4 + (-)2(\mu_K/m_b)a_6$, μ_K being the chiral scale associated with the kaon. The annihilation effect is then less conspicuous in the former than in the latter. If the penguin annihilation is real, both decays will exhibit small direct CP asymmetries, i.e., $A_{CP}(B^{\pm} \to K^{\pm}\pi^0) \approx A_{CP}(B^{\pm} \to K^{\pm}\rho^0) \approx 0$. If imaginary, $A_{CP}(B^{\pm} \to K^{\pm}\rho^0)$ will be larger. The current data $A_{CP}(B^{\pm} \to K^{\pm}\pi^0) = 0.050 \pm 0.025$ and $A_{CP}(B^{\pm} \to K^{\pm}\rho^0) = 0.31^{+0.11}_{-0.10}$ [11] favor an imaginary penguin annihilation. We emphasize that strong phases, generated by subleading corrections, are the leading effect for direct CP asymmetries of B meson decays. For example, the prediction for the direct CP asymmetry $A_{CP}(B^{\pm} \to K^{\pm}\pi^0)$ is sensitive to the strong phase of the ratio C/T [12,13], where C (T) is the color-suppressed (color-allowed) tree amplitude, though the branching ratio $B(B^{\pm} \to K^{\pm}\pi^0)$ is not. Assuming this ratio to be real as in the leading-power SCET [5], it is difficult to explain the data. Therefore, subleading corrections must be handled carefully in order to have a reliable evaluation of strong phases.

It will be explained that the different penguin annihilation effects observed in PQCD and SCET arise from how to treat parton transverse momenta k_T or other formally power-suppressed intrinsic mass scales which appear in

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propagators. In other words, we raise a question about whether the expansion in powers of Λ/m_b is reliable for estimating strong phases at finite physical m_b. If the small scales are neglected or expanded, the internal particles are on their mass shell only at the end points of parton momentum fractions, where meson distribution amplitudes usually vanish, or the zero-bin subtraction suppresses the end-point contribution. An annihilation amplitude is then real. If k_T or a small scale is kept, the on-shell condition of internal particles does not occur at the end points, so there is a potential to generate a sizable strong phase. A strong phase from the known Bander-Silverman-Soni mechanism [14] is also a consequence of the on-shell condition of internal particles. Note that this on-shell condition does not break the factorization theorem, since it belongs to the so-called nonpinched singularities [15]. They differ from the pinched singularities, which are absorbed into nonperturbative distribution amplitudes. When m_b approaches infinity, the on-shell region coincides with the end points, and the same vanishing results for strong phases will be derived, irrespective of whether the small scales are expanded into a power series. For the physical value of m_b, however, a formally power-suppressed correction, if not expanded, may have a significant numerical effect on strong phases and lead to large direct CP asymmetries in B meson decays.

We illustrate why a formally power-suppressed correction of $O(\Lambda/m_b)$ could produce a sizable strong phase in an annihilation amplitude using a simple example. Suppose that we have a kernel of the form using the principal-value prescription

$$\frac{1}{x - r + i\epsilon} = P\frac{1}{x} - i\pi\delta(x), \tag{1}$$

where x is the momentum fraction and the subleading term r/x, $r \sim \Lambda/m_b$, is neglected at leading power. The effect of the subleading terms can be included systematically order by order. Equation (1) holds in principle as long as the contribution from the small x region is suppressed by a meson distribution amplitude, namely, as the main contribution comes from the region with $r/x \sim O(\Lambda/m_b)$. The imaginary part is proportional to $\delta(x)$, so it receives contribution from the end point $x = 0$. The suppression at the end point from a distribution amplitude then leads to vanishing of the strong phase. The expansion in Eq. (1) has been adopted in QCDF and SCET. In the former, if the evaluation of the convolution develops a singularity from the end-point contribution, a strong phase has to be parametrized, and there is no constraint on its magnitude. In fact, the end-point singularity should imply breakdown of the expansion in QCDF. In SCET, if the end-point singularity exists, it is removed by the zero-bin subtraction [7], which is introduced to avoid double counting of soft degrees of freedom in the collinear ones. The imaginary part, proportional to $\delta(x)$ as in Eq. (1), is then also removed by the zero-bin subtraction. That is, once the terms of order r are dropped from the beginning, the imaginary part is suppressed either by the end-point behavior of a distribution function or by the zero-bin subtraction in SCET. Therefore, the strong phase is small and appears only at higher orders in α_s or at higher powers in Λ/m_b (though the numerical size of these higher-order contributions needs to be studied explicitly).

On the other hand, writing the kernel without expansion, Eq. (1) becomes

$$\frac{1}{x - r + i\epsilon} = P\frac{1}{x - r} - i\pi\delta(x - r), \tag{2}$$

and an imaginary part arises in the region away from the end point by $r \sim \Lambda/m_b$. Note that the delta function $\delta(x - r)$ in the above expression cannot be obtained by expanding the denominator to any fixed power in r/x (it can by expanding the denominator to all powers). Hence, it is rather a result of all-order summation of the subleading terms. In the languages of QCDF or SCET, it means that the above imaginary part cannot be derived by including subleading operators to some given powers of Λ/m_b. The treatment of the subleading terms as in Eq. (2) is employed in the PQCD approach based on the k_T factorization theorem. As argued in Ref. [16], a parton, carrying a transverse momentum k_T as small as Λ initially, accumulates its k_T after emitting infinitely many collinear gluons. When the parton participates in a hard scattering eventually, k_T can become as large as the hard scale. Such an accumulation is described by the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi evolution [17] for a parton distribution function in inclusive processes and by the Sudakov evolution [18] for a hadron wave function in exclusive processes. For two-body nonleptonic B meson decays, k_T^2 of internal particles in a hard kernel reaches the hard scale of $O(m_b\Lambda)$. That is, the effect resulting from k_T^2 is suppressed by a power of $k_T^2/m_b^2 \sim O(\Lambda/m_b)$ and could be represented by r in Eq. (2).

If a meson distribution amplitude convoluted with Eq. (2) diminishes linearly near the end point, the imaginary part will be of order $r \sim \Lambda/m_b$. Nevertheless, it can be numerically significant due to the constant factor π. For instance, with the distribution amplitude $\phi(x) = 6x(1 - x)$, the real parts from Eqs. (1) and (2) differ by only 15%. The imaginary part from Eq. (1) is zero, but that from Eq. (2) reaches half of the real part for a typical value of $r = 0.1$. Obviously, in order that the imaginary part becomes negligible, i.e., about 5% of the real part, r must decrease to 0.01 (or m_b increases up to 50 GeV). The lessons we learn from this simple example are (i) for very small r, the expansion in powers of r is justified and the imaginary part, if any, is suppressed; (ii) if r is small but away from the end point, there can be an appreciable imaginary part resulting from subleading terms; (iii) in any case, r is expected to give a minor effect on branching ratios but a larger effect on direct CP asymmetries in B meson decays.

The above comparison mimics the different treatments of subleading corrections to B meson decays in SCET and in PQCD: All of the contributions are expanded in Λ/m_b and in α_s to fixed order for a given accuracy in SCET, while part of the subleading corrections is summed to all orders in PQCD. For decay rates, the two approaches agree roughly, but some observables such as the strong phase have different values as stated before. In order to see if there is a possibility to have an imaginary part in the annihilation channel in SCET, we apply the toy model in Eq. (2) to SCET with the zero-bin subtraction [7], keeping a small scale in particle propagators, such as an averaged parton transverse momentum in PQCD or the hard-collinear scale in SCET. Note that subleading terms are always expanded in SCET to give higher-dimensional operators and contribute through the matrix elements of these operators. Therefore, retaining a subleading term without expansion causes double counting and spoils the power counting rules of SCET. To avoid these problems, the leading SCET formalism will be employed here. We would raise the question by means of this toy model about handling subleading terms properly as far as the imaginary part is concerned.

We compute the branching ratio and the direct CP asymmetry of the $B^0 \to K^{\mp}\pi^{\pm}$ decays explicitly in our toy model. Let the momenta of the outgoing quark u and antiquark $\bar{u}$ in opposite directions be $k_2 = (0, yP_2^-, \mathbf{0}_T)$ and $k_3 = (\bar{x}P_3^+, 0, \mathbf{0}_T)$, respectively, for the $\bar{B}^0 \to K^-\pi^+$ mode, where P_2 (P_3) is the pion (kaon) momentum and $\bar{x} = 1 - x$. The expression for the penguin annihilation amplitude in the SCET formalism with the zero-bin subtraction is quoted from Ref. [8]:

$$\begin{aligned} A_{\mathrm{Lann}}(K^-\pi^+) = &-\frac{G_F f_B f_K f_\pi}{\sqrt{2}}(\lambda_c^{(s)} + \lambda_u^{(s)})\frac{4\pi\alpha_s(\mu_h)}{9}\bigg\{\left(\frac{C_9}{6} - \frac{C_3}{3}\right)[\langle \bar{x}^{-2}\rangle^K \langle y^{-1}\rangle^\pi - \langle [y(x\bar{y} - 1)]^{-1}\rangle^{\pi K}] \\ &- \frac{2\mu_\pi}{3m_b}\left(C_6 - \frac{C_8}{2} + \frac{C_5}{3} - \frac{C_7}{6}\right)[\langle y^{-2}\bar{y}^{-1}\rangle^\pi_{pp}(\langle \bar{x}^{-2}\rangle^K + \langle \bar{x}^{-1}\rangle^K) - \frac{2\mu_\pi}{3m_b}\left(\frac{C_5}{3} - \frac{C_7}{6}\right)\langle [(1 - x\bar{y})\bar{x}y^2]^{-1}\rangle^{\pi K}_{pp} \\ &+ \frac{2\mu_K}{3m_b}\left(\frac{C_5}{3} - \frac{C_7}{6}\right)\langle [(1 - x\bar{y})\bar{x}^2 y]^{-1}\rangle^{K\pi}_{pp} - \frac{2\mu_K}{3m_b}\left(C_6 - \frac{C_8}{2} + \frac{C_5}{3} - \frac{C_7}{6}\right)[(\langle y^{-2}\rangle^\pi + \langle y^{-1}\rangle^\pi)\langle x^{-1}\bar{x}^{-2}\rangle^K_{pp}]\bigg\}, \end{aligned} \tag{3}$$

where G_F is the Fermi constant, $f_{B,K,\pi}$ the meson decay constants, $\lambda_{u,c}^{(s)}$ the products of the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, $\mu_h \sim m_b$ the hard scale, C_i the Wilson coefficients, and μ_π the chiral scale associated with the pion. The logarithmic terms $\ln\mu_\pm$ in Ref. [8], being cancelled in the calculation of the Wilson coefficients and the zero-bin subtraction, have been dropped. There is no logarithmic enhancement at higher orders in α_s, since physical quantities should be independent of the scales $\mu_\pm$. Because of the large theoretical uncertainty shown below, the constant κ resulting from the above logarithmic cancellation will be neglected [8]. The three-parton twist-3 contribution to the penguin annihilation, being numerically smaller by 1 order of magnitude than Eq. (3) [19], is not included.

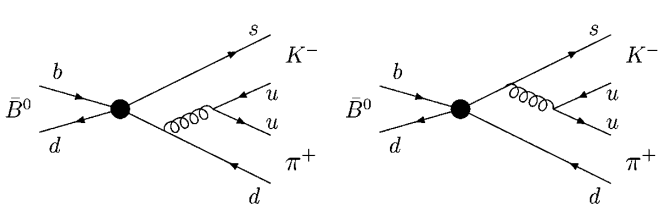

FIG. 1. "Factorizable" annihilation diagrams in the $\bar{B}^0 \to K^-\pi^+$ decay, where the black dots denote a scalar penguin operator in the effective weak Hamiltonian.

We introduce a small constant r into internal quark propagators involved in the factorizable piece of Eq. (3), corresponding to Fig. 1. Inserting r into gluon propagators generates a strong phase down by a factor of 3. The strong phase from the nonfactorizable annihilation amplitude is smaller by 2 orders of magnitude. Applying the principle-value prescription, we obtain the extra imaginary pieces via the following substitutions:

$$\begin{aligned} &\langle \bar{x}^{-2}\rangle^M \to \langle \bar{x}^{-2}\rangle^M + i\,\mathrm{Im}\langle \bar{x}^{-2}\rangle^M, \\ &\mathrm{Im}\langle \bar{x}^{-2}\rangle^M = -\pi\int_0^1 dx\frac{\phi_M(x) + \bar{x}\phi_M'(1)}{\bar{x}}\delta(\bar{x} - r), \end{aligned} \tag{4}$$

$$\begin{aligned} &\langle y^{-2}\bar{y}^{-1}\rangle^M_{pp} \to \langle y^{-2}\bar{y}^{-1}\rangle^M_{pp} + i\,\mathrm{Im}\langle y^{-2}\bar{y}^{-1}\rangle^M_{pp}, \\ &\mathrm{Im}\langle y^{-2}\bar{y}^{-1}\rangle^M_{pp} = -\pi\int_0^1 dy\left[\frac{\phi^M_{pp}(y)}{y(1-y)} - \frac{y\phi^{M\prime}_{pp}(0)}{y}\right]\delta(y - r). \end{aligned} \tag{5}$$

Employing the parametrizations for the leading-twist distribution amplitude $\phi_M(x)$ and for the two-parton twist-3 distribution amplitudes $\phi^M_{pp}(x)$ [8,19]

$$\phi_M(x) = 6x(1-x)[1 + a_1^M(6x-3) + 6a_2^M(1-5x+5x^2) - 10a_3^M(1-9x+21x^2-14x^3) + 15a_4^M(1-14x+56x^2-84x^3+42x^4) + \cdots],$$
$$\phi_{pp}^M(x) = 6x(1-x)[1 + a_{1pp}^M(6x-3) + 6a_{2pp}^M(1-5x+5x^2) + \cdots], \qquad (6)$$

with $M = \pi, K$, it is easy to find that both Eqs. (4) and (5) are proportional to r as expected.

The importance of the penguin annihilation contribution relative to the full penguin one has been estimated in SCET [8] and found to be about 10% with large uncertainty in the $B^0 \to K^\mp\pi^\pm$ decays. The full penguin contribution does not come from an explicit evaluation in the same SCET framework but from a fitting to the $B \to K\pi$ data. We can certainly take this approach. However, the factorization formulas for the emission amplitudes have been available in Ref. [7], so they will be adopted in the numerical analysis below. The feature of generating strong phases does not depend on how we estimate the emission amplitudes. Besides, we shall not consider the free parameters associated with the long-distance charming penguin, which is not factorizable in SCET. As demonstrated later, a decay amplitude under the zero-bin subtraction is very sensitive to higher Gegenbauer moments a_n^M and a_{npp}^M in Eq. (6) [20], which are mostly unknown. Hence, we shall determine these moments by fitting the SCET formulas to data of branching ratios, which are then used to predict direct CP asymmetries. If a strong phase from the source considered here is sizable, the whole CP asymmetry cannot be attributed to the nonperturbative charming penguin alone.

At lowest order in $\alpha_s(m_b)$ with the Wilson coefficients $T^{(+)} = 1$ and $C_J^{(+)} = 1$ in SCET$_\mathrm{I}$ [4], the $B \to \pi$ transition form factor is decomposed into

$$f_+(E) = \zeta^{B\pi}(E) + \zeta_J^{B\pi}(E). \qquad (7)$$

The second term is factorizable, written as

$$\zeta_J^{B\pi}(E) = \frac{f_B f_\pi m_B}{4E^2}\frac{4\pi\alpha_s(\mu_i)}{9}\left(\frac{2E}{m_B} + \frac{2E}{m_b} - 1\right) \times \int_0^1 dy \frac{\phi_\pi(y)}{y}\int_0^\infty dk^+ \frac{\phi_B^+(k^+)}{k^+}, \qquad (8)$$

where $\mu_i \sim \sqrt{m_b\Lambda}$ is the intermediate scale and k^+ the momentum of the spectator quark in the B meson. For charmless two-body nonleptonic B meson decays, we take the pion energy $E = m_B/2$, m_B being the B meson mass. The first term also factorizes after implementing the zero-bin subtraction for the end-point singularity [7]

$$\zeta^{B\pi}(E) = \frac{f_B f_\pi m_B}{4E^2}\frac{4\pi\alpha_s(\mu_i)}{9} \times \int_0^1 dy \int_0^\infty dk^+ \left\{\frac{(1+y)\phi_\pi(y)}{(y^2)_\emptyset}\frac{\phi_B^-(k^+)}{(k^+)_\emptyset} + \mu_\pi \frac{(\phi_\pi^p + \frac{1}{6}\phi_\pi^{\sigma\prime})(y)}{(y^2)_\emptyset}\frac{\phi_B^+(k^+)}{(k^{+2})_\emptyset}\right\}, \qquad (9)$$

where only the terms from the two-parton pion distribution amplitudes are retained. The relation among ϕ_π^p, ϕ_π^σ, and ϕ_{pp}^π can be found in Ref. [8]. The formulas for the $B \to K$ form factor in SCET are similar. We multiply Eq. (7) by the appropriate CKM matrix elements and Wilson coefficients, including a part of next-to-leading-order corrections [21], to obtain the emission contributions from both the tree and penguin operators. The Wilson coefficient a_6 was neglected in the previous SCET analysis, since the associated penguin contribution is power-suppressed. However, it is enhanced by the chiral scale and numerically large. Furthermore, the power-suppressed annihilation has been formulated into SCET, so there is no reason for ignoring a_6 [21].

The zero-bin subtraction for the logarithmic end-point singularity associated with the pion distribution amplitude ϕ_π in the first term of Eq. (9) is referred to in Ref. [7], where the term proportional to y in $(1+y)$ does not require subtraction. We also need the zero-bin subtraction for the linear end-point singularity present in the second term of Eq. (9) [22]:

$$\begin{aligned}\int_0^1 dy \frac{\phi_\pi^p(y)}{(y^2)_\emptyset} &\equiv \int_0^1 dy \frac{\phi_\pi^p(y) - \phi_\pi^p(0) - y\phi_\pi^{p\prime}(0)}{y^2} - \int_1^\infty dy y^\epsilon (y-1)^\epsilon \frac{\phi_\pi^p(0) + y\phi_\pi^{p\prime}(0)}{y^2} \times \left(\frac{\bar{n}\cdot P_2}{\mu_-}\right)^{2\epsilon} \\ &= \int_0^1 dy \frac{\phi_\pi^p(y) - \phi_\pi^p(0) - y\phi_\pi^{p\prime}(0)}{y^2} - \phi_\pi^p(0) + \ln\left(\frac{\bar{n}\cdot P_2}{\mu_-}\right)\phi_\pi^{p\prime}(0), \end{aligned} \qquad (10)$$

where the lightlike vector $\bar{n}$ along a Wilson line is involved in the definition for the pion distribution amplitudes and $\bar{n}\cdot P_2 = 2E$. The subtraction associated with the derivative of the two-parton twist-3 pion distribution amplitude $\phi_\pi^{\sigma\prime}$ is similar.

We consider the models for the B meson distribution amplitudes $\phi_B^\pm$ proposed by Kawamura *et al.* (KKQT) [23] and by Grozin and Neubert (GN) [24]. The associated zero-bin subtraction is defined by

$$\int_0^\infty dk^+ \frac{\phi_B^-(k^+)}{(k^+)_\emptyset} \equiv \int_0^\infty dk^+ \frac{\phi_B^-(k^+)}{k^+} - \int_0^{\bar\Lambda} dk^+ \frac{\phi_B^-(0)}{k^+} + \ln\Big(\frac{n\cdot v\bar\Lambda}{\mu_+}\Big)\phi_B^-(0)$$
$$= \begin{cases} -\frac{1}{\bar\Lambda}(1-\ln 2) + \ln(\frac{n\cdot v\bar\Lambda}{\mu_+})\phi_B^-(0) & \text{for KKQT,} \\ -\frac{1}{\omega_0}(\gamma_E + \ln\frac{\bar\Lambda}{\omega_0}) + \ln(\frac{n\cdot v\bar\Lambda}{\mu_+})\phi_B^-(0) & \text{for GN,} \end{cases} \tag{11}$$

$$\int_0^\infty dk^+ \frac{\phi_B^+(k^+)}{(k^{+2})_\emptyset} \equiv \int_0^\infty dk^+ \frac{\phi_B^+(k^+)}{k^{+2}} - \int_0^{\bar\Lambda} dk^+ \frac{\phi_B^{+\prime}(0)}{k^+} + \ln\Big(\frac{n\cdot v\bar\Lambda}{\mu_+}\Big)\phi_B^{+\prime}(0)$$
$$= \begin{cases} \frac{1}{2\bar\Lambda^2}\ln 2 + \ln(\frac{n\cdot v\bar\Lambda}{\mu_+})\phi_B^{+\prime}(0) & \text{for KKQT,} \\ -\frac{1}{\omega_0^2}(\gamma_E + \ln\frac{\bar\Lambda}{\omega_0}) + \ln(\frac{n\cdot v\bar\Lambda}{\mu_+})\phi_B^{+\prime}(0) & \text{for GN,} \end{cases} \tag{12}$$

with the parameter relation $\omega_0 = 2\bar\Lambda/3$, $\bar\Lambda$ being the B meson and the b quark mass difference. In the above expressions, n is a lightlike vector along the Wilson line in the definition for the B meson distribution amplitudes, and v is the B meson velocity. The terms containing $\ln\mu_\pm$ in Eqs. (10)–(12) are also dropped.

For the numerical analysis, we assume the Gegenbauer moments of the pion and kaon distribution amplitudes $a_1^\pi = 0$, $a_1^K = -0.05$ consistent with the results in Refs. [25,26], $a_2^K = a_2^\pi = 0.2$ [26–28], $a_3^\pi = 0$, $a_4^K = a_4^\pi$, $a_{1pp}^\pi = a_{1pp}^K = 0$, and $a_{2pp}^K = a_{2pp}^\pi$, among which a_4^M and a_{2pp}^M are most uncertain. To simplify the formulas, we do not consider the Gegenbauer moment a_3^K for the twist-2 kaon distribution amplitude. That is, we keep one most uncertain parameter from each of ϕ_M and ϕ_{pp}^M, whose variation is sufficient for our purpose. The hard and intermediate scales are fixed at $\mu_h = m_b$ and $\mu_i = \sqrt{m_b\bar\Lambda}$, respectively, with $\bar\Lambda = 0.55$ GeV and $m_b = m_b^{1S} = 4.7$ GeV. Other relevant heavy-quark masses are taken to be $m_c = m_c^{1S} = 1.4$ GeV and $\bar m_b = \bar m_b^{\overline{\rm MS}}(\bar m_b) = 4.2$ GeV. We obtain the chiral scales $\mu_\pi(\mu_h) = 2.4$ GeV, $\mu_K(\mu_h) = 3.0$ GeV, $\mu_\pi(\mu_i) = 1.8$ GeV, and $\mu_K(\mu_i) = 2.3$ GeV from the two-loop running for the strong coupling constant with $\alpha_s(M_Z = 91.1876\ \text{GeV}) = 0.118$ and for the light-quark masses with $m_{u,d}(2\ \text{GeV}) = 5$ MeV and $m_s(2\ \text{GeV}) = 95$ MeV. We take the Wilson coefficients for four-fermion operators evaluated at $\mu_h = m_b$ and at next-to-leading-logarithmic level: $C_1 = 1.078$, $C_2 = -0.177$, $C_3 = 0.014$, $C_4 = -0.034$, $C_5 = 0.009$, $C_6 = -0.040$, $C_7 = 0.7\times10^{-4}$, $C_8 = 4.5\times10^{-4}$, $C_9 = -9.9\times10^{-3}$, and $C_{10} = 1.8\times10^{-3}$. Those for dipole operators at leading-logarithmic level are $C_{7\gamma} = -0.314$ and $C_{8G} = -0.149$ [29]. We also take the Fermi constant $G_F = 1.16639\times10^{-5}\ \text{GeV}^{-2}$, the decay constants $f_B = 0.22$ GeV, $f_K = 0.16$ GeV, and $f_\pi = 0.131$ GeV, the meson masses $m_B = 5.28$ GeV, $m_K = 0.497$ GeV, and $m_\pi = 0.14$ GeV, the B meson lifetime $\tau_B^0 = 1.530\times10^{-12}$ sec, and the CKM matrix elements $V_{us} = 0.2257$, $V_{ub} = (4.2\times10^{-3})\exp(-i\phi_3)$, $V_{cs} = 0.957$, and $V_{cb} = 0.0416$ with the weak phase $\phi_3 = 74°$ [30].

Adopting the above parameters, the two pieces $\zeta^{B\pi}$ and $\zeta_J^{B\pi}$ of the $B\to\pi$ form factor are written as

$$\zeta^{B\pi} = \begin{cases} 0.01 + 0.75a_2^\pi + 2.57a_4^\pi + 0.43a_{2pp}^\pi & \text{for KKQT,} \\ 0.09 + 0.65a_2^\pi + 2.23a_4^\pi - 2.73a_{2pp}^\pi & \text{for GN,} \end{cases} \tag{13}$$

$$\zeta_J^{B\pi} = \begin{cases} 0.016(1.0 + a_2^\pi + a_4^\pi) & \text{for KKQT,} \\ 0.024(1.0 + a_2^\pi + a_4^\pi) & \text{for GN.} \end{cases} \tag{14}$$

Note that the coefficients in Eq. (13) grow quadratically with the order n of the Gegenbauer moments a_n^π [20]. This sensitivity is attributed to the increasing slope of the higher Gegenbauer polynomials at the end points of the momentum fraction x. The sign flip of the a_{2pp}^π terms indicates that $\zeta^{B\pi}$ also depends strongly on the models of the B meson distribution amplitudes in SCET. We mention that the PQCD approach does not suffer such sensitivity, because the end-point singularity is smeared by including parton transverse momenta k_T, whose order of magnitude is governed by the Sudakov factor.

The strong dependence on the higher Gegenbauer moments also appears in the penguin annihilation amplitude

$$\begin{aligned} 10^4\hat P_{K\pi}^{\rm ann} &\equiv -10^4\frac{\sqrt2}{G_F m_B^2}\frac{A_{\rm Lann}(K^-\pi^+)}{(1\ \text{GeV})} \\ &= 2.76(0.07 + a_4^\pi)(1.20 + a_4^\pi) \\ &\quad + a_{2pp}^\pi(27.0 + 413.1a_4^\pi) \\ &\quad - i\pi r a_{2pp}^\pi(53.2 + 1747a_4^\pi), \end{aligned} \tag{15}$$

with a significant growth of the coefficients of a_4^π. The imaginary contribution is proportional to the second moment a_{2pp}^π. In fact, it could depend on the zeroth moment, i.e., the normalization of ϕ_{pp}^M, if the denominator $1-y$ is not replaced by 1 in the subtraction term in Eq. (5). The choice of the denominators $1-y$ and 1 correspond to different zero-bin subtraction schemes pointed out before. That is, the size of the imaginary part depends on the amount of the subtracted contribution, i.e., on zero-bin subtraction schemes, since it is generated at $\bar x\sim\Lambda/m_b$ or $y\sim\Lambda/m_b$ as shown in Eqs. (4) and (5). The dependence on subtraction schemes also exists in all other definitions like Eqs. (10)–(12), which will not be discussed in this work.

For the range of a_4^π, the crude bound $a_4^\pi \geq -0.07$ has been determined in Ref. [28]. The analysis based on the data of the pion transition form factor suggests $a_4^\pi \approx -0.05$ in Ref. [31] and the constraint $a_2^\pi + a_4^\pi = -0.03 \pm 0.14$ in Ref. [32], both of which prefer a negative value of a_4^π (considering $a_2^\pi \approx 0.2$). The range of a_{2pp}^π is basically undetermined. We shall regard these two parameters as being free and fix them by the strategy stated before: Adjust a_4^π and a_{2pp}^π, such that the $B \to \pi$ form factor has the value around $f_+ = 0.24 \pm 0.05$ [13], and the $B^0 \to K^\mp \pi^\pm$ decays have the branching ratio close to the data $B(B^0 \to K^\mp \pi^\pm) = (19.4 \pm 0.6) \times 10^{-6}$ [11]. Because the last two terms in $\zeta^{B\pi}$ for the KKQT model are of the same sign, and the coefficient of a_4^π is large, the constraint from the form factor value leads to a smaller a_4^π. Equation (15) then implies that the coefficient of r, i.e., the imaginary part of the annihilation amplitude, is smaller and that the strong phase is less sensitive to the variation of r. On the contrary, the last two terms in $\zeta^{B\pi}$ for the GN model have the coefficients with the same order of magnitude but in opposite signs. Hence, a_4^π (and also a_{2pp}^π) is larger, and the strong phase is more sensitive to the variation of r in this case.

Employing the KKQT model for the B meson distribution amplitudes, we obtain $a_4^\pi \approx 0.01$ and $a_{2pp}^\pi \approx 0.23$, corresponding to which the $B \to \pi$ form factor, the $B^0 \to K^\mp \pi^\pm$ branching ratio, and the predicted direct CP asymmetry are given by

$$\zeta^{B\pi} = 0.29, \qquad \zeta_J^{B\pi} = 0.02,$$
$$B(B^0 \to K^\mp \pi^\pm) = \begin{cases} 20.5 \times 10^{-6} & \text{for } r = 0.0, \\ 20.0 \times 10^{-6} & \text{for } r = 0.1, \\ 19.8 \times 10^{-6} & \text{for } r = 0.2, \end{cases}$$
$$A_{CP}(B^0 \to K^\mp \pi^\pm) = \begin{cases} 0.08 & \text{for } r = 0.0, \\ 0.05 & \text{for } r = 0.1, \\ 0.02 & \text{for } r = 0.2. \end{cases} \tag{16}$$

We do not attempt a fine-tuning here but accept the values of a_4^π and a_{2pp}^π as solutions, when they produce the $B \to \pi$ form factor and the $B^0 \to K^\mp \pi^\pm$ branching ratio close to the designated ranges. The results shift with the slight variation of a_4^π and a_{2pp}^π, but the behavior for different r in Eq. (16) has the same pattern. In principle, $\zeta^{B\pi}$ and $\zeta_J^{B\pi}$ have the same scaling law in α_s and in $1/m_b$ [33,34]. The numerical hierarchy $\zeta^{B\pi} \gg \zeta_J^{B\pi}$ in Eq. (16), consistent with the PQCD results [33], may be altered in different zero-bin subtraction schemes. It is obvious that the additional dependence on r has a negligible effect on the branching ratio. However, it affects the strong phase: $A_{CP}(B^0 \to K^\mp \pi^\pm)$ decreases by 40% from $r = 0$ to $r = 0.1$. Since the imaginary part is proportional to r, it is difficult to accommodate the data $A_{CP}(B^0 \to K^\mp \pi^\pm) = -0.097 \pm 0.012$ [11] with reasonable values of r using the KKQT model.

For the GN model, we find two sets of solutions corresponding to $a_4^\pi \approx 0.18$ and $a_{2pp}^\pi \approx 0.15$:

$$\zeta^{B\pi} = 0.21, \qquad \zeta_J^{B\pi} = 0.03,$$
$$B(B^0 \to K^\mp \pi^\pm) = \begin{cases} 20.1 \times 10^{-6} & \text{for} r = 0.0, \\ 20.4 \times 10^{-6} & \text{for} r = 0.1, \\ 25.1 \times 10^{-6} & \text{for} r = 0.2, \end{cases}$$
$$A_{CP}(B^0 \to K^\mp \pi^\pm) = \begin{cases} 0.06 & \text{for} r = 0.0, \\ -0.06 & \text{for} r = 0.1, \\ -0.14 & \text{for} r = 0.2 \end{cases} \tag{17}$$

and to $a_4^\pi \approx -0.22$ and $a_{2pp}^\pi \approx -0.20$:

$$\zeta^{B\pi} = 0.28, \qquad \zeta_J^{B\pi} = 0.02,$$
$$B(B^0 \to K^\mp \pi^\pm) = \begin{cases} 18.6 \times 10^{-6} & \text{for } r = 0.0, \\ 19.4 \times 10^{-6} & \text{for } r = 0.1, \\ 26.5 \times 10^{-6} & \text{for } r = 0.2, \end{cases}$$
$$A_{CP}(B^0 \to K^\mp \pi^\pm) = \begin{cases} 0.08 & \text{for } r = 0.0, \\ -0.10 & \text{for } r = 0.1, \\ -0.20 & \text{for } r = 0.2. \end{cases} \tag{18}$$

The existence of the two sets of solutions with opposite signs is understandable. Because the term proportional to a_4^π in the imaginary part of Eq. (15) dominates over the constant term as $|a_4^\pi|$ reaches about 0.2, the product $a_{2pp}^\pi a_4^\pi$ matters, and a_4^π and a_{2pp}^π can flip sign simultaneously.

As indicated by Eqs. (17) and (18), the branching ratio is stable, while the strong phase is very sensitive to the variation of r, so that we easily accommodate the data of $A_{CP}(B^0 \to K^\mp \pi^\pm)$ with a typical value of $r = 0.1 \sim 0.15$. The predicted $A_{CP}(B^0 \to K^\mp \pi^\pm)$ for $r = 0$, i.e., real penguin annihilation ($r = 0.1$, i.e., complex penguin annihilation) is close to that from QCDF in the default scenario [35] (PQCD [2,13]). Therefore, the strong phases resulting from the power-suppressed source in the penguin annihilation could be numerically crucial for the estimation of direct CP asymmetries. We then understand the opposite conclusions on the effect of the penguin annihilation drawn in SCET and in PQCD: The almost real annihilation amplitude in the former and the almost imaginary annihilation amplitude in the latter are attributed to the different treatments of the formally power-suppressed terms at the physical b quark mass. Note that the solutions of a_4^π and a_{2pp}^π in Eqs. (16)–(18) will be changed, if higher Gegenbauer moments in Eq. (6) are taken into account, which cause even larger variation of the decay amplitudes. However, the strong dependence of $A_{CP}(B^0 \to K^\mp \pi^\pm)$ on r will persist.

As stated before, SCET provides a systematical expansion in powers of Λ/m_b, but it is somewhat twisted here by keeping subleading terms in particle propagators in order to trace a possible source for generating strong phases. Note that the zero-bin subtraction is not essential for generating strong phases, though it may change their numerical values. The twist actually violates the power counting rules and other aspects of SCET. Hence, our analysis does not imply the breakdown of SCET in its application to B meson decays but helps clarify why there are discrepancies between the studies of direct CP asymmetries in SCET and PQCD. It hints that more caution is necessary for fixed-power evaluations of direct CP asymmetries at the physical mass m_b. The expansion would be reliable for decay rates and direct CP asymmetries, if the b quark mass was 10 times heavier. In that case, the contribution from the on-shell region of internal particles can be really suppressed by hadron distribution amplitudes or excluded by the zero-bin subtraction. For $m_b \approx 5$ GeV, the imaginary part, despite being down by Λ/m_b, can be numerically large due to the factor π. In this case, a novel method to generate strong phases with finite physical m_b should be developed.

We have shown that introducing a small scale into denominators of internal quark propagators accommodates both the measured branching ratio and the direct CP asymmetry of the $B^0 \to K^{\mp}\pi^{\pm}$ decays. Retaining a small quantity in denominators without expansion is equivalent to summation of the associated power corrections to all orders. It is similar to resummation of part of higher-order corrections in α_s for many QCD processes. It is claimed in the PQCD approach that the parton transverse momenta can be maintained in denominators consistently in the k_T factorization theorem [36,37]. This treatment is justified by different power counting rules, which hold in the region of small parton momenta [37]. The alternative power expansion postulated in the k_T factorization theorem has led to strong phases in agreement with the indication of data in B meson decays. In SCET, strong phases from penguin annihilation channels are indeed suppressed in the heavy-quark limit. The point of our paper is to speculate on the effects of on-shell conditions with subleading terms being included, which are not diminished sufficiently by the end-point behavior of distribution amplitudes or by the zero-bin subtraction for finite heavy-quark mass. No matter whether improved experimental data turn out to favor small or large strong phases, our concern remains: One always needs to estimate contributions of subleading terms along with the size of the nonperturbative charming penguins in SCET.

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