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Measuring the top quark mass with the m_{T2} variable

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We investigate the possibility of measuring the top quark mass using the collider variable m_{T2} at the CERN LHC experiment. Monte Carlo studies of m_{T2} are performed with the events corresponding to the dilepton decays of $t\bar{t}$ produced at the LHC with 10 fb^{-1} integrated luminosity. Our analysis suggests that the top quark mass can be determined by the m_{T2} variable with a reasonable accuracy, though the precision will be determined by systematic errors for which a more complete analysis would need to be performed.

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I. INTRODUCTION

When the CERN Large Hadron Collider (LHC) is turned on, it will serve as a "top quark factory" [1,2]. The cross section for $t\bar{t}$ pair production at the LHC is estimated to be 833 pb at the next leading order calculation [3], implying roughly 8×10^6 $t\bar{t}$ pairs per year at low luminosity run (10 fb^{-1}/year). Such a large number of $t\bar{t}$ events will enable us to measure the top quark mass with high precision.

Precision measurement of the top quark mass m_t is desirable in many respects. For example, it would help to constrain the allowed Higgs boson mass in the standard model (SM). In general, it would affect the constraints on the allowed parameter space of various models of new physics at the TeV scale, including the minimal supersymmetric standard model and technicolorlike models. The top quark mass measurement can be performed through various methods in different channels, which have their own advantage/disadvantage with different systematic uncertainties. Overall, the accuracy of m_t measured at the LHC is expected to be around 1 GeV [4].

In the SM, top quark decays mostly into a b quark and a W boson. The W boson then decays hadronically ($W \to qq'$) or leptonically ($W \to l\nu$). Depending on the W boson decay mode, the $t\bar{t}$ events are divided into three channels, *i.e.*, the dilepton channel (both W bosons decay leptonically), the lepton plus jets channel (one W boson decays leptonically and the other hadronically), and the pure hadronic channel (both W bosons decay hadronically).

The dilepton channel has a small branching fraction compared to the lepton plus jets channel and the pure hadronic channel. It also involves two missing neutrinos, which makes a direct event-by-event measurement of m_t not possible. However, it has a cleaner environment, e.g. less combinatorial background and less jet energy scale dependence, compared to other channels, therefore various approaches for an indirect measurement of m_t with dilepton channel have been investigated [4].

It has been shown that the collider variable m_{T2} [5] can be useful for the determination of new particle masses in the process in which new particles are pair produced at the hadron collider and each of them decays into one invisible particle and one or more visible particles [5–9]. In this paper, we examine the possibility to determine the top quark mass using m_{T2} at the LHC experiment. For this, we perform three Monte Carlo studies of m_{T2} for the process $t\bar{t} \to bl^+\nu\bar{b}l^-\nu$: the first which determines the end point value of the m_{T2} distribution for the neutrino mass $m_\nu = 0$, the second to examine the functional dependence of $m_{T2}^{\max}$ on the trial neutrino mass $\tilde{m}_\nu \neq 0$, which would determine m_t for a given value of the W boson mass m_W, and the third which fits the m_{T2} distribution to "template" distributions. Our analysis suggests that the top quark mass can be determined by the m_{T2} variable alone with a reasonable error, though a more complete analysis of systematic uncertainties would need to be performed.

In Sec. II, we briefly introduce the m_{T2} variable for the dilepton decay of $t\bar{t}$. The results of Monte Carlo studies are presented in Sec. III, and Sec. IV is the conclusion.

II. TRANSVERSE MASS AND m_{T2} FOR TOP QUARK

Let us consider a $t\bar{t}$ pair production and its subsequent decay at the LHC:

$$pp \to t\bar{t} \to bW^+\bar{b}W^-. \tag{1}$$

In the case that one of the W bosons decays into leptons, one can consider the associated transverse mass of $t \to bl\nu$, which is defined as

$$m_T^2 = m_{bl}^2 + m_\nu^2 + 2(E_T^{bl}E_T^\nu - \mathbf{p}_T^{bl} \cdot \mathbf{p}_T^\nu), \tag{2}$$

where m_{bl} and $\mathbf{p}_T^{bl}$ denote the invariant mass and transverse momentum of the bl system, respectively, while m_ν and $\mathbf{p}_T^\nu$ are the mass and transverse momentum of the missing neutrino, respectively. The transverse energies of the bl system and neutrino are defined as

$$E_T^{bl} \equiv \sqrt{|\mathbf{p}_T^{bl}|^2 + m_{bl}^2} \quad \text{and} \quad E_T^\nu \equiv \sqrt{|\mathbf{p}_T^\nu|^2 + m_\nu^2}. \tag{3}$$

