Form factors of $B_{u,d,s}$ decays into p-wave axial-vector mesons in the light-cone sum rule approach

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We calculate the vector and axial-vector form factors of $B_{u,d,s}$ decays into P-wave axial-vector mesons in the light-cone sum rule approach. For the sum rule results, we have included corrections of order m_A/m_b, where m_A is the mass of the axial-vector meson A. The results are relevant to the light-cone distribution amplitudes of the axial-vector mesons. It is important to note that, owing to the G parity, the chiral-even two-parton light-cone distribution amplitudes of the $^3P_1$ ($^1P_1$) mesons are symmetric (antisymmetric) under the exchange of quark and antiquark momentum fractions in the SU(3) limit. For chiral-odd light-cone distribution amplitudes, it is the other way around. The predictions for decay rates of $B_{u,d,s} \to A e \nu_e$ are also presented.

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I. INTRODUCTION

The inclusive and exclusive B decays provide potentially stringent test of the standard model. Although the inclusive rare decays are theoretically clean, they are a challenge for measurements at B factories. The exclusive processes may easily be accessible for experiments, but knowledge of form factors is required. The production of the axial-vector mesons has been seen in charmful B decays $B \to J/\psi K_1(1270)$ and $B \to D a_1(1260)$ [1]. As for charmless hadronic B decays, $B^0 \to a_1^{\pm}(1260)\pi^{\mp}$ are the first modes measured by BABAR and Belle [2–4]. Information for weak phase $\alpha \equiv \arg(-V_{\rm td}V_{\rm tb}^*/V_{\rm ud}V_{\rm ub}^*)$ can be extracted from their time-dependent measurement or by relating these decays with corresponding $\Delta S = 1$ decays. BABAR has further reported the observation of the decays $\bar{B}^0 \to b_1^{\pm}\pi^{\mp}$, $b_1^+K^-$, $K_1^-(1270)\pi^+$, $K_1^-(1400)\pi^+$, $a_1^+K^-$ and $B^- \to b_1^0\pi^-$, $b_1^0K^-$, $a_1^0\pi^-$, $a_1^-\pi^0$, $a_1^-\bar{K}^0$, $f_1(1285)K^-$, $f_1(1420)K^-$ [5,6]. Very recently, $B^- \to K_1^-(1270)\phi$, $K_1^-(1400)\phi$ have been observed by BABAR [7]. Using the QCD factorization approach, we have studied charmless two-body B decays involving one or two axial-vector meson(s) in the final state [8–10].

In the quark model, two lowest nonets of $J^P = 1^+$ axial-vector mesons are expected as the orbitally excited $q\bar{q}'$ states. In terms of the spectroscopic notation $n^{2S+1}L_J$, where the radial excitation is denoted by the principle number n, there are two types of the lowest p-wave mesons, namely, 1^3P_1 and 1^1P_1. These two nonets have distinctive C quantum numbers, $C = +$ and $C = -$, respectively. Experimentally, the $J^{PC} = 1^{++}$ nonet consists of $a_1(1260)$, $f_1(1285)$, $f_1(1420)$, and K_{1A}, while the 1^{+-} nonet contains $b_1(1235)$, $h_1(1170)$, $h_1(1380)$, and K_{1B}. The physical mass eigenstates $K_1(1270)$ and $K_1(1400)$ are mixtures of K_{1A} and K_{1B} states owing to the mass difference of the strange and nonstrange light quarks.

In QCD language, a real hadron should be described in terms of a set of Fock states for which each state has the same quantum number as the hadron. Because of G parity, the decay constant for the *local* axial-vector (*local* tensor) current coupling to the $^1P_1$ ($^3P_1$) state vanishes in the SU (3) limit. However the constituent partons within a hadron are actually nonlocalized. Projecting the axial-vector meson along the light cone, due to the G parity the chiral-even light-cone distribution amplitudes (LCDAs) of a 1^1P_1 (1^3P_1) meson defined by the *nonlocal* axial-vector current is antisymmetric (symmetric) under the exchange of *quark* and *antiquark* momentum fractions in the SU(3) limit, whereas the chiral-odd LCDAs defined by the nonlocal tensor current are symmetric (antisymmetric) [11,12]. The large magnitude of the first Gegenbauer moment of the mentioned antisymmetric LCDAs can have large impact on B decays involving a 1^3P_1 or/and 1^1P_1 meson(s). The related phenomenologies are thus interesting [8,9,13].

In this paper, we present the first complete analysis for the form factors of the $B_{u,d,s}$ decays into light axial-vector mesons (A) via the vector/axial-vector current in the light-cone sum rule approach, where A is the light P-wave meson, which can be the 1^3P_1, 1^1P_1 or their mixture state. The method of light-cone sum rules has been widely used in the studies of nonperturbative processes, including weak baryon decays [14], heavy meson decays [15], and heavy to light transition form factors [16–18]. Using the traditional QCD sum rule approach [19], where the three-point correlation function is considered, $B \to a_1$ form factors were calculated in Ref. [20]. The BABAR measurement of $\bar{B}^0 \to a_1^+\pi^-$ [4] favors $V_0^{Ba_1}(0) \approx 0.30$ [8,9], which is in good agreement with the light-cone sum rule result that we obtain here. The value given in Ref. [20] is a little small but still consistent with the data within the errors. $B \to A$ form factors were studied in Ref. [21] by using the light-front quark model. Nevertheless, it is found to be $V_0^{Ba_1}(0) = 0.13$ in the light-front quark model calculation. It is interesting to note that very recently Wang [22] used the B meson light-cone sum rule approach to calculate $B \to a_1$ form factors.

The rest of the paper is organized as follows: The definitions of decay constants and $B_q \to A$ form factors are given in Sec. II. In Sec. III, we derive the light-cone sum rules for the relevant form factors. The numerical results for form factors are given in Sec. IV, where we also give the predictions for decay rates of $B_{u,d,s} \to A e \nu_e$. A brief summary is given in Sec. V. The relevant expressions for two-parton and three-parton LCDAs are collected in Appendix A, an alternative definition for the form factors is given in Appendix B, and the formula for semileptonic $B_{u,d,s} \to A e \nu_e$ decays is presented in Appendix C.

II. DEFINITIONS OF DECAY CONSTANTS AND FORM FACTORS

The G parity[1] conserving decay constants of the axial-vector mesons are defined as

$$\langle 1^3P_1(P,\lambda)|\bar{q}_1(0)\gamma_\mu\gamma_5 q_2(0)|0\rangle = i f_{3P_1} m_{3P_1}\epsilon^{*(\lambda)}_\mu, \quad (2.1)$$

$$\langle 1^1P_1(P,\lambda)|\bar{q}_1(0)\sigma_{\mu\nu}\gamma_5 q_2(0)|0\rangle = f^\perp_{1P_1}(\epsilon^{*(\lambda)}_\mu P_\nu - \epsilon^{*(\lambda)}_\nu P_\mu), \quad (2.2)$$

where f_{3P_1} is scale independent, but $f^\perp_{1P_1}$ is scale dependent [11,12]. On the other hand, we define the G-parity violating decay constants to be

$$\langle 1^3P_1(P,\lambda)|\bar{q}_1(0)\sigma_{\mu\nu}\gamma_5 q_2(0)|0\rangle = f_{3P_1} a_0^{\perp,{}^3P_1}(\epsilon^{*(\lambda)}_\mu P_\nu - \epsilon^{*(\lambda)}_\nu P_\mu), \quad (2.3)$$

$$\langle 1^1P_1(P,\lambda)|\bar{q}_1(0)\gamma_\mu\gamma_5 q_2(0)|0\rangle = i f^\perp_{1P_1}(1\ \text{GeV}) a_0^{\parallel,{}^1P_1} m_{1P_1}\epsilon^{*(\lambda)}_\mu, \quad (2.4)$$

where $a_0^{\perp,{}^3P_1}$ and $a_0^{\parallel,{}^1P_1}$, which are, respectively, the zeroth Gegenbauer moments of $\Phi_\perp^{{}^3P_1}$ and $\Phi_\parallel^{{}^1P_1}$, are zero in the SU (3) limit. The definitions for the LCDAs $\Phi_\perp^{{}^3P_1}$ and $\Phi_\parallel^{{}^1P_1}$ are collected in Appendix A. We further define $f^\perp_{{}^3P_1} = f_{{}^3P_1}$ and $f_{{}^1P_1} = f^\perp_{{}^1P_1}(\mu = 1\ \text{GeV})$ in the present study as did in Ref. [11]. In the present work, the G-parity violating parameters, e.g. $a_1^{\parallel,K_{1A}}$, $a_{0,2}^{\perp,K_{1A}}$, $a_1^{\perp,K_{1B}}$ and $a_{0,2}^{\parallel,K_{1B}}$, are considered for mesons containing a strange quark. Note that for G-parity violating quantities, their signs have to be flipped from mesons to antimesons.

The semileptonic form factors for the $\bar{B}_q \to A$ transition are defined as

$$\langle A(P,\lambda)|A_\mu|\bar{B}_q(p_B)\rangle = i\frac{2}{m_{B_q}+m_A}\epsilon_{\mu\nu\alpha\beta}\epsilon^{*\nu}_{(\lambda)} p_B^\alpha P^\beta A^{B_qA}(q^2),$$
$$\langle A(P,\lambda)|V_\mu|\bar{B}_q(p_B)\rangle = -\Big\{(m_{B_q}+m_A)\epsilon^{(\lambda)*}_\mu V_1^{B_qA}(q^2) - (\epsilon^{(\lambda)*}\cdot p_B)(p_B+P)_\mu\frac{V_2^{B_qA}(q^2)}{m_B+m_A} - 2m_A\frac{\epsilon^{(\lambda)*}\cdot p_B}{q^2}q^\mu[V_3^{B_qA}(q^2) - V_0^{B_qA}(q^2)]\Big\}, \quad (2.5)$$

where $q = p_B - P$, $V_3^{B_qA}(0) = V_0^{B_qA}(0)$,

$$V_3^{B_qA}(q^2) = \frac{m_B+m_A}{2m_A}V_1^{B_qA}(q^2) - \frac{m_B-m_A}{2m_A}V_2^{B_qA}(q^2), \quad (2.6)$$

and we adopt the convention $\epsilon^{0123} = -1$. An alternative definition for the form factors is given in Appendix B.

III. THE LIGHT-CONE SUM RULES

We consider the following two-point correlation function, which is sandwiched between the vacuum and transversely polarized A meson (in this section $A \equiv$ a pure 1^3P_1 or 1^1P_1 state), to calculate the form factors

$$i\int d^4x e^{iqx}\langle A(P,\perp)|T[\bar{q}_1(x)\gamma_\mu(1-\gamma_5)b(x)j^\dagger_{B_{q_2}}(0)]|0\rangle = -\mathbf{V_1}(q^2)\epsilon^{*(\perp)}_\mu + \mathbf{V_2}(q^2)(\epsilon^{*(\perp)}q)(2P+q)_\mu + \mathbf{V}(q^2)\frac{\epsilon^{*(\perp)}q}{q^2}q_\mu - i\mathbf{A}(q^2)\epsilon_{\mu\nu\rho\sigma}\epsilon^{*\nu}_{(\perp)}q^\rho P^\sigma, \quad (3.1)$$

where $p_B^2 = (P+q)^2$, P is the momentum of the A meson, and $j_B = i\bar{q}_2\gamma_5 b$ (with $q_{1(2)} \equiv u$, d or s) is the interpolating current for the B_{q_2} meson, so that

$$\langle 0|j_{B_{q_2}}(0)|\bar{B}_{q_2}(p_B)\rangle = \frac{f_{B_{q_2}}m^2_{B_{q_2}}}{m_b+m_{q_2}}. \quad (3.2)$$

In the region of sufficiently large virtualities $m_b^2 - p_B^2 \gg \Lambda_{\text{QCD}}m_b$ with q^2 being small and positive, the operator product expansion is applicable in Eq. (3.1), so that for an energetic A meson the correlation function in Eq. (3.1) can be represented in terms of the LCDAs of the A meson

[1]Here the idea of G parity is extended to U-spin and V-spin multiplets.

$$i\int d^4x e^{iqx}\langle A(P,\perp)|T[\bar q_1(x)\gamma_\mu(1-\gamma_5)b(x)j_B^\dagger(0)]|0\rangle = \int_0^1 du \frac{-i}{(q+k)^2-m_b^2}\mathrm{Tr}[\gamma_\mu(1-\gamma_5) \times (\not q+\not k+m_b)\gamma_5 M_\perp^A]|_{k=uEn_-} + \mathcal{O}\left(\frac{m_A^2}{E^2}\right), \quad (3.3)$$

where $E=|\vec P|$, $P^\mu = En_-^\mu + m_A^2 n_+^\mu/(4E) \simeq En_-^\mu$ with two lightlike vectors $n_-^\mu=(1,0,0,-1)$ and $n_+^\mu=(1,0,0,1)$ satisfying $n_-n_+=2$ and $n_-^2=n_+^2=0$. Here, $E\sim m_b$, and we have assigned the momentum of the q_1 quark in the A meson to be

$$k^\mu = uEn_-^\mu + k_\perp^\mu + \frac{k_\perp^2}{4uE}n_+^\mu, \quad (3.4)$$

where $k_\perp$ is of order Λ_{QCD}. Here, u is the momentum fraction carried by the quark in the axial-vector meson. In Eq. (3.3), to calculate contributions in the momentum space, we have used the following substitution

$$x^\mu \to -i\frac{\partial}{\partial k_\mu} \simeq -i\left(\frac{n_+^\mu}{2E}\frac{\partial}{\partial u}+\frac{\partial}{\partial k_{\perp\mu}}\right), \quad (3.5)$$

to the Fourier transform for

$$\langle A(P,\lambda)|\bar q_{1\alpha}(x)q_{2\delta}(0)|0\rangle = -\frac{i}{4}\int_0^1 du e^{iuPx}\Big\{f_A m_A\Big[\not P\gamma_5\frac{\epsilon^*_{(\lambda)}x}{Px}\left(\Phi_\parallel(u)+\frac{m_A^2x^2}{16}\mathbf{A}_\parallel^2\right)+\left(\not\epsilon^*-\not P\frac{\epsilon^*_{(\lambda)}z}{Px}\right)\gamma_5 g_\perp^{(a)}(u) - \not x\gamma_5\frac{\epsilon^*_{(\lambda)}x}{2(Px)^2}m_A^2\bar g_3(u)+\epsilon_{\mu\nu\rho\sigma}\epsilon^{*\mu}_{(\lambda)}p^\rho x^\sigma\gamma^\mu\frac{g_\perp^{(v)}(u)}{4}\Big]+f_A^\perp\Big[\frac{1}{2}(\not P\not\epsilon^*_{(\lambda)}-\not\epsilon^*_{(\lambda)}\not P)\gamma_5 \times\left(\Phi_\perp(u)+\frac{m_A^2x^2}{16}\mathbf{A}_\perp^2\right)-\frac{1}{2}(\not P\not x-\not x\not P)\gamma_5\frac{\epsilon^*_{(\lambda)}x}{(Px)^2}m_A^2\bar h_\parallel^{(t)}(u)-\frac{1}{4}(\not\epsilon^*_{(\lambda)}\not x-\not x\not\epsilon^*_{(\lambda)})\gamma_5 \times\frac{m_A^2}{Px}\bar h_3(u)+i(\epsilon^*_{(\lambda)}x)m_A^2\gamma_5\frac{h_\parallel^{(p)}(u)}{2}\Big]\Big\}_{\delta\alpha}, \quad (3.6)$$

where $x^2\neq 0$,

$$\bar g_3(u) = g_3(u)+\Phi_\parallel - 2g_\perp^{(a)}(u),\quad \bar h_\parallel^{(t)} = h_\parallel^{(t)}-\frac{1}{2}\Phi_\perp(u)-\frac{1}{2}h_3(u),\quad \bar h_3(u) = h_3(u)-\Phi_\perp(u). \quad (3.7)$$

The detailed definitions for the relevant two-parton LCDAs are collected in Appendix A. In Eq. (3.5), the term of order $k_\perp^2$ is omitted in the calculation. Consequently, we can obtain the light-cone projection operator of the A meson in the momentum space

$$M_{\delta\alpha}^A = M_{\delta\alpha\parallel}{}^A + M_{\delta\alpha\perp}{}^A, \quad (3.8)$$

where $M_{\delta\alpha\parallel}^A$ and $M_{\delta\alpha\perp}^A$ are the longitudinal and transverse projectors, respectively. The longitudinal projector, which projects the longitudinal component of the axial-vector meson, is given by

$$M_\parallel^A = -i\frac{f_A}{4}\frac{m_A(\epsilon^*_{(\lambda)}n_+)}{2}\Big\{\not n_-\gamma_5\Phi_\parallel(u)-\frac{f_A^\perp}{f_A}\frac{m_A}{E}\times\Big[-\frac{i}{2}\sigma_{\mu\nu}\gamma_5 n_-^\mu n_+^\nu h_\parallel^{(t)}(u)-iE\int_0^u dv(\Phi_\perp(v)-h_\parallel^{(t)}(v))\sigma_{\mu\nu}\gamma_5 n_-^\mu\frac{\partial}{\partial k_{\perp\nu}}+\gamma_5\frac{h_\parallel^{\prime(p)}(u)}{2}\Big]\Big|_{k=up}+\mathcal{O}\left(\frac{m_A^2}{E^2}\right)\Big\}, \quad (3.9)$$

and the transverse projector reads

$$M_\perp^A = i\frac{f_A^\perp}{4}E\Big\{\not\epsilon_\perp^{*(\lambda)}\not n_-\gamma_5\Phi_\perp(u)-\frac{f_A}{f_A^\perp}\frac{m_A}{E}\Big[\not\epsilon_\perp^{*(\lambda)}\gamma_5 g_\perp^{(a)}(u) - E\int_0^u dv(\Phi_\parallel(v)-g_\perp^{(a)}(v))\not n_-\gamma_5\epsilon_{\perp\mu}^{*(\lambda)}\frac{\partial}{\partial k_{\perp\mu}} + i\varepsilon_{\mu\nu\rho\sigma}\gamma^\mu\epsilon_\perp^{*(\lambda)\nu}n_-^\rho\left(n_+^\sigma\frac{g_\perp^{(v)\prime}(u)}{8}-E\frac{g_\perp^{(v)}(u)}{4}\times\frac{\partial}{\partial k_{\perp\sigma}}\right)\Big]\Big|_{k=up}+\mathcal{O}\left(\frac{m_A^2}{E^2}\right)\Big\}, \quad (3.10)$$

where the *exactly* longitudinal and transverse polarization vectors of the axial-vector meson, independent of the coordinate variable x, are defined as

$$\epsilon^{*(0)\mu} = \frac{E}{m_A}\left[\left(1-\frac{m_A^2}{4E^2}\right)n_-^\mu-\frac{m_A^2}{4E^2}n_+^\mu\right],$$
$$\epsilon_\perp^{*(\lambda)\mu} \equiv \left(\epsilon^{*(\lambda)\mu}-\frac{\epsilon^{*(\lambda)}n_+}{2}n_-^\mu-\frac{\epsilon^{*(\lambda)}n_-}{2}n_+^\mu\right)\delta_{\lambda,\pm1}. \quad (3.11)$$

Here, we have assumed that the meson moves along the n_-^μ direction. A similar discussion about the projection operator for the vector meson can be found in Ref. [23]. From the expansion of the transverse projection operator, one can find that contributions arising from $g_\perp^{(a)}$, $\Phi_\parallel - g_\perp^{(a)}$, $g_\perp^{(v)\prime}$, and $g_\perp^{(v)}$ are suppressed by m_A/E as compared with the term with $\Phi_\perp$. Note that in Eq. (3.3) the derivative with respect to the transverse momentum acts on the hard scattering amplitude before the collinear approximation is taken. Note also that applying equations of motion, the twist-three two-parton LCDAs can be expressed in terms of leading-twist and twist-three three-parton light-cone distribution amplitudes. One can thus understand from Eqs. (3.9) and (3.10) that the expansion parameter in the light-cone sum rules should be m_A/m_b, instead of twist.

At the quark-gluon level, after performing the integration of Eq. (3.3), the results up to $\mathcal{O}(m_A/m_b)$ read

$$\mathbf{V_1}^{\rm QCD} = -\frac{m_b^2 f_A^\perp}{2}\int_0^1 \frac{du}{u}\left\{\frac{1}{m_b^2 - u p_B^2 - \bar{u} q^2}\times\left[\frac{m_b^2-q^2}{m_b^2}\Phi^\perp(u) - \left(\frac{m_A f_A}{m_b f_A^\perp}\right)2u g_\perp^{(a)}(u)\right]\right\}, \tag{3.12}$$

$$\mathbf{V_2}^{\rm QCD} = -\frac{m_b^2 f_A^\perp}{2}\int_0^1 du\left\{\frac{1}{m_b^2 - u p_B^2 - \bar{u} q^2}\frac{\Phi^\perp(u)}{m_b^2} - \frac{m_A f_A}{m_b f_A^\perp}\frac{2\Phi_a(u)}{(m_b^2 - u p_B^2 - \bar{u} q^2)^2}\right\}, \tag{3.13}$$

$$\mathbf{V}^{\rm QCD} = \frac{q^2 m_b^2 f_A^\perp}{2}\int_0^1 \frac{du}{u}\left\{\frac{1}{m_b^2 - u p_B^2 - \bar{u} q^2}\frac{\Phi^\perp(u)}{m_b^2} - \frac{m_A f_A}{m_b f_A^\perp}\frac{2\Phi_a(u)}{(m_b^2 - u p_B^2 - \bar{u} q^2)^2}\right\}, \tag{3.14}$$

$$\mathbf{A}^{\rm QCD} = -m_b^2 f_A^\perp\int_0^1 du\left\{\frac{1}{m_b^2 - u p_B^2 - \bar{u} q^2}\frac{\Phi^\perp(u)}{m_b^2} - \frac{m_A f_A}{m_b f_A^\perp}\frac{g_\perp^{(v)}(u)/2}{(m_b^2 - u p_B^2 - \bar{u} q^2)^2}\right\}, \tag{3.15}$$

where $\Phi_a(u) \equiv \int_0^u dv(\Phi_\parallel(v) - g_\perp^{(a)}(v))$ and $\bar{u} \equiv 1-u$. Note that here the contributions due to the *explicit* three-parton LCDAs are suppressed by $\mathcal{O}(m_A^2/m_b^2)$ as compared with the term involving $\Phi_\perp$.

We have given the results for $\mathbf{V_1}$, $\mathbf{V_2}$, and $\mathbf{A}$ from the hadron and quark-gluon points of view. Thus, for instance, for the form factor V_1 the contribution due to the lowest-lying A meson can be further approximated with the help of quark-hadron duality

$$V_1(q^2)\cdot\frac{m_{B_{q_2}}+m_A}{m_{B_{q_2}}^2 - p_B^2}\cdot\frac{m_{B_{q_2}}^2 f_B}{m_b + m_{q_2}} = \frac{1}{\pi}\int_{m_b^2}^{s_0}\frac{{\rm Im}\mathbf{V_1}^{\rm QCD}(s,q^2)}{s-p_B^2}ds, \tag{3.16}$$

where s_0 is the excited-state threshold. After applying the Borel transform $p_B^2 \to M^2$ [15,17,19] to the above equation, we obtain

$$V_1(q^2) = \frac{(m_{B_{q_2}}+m_A)(m_b+m_{q_2})}{m_{B_{q_2}}^2 f_B}e^{-m_{B_{q_2}}^2/M^2}\frac{1}{\pi}\times\int_{m_b^2}^{s_0} e^{s/M^2}{\rm Im}\mathbf{V_1}^{\rm QCD}(s,q^2)ds. \tag{3.17}$$

We obtain the light-cone sum rule results

$$V_1^{B_{q_2}A}(q^2) = -\frac{(m_b+m_{q_2})m_b^2 f_A^\perp}{(m_{B_{q_2}}+m_A)m_{B_{q_2}}^2 f_{B_{q_2}}}e^{(m_{B_{q_2}}^2 - m_b^2)/M^2}\times\int_0^1 du\left\{\frac{1}{u}e^{\bar{u}(q^2-m_b^2)/(uM^2)}\theta[c(u,s_0)]\times\left(\Phi^\perp(u)\frac{m_b^2-q^2}{2um_b^2} - \frac{m_{B_{q_2}}+m_A}{m_{B_{q_2}}}\times\frac{m_A f_A}{m_b f_A^\perp}g_\perp^{(a)}(u)\right)\right\}, \tag{3.18}$$

$$V_2^{B_{q_2}A}(q^2) = -\frac{(m_b+m_{q_2})(m_{B_{q_2}}+m_A)f_A^\perp}{2m_{B_{q_2}}^2 f_{B_{q_2}}}e^{(m_{B_{q_2}}^2 - m_b^2)/M^2}\times\int_0^1 du\left\{\frac{1}{u}e^{\bar{u}(q^2-m_b^2)/(uM^2)}\left[\Phi^\perp(u)\theta[c(u,s_0)] - \frac{m_{B_{q_2}}}{m_{B_{q_2}}+m_A}\frac{2m_A m_b f_A}{uM^2 f_A^\perp}\Phi_a(u)(\theta[c(u,s_0)] + uM^2\delta[c(u,s_0)])\right]\right\}, \tag{3.19}$$

$$V_0^{B_{q_2}A}(q^2) = V_3^{B_{q_2}A}(q^2) - \frac{q^2(m_b+m_{q_2})f_A^\perp}{4m_{B_{q_2}}^2 m_A f_{B_{q_2}}}e^{(m_{B_{q_2}}^2 - m_b^2)/M^2}\times\int_0^1 du\left\{\frac{1}{u}e^{\bar{u}(q^2-m_b^2)/(uM^2)}\left[\Phi^\perp(u)\theta[c(u,s_0)] - \frac{2m_A m_b f_A}{uM^2 f_A^\perp}\Phi_a(u)\left(\theta[c(u,s_0)] + uM^2\delta[c(u,s_0)]\right)\right]\right\}, \tag{3.20}$$

$$A^{B_{q_2}A}(q^2) = -\frac{(m_b+m_{q_2})(m_{B_{q_2}}+m_A)f_A^\perp}{2m_{B_{q_2}}^2 f_{B_{q_2}}}e^{(m_{B_{q_2}}^2 - m_b^2)/M^2}\times\int_0^1 du\left\{\frac{1}{u}e^{\bar{u}(q^2-m_b^2)/(uM^2)}\left[\Phi^\perp(u)\theta[c(u,s_0)] - \frac{m_{B_{q_2}}}{m_{B_{q_2}}+m_A}\frac{m_A m_b f_A}{2uM^2 f_A^\perp}g_\perp^{(v)}(u)\left(\theta[c(u,s_0)] + uM^2\delta[c(u,s_0)]\right)\right]\right\}, \tag{3.21}$$

where $V_3^{B_{q_2}A}(q^2)$ is given by Eq. (2.6), and $c(u,s_0) = us_0 - m_b^2 + (1-u)q^2$.

IV. RESULTS

A. Input parameters

In this subsection, we shall briefly summarize the relevant input parameters, which are collected in Tables I, II, and III. The masses of u and d quarks are neglected.

The physical states $K_1(1270)$ and $K_1(1400)$ are the mixtures of the K_{1A} and K_{1B}. K_{1A} and K_{1B} are not mass eigenstates, and they can be mixed together due to the strange and nonstrange light quark mass difference. Their relations can be written as

$$|\bar{K}_1(1270)\rangle = |\bar{K}_{1A}\rangle \sin\theta_{K_1} + |\bar{K}_{1B}\rangle \cos\theta_{K_1},$$
$$|\bar{K}_1(1400)\rangle = |\bar{K}_{1A}\rangle \cos\theta_{K_1} - |\bar{K}_{1B}\rangle \sin\theta_{K_1}. \quad (4.1)$$

The sign ambiguity for θ_{K_1} is due to the fact that one can add arbitrary phases to $|\bar{K}_{1A}\rangle$ and $|\bar{K}_{1B}\rangle$. This sign ambiguity can be removed by fixing the signs for $f_{K_{1A}}$ and $f^{\perp}_{K_{1B}}$, which do not vanish in the SU(3) limit. Following Ref. [11], we adopt the convention $f_{K_{1A}} > 0$, $f^{\perp}_{K_{1B}} > 0$. Combining the analyses for the data of the decays $B \to K_1\gamma$ and $\tau^- \to K_1^-(1270)\nu_\tau$ [1,24], the mixing angle was found to be $\theta_{K_1} = -(34 \pm 13)^\circ$ [13].

Analogous to the $\eta - \eta'$ mixing in the pseudoscalar sector, the 1^3P_1 states $f_1(1285)$ and $f_1(1420)$ have mixing via

$$|f_1(1285)\rangle = |f_1\rangle \cos\theta_{^3P_1} + |f_8\rangle \sin\theta_{^3P_1},$$
$$|f_1(1420)\rangle = -|f_1\rangle \sin\theta_{^3P_1} + |f_8\rangle \cos\theta_{^3P_1}, \quad (4.2)$$

and likewise the 1^1P_1 states, $h_1(1170)$, and $h_1(1380)$ can be mixed in terms of the pure octet h_8 and singlet h_1

$$|h_1(1170)\rangle = |h_1\rangle \cos\theta_{^1P_1} + |h_8\rangle \sin\theta_{^1P_1},$$
$$|h_1(1380)\rangle = -|h_1\rangle \sin\theta_{^1P_1} + |h_8\rangle \cos\theta_{^1P_1}. \quad (4.3)$$

Using the Gell-Mann-Okubo mass formula [11], we obtain the mixing angles $\theta_{^1P_1}$ and $\theta_{^3P_1}$ to be

$$\theta_{^3P_1} = (23.6^{+17.0}_{-23.6})^\circ,$$
$$\theta_{^1P_1} = (28.1^{+9.8}_{-17.2})^\circ, \quad \text{for } \theta_{K_1} = -(34 \pm 13)^\circ. \quad (4.4)$$

For $^3P_1$ states the decay constants $f^q_{f_1(1285)}$ and $f^q_{f_1(1420)}$ are defined by

$$\langle 0|\bar{q}\gamma_\mu\gamma_5 q|f_1(1285)(P,\lambda)\rangle = -i m_{f_1(1285)} f^q_{f_1(1285)} \epsilon^{(\lambda)}_\mu,$$
$$\langle 0|\bar{q}\gamma_\mu\gamma_5 q|f_1(1420)(P,\lambda)\rangle = -i m_{f_1(1420)} f^q_{f_1(1420)} \epsilon^{(\lambda)}_\mu, \quad (4.5)$$

and for 1^1P_1 states the tensor decay constants are

$$\langle 0|\bar{q}\sigma_{\mu\nu} q|h_1(1170)(P,\lambda)\rangle = i f^{\perp,q}_{h_1(1170)} \epsilon_{\mu\nu\alpha\beta} \epsilon^\alpha_{(\lambda)} P^\beta,$$
$$\langle 0|\bar{q}\sigma_{\mu\nu} q|h_1(1380)(P,\lambda)\rangle = i f^{\perp,q}_{h_1(1380)} \epsilon_{\mu\nu\alpha\beta} \epsilon^\alpha_{(\lambda)} P^\beta. \quad (4.6)$$

The reader is referred to [8,11] for details.

TABLE I. Masses and decay constants for 1^3P_1 and 1^1P_1 states obtained in the QCD sum rule calculation [11].

State	Mass [GeV]	Decay constant $f_{^{3(1)}P_1}$ [MeV]
$a_1(1260)$	1.23 ± 0.06	238 ± 10
$f_1(1^3P_1)$	1.28 ± 0.06	245 ± 13
$f_8(1^3P_1)$	1.29 ± 0.05	239 ± 13
K_{1A}	1.32 ± 0.06	250 ± 13
$b_1(1235)$	1.21 ± 0.07	180 ± 8
$h_1(1^1P_1)$	1.23 ± 0.07	180 ± 12
$h_8(1^1P_1)$	1.37 ± 0.07	190 ± 10
K_{1B}	1.34 ± 0.08	190 ± 10

TABLE II. Gegenbauer moments of $\Phi_\perp$ and $\Phi_\parallel$ for 1^3P_1 and 1^1P_1 mesons, respectively, where $a_0^{\perp,K_{1A}}$ and $a_0^{\parallel,K_{1B}}$ are updated from the $B \to K_1\gamma$ analysis, and $a_1^{\parallel,K_{1A}}$, $a_2^{\perp,K_{1A}}$, $a_2^{\parallel,K_{1B}}$, and $a_1^{\perp,K_{1B}}$ are then obtained from Eq. (141) in Ref. [11].

μ	$a_2^{\parallel,a_1(1260)}$	$a_2^{\parallel,f_1^{^3P_1}}$	$a_2^{\parallel,f_8^{^3P_1}}$	$a_2^{\parallel,K_{1A}}$	$a_1^{\parallel,K_{1A}}$	
1 GeV	-0.02 ± 0.02	-0.04 ± 0.03	-0.07 ± 0.04	-0.05 ± 0.03	$-0.30^{+0.00}_{-0.20}$	
2.2 GeV	-0.01 ± 0.01	-0.03 ± 0.02	-0.05 ± 0.03	-0.04 ± 0.02	$-0.25^{+0.00}_{-0.17}$	
μ	$a_1^{\perp,a_1(1260)}$	$a_1^{\perp,f_1^{^3P_1}}$	$a_1^{\perp,f_8^{^3P_1}}$	$a_1^{\perp,K_{1A}}$	$a_0^{\perp,K_{1A}}$	$a_2^{\perp,K_{1A}}$
1 GeV	-1.04 ± 0.34	-1.06 ± 0.36	-1.11 ± 0.31	-1.08 ± 0.48	$0.27^{+0.03}_{-0.17}$	0.02 ± 0.21
2.2 GeV	-0.85 ± 0.28	-0.86 ± 0.29	-0.90 ± 0.25	-0.88 ± 0.39	$0.25^{+0.03}_{-0.16}$	0.01 ± 0.15
μ	$a_1^{\parallel,b_1(1235)}$	$a_1^{\parallel,h_1^{^1P_1}}$	$a_1^{\parallel,h_8^{^1P_1}}$	$a_1^{\parallel,K_{1B}}$	$a_0^{\parallel,K_{1B}}$	$a_2^{\parallel,K_{1B}}$
1 GeV	-1.95 ± 0.35	-2.00 ± 0.35	-1.95 ± 0.35	-1.95 ± 0.45	-0.19 ± 0.07	$0.10^{+0.15}_{-0.19}$
2.2 GeV	-1.61 ± 0.29	-1.65 ± 0.29	-1.61 ± 0.29	-1.57 ± 0.37	-0.19 ± 0.07	$0.07^{+0.11}_{-0.14}$
μ	$a_1^{\perp,b_1(1235)}$	$a_1^{\perp,h_1^{^1P_1}}$	$a_1^{\perp,h_8^{^1P_1}}$	$a_1^{\perp,K_{1B}}$	$a_0^{\perp,K_{1B}}$	
1 GeV	0.03 ± 0.19	0.18 ± 0.22	0.14 ± 0.22	-0.02 ± 0.22	$0.30^{+0.00}_{-0.33}$	
2.2 GeV	0.02 ± 0.15	0.14 ± 0.17	0.11 ± 0.17	-0.02 ± 0.17	$0.24^{+0.00}_{-0.27}$	

TABLE III. Input parameters for quark masses, CKM matrix element, and *effective* $B_{(s)}$ decay constants, and for twist-3 3-parton LCDAs of the K_{1A} and K_{1B} states [1,11]. The G-parity violating parameters are updated due to new values for $a_0^{\perp,K_{1A}}$ and $a_0^{\parallel,K_{1B}}$ given in Ref. [13].

Strange quark mass (GeV), pole b-quark mass (GeV), and couplings			
m_s (2 GeV)	$m_{b,\mathrm{pole}}$	α_s (1 GeV)	α_s (2.2 GeV)
0.09 ± 0.01	4.85 ± 0.05	0.495	0.287

The CKM matrix element and the *effective* $B_{(s)}$ decay constants		
$\lvert V_{ub}\rvert$	$f_B(\alpha_s = 0)$ [MeV]	$f_{B_s}(\alpha_s = 0)$ [MeV]
$(4.31 \pm 0.30) \times 10^{-3}$	145 ± 10	165 ± 10

G-parity conserving parameters of twist-3 3-parton LCDAs at the scale 2.2 GeV			
	$f^V_{3,{}^3P_1}$ [GeV^2]	$\omega^V_{{}^3P_1}$	$f^A_{3,{}^3P_1}$ [GeV^2]
a_1	0.0036 ± 0.0018	-2.9 ± 0.9	0.0012 ± 0.0005
f_1	0.0036 ± 0.0018	-2.8 ± 0.9	0.0012 ± 0.0005
f_8	0.0035 ± 0.0018	-3.0 ± 1.0	0.0015 ± 0.0005
K_{1A}	0.0034 ± 0.0018	-3.1 ± 1.1	0.0014 ± 0.0007
	$f^A_{3,{}^1P_1}$ [GeV^2]	$\omega^A_{{}^1P_1}$	$f^V_{3,{}^1P_1}$ [GeV^2]
b_1	-0.0036 ± 0.0014	-1.4 ± 0.3	0.0030 ± 0.0011
h_1	-0.0033 ± 0.0014	-1.7 ± 0.4	0.0027 ± 0.0012
h_8	-0.0035 ± 0.0014	-2.9 ± 0.8	0.0027 ± 0.0012
K_{1B}	-0.0041 ± 0.0018	-1.7 ± 0.4	0.0029 ± 0.0012

G-parity violating parameters of twist-3 3-parton LCDAs of the K_{1A} at $\mu = 2.2$ GeV		
$\sigma^V_{K_{1A}}$	$\lambda^A_{K_{1A}}$	$\sigma^A_{K_{1A}}$
0.01 ± 0.04	-0.12 ± 0.22	-1.9 ± 1.1

G-parity violating parameters of twist-3 3-parton LCDAs of the K_{1B} at $\mu = 2.2$ GeV		
$\lambda^V_{K_{1B}}$	$\sigma^V_{K_{1B}}$	$\sigma^A_{K_{1B}}$
-0.23 ± 0.18	1.3 ± 0.8	0.03 ± 0.03

B. Numerical results for the form factors

We numerically analyze the light-cone sum rules for the transition form factors, where the pole b quark mass is adopted in the calculation. The parameters appearing in the sum rules are evaluated at the factorization scale $\mu_f = \sqrt{m_{B_q}^2 - m_{b,\mathrm{pole}}^2}$.

We find that for $s_0 \simeq (34 \sim 37)$ GeV^2 the V_1 sum rule can be stable within the Borel mass range $6.0\ \mathrm{GeV}^2 < M^2 < 12.0\ \mathrm{GeV}^2$. Therefore, we choose the Borel windows to be $6.0\ \mathrm{GeV}^2 < M^2 < 12.0\ \mathrm{GeV}^2$, $(6.0 + \delta_{V_2})\ \mathrm{GeV}^2 < M^2 < (12.0 + \delta_{V_2})\ \mathrm{GeV}^2$, and $(6.0 + \delta_A)\ \mathrm{GeV}^2 < M^2 < (12.0 + \delta_A)\ \mathrm{GeV}^2$ for V_1, V_2, and A, respectively, where the correction originating from higher resonance states amounts to 8% to 20%. $V_0(0)$ equals to $V_3(0)$, where the latter can be obtained from $V_1(0)$ and $V_2(0)$. As for $q^2 \neq 0$, the Borel windows for $V_3(q^2) - V_0(q^2)$ is $(6.0 + \delta_V)\ \mathrm{GeV}^2 < M^2 < (12.0 + \delta_V)\ \mathrm{GeV}^2$. The excited threshold s_0 is determined when the most stable plateau of the V_1 sum rule result is obtained within the Borel window. Using the same s_0, we can then determine δ_{V_1}, δ_A, and δ_V, so that the sum rule results for V_2, A, and $V_3 - V_0$ are stable within the Borel windows. In Table IV, we show that, for $q^2 = 0$ and $m_{b,\mathrm{pole}} = 4.85$ GeV, the corresponding s_0's lie in the interval 34–35 GeV^2. The values of $\delta_{V_2,A,V}$, corresponding to $m_{b,\mathrm{pole}} = 4.85$ GeV, are also collected in Table IV. In the present study, the excited thresholds change slightly for larger q^2. However, for simplicity the values of s_0 and $\delta_{V_2,A,V}$ are chosen to be independent of q^2.

In the numerical analysis, we use the *effective* B decay constant $f_B(\alpha_s = 0) = 145 \pm 10$ MeV, which is in agreement with the QCD sum rule result without radiative corrections [25]. We have checked that, using this value of f_B and $m_b = 4.85$ GeV in the light-cone sum rules of $B \to \rho$ transition form factors of the same order of α_s and m_ρ/m_b, we can get results $A_1^{B\rho}(0) \simeq 0.23$, $A_2^{B\rho}(0) \simeq 0.22$, and $V^{B\rho}(0) \simeq 0.32$, in good agreement with that given in Ref. [18], where the radiative corrections are included. In the literature, it was found that the contributions due to radiative corrections in the form factor sum rules can be canceled if one adopts the f_B sum rule result with the same

TABLE IV. Parameters (in units of GeV^2) relevant to the excited-state thresholds of the light-cone sum rules, where $m_{m_b,\text{pole}} = 4.85$ GeV is used. Here, parameters correspond to $q^2 = 0$.

	$B \to a_1$	$B \to f_1$	$B \to f_8$	$B \to K_{1A}$	$B \to b_1$	$B \to h_1$	$B \to h_8$	$B \to K_{1B}$
s_0	34.18	34.17	34.10	34.14	34.25	34.90	35.02	34.23
δ_{V_2}	3.05	3.15	3.12	3.55	4.38	7.48	5.64	3.75
δ_V	3.05	3.15	3.12	3.55	4.38	7.48	5.64	3.75
δ_A	1.61	1.55	1.56	−0.46	3.25	3.42	1.82	2.55

order of α_s corrections in the calculation [18,26]. Therefore, radiative corrections might be negligible in the present analysis.

Including the terms up to order of m_A/m_b in the light-cone expansion, the three-parton distribution amplitudes do not contribute directly to the sum rules, but they enter the sum rules, since $g_\perp^{(a)}$ and $g_\perp^{(v)}$ can be represented in terms of the leading-twist (two-parton) and twist-3 three-parton LCDAs. To estimate the theoretical uncertainties of the sum rule results due to higher-twist effects, we put all parameters related to twist-3 three-parton LCDAs to be zero and find that the changes of the resulting form factors are less than 3%. We thus conclude that the higher-twist effects might be negligible.

The form factors results in the light-cone sum rule calculation are exhibited in Table V, where the momentum dependence is parameterized in the three-parameter form

$$F^{B_qA}(q^2) = \frac{F^{B_qA}(0)}{1 - a(q^2/m_{B_q}^2) + b(q^2/m_{B_q}^2)^2}, \qquad (4.7)$$

with $F^{B_qA} \equiv V_{0,1,2}^{B_qA}$ or A^{B_qA}. For simplicity, we do not show the theoretical errors for the parameters a and b. As $q^2 \gtrsim 10$ GeV^2, the sum rule results become less stable. To get reliable estimate for the q^2 dependence of the form factors, the present results are fitted in the range $0 \le q^2 \le 6$ GeV^2. The theoretical errors for $F(0)$ are due to variation of the Borel mass, the Gegenbauer moments, the decay constants, the strange quark mass, and the pole b quark mass, which are then added in quadrature. The errors are dominated by variation of the pole b quark mass. It should be stressed that in the convention of the present work, the decay constants of 1^1P_1 and 1^3P_1 axial-vector mesons are of the same sign, so that the form factors for $B \to 1^1P_1$ and $B \to 1^3P_1$ transitions have opposite signs. The sign convention is the other way around in the light-front quark model [21] and perturbative QCD [27] calculations.

For the numerical analysis of $B_s \to$ form factors, we adopt the *effective* decay constant $f_{B_s}(\alpha_s = 0) \simeq 1.14 \times f_{B_d}(\alpha_s = 0) \simeq 165 \pm 11$ MeV, which is estimated by using the relevant QCD sum rule result. Finally, we obtain the

TABLE V. Form factors for $B_{u,d} \to a_1$, b_1, K_{1A}, K_{1B}, f_1, f_8, h_1, h_8 transitions obtained in the light-cone sum rule calculation are fitted to the 3-parameter form in Eq. (4.7). Here, because the decay constants $f_{^3P_1}$ and $f_{^1P_1}^\perp$, which are defined in Eqs. (2.1) and (2.2), are of the same sign, the form factors for $B \to 1^1P_1$ and $B \to 1^3P_1$ transitions have opposite signs.

F	$F(0)$	a	b	F	$F(0)$	a	b
$V_1^{Ba_1}$	0.37 ± 0.07	0.645	0.250	$V_1^{Bb_1}$	-0.20 ± 0.04	0.748	0.063
$V_2^{Ba_1}$	0.42 ± 0.08	1.48	1.00	$V_2^{Bb_1}$	-0.09 ± 0.02	0.539	1.76
$V_0^{Ba_1}$	0.30 ± 0.05	1.77	0.926	$V_0^{Bb_1}$	-0.39 ± 0.07	1.22	0.426
A^{Ba_1}	0.48 ± 0.09	1.64	0.986	A^{Bb_1}	-0.25 ± 0.05	1.69	0.910
$V_1^{BK_{1A}}$	0.34 ± 0.07	0.635	0.211	$V_1^{BK_{1B}}$	$-0.29^{+0.08}_{-0.05}$	0.729	0.074
$V_2^{BK_{1A}}$	0.41 ± 0.08	1.51	1.18	$V_2^{BK_{1B}}$	$-0.17^{+0.05}_{-0.03}$	0.919	0.855
$V_0^{BK_{1A}}$	0.22 ± 0.04	2.40	1.78	$V_0^{BK_{1B}}$	$-0.45^{+0.12}_{-0.08}$	1.34	0.690
$A^{BK_{1A}}$	0.45 ± 0.09	1.60	0.974	$A^{BK_{1B}}$	$-0.37^{+0.10}_{-0.06}$	1.72	0.912
$V_1^{Bf_1}$	0.23 ± 0.04	0.640	0.153	$V_1^{Bh_1}$	-0.13 ± 0.03	0.612	0.078
$V_2^{Bf_1}$	0.26 ± 0.05	1.47	0.956	$V_2^{Bh_1}$	-0.07 ± 0.02	0.500	1.63
$V_0^{Bf_1}$	0.18 ± 0.03	1.81	0.880	$V_0^{Bh_1}$	-0.24 ± 0.04	1.16	0.294
A^{Bf_1}	0.30 ± 0.05	1.63	0.900	A^{Bh_1}	-0.17 ± 0.03	1.54	0.848
$V_1^{Bf_8}$	0.16 ± 0.03	0.644	0.209	$V_1^{Bh_8}$	-0.11 ± 0.02	0.623	0.094
$V_2^{Bf_8}$	0.19 ± 0.03	1.49	1.09	$V_2^{Bh_8}$	-0.06 ± 0.01	0.529	1.53
$V_0^{Bf_8}$	0.12 ± 0.02	1.84	0.749	$V_0^{Bh_8}$	-0.18 ± 0.03	1.22	0.609
A^{Bf_8}	0.22 ± 0.04	1.64	0.919	A^{Bh_8}	-0.13 ± 0.02	1.56	0.827

TABLE VI. Form factors for $B_s \to K_{1A}, K_{1B}, f_1, f_8, h_1, h_8$ transitions obtained in the light-cone sum rule calculation are fitted to the 3-parameter form in Eq. (4.7). Here, because the decay constants, $f_{^3P_1}$ and $f^{\perp}_{^1P_1}$, are of the same sign, the form factors for $B_s \to 1^1P_1$ and $B_s \to 1^3P_1$ transitions have opposite signs.

F	$F(0)$	a	b	F	$F(0)$	a	b
$V_1^{B_sK_{1A}}$	0.30 ± 0.06	0.635	0.211	$V_1^{B_sK_{1B}}$	$-0.25^{+0.07}_{-0.04}$	0.729	0.074
$V_2^{B_sK_{1A}}$	0.36 ± 0.07	1.51	1.18	$V_2^{B_sK_{1B}}$	$-0.15^{+0.04}_{-0.03}$	0.919	0.855
$V_0^{B_sK_{1A}}$	0.19 ± 0.04	2.40	1.78	$V_0^{B_sK_{1B}}$	$-0.40^{+0.11}_{-0.07}$	1.34	0.690
$A^{B_sK_{1A}}$	0.40 ± 0.08	1.60	0.974	$A^{B_sK_{1B}}$	$-0.33^{+0.09}_{-0.05}$	1.72	0.912
$V_1^{B_sf_1}$	0.20 ± 0.04	0.640	0.153	$V_1^{B_sh_1}$	-0.11 ± 0.03	0.612	0.078
$V_2^{B_sf_1}$	0.23 ± 0.04	1.47	0.956	$V_2^{B_sh_1}$	-0.06 ± 0.02	0.500	1.63
$V_0^{B_sf_1}$	0.16 ± 0.03	1.81	0.880	$V_0^{B_sh_1}$	-0.21 ± 0.04	1.16	0.294
$A^{B_sf_1}$	0.26 ± 0.04	1.63	0.900	$A^{B_sh_1}$	-0.15 ± 0.03	1.54	0.848
$V_1^{B_sf_8}$	-0.28 ± 0.05	0.644	0.209	$V_1^{B_sh_8}$	0.19 ± 0.04	0.623	0.094
$V_2^{B_sf_8}$	-0.33 ± 0.05	1.49	1.09	$V_2^{B_sh_8}$	0.11 ± 0.02	0.529	1.53
$V_0^{B_sf_8}$	-0.21 ± 0.04	1.84	0.749	$V_0^{B_sh_8}$	0.32 ± 0.05	1.22	0.609
$A^{B_sf_8}$	-0.39 ± 0.07	1.64	0.919	$A^{B_sh_8}$	0.23 ± 0.04	1.56	0.827

relations (with $F \equiv V_{0,1,2}$ or A)

$$\frac{F^{BK_{1A}}(q^2)}{F^{B_sK_{1A}}(q^2)} \simeq \frac{F^{Bf_1}(q^2)}{F^{B_sf_1}(q^2)} \simeq -\frac{2F^{Bf_8}(q^2)}{F^{B_sf_8}(q^2)} \simeq \frac{F^{BK_{1B}}(q^2)}{F^{B_sK_{1B}}(q^2)} \simeq \frac{F^{Bh_1}(q^2)}{F^{B_sh_1}(q^2)} \simeq -\frac{2F^{Bh_8}(q^2)}{F^{B_sh_8}(q^2)} \simeq 1.14. \tag{4.8}$$

In Table VI, we show the form factor results at the maximum recoil (i.e., at $q^2 = 0$).

C. Branching ratios

Our results for the semileptonic decay rates $B_{u,d,s} \to A e \bar{\nu}_e$ are listed in Table VII. Most branching ratios are of order 10^{-4}. For $B_{u,d}$ decays involving the a_1 or $f_1(1285)$ we obtain $\Gamma_L/\Gamma_T \simeq 0.6$, whereas for decays containing 1^1P_1 mesons we find that Γ_L/Γ_T is close to 2. In short, the polarization fractions follow the relations $\Gamma_- > \Gamma_L \gg \Gamma_+$ for the former, and $\Gamma_L > \Gamma_- \gg \Gamma_+$ for the latter. On the other hand, we have $\Gamma_L/\Gamma_T \sim 0.6 - 0.7$, 1.1, $1.4 - 1.8$ for the semileptonic B_s decays involving the f_1, h_1, and K_1, respectively. These results are sensitive to the values of form factors. Moreover, we have the salient patterns

$$\mathcal{B}^u[a_1(1260)] > \mathcal{B}^u[b_1(1235)] > \mathcal{B}^u[f_1(1285)] > \mathcal{B}^u[h_1(1170)] \gg \mathcal{B}^u[f_1(1420)] \gtrsim \mathcal{B}^u[h_1(1380)], \tag{4.9}$$

$$\mathcal{B}^s[K_1(1270)] \gtrsim \mathcal{B}^s[K_1(1400)] > \mathcal{B}^s[f_1(1420)] > \mathcal{B}^s[h_1(1380)] \gg \mathcal{B}^s[f_1(1285)] \gtrsim \mathcal{B}^s[h_1(1170)], \tag{4.10}$$

TABLE VII. Decay rates of $B_{u,d,s} \to A e \bar{\nu}_e$ obtained in this work, where $\Gamma_{\pm,L}$ are in units of $10^6\ \mathrm{s}^{-1}$, and the branching ratios are in units of 10^{-4}. Γ_L stands for the portion of the rate with a longitudinal polarization A, Γ_+ with a positive helicity A, and Γ_- with a negative helicity A. $\Gamma_T = \Gamma_+ + \Gamma_-$. Here, we use $\theta_{K_1} = -(34 \pm 13)^\circ$, $\theta_{^3P_1} = (23.6^{+17.0}_{-23.6})^\circ$, and $\theta_{^1P_1} = (28.1^{+9.8}_{-17.2})^\circ$. The first error comes from the variation of form factors, and the second from the mixing angles.

A	Γ_+	Γ_-	Γ_L	Γ_L/Γ_T	$\mathcal{B}(\bar{B}^0 \to A^+ e^- \bar{\nu}_e)$	$\mathcal{B}(B^- \to A^0 e^- \bar{\nu}_e)$
$a_1(1260)$	$7.6^{+3.2}_{-2.6}$	115^{+48}_{-39}	$74.8^{+30.7}_{-25.4}$	$0.61^{+0.00}_{-0.00}$	$3.02^{+1.03}_{-1.03}$	$3.24^{+1.33}_{-1.13}$
$f_1(1285)$	$4.2^{+1.7+0.0}_{-1.4-1.1}$	$62.1^{+23.6+2.1}_{-19.8-18.9}$	$40.4^{+14.9+0.5}_{-12.5-11.3}$	$0.61^{+0.01+0.02}_{-0.00-0.01}$	$1.63^{+0.60+0.04}_{-0.51-0.48}$	$1.75^{+0.65+0.04}_{-0.55-0.52}$
$f_1(1420)$	$0.1^{+0.1+1.2}_{-0.0-0.0}$	$2.2^{+1.0+15.7}_{-0.8-1.7}$	$1.1^{+0.9+10.3}_{-1.0-0.5}$	$0.48^{+0.10+0.60}_{-0.14-0.00}$	$0.05^{+0.03+0.42}_{-0.02-0.03}$	$0.06^{+0.03+0.44}_{-0.03-0.04}$
$b_1(1235)$	$3.3^{+1.4}_{-1.2}$	$37.7^{+16.6}_{-13.5}$	$85.5^{+36.5}_{-30.2}$	$2.08^{+0.03}_{-0.01}$	$1.93^{+0.84}_{-0.68}$	$2.07^{+0.90}_{-0.73}$
$h_1(1170)$	$1.9^{+1.2+0.1}_{-0.9-0.4}$	$24.7^{+10.8+1.0}_{-8.8-4.8}$	$54.7^{+24.8+2.4}_{-20.2-11.2}$	$2.05^{+0.01+0.01}_{-0.01-0.01}$	$1.24^{+0.57+0.06}_{-0.45-0.25}$	$1.33^{+0.60+0.06}_{-0.49-0.27}$
$h_1(1380)$	$0.1^{+0.2+0.5}_{-0.0-0.1}$	$0.8^{+0.1+3.7}_{-0.2-0.8}$	$1.8^{+0.5+7.8}_{-0.5-1.7}$	$1.97^{+0.03+0.12}_{-0.14-0.07}$	$0.04^{+0.01+0.18}_{-0.01-0.00}$	$0.04^{+0.02+0.20}_{-0.01-0.00}$
A	Γ_+	Γ_-	Γ_L	Γ_L/Γ_T	$\mathcal{B}(B_s \to A^+ e^- \bar{\nu}_e)$	
$f_1(1285)$	$0.4^{+1.0+2.1}_{-0.2-0.3}$	$4.4^{+9.0+32.8}_{-4.2-3.3}$	$3.3^{+8.2+20.1}_{-3.2-2.9}$	$0.68^{+0.07+0.00}_{-0.42-0.36}$	$0.12^{+0.27+0.00}_{-0.12-0.10}$	
$f_1(1420)$	$6.2^{+0.9+0.4}_{-0.8-2.1}$	$90.8^{+17.9+2.8}_{-16.3-27.3}$	$57.6^{+12.3+2.6}_{-11.2-18.3}$	$0.59^{+0.01+0.02}_{-0.01-0.01}$	$2.27^{+0.45+0.08}_{-0.42-0.70}$	
$h_1(1170)$	$0.1^{+0.1+0.2}_{-0.0-0.0}$	$0.1^{+0.2+5.7}_{-0.1-0.0}$	$0.2^{+0.3+11.6}_{-0.2-0.0}$	$1.37^{+0.02+0.71}_{-0.48-0.00}$	$0.01^{+0.00+0.25}_{-0.00-0.00}$	
$h_1(1380)$	$3.7^{+2.2+0.0}_{-1.7-0.3}$	$37.6^{+17.4+0.0}_{-14.1-4.4}$	$74.2^{+37.7+0.0}_{-30.0-8.2}$	$1.80^{+0.04+0.00}_{-0.06-0.01}$	$1.69^{+0.84+0.00}_{-0.67-0.19}$	
$K_1(1270)$	$9.2^{+3.7+0.6}_{-4.0-1.4}$	141^{+54+9}_{-61-22}	159^{+57+0}_{-72-10}	$1.06^{+0.00+0.15}_{-0.05-0.12}$	$4.53^{+1.67+0.00}_{-2.00-0.44}$	
$K_1(1400)$	$9.4^{+3.4+0.6}_{-4.1-1.5}$	119^{+45+8}_{-51-19}	135^{+49+0}_{-61-7}	$1.05^{+0.00+0.13}_{-0.04-0.11}$	$3.86^{+1.43+0.03}_{-1.70-0.40}$	

where $\mathcal{B}^q[a_1(1260)] \equiv \mathcal{B}(B_q \to Ae\bar{\nu}_e)$. These patterns are sensitive to the mixing angles and thus can offer deeper insights for the quark contents of the P-wave mesons.

V. SUMMARY

We have calculated the vector and axial-vector form factors of B decays into P-wave axial-vector mesons in the light-cone sum rule approach. Owing to the G-parity, the chiral-even two-parton light-cone distribution amplitudes of the 1^3P_1 and 1^1P_1 mesons are, respectively, symmetric and antisymmetric under the exchange of quark and antiquark momentum fractions in the SU(3) limit. For chiral-odd light-cone distribution amplitudes, it is the other way around. The sum rule results for form factors are sensitive to the light-cone distribution amplitudes of the axial-vector mesons. To extract the relevant form factors, the polarization of the axial-vector meson is chosen to be transversely polarized in the light-cone sum rule calculation. