Pion electromagnetic form factor, perturbative QCD, and large-N_c Regge models

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We present a construction of the pion electromagnetic form factor where the transition from large-N_c Regge vector-meson dominance models with infinitely many resonances to perturbative QCD is built in explicitly. The construction is based on an appropriate assignment of residues to the Regge poles, which fulfills the constraints of the parton-hadron duality and perturbative QCD. The model contains a slowly falling off nonperturbative contribution, which dominates over the perturbative QCD radiative corrections for the experimentally accessible momenta. The leading order and next-to-leading order calculations show a converging pattern that describes the available data within uncertainties, while the onset of asymptotic QCD takes place at extremely high momenta $Q \sim 10^3$–10^4 GeV. The method can be straightforwardly extended to study other form factors where the perturbative QCD result is available.

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I. INTRODUCTION

The composite nature of hadrons can be best seen by studying their electromagnetic form factors at a sufficiently large momentum transfer [1]. The pion, being the lowest u and d quark-antiquark excitation of the vacuum and identified with the would-be Goldstone massless mode of the spontaneously broken chiral symmetry, provides the simplest candidate to test our present knowledge on hadronic interactions. Because of relativistic and gauge invariance the pion charge form factor (we take π^+ for definiteness) can be written as

$$\langle \pi^+(p')|J_\mu^{\rm em}(0)|\pi^+(p)\rangle = (p'^\mu + p^\mu)F(q^2), \qquad (1)$$

with $q = p' - p$, and $J_\mu^{\rm em}(x) = \sum_{q=u,d,s,\ldots} e_q \bar{q}(x)\gamma_\mu q(x)$ is the electromagnetic current, with e_q denoting the quark charge in units of the elementary charge. The charge normalization requires

$$F(0) = 1. \qquad (2)$$

The pion charge form factor has been the subject of intense experimental efforts [2–9]. Moreover, it is expected to be measured at TJLAB in the spacelike range of 1 GeV$^2 \le -t \le 6$ GeV^2 with unprecedented high precision $\Delta(-tF(t)) \sim 0.02$ GeV^2. The results might be used as a stringent test of the perturbative QCD (pQCD) radiative corrections. Actually, in the spacelike region where $t = -Q^2$, $F(t)$ is real and at large Q^2 values the pQCD methods can be applied, yielding asymptotically [10–15]

$$F(-Q^2) = \frac{16\pi f_\pi^2 \alpha(Q^2)}{Q^2}\left[1 + 6.58\frac{\alpha(Q^2)}{\pi} + \ldots\right], \quad Q^2 \gg M^2, \qquad (3)$$

with $f_\pi = 92.3$ MeV denoting the pion weak decay constant, and M the lowest vector-meson mass. Further higher-order power corrections are of the order $\mathcal{O}(1/Q^4)$ and correspond to higher twist operators [16,17]. The form factor depends logarithmically on the scale through the running coupling constant

$$\alpha(Q^2) = \frac{4\pi}{\beta_0 \log(Q^2/\Lambda^2)}, \qquad (4)$$

$$\beta_0 = \tfrac{11}{3}N_c - \tfrac{2}{3}N_f. \qquad (5)$$

We use the $\overline{\rm MS}$ scheme and the factorization scale coinciding with the renormalization scale. Also, the asymptotic form of the pion parton distribution amplitude $\phi_\pi(x) = 6x(1-x)$ is used. Details of the complete analysis may be found in Ref. [18]. The second term in brackets in Eq. (3) is the next-to-leading (NLO) correction. It is at an acceptable 20% level when $\alpha \sim 0.1$, which suggests that one might observe this radiative correction at relatively large scales $Q^2 \sim M_Z^2$.

In the intermediate energy region the form factor behaves to a very good accuracy as

$$F(-Q^2) = \frac{M_V^2}{Q^2 + M_V^2}, \qquad Q^2 \le M_V^2, \qquad (6)$$

with $M_V = 720$ MeV, complying to the old vector-meson dominance models (VMD) (see e.g. Ref. [19] and references therein) and showing no obvious trace of the pQCD behavior. In the region close to the zero momentum transfer chiral corrections become important [20,21]. For timelike momenta the pion form factor becomes complex and

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can be related by crossing to the $e^+e^- \to \pi^+\pi^-$ annihilation amplitude $\langle \pi^+\pi^- | J_\mu^{\rm em} | 0 \rangle = F(s)(p^\mu + p'^\mu)$, where the final state interactions due to $\pi\pi$ scattering and unitarity play a crucial role [22]. While both the timelike and the spacelike regions are related by an unsubtracted dispersion relation [23]

$$F(t) = \frac{1}{\pi} \int_{t_0}^{\infty} \frac{{\rm Im} F(t')}{t' - t - i\epsilon} dt', \qquad (7)$$

the well-known timelike region does not determine unambiguously when the onset of the pQCD takes place. Actually, the single VMD model shows that even in the spacelike region as low as $Q \sim m_\rho$ the traces of chiral logs and final-state interactions are meager.

Given the fact that the pQCD effects cannot directly be observed at presently available energies, numerous phenomenological QCD-based approaches and model calculations have been suggested in order to understand the transition from the soft to hard scales. They include standard QCD sum rules [24], local-duality QCD sum rules [25,26], light-cone QCD sum rules [27], nonlocal condensates [28,29], Schwinger-Dyson equations [30], instanton-based models [31,32], constituent quark models [33], nonlocal quark models [34,35], etc. The scale of the onset of pQCD provoked heated debates in the past. The problem is crucial, as it provides a decisive fingerprint of the underlying quark-gluon substructure of the pion. We note that the upcoming lattice QCD calculations extending the work reported in [36–40] can directly verify this issue without necessarily spanning such a wide energy window as in the experiment. The reason is that a lot of progress has been achieved in extrapolating the lattice data to the chiral limit, which incorporates the enhancement and nonlinearities triggered by the chiral logs.

The class of calculations listed above contains quarks and gluons as explicit dynamical degrees of freedom, and hence requires a detailed knowledge of the pion wave function. On the other hand, the parton-hadron duality implies that any hadronic property be describable in the purely hadronic language without an explicit reference to the basic fundamental fields. For instance, the success of the simple VMD fit for the pion charge form factor suggests the inclusion of further radially excited $I^G J^{PC} = 1^+1^{--}$ states ρ', ρ'', ρ''' ...

$$F(t) = \sum_{V=\rho,\rho',\ldots}^{V_{\rm max}} \frac{c_V M_V^2}{M_V^2 - t}. \qquad (8)$$

This finite sum involves states with a mass below $M_{V_{\rm max}}$, the highest allowed vector-meson mass, which acts as a high-energy cutoff. Thus, it could reliably reproduce the data (see below) in a region where $Q^2 < M^2_{V_{\rm max}}$, and will only produce inverse integer powers of Q^2 asymptotically when $Q^2 \gg M^2_{V,{\rm max}}$. This is in formal contradiction with Eq. (3), where there is no high-energy cutoff and the behavior $1/(Q^2 \log Q^2)$ is obtained. Thus, infinitely many states are clearly needed. This complies to the 't Hooft large-N_c limit [41], where any hadronic amplitude can be written in terms of tree diagrams with (infinitely many) mesons and glueballs. In particular, in the large-N_c limit the pion form factor can be written in the form (8) with *infinitely many* resonances.

Based on the success of the Veneziano-Lovelace-Shapiro dual resonance model (see e.g. [42,43] and references therein) Suura [44] and Frampton [45] proposed analytic models, which have recently been resurrected and further elaborated by Dominguez [46,47]. Incidentally, the resulting expressions for the pion charge form factor turn out to be quite similar to the AdS/CFT hard-wall and soft-wall calculations carried out in [48–50]. Despite the successful fit to the data, these calculations do not reproduce the formal asymptotic pQCD behavior, a fact which has been interpreted as an intrinsic limitation of the approach [49]. This poses an intriguing puzzle: how do hadronic large-N_c models satisfy the QCD constraints, including the presence of logarithms? Quite generally, pQCD predicts integer powers and logarithms of Q^2, whereas the models of Refs. [44–47] are able to generate fractional powers.

In the present paper, we analyze the problem for the case of the pion charge form factor and show how the pQCD constraints can judiciously be implemented in a large-N_c Regge model in an exact manner and at the same time preserve the good description of the experimental data. The essence of the approach is a careful assignment of coupling constants to the infinitely many resonances. As a result, the form (3) emerges from the infinite sum (8). We term the mechanism the *power-to-log transmutation*, which essentially corresponds to a suitable superposition of fractional twist operators in the Regge model of Refs. [44–47]. The present study follows our investigation of the two-point functions [51,52]. In a previous paper [53], we have shown how the large-N_c Regge models can be used to deal with the $\gamma^* \pi^0 \to \gamma$ transition form factor, where the radiative pQCD corrections characterized by the relevant anomalous dimensions are generated with the suitable QCD evolution equations.

II. MESON DOMINANCE

In this preparatory section, we introduce the basic definitions and notation for the pion form factor in VMD models. The electromagnetic current is written as $J^{\mu,{\rm em}}(x) = B^\mu(x)/2 + J_V^{\mu,3}(x)$ with $B^\mu(x) = \sum_q \bar{q}(x)\gamma^\mu q(x)/N_c$ being the baryon current and $J_V^{\mu a}(x) = \sum_q \bar{q}(x)\tau^a \gamma^\mu q(x)/2$ the isovector current. Using the isospin invariance, assumed throughout, we have

$$\langle \pi^a(p') | J_V^{\mu,b}(0) | \pi^c(p) \rangle = \epsilon^{abc}(p'^\mu + p^\mu) F(q^2), \qquad (9)$$

with $|\pi^i(p)\rangle$ denoting a pion state, and a, b, c the Cartesian isospin indices. In the large-N_c limit the *meson dominance* of the pion charge form factor is the statement that one can parameterize the (isovector) current as a superposition of vector-meson fields, $\rho^a_{n,\mu}(x)$

$$J_V^{\mu,a}(x) = \sum_n F_V(n) M_V(n) \rho_n^{\mu,a}(x), \tag{10}$$

where $n = 0$ corresponds to the ground state $\rho(770)$ meson, and higher values of n to excited states. Correspondingly, the matrix element between the vacuum and the one-vector-meson state is

$$\langle 0|J_V^{a\mu}(0)|\rho_n^b, \epsilon\rangle = \delta^{ab} M_V(n) F_V(n) \epsilon^\mu, \tag{11}$$

with ϵ_μ denoting the vector-meson polarization. The coupling constants may be determined from the electromagnetic decay $\rho_n \to e^+e^-$ using the formula

$$\Gamma(\rho_n \to e^+e^-) = \frac{4\pi\alpha^2}{3} \frac{F_V(n)^2}{M_V(n)} \tag{12}$$

for the partial decay rate. For $M_V = m_\rho = 770$ MeV and $\Gamma(\rho \to e^+e^-) = 6.5$ keV one gets $F_V = F_\rho = 150$ MeV.

The two-point vector current-vector current correlator is defined as

$$\begin{aligned}\Pi_V^{\mu a,\nu b}(q) &= i \int d^4x e^{-iq\cdot x} \langle 0|T\{J_V^{\mu a}(x) J_V^{\nu b}(0)\}|0\rangle \\ &= \Pi_V(q^2)(q^\mu q^\nu - g^{\mu\nu} q^2)\delta^{ab},\end{aligned} \tag{13}$$

where

$$\Pi_V(t) = \sum_n \frac{F_V(n)^2}{M_V(n)^2 - t}. \tag{14}$$

The quark-hadron duality for large values of t in (14) requires the Regge model—parton model matching condition [51,52] for asymptotically large values of the radial quantum number n

$$\frac{F_V(n)^2}{dM_V^2(n)/dn} \sim \frac{N_c}{24\pi^2}. \tag{15}$$

For the radial Regge model (see next section) $dM_V^2(n)/dn = a = \text{const.}$; hence, at large n we must have $F_V(n) = \text{const}$ [51,52].

The vector-meson-pion-pion amplitude is

$$\langle \pi^a(p')|\rho_{n,\mu}^b(0)|\pi^c(p)\rangle = (p'_\mu + p_\mu)\frac{\epsilon^{abc} g_{V\pi\pi}(n)}{M_V(n)^2 - t}, \tag{16}$$

with $g_{V\pi\pi}(n)$ the coupling constant. This yields the $\rho_n \to \pi\pi$ partial decay rate

$$\Gamma(\rho_n \to \pi\pi) = \frac{g_{V\pi\pi}^2 M_V}{48\pi}\left(1 - \frac{4m_\pi^2}{M_V^2}\right)^{3/2}. \tag{17}$$

For the $\rho(770)$ meson one gets $g_{\rho\pi\pi} \simeq 6$ for $\Gamma(\rho \to 2\pi) = 150$ MeV.

For the pion electromagnetic form factor we have

$$F(t) = \sum_n \frac{c_n M_V(n)^2}{M_V(n)^2 - t} \qquad c_n = \frac{F_V(n) g_{V\pi\pi}(n)}{M_V(n)}. \tag{18}$$

With the adopted conventions we note that $F_V(n)$ has the dimension of energy, while $g_{V\pi\pi}(n)$ and c_n are dimensionless. Note that the signs of the residues appearing in the form factor (18) may *a priori* be positive or negative, while all contributions to the two-point correlator (14) are positive. The possibility of different signs in Eq. (18) provides a mechanism for cancellation. The form factor satisfies the dispersion relation (7) with the spectral density

$$\frac{1}{\pi}\,\mathrm{Im}F(t) = \sum_n c_n M_V(n)^2 \delta(t - M_V(n)^2). \tag{19}$$

Note that with the previously listed parameters for the lowest $\rho(770)$ resonance one has $c_\rho = g_{\rho\pi\pi} F_\rho / m_\rho = 1.17$. Because of charge conservation this requires higher states with negative c_n coefficients. In fact, taking Eq. (8) with the physical vector-meson masses $m_\rho = 770$ MeV, $m_{\rho'} = 1459(10)$ MeV, $m_{\rho''} = 1720(20)$ MeV, and $m_{\rho'''} = 2000(30)$ MeV and using the coupling constants c_n as fit parameters to the electromagnetic form factor data in the intermediate Q^2 range 0.6 GeV$^2 < Q^2 < 2.4$ GeV^2 yields $c_\rho = 1.25$, $c_{\rho'} = -0.17$ for two resonances, $c_\rho = 1.39$, $c_{\rho'} = -0.53$, $c_{\rho''} = 0.26$ for three resonances, and $c_\rho = 1.39$ and $c_{\rho'} = -0.53$, $c_{\rho''} = 0.26$, $c_{\rho'''} = -0.004$ for four resonances. Such an approach, although phenomenologically appealing and numerically stable for the lowest energy states, can only yield an integer power falloff and, as already mentioned, does not match to pQCD, Eq. (3), at high energies.

Strictly speaking one should consider in Eq. (18) the leading large-N_c contributions to the vector-meson parameters. According to Ref. [41], one has $M_V(n) \sim N_c^0$, $F_V(n) \sim \sqrt{N_c}$, and $g_{V\pi\pi}(n) \sim 1/\sqrt{N_c}$, such that $F(t) \sim N_c^0$. Corrections to this behavior are generally $1/N_c$ suppressed relative to the leading order and hence we expect at worse a 30% detuning of the physical values. The large-N_c dependence of meson parameters has been studied in unitarized chiral perturbation theory approaches yielding a larger value for the vector-meson mass when $m_\rho^{N_c} \to m_\rho^\infty \sim 1.2 m_\rho^{N_c=3}$ [54,55]. Chiral quark models at the one loop level are large N_c motivated. The spectral quark model [56] reproduces by construction the simple VMD result, Eq. (6), for the pion form factor providing, in addition, the value $m_\rho^2 = 24\pi^2 f_\pi^2/N_c$, which for $f_\pi = 92.3$ MeV yields $m_\rho \sim 820$ MeV, a larger value than the physical mass. The trend to an increased value of the ρ-meson mass can also be traced when in a fit of the two resonance version of the generalized VMD, Eq. (8), the lowest mass state is allowed to vary. Keeping $m_{\rho'} = 1460$ MeV this yields $c_\rho = 1.29$, $c_{\rho'} = 1 - c_\rho = -0.29$, and $m_\rho = 864$ MeV.

III. REGGE MODELS

We now proceed to review the Regge models in the scope necessary for our analysis, in particular, regarding the pion electromagnetic form factor. The radial Regge trajectories are

$$M_n^2 = M^2 + an. \tag{20}$$

The slope of the radial Regge trajectory a may be identified with the string tension $\sigma = a/(2\pi)$, which for heavy quarks corresponds to the confining potential $V(r) = \sigma r$. Acceptable values are in the range $\sigma = 420$–500 MeV [57]. In this work we use for definiteness

$$\sigma = 450 \text{ MeV}, \qquad M = 820 \text{ MeV}. \tag{21}$$

As mentioned above, these parameters need not exactly reproduce the physical values, as the accuracy of the present large-N_c Regge approach is not expected to be better than the large-N_c expansion itself. Fortunately, the pion electromagnetic form factor turns out not to be very sensitive to the details of the radial Regge trajectory.

Following Refs. [44–47], we consider the function

$$f_b(t) = \frac{B(b-1, \frac{M^2-t}{a})}{B(b-1, \frac{M^2}{a})}, \tag{22}$$

with $B(x, y) = \Gamma(x)\Gamma(y)/\Gamma(x+y)$ denoting the Euler Beta function. The function (22) fulfills the normalization condition

$$f_b(0) = 1. \tag{23}$$

For $x \gg y$ one has $B(x, y) \sim \Gamma(y)x^{-y}$, hence in the asymptotic region of $M^2 - t \gg (b-1)a$ we find

$$f_b(t) \sim \frac{\Gamma(\frac{M^2}{a} + b - 1)}{\Gamma(\frac{M^2}{a})}\left(\frac{a}{M^2 - t}\right)^{b-1}. \tag{24}$$

Moreover, this function is positive on the real axis $t < 0$, and has single poles at $t = M_n^2 = M^2 + an$, with residua read off from the expansion

$$f_b(t) = \frac{a}{B(b-1, \frac{M^2}{a})} \sum_{n=0}^{\infty} \frac{\Gamma(2-b+n)}{\Gamma(n+1)\Gamma(2-b)} \frac{1}{an + M^2 - t}. \tag{25}$$

The function depends on three parameters: the lowest-lying meson mass M, the string tension $\sigma = a/(2\pi)$, and the asymptotic falloff parameter b. An interesting feature is the fact that for noninteger values of b a large-t expansion in powers of $1/t$ has zero coefficients. For integer $b = N + 1$ the formula corresponds to exactly N mesons

$$f_{N+1}(t) = \prod_{n=0}^{N}\left(\frac{M^2 + an}{M^2 + an - t}\right). \tag{26}$$

Particular expressions corresponding to $b = 2, 3, 4$ are[1]

$$f_2(t) = \frac{M^2}{M^2 - t}, \qquad f_3(t) = \frac{M^2(M^2 + a)}{(M^2 - t)(M^2 + a - t)},$$
$$f_4(t) = \frac{M^2(M^2 + a)(M^2 + 2a)}{(M^2 - t)(M^2 + a - t)(M^2 + 2a - t)}. \tag{27}$$

At asymptotic values of Q^2 Eq. (25) yields

$$f_b(t = -Q^2) = \frac{\Gamma(\frac{M^2}{a} + b - 1)}{\Gamma(\frac{M^2}{a})}\left(\frac{a}{Q^2}\right)^{b-1}. \tag{28}$$

Thus, the value of b controls the asymptotic falloff in the Q^2 variable.

IV. FROM POWERS TO LOGARITHMS

In this section, we carry out the construction of the pion electromagnetic form factor, which complies to the asymptotic pQCD constraints. For the pion charge form factor one has the leading power behavior

$$F(Q^2) = \frac{16\pi f_\pi^2 \alpha(Q^2)}{Q^2} \sum_{n=0}^{\infty} c_n \alpha(Q^2)^n, \tag{29}$$

with the coefficients c_n calculable in pQCD (albeit this perturbative series may diverge). We have at LO $c_0 = 1$, which is stable in the large-N_c limit, as $\alpha \sim 1/N_c$ and $f_\pi \sim \sqrt{N_c}$. For large momenta $F(Q^2)$ is bounded as follows:

$$\frac{C}{Q^4} < F(Q^2) < \frac{C'}{Q^2}. \tag{30}$$

Thus, according to (28), the admissible possible power dependence effectively corresponds to $2 < b < 3$. A fit to

[1]One might think that taking $N \to \infty$ the general result would be recovered, but according to the product formula for the gamma function

$$\Gamma(z) = \lim_{N\to\infty} N^z \prod_{k=1}^{N} \frac{k}{(k+z)},$$

we see that this is not the case, since

$$\lim_{N\to\infty} \prod_{n=1}^{N} \frac{M^2 + an}{M^2 + an - t} = \lim_{N\to\infty} N^{t/a} \frac{\Gamma((M^2 - t)/a)}{\Gamma(M^2/a)},$$

and the result is ambiguous. This function has the poles located at the same place as in (22). The ambiguity is manifest in the choice of the parameter b. Generally speaking, the result for noninteger $N < b < N + 1$ has infinitely many resonances, but it is closer to the case of finite N rather than to $N \to \infty$. This suggests that a method based on truncating the tower of mesons is not expected to be convergent for increasing t values.

the data yields with $b = 2.3(1)$ [47] in agreement with the above expectations. This is a remarkable result, for it indicates on the one hand that even at energies where pQCD does not clearly set in, there seems to be some indirect information on the best possible fractional power behavior. On the other hand, note that in order to have a fractional power from the leading twist pQCD result (29) we need a nonanalytic dependence of the form $e^{-c/\alpha(Q^2)} \sim (Q^2/\Lambda^2)^{-4\pi c/\beta_0}$, which is clearly out of reach for standard perturbation theory. The previous considerations suggest that pQCD and large-N_c Regge models are mutually incompatible. As we discuss shortly this is not necessarily so.

Now we come to the core of our construction. In order to generate the asymptotic dependence (29) we superpose the Regge model formula (22) over the values of b

$$F(t) = \int_2^\infty db \rho(b) f_b(t), \tag{31}$$

where the density is given by

$$\rho(b) = \rho_{\rm high}(b) + \rho_{\rm low}(b), \tag{32}$$

and

$$\rho_{\rm high}(b) = \frac{4\pi}{\beta_0} \frac{16\pi f_\pi^2}{\Lambda^2} \left(\frac{a}{\Lambda^2}\right)^{1-b} \frac{\Gamma(\frac{M^2}{a})}{\Gamma(\frac{M^2}{a} + b - 1)} \sum_{n=0}^\infty \frac{c_n}{n!} \times \left(\frac{4\pi}{\beta_0}\right)^n (b-2)^n. \tag{33}$$

The coefficients c_n are precisely the same as in Eq. (29). Note that Eq. (33) corresponds to a Borel transformation of the original perturbative series, a feature which is welcome in view that the pQCD series is generally believed to be divergent but Borel-summable (see, e.g. Ref. [58] and references therein). The formula

$$\int_0^\infty d\epsilon \epsilon^n x^{-\epsilon} = \frac{n!}{\log^{n+1} x} \tag{34}$$

is the key ingredient in the *power-to-log transmutation*, where $\epsilon = b - 2$. Note that by taking the spectral density (33) we get the right pQCD asymptotics when the large-Q^2 behavior of the Regge model is used. The lower limit of integration in Eq. (31) controls the power of Q^2 in front of the right-hand side of Eq. (29). In fact, it is the behavior of $\rho(b)$ in the vicinity of $b = 2$ that determines the asymptotic behavior of $F(Q^2)$, thus $\rho(b)$ is not determined uniquely away from $b = 2$. One could attempt to use the form (33) for all values of b. However, according to the charge conservation we have to fulfill the normalization condition

$$F(0) = \int_0^\infty db \rho(b) = 1. \tag{35}$$

Fixing the scale $\Lambda_{\rm QCD} = 250$ MeV, we get both at LO and NLO

$$Z_{\rm high} = \int_2^\infty db \rho_{\rm high}(b) < 1. \tag{36}$$

To account for the missing strength we add an extra nonperturbative contribution $\rho_{\rm low}(b)$, which has support away from $b = 2$. For simplicity is taken in the form of a delta function

$$\rho_{\rm low}(b) = (1 - Z_{\rm high})\delta(b - b_0), \tag{37}$$

with $b_0 = 2.3$, as in the fit of Ref. [47], although other less singular distributions could also be used. Certainly, the presence of $\rho_{\rm low}(b)$ is not affecting the asymptotics of $F(Q^2)$, which is governed by the behavior near $b = 2$, but it modifies $F(Q^2)$ at lower momenta.

Explicitly, at LO and NLO we use

$$\rho^{\rm LO}_{\rm high}(b) = \frac{4\pi}{\beta_0} \frac{16\pi f_\pi^2}{\Lambda^2} \left(\frac{a}{\Lambda^2}\right)^{1-b} \frac{\Gamma(\frac{M^2}{a})}{\Gamma(\frac{M^2}{a} + b - 1)},$$
$$\rho^{\rm NLO}_{\rm high}(b) = \rho^{\rm LO}_{\rm high}(b)\left[1 + \frac{4\pi}{\beta_0} \frac{6.58}{\pi} \frac{(b-2)}{2!}\right], \tag{38}$$

which with parameters (21) and $N_f = 3$ yield

$$Z^{\rm LO}_{\rm high} = 0.27, \qquad Z^{\rm NLO}_{\rm high} = 0.39. \tag{39}$$

The spectral densities (38) are plotted in Fig. 1, with the dashed and solid lines representing the LO and NLO formulas, respectively. The $\rho^{\rm low}(b)$ contribution is represented by the vertical line at $b = b_0 = 2.3$. We note that the strength of the spectral density is practically contained in the interval between 2 and 3, and at large b we have a very fast falloff $\rho_{\rm high}(b) \sim b^{3/2 - M^2/a}(a/\Lambda^2)^{-b} e^{-b\log(b/e)}$. More generally, we might also include a finite upper limit of integration using the formula

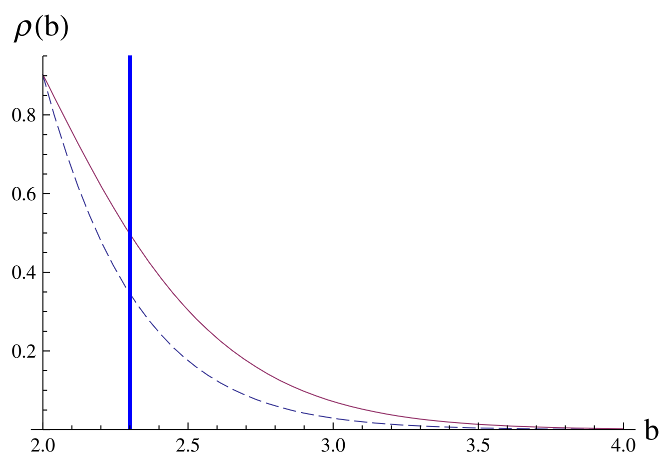

FIG. 1 (color online). The density $\rho^{\rm high}(b)$ at LO (dashed line) and NLO (solid line). The $\rho^{\rm low}(b)$ contribution is represented by the vertical line at $b = b_0 = 2.3$.

$$\int_{b_1}^{b_2} db\left(\frac{M_V^2-t}{a}\right)^{1-b} = \frac{1}{\log(M_V^2-t)/a}\left[\left(\frac{M_V^2-t}{a}\right)^{1-b_1} - \left(\frac{M_V^2-t}{a}\right)^{1-b_2}\right]. \quad (40)$$

The resulting value for Z_{high} changes by a small amount for $b_1 > 3$, depending on its precise value. Actually, by extending the integration to infinity we are maximizing the impact of perturbative corrections, and as we see, they are not large. Thus, not much change is expected from cutting off the integral above $b = 3$.

V. POLE-RESIDUE ASSIGNMENT

Our procedure is equivalent to imposing pQCD constraints for the pole-residue assignment in the spectral representation of the pion charge form factor. Looking formally at the problem, we need to form the spectral density (18) in such a manner that the asymptotic pQCD constraints are satisfied (apart for other constraints, such as normalization). In the preceding section, we have demonstrated explicitly that it is possible to accomplish this goal. More generally, in the large-N_c Regge model we have to choose the location of poles and fix their residues. Admittedly, there is a redundancy between shifting the poles or the residues. We decide to keep the poles fixed at their original location (20), because they are phenomenologically well described by the Regge trajectories. For the residues the prescription of the previous section is equivalent to taking

$$c_n = \int_2^\infty db\rho(b)\frac{a}{B(b-1,\frac{M^2}{a})}\frac{\Gamma(2-b+n)}{\Gamma(n+1)\Gamma(2-b)}. \quad (41)$$

In Fig. 2, we show the values of c_n for the three considered models: the model with fixed b of Dominguez [47], with $\rho(b) = \delta(b-2.3)$ (circles), and our model for the LO (squares) and NLO (diamonds) cases. We note a strong similarity between all cases. In particular, the first residue

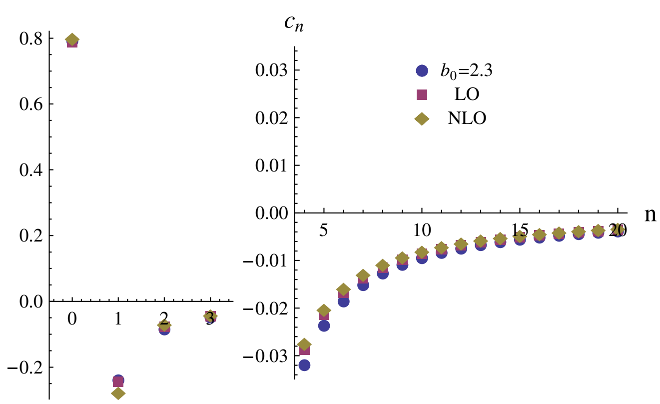

FIG. 2 (color online). Residues $c_n = F_V(n)g_{V\pi\pi}(n)/M_V(n)$ for the Regge poles of the pion charge form factor at $M_V(n)^2 = M^2 + an$ for $n = 0, 1, 2, 3$ (left) and for $n = 4, 5, \ldots$ (right).

c_0 is positive, and the remaining residues are negative, which leads to the desired cancellation. At very large values of n (not displayed) the LO and NLO residues have a larger magnitude than for the model with fixed b. Despite this similarity, we note that our LO and NLO models do satisfy the asymptotic pQCD constraints, while the fixed-b model does not. This reflects in the subtlety of the cancellation in the power-to-log transmutation mechanism. We stress that within our scheme we may achieve the goal of reproducing pQCD without modifying the spectrum; our spectral method features an effective way of implementing QCD radiative corrections by appealing to a modification of the meson wave functions.

At this point it is also interesting to display the values of the resulting $g_{V\pi\pi}(n)$ couplings. This requires some knowledge on the vector-meson-photon coupling $F_V(n)$. As mentioned above, quark-hadron duality for large t at the level of the two-point vector correlator requires the Regge model—parton model matching condition, Eq. (15), which for the mass formula, Eq. (20), becomes

$$2\pi\sigma = 24\pi^2 F_V^2/N_c. \quad (42)$$

This formula works reasonably well already for the lowest $\rho(770)$ state, where $F_\rho = 150$ MeV yields $\sqrt{\sigma} = 530$ MeV, while we expect $\sigma = 420$–500 MeV [57]. Following previous works [51,52], the formula (42) will be assumed to be valid for all n disregarding possible nonlinearities, which are not very relevant within the present context.[2] With $F_V = 150$ MeV and c_n from Fig. 2 with Eq. (18) we get

$$g_{\rho\pi\pi} = 4.3(4.4), \qquad g_{\rho'\pi\pi} = -2.3(-2.6),$$
$$g_{\rho''\pi\pi} = -0.9(-0.8), \qquad g_{\rho'''\pi\pi} = -0.6(-0.6), \quad (43)$$

where the first values are for the LO model, and the values in parenthesis for the NLO model.

VI. PION CHARGE FORM FACTOR RESULTS

In Fig. 3, we display the results of our model for the pion charge form factor and compare them to the TJLAB [6–8] and Cornell [3] data. The three lines close to one another and to the data are the NLO model (solid line), the LO model (dashed line), and the model with fixed $b = b_0 = 2.3$. The two lower curves correspond to the NLO (solid) and (LO) asymptotic pQCD results. We note that in the range of momenta accessible to experiments all the considered models yield very close predictions and describe the data well. These predictions depart from one another at very high values of Q^2, as can be seen from Fig. 4, where

[2] The sensitivity to details of the Regge trajectory depends on the computed observable. While for the pion form factor analyzed here there is some freedom, condensates with proper signs are crucially dependent on these details, as shown in Refs. [51,52].

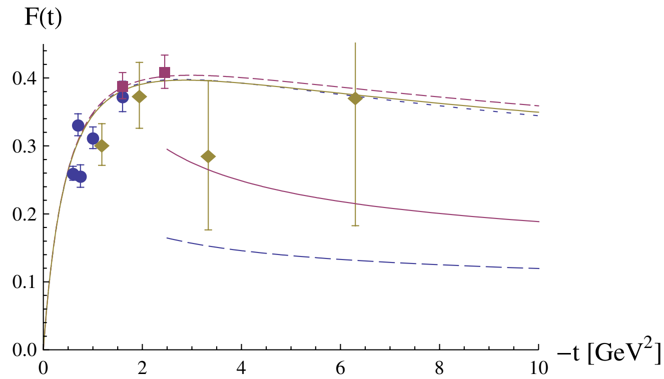

FIG. 3 (color online). The pion charge form factor at LO and NLO in the spacelike region $t < 0$: NLO (solid), LO (dashed), and $b_0 = 2.3$ (dotted). We plot $-tF(t)$ in the region up to $t = -10\ \mathrm{GeV}^2$ and compare with the TJLAB [6–8] (circles and squares) and Cornell [3] (diamonds) data. The two lower curves correspond to the NLO (solid) and (LO) asymptotic pQCD results.

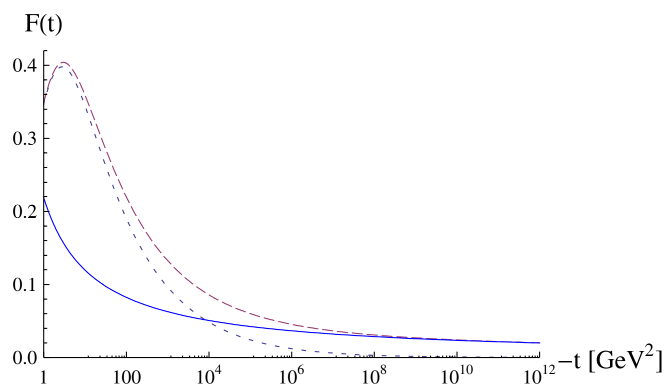

FIG. 4 (color online). The pion charge form factor in the LO model in the spacelike region $t < 0$. We plot $-tF(t)$ in the region up to very high $t = -10^{12}\ \mathrm{GeV}^2$ (on a log scale). The solid line represents the asymptotic LO pQCD result. The dashed line is the LO model. The short-dashed line is the model with $b_0 = 2.3$.

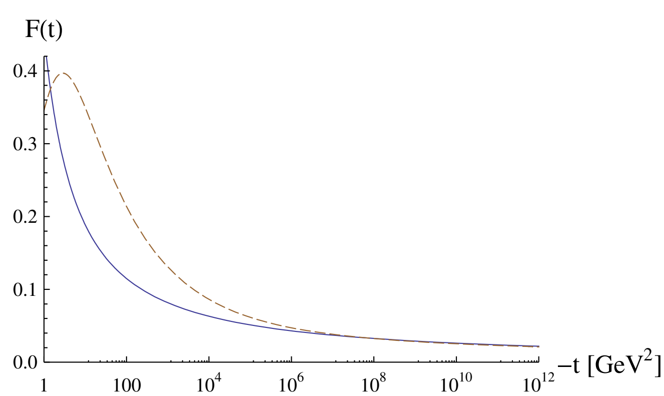

FIG. 5 (color online). Same as Fig. 4 for the NLO case.

we plot the LO result (dashed line) and the asymptotic LO pQCD expression (solid line). We note that the curves meet at $Q^2 \sim 10^8\ \mathrm{GeV}^2$, which is a very high scale. For comparison we also plot the result of the fixed b model (dotted line), which with the chosen value $b_0 = 2.3$ decays as $(1/Q^2)^{1.3}$. Figure 5 shows the same study for the NLO calculation, with the model denoted by the dashed, and the NLO pQCD calculation by the solid lines, respectively. The two curves meet at somewhat lower scales $Q^2 \sim 10^7\ \mathrm{GeV}^2$ than for the LO case of Fig. 4.

VII. CONCLUSIONS

There have been countless attempts to understand the delayed onset of pQCD in the pion charge form factor. The standard VMD model is known to fit the available data remarkably well, but shows no obvious link to pQCD. In the present paper, we have approached the problem from the viewpoint of the large-N_c Regge models. Our approach exploits explicitly the quark-hadron duality at a nonperturbative level and has the genuine advantage that much of the discussion can be carried out without an explicit reference to the light-cone wave functions and/or parton distribution amplitudes; many uncertainties in current calculations seem related to our lack of the detailed knowledge of these nonperturbative objects used in the description of exclusive processes. Our generalized VMD model includes infinitely many resonances, describes the data, and simultaneously complies to the known short-distance pQCD constraints. The present framework requires a suitable modeling both of the spectrum and the vector-meson coupling to the electromagnetic current. While it describes the so far experimentally explored spacelike momentum region, it is rather hard to provide estimates of the systematic error of the calculation. The important feature which has been clearly identified several times in the past in the analysis of the data is that at large Q^2 the pion form factor seems to have a noninteger power falloff, which actually turns out to be in the expected range for the best possible pQCD power-log behavior, but still is qualitatively different from the theoretical expectations based on pQCD. We have shown that there is no contradiction between both behaviors. Actually, we have spelled out a simple mechanism where a suitable superposition of noninteger power falloffs may transmute into the desired asymptotic pQCD behavior, including the presence of logarithms. We have shown that such a procedure does not spoil the good agreement in the so far experimentally accessible region down the low energy region where chiral corrections cause sizable distortions from any large-N_c calculation. Moreover, we are able to reproduce simultaneously the high-energy pQCD behavior, providing some confidence on the range where pQCD sets in. We find that about $1/4$ for the LO and about $1/3$ for the NLO case of the pion charge is due to the high-energy pQCD tail in our approach. Finally, the present calculations suggest that nonperturbative contributions dominate in the region corresponding to the present and planned experimental data, and would saturate the full result only at extremely high values $Q^2 \sim 10^7$–10^8 GeV^2.

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