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If the other W boson decays into hadrons, i.e. for the process $t\bar{t} \to bl\nu\bar{b}qq'$, $\mathbf{p}_T^\nu$ can be read off from the total missing transverse momentum $\mathbf{p}_T^{\rm miss}$. One might then construct the m_T distribution of $t \to bl\nu$ from data, which can be used to determine the top quark mass m_t as its shape and end point depend on m_t. However, to determine $\mathbf{p}_T^\nu$ in the process $t\bar{t} \to bl\nu\bar{b}qq'$, one needs to measure the full final state momenta of $\bar{t} \to \bar{b}qq'$, which by itself would determine m_t in an event-by-event basis. At any rate, if one uses information from $\bar{t} \to \bar{b}qq'$ to determine m_t, the procedure involves more jets, which would result in larger uncertainties in the determined value of m_t.

A method to determine m_t without using the hadronic decay of W is to construct m_{T2} for the dilepton decay

$$t\bar{t} \equiv t^{(1)}t^{(2)} \to b^{(1)}l^{(1)}\nu^{(1)}b^{(2)}l^{(2)}\nu^{(2)}. \qquad (4)$$

Although each neutrino momentum cannot be measured in this case, still the total missing transverse momentum $\mathbf{p}_T^{\rm miss} = \mathbf{p}_\mathbf{T}^{\nu(\mathbf{1})} + \mathbf{p}_\mathbf{T}^{\nu(\mathbf{2})}$ can be determined experimentally. The m_{T2} variable of each event is defined as

$$m_{T2} \equiv \min_{\mathbf{p}_T^{\nu(1)}+\mathbf{p}_T^{\nu(2)}=\mathbf{p}_T^{\rm miss}} [\max\{m_T^{(1)}, m_T^{(2)}\}], \qquad (5)$$

where $m_T^{(i)}$ $(i = 1, 2)$ is the transverse mass of $t^{(i)} \to b^{(i)}l^{(i)}\nu^{(i)}$, and the minimization is performed over the *trial* neutrino momenta $\mathbf{p}_T^{\nu(i)}$ constrained as

$$\mathbf{p}_T^{\nu(1)} + \mathbf{p}_T^{\nu(2)} = \mathbf{p}_T^{\rm miss}. \qquad (6)$$

The above definition of m_{T2} indicates that m_{T2} for $m_\nu = 0$ is bounded above by m_t in the approximation ignoring the decay width of top quark. One might then determine m_t as

$$m_t = m_{T2}^{\max}(m_\nu = 0) \equiv \max[m_{T2}(m_{bl}^{(1)}, \mathbf{p}_T^{bl(1)}, m_{bl}^{(2)}, \mathbf{p}_T^{bl(2)}, m_\nu = 0)]. \qquad (7)$$

In fact, because of nonzero decay width, there can be certain amount of events which give m_{T2} exceeding the physical top quark mass m_t. Our Monte Carlo study suggests that such events do not spoil the sharp edge structure of the m_{T2} distribution with which one can determine m_t rather precisely. Figure 1 shows the top quark m_{T2} distribution for $m_\nu = 0$ obtained from a parton level Monte Carlo simulation[1] using the PYTHIA event generator [10] with an input top mass of $m_t = 170.9$ GeV. In the figure, the hatched and unhatched histograms present the cases without and with the top decay width, respectively. One can see that m_{T2} tends to zero rapidly near the input top mass with a minor but long tail beyond the input mass which is mainly due to the nonzero top decay width.[2]

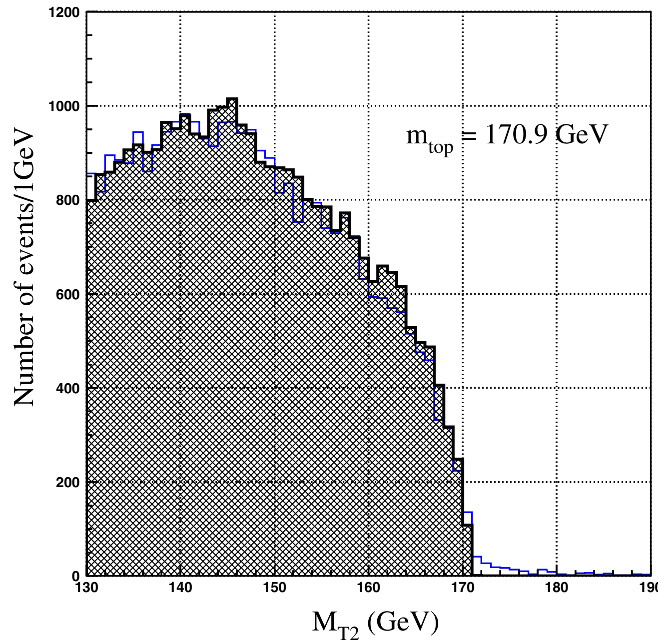

FIG. 1 (color online). m_{T2} distribution obtained from partonic-level simulation. The input top quark mass of 170.9 GeV is used for the simulation. The hatched and unhatched histograms correspond to the cases with zero and usual decay width of the top quark, respectively. One can find a sharp edge at the input top mass, with a small tail which is mainly due to the finite top quark decay width.

One can consider the top quark m_{T2} defined as above for the arbitrary trial neutrino mass which is *not* the same as the true neutrino mass. In such a case, m_{T2} is not only a function of the observable kinematic variables $m_{bl}^{(i)}$ and $\mathbf{p}_T^{bl(i)}$ $(i = 1, 2)$, but also of the trial neutrino mass. Let $\tilde{m}_\nu$ denote the trial neutrino mass to distinguish it from the true neutrino mass $m_\nu = 0$. The end point value of m_{T2} for generic $\tilde{m}_\nu$,

$$m_{T2}^{\max}(\tilde{m}_\nu) = \max[m_{T2}(m_{bl}^{(1)}, \mathbf{p}_T^{bl(1)}, m_{bl}^{(2)}, \mathbf{p}_T^{bl(2)}, \tilde{m}_\nu)], \qquad (8)$$

appears to be a function of $\tilde{m}_\nu$, and its functional form provides a relation between m_t, the W boson mass m_W, and the b quark mass m_b. Using the result of Ref. [6], one easily finds that $m_{T2}^{\max}$ as a function of $\tilde{m}_\nu$ is given by

$$m_{T2}^{\max}(\tilde{m}_\nu) = \frac{m_t^2 + (m_{bl}^{\max})^2}{2m_t} + \sqrt{\left(\frac{m_t^2 - (m_{bl}^{\max})^2}{2m_t}\right)^2 + \tilde{m}_\nu^2}, \qquad (9)$$

where

$$(m_{bl}^{\max})^2 = m_b^2 + \tfrac{1}{2}(m_t^2 - m_W^2 - m_b^2) + \tfrac{1}{2}\sqrt{(m_t^2 - m_W^2 - m_b^2)^2 - 4m_W^2 m_b^2}. \qquad (10)$$

[1]For simplicity, here we switched off the initial and final state radiations as well as the quark fragmentation process.

[2]Such a sharp edge structure of m_{T2} distribution at the input mass of the mother particle can be confirmed also in the m_{T2} distribution for $W^+W^- \to l^+\nu l^-\nu$.

This analytic expression of $m_{T2}^{\max}(\tilde{m}_\nu)$ provides another way to determine m_t, i.e. one can determine m_t by fitting $m_{T2}^{\max}(\tilde{m}_\nu)$ obtained from data to this analytic expression with the known values of m_W and m_b. As Eq. (9) can be considered as a constraint which should be satisfied by the maximum of the m_{T2} distribution for generic value of $\tilde{m}_\nu$, it might help to read the correct end point positions. It can be also used to check the effect of initial state radiation (ISR) on the m_{T2} variable constructed from real data. Since the analytic expression (9) is derived under the assumption of no ISR, any deviation of $m_{T2}^{\max}(\tilde{m}_\nu)$ from Eq. (9) would indicate the significance of the ISR effect.

III. EXPERIMENTAL FEASIBILITY

Measuring the top mass using m_{T2}, when applied to real data, will suffer from a variety of uncertainty factors such as backgrounds, event selection cuts, finite jet energy resolution, and combinatorial background. In order to check the feasibility of the m_{T2} method at the LHC, we have generated Monte Carlo samples of $t\bar{t}$ events by PYTHIA [10] with the CTEQ5L parton distribution function (PDF) [11]. The event sample corresponds to 10 fb^{-1} integrated luminosity.

The generated events have been further processed with a modified version of fast detector simulation program PGS [12], which approximate an ATLAS or CMS-like detector with reasonable efficiencies and fake rates. The PGS program uses a cone algorithm for jet reconstruction, with default value of cone size $\Delta R = 0.5$, where ΔR is a separation in the azimuthal angle and pseudorapidity plane. And the b jet tagging efficiency ϵ_b is introduced as a function of the jet transverse energy and pseudorapidity, with a typical value of $\epsilon_b \sim 50\%$ in the central region for high energy jets.

In the PGS, isolated leptons (electron and muon) are identified with some isolation cuts on the calorimeter activity around the lepton track [13]. For electrons, the isolation cuts are (i) $ETISO/E_T < 0.1$, where $ETISO$ is the total transverse calorimeter energy in a 3×3 grid around the electron candidate (excluding the candidate cell) and E_T is the transverse energy of the electron candidate, (ii) $PTISO < 5$ GeV, where $PTISO$ is the total p_T of tracks (except the electron track) with $p_T > 0.5$ GeV within a $\Delta R < 0.4$ cone around the electron candidate, and (iii) $0.5 < EP < 1.5$, where EP is the ratio of the calorimeter cell energy to the p_T of the candidate track. For isolated muons, (i) $PTISO < 5$ GeV and (ii) $ETRAT < 0.1125$, where $ETRAT$ is the ratio of E_T in a 3×3 calorimeter array around the muon (including the muon's cell) to the p_T of the muon.

The dilepton events are selected by requiring (A) only two isolated leptons of opposite charge with $p_T > 25$ GeV and $|\eta| < 2.5$, (B) dilepton invariant mass with $|m_{ll} - m_Z| > 5$ GeV, (C) large missing transverse energy $E_T^{\text{miss}} > 40$ GeV, and (D) at least two b-tagged jets with $p_T > 30$ GeV and $|\eta| < 3.0$. After this selection, 5133 events are survived among the 5.5×10^6 generated $t\bar{t}$ events[3] (in which 1.8×10^5 are the dilepton events, considering only electrons and muons), leading to a selection efficiency of about 2.8% for the dilepton channel signal events.

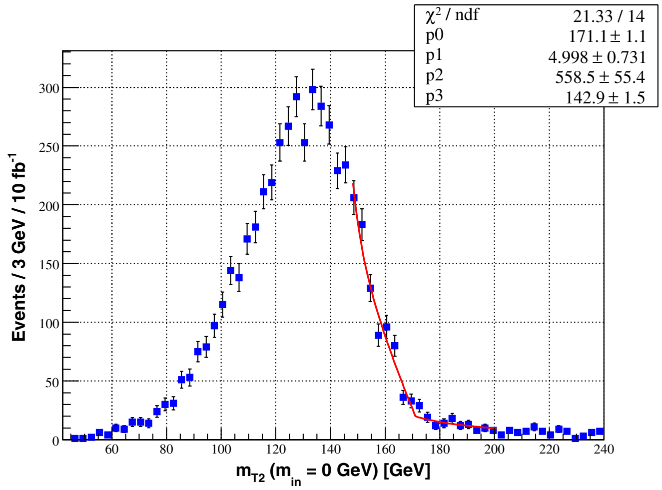

FIG. 2 (color online). m_{T2} distribution after event selection. The input value of top quark is $m_t = 170.9$ GeV. A fit to the distribution near end point region is also shown, providing a fit value of $m_t = 171.1 \pm 1.1$ GeV.

The main backgrounds might come from $Z/\gamma^*/W$ production with additional jets, diboson events with additional jets, and $b\bar{b}$ events with misidentified leptons. We have generated the main background events using PYTHIA, ALPGEN [14], and ACERMC [15], and required the same selection cuts as the $t\bar{t}$ dilepton events. After the cuts, it turns out that those backgrounds are reduced to a negligible level. For instance, among 5.3×10^5 events (for 10 fb^{-1}) of $Zb\bar{b}$, which is one of the most dominant SM backgrounds, only 28 events remain after the selection cuts. We will not include the background events in our further analysis, for simplicity.

With two b jets and two leptons in each selected event, there are two possible combinations for bl pairing. We calculated m_{T2} variable for each of the two possible bl combinations, and chose the smaller one as the final m_{T2}. This procedure closely follows the idea proposed in Ref. [8].

Figure 2 shows the resulting m_{T2} distribution for the selected events. As anticipated, one can find an edge structure around $m_{T2} = 170$ GeV, on the distribution. We employ three methods to precisely determine the top quark mass from the m_{T2} distribution, which will be discussed in the following three subsections.

[3] This number of events corresponds to the cross section of 550 pb at the leading order calculation.

A. A fit near the end point

Figure 2 shows the m_{T2} distribution obtained from the selected events for the neutrino mass $m_\nu = 0$. It is fitted with an empirical function which consists of a linear function for signal distribution and an inverse linear function for background distribution. The fit range was chosen within $\pm\mathcal{O}(10)$ bins around a plausible end point. Such fitting of the m_{T2} distribution results in

$$m_t = 171.1 \pm 1.1 \text{ GeV}, \tag{11}$$

which reproduces the input top quark mass of 170.9 GeV with a precision at the level of 1 GeV. However, notice that the above result (11) does not include systematic uncertainties from various sources such as the choice of binning of m_{T2} distribution.

To estimate possible systematic error associated with the fitting procedure, we have repeated the fitting with two linear functions for both signal and background distributions. The resulting top quark mass is then given by $m_t = 169.9 \pm 1.8$ GeV, showing a mass shift of 1.2 GeV. Systematic error from the fitting procedure might be improved by considering a template binned likelihood fit, which will be discussed in Sec. III C.

Absolute jet energy scale also affects the determination of the top mass. The b-jet energy scale is assumed to be known within 1% accuracy. It is found that the 1% variation of the jet scale leads to a shift of the resulting top mass of 0.5 GeV.

Uncertainty due toISR is estimated by comparing the nominal data (with ISR switched on) to the one which is generated while switching off ISR. The 20% of the resulting top mass shift is found to be 0.4 GeV, which is taken as the systematic error from ISR uncertainty [4]. The same approach to final state radiation induces a systematic error of 0.7 GeV.

For systematic error from PDF uncertainty, it is found that the use of CTEQ3L (GRV94L) PDF, instead of the default CTEQ5L PDF, leads to a shift of the central top mass of 0.3 (1.3) GeV, with a suitable choice of fit range.

B. End point as a function of trial neutrino mass

As we have discussed in Sec. II, the end point of m_{T2} distribution can be considered as a function of a trial neutrino mass, if we use a trial neutrino mass $\tilde{m}_\nu \neq 0$ for the m_{T2} calculation. Using the selected dilepton decays of $t\bar{t}$, we constructed the m_{T2} distributions for different choices of $\tilde{m}_\nu$. Figure 3(a) shows the m_{T2} distribution for $\tilde{m}_\nu = 80$ GeV. Here we also performed a fit to the m_{T2} distribution with a linear function for signal and an inverse linear function for background. The maximum of m_{T2} is then determined to be $m_{T2}^{\max} = 232.6 \pm 1.5$ GeV for $\tilde{m}_\nu = 80$ GeV. The $m_{T2}^{\max}$ as a function of $\tilde{m}_\nu$ is shown in Fig. 3(b). Fitting the data points to the theoretical curve (9) considering m_t as a free parameter while using $m_W = 80.45$ GeV and $m_b = 4.7$ GeV, we obtain

$$m_t = 170.5 \pm 0.5 \text{ GeV}, \tag{12}$$

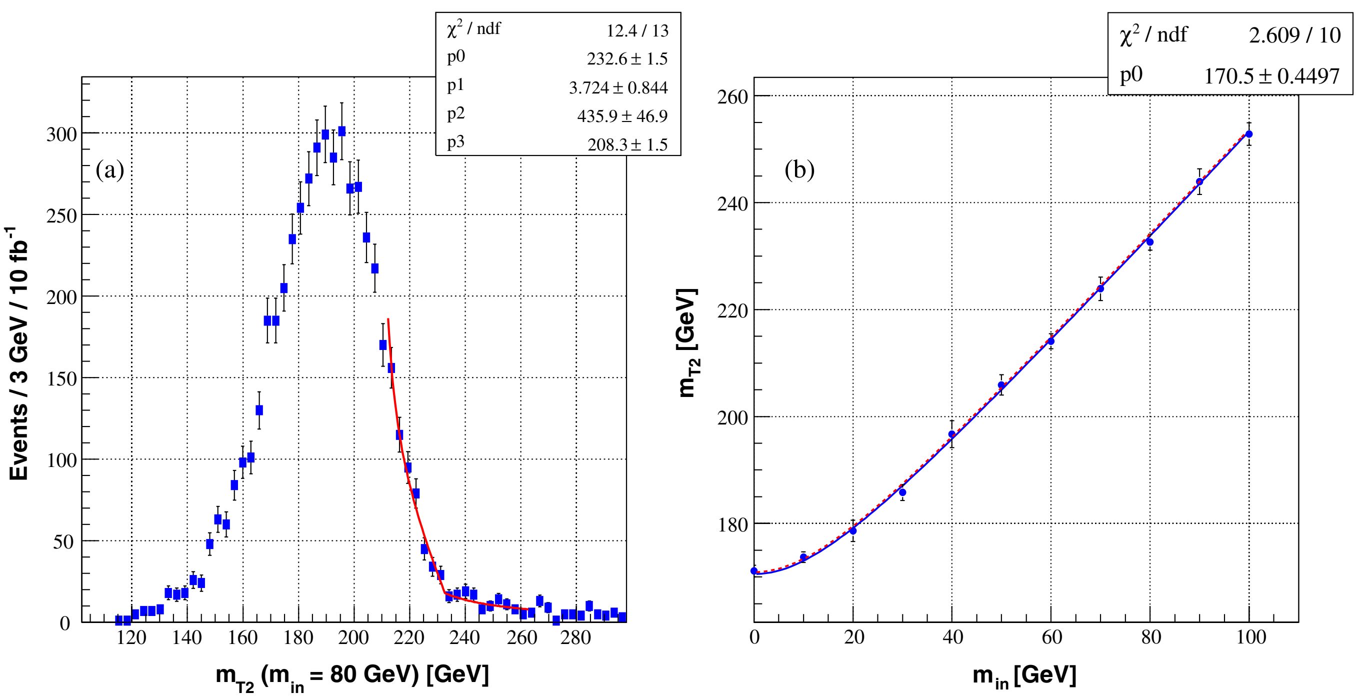

FIG. 3 (color online). (a) An example of m_{T2} distribution with a trial neutrino mass. Here, the trial mass is set to $\tilde{m}_\nu = 80$ GeV. (b) The maximum of m_{T2} as a function of trial neutrino mass $\tilde{m}_\nu$. Also shown is the fit of the data points to theoretical curve (2.7) considering m_t as a free parameter.

which is quite close to the input top quark mass $m_t = 170.9$ GeV. However, this result does not include systematic uncertainties such as the choice of binning of m_{T2} distribution. The uncertainty due to a variation of m_b is negligible as it is of $\mathcal{O}(m_b \delta m_b / m_t)$. To check the effect of the W boson mass, we repeated the fitting procedure while varying m_W by ± 0.5 GeV. The resulting shift of m_t turns out to be negligible.

C. Template binned likelihood fit

Perhaps the most reliable way to determine m_t using m_{T2} is to employ the template binned likelihood fit. For this, we attempted to fit the m_{T2} distribution of the "nominal data" (which was generated with $m_t = 170.9$ GeV) to "templates". Here, a template means a simulated m_{T2} distribution with an input top quark mass different from 170.9 GeV. The templates were generated with input top quark mass between 166 and 176 GeV, in steps of 1 or 0.5 GeV, using the same PYTHIA + PGS Monte Carlo programs as the case of nominal data sample.

Figure 4(a) shows three representative m_{T2} distributions for the nominal data (points) and two templates with $m_t = 166$ GeV (blue solid line) and $m_t = 176$ GeV (red solid line), respectively. Each template distribution is normalized to make the total number of events the same as that of the nominal data. One can notice that those three m_{T2} distributions are well separated from each other, showing the sensitivity of the m_{T2} distribution to the input top quark mass.

Each template distribution is compared to the nominal data distribution for a calculation of the logarithm of the binned likelihood. The binned likelihood is defined as the product of the Poisson probability for each bin over the N bins in the fit range:

$$\mathcal{L} = \prod_{i=1}^{N} \frac{e^{-m_i} m_i^{n_i}}{n_i!}, \tag{13}$$

where n_i and m_i are the event numbers at the i-th bin in the distributions of the nominal data and the normalized template, respectively. The minimum of $-\ln\mathcal{L}$ gives the best fit value of the top quark mass. We have chosen the 1σ deviated value of the top quark mass as the one increasing $-\ln\mathcal{L}$ by $1/2$.

We fit the m_{T2} distribution of nominal data to templates in the range $100\ \text{GeV} < m_{T2} < 180\ \text{GeV}$. The result of the likelihood fit for m_{T2} distributions is shown in Fig. 4(b), where the negative logarithm of the likelihood ratio $\mathcal{L}/\mathcal{L}_{\max}$ as a function of m_t is depicted. The $\mathcal{L}_{\max}$ is the maximum likelihood which was determined as the minimum of a parabola fit to the $-\ln\mathcal{L}$ distribution. The top quark mass resulting from our template likelihood fit is given by

$$m_t = 170.3 \pm 0.3\ \text{GeV}, \tag{14}$$

which reproduces well the input top quark mass with a small statistical error.

Again, the above result does not include systematic error. Although a detailed analysis of systematic uncertain-

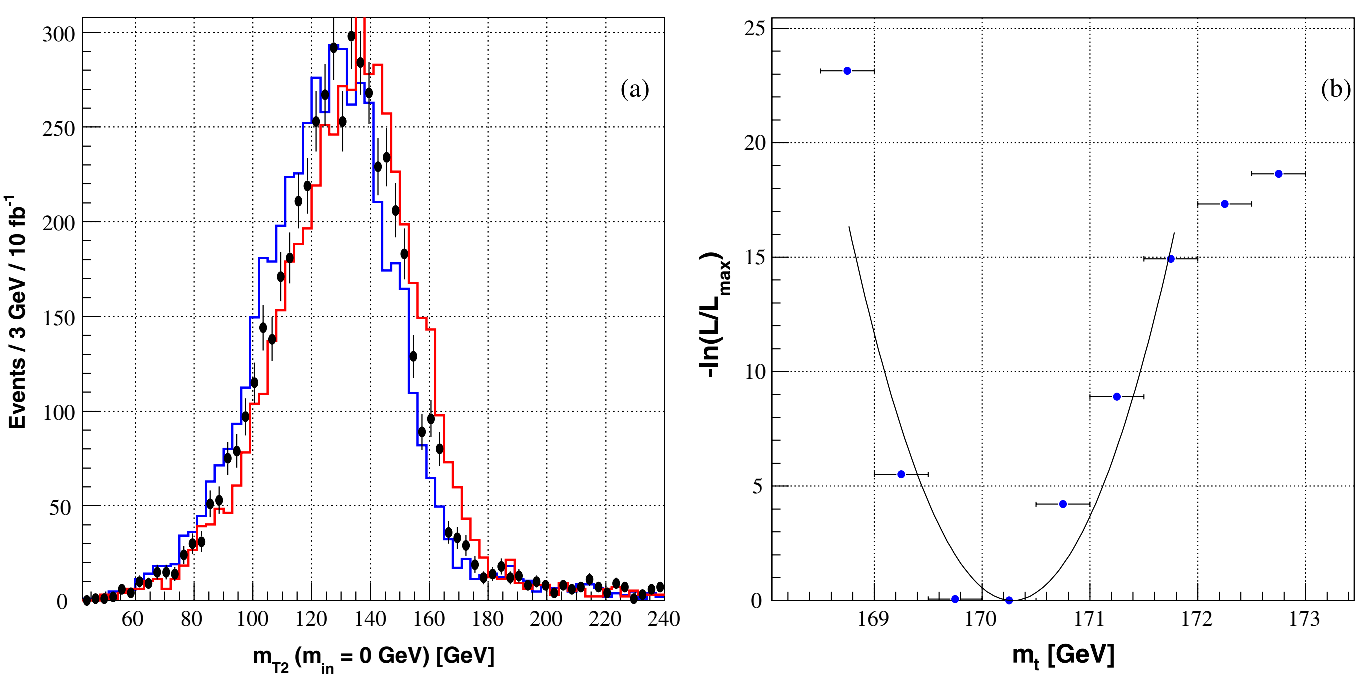

FIG. 4 (color online). (a) Three representative m_{T2} distributions for the nominal data (points) and two templates with $m_t = 166$ GeV (blue solid line) and $m_t = 176$ GeV (red solid line), respectively. (b) The negative logarithm of the likelihood ratio $\mathcal{L}/\mathcal{L}_{\max}$ as a function of m_t for the m_{T2} fit.

ties in the template fit method is beyond the scope of this work, we expect that systematic errors from b-jet energy scale, ISR/FSR and PDF are also at the level of 1 GeV as those in the end point fit method discussed in Sec. III A.

IV. CONCLUSION

We have examined the possibility of determining the top quark mass using the m_{T2} distribution of the dileptonic decay channel of $t\bar{t}$ events at the LHC. For this, we have performed three Monte Carlo studies for the events produced at the LHC with 10 fb^{-1} integrated luminosity: the first to fit the m_{T2} distribution near the end point (for the neutrino mass $m_\nu = 0$) with an empirical function, the second to fit the functional dependence of $m_{T2}^{\max}$ on the trial neutrino mass $\tilde{m}_\nu \neq 0$, and the third to perform a template binned likelihood fitting. Our analysis suggests that the top quark mass can be extracted by the m_{T2} variable with a reasonable accuracy, although more complete analysis of systematic errors is required to estimate the precision more accurately.

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[1] ATLAS Collaboration, Report No. CERN-LHCC-94-43.
[2] CMS Collaboration, Report No. CERN-LHCC-2006-021.
[3] R. Bonciani *et al.*, Nucl. Phys. **B529**, 424 (1998).
[4] I. Borjanovic *et al.*, Eur. Phys. J. C **39S2**, 63 (2005).
[5] C. G. Lester and D. J. Summers, Phys. Lett. B **463**, 99 (1999); A. Barr, C. Lester, and P. Stephens, J. Phys. G **29**, 2343 (2003).
[6] W. S. Cho, K. Choi, Y. G. Kim, and C. B. Park, Phys. Rev. Lett. **100**, 171801 (2008); J. High Energy Phys. 02 (2008) 035.
[7] B. Gripaios, J. High Energy Phys. 02 (2008) 053.
[8] A. J. Barr, B. Gripaios, and C. G. Lester, J. High Energy Phys. 02, (2008) 014.
[9] G. Ross and M. Serna, Phys. Lett. B **665**, 212 (2008); M. Nojiri, Y. Shimizu, S. Okada, and K. Kawagoe, J. High Energy Phys. 06 (2008) 035.
[10] T. Sjostrand, P. Eden, C. Friberg, L. Lonnblad, G. Miu, S. Mrenna, and E. Norrbin, Comput. Phys. Commun. **135**, 238 (2001); T. Sjostrand, S. Mrenna, and P. Skands, Report No. LU TP 06-13/FERMILAB-PUB-06-052-CD-T.
[11] H. L. Lai *et al.* (CTEQ Collaboration), Eur. Phys. J. C **12**, 375 (2000).
[12] http://www.physics.ucdavis.edu/~conway/research/software/pgs/pgs4-general.htm.
[13] http://v1.jthaler.net/olympicswiki/doku.php.
[14] M. L. Mangano, M. Moretti, F. Piccinini, R. Pittau, and A. D. Polosa, J. High Energy Phys. 07 (2003) 001.
[15] B. P. Kersevan and E. Richter-Was, arXiv:hep-ph/0405247; Comput. Phys. Commun. **149**, 142 (2003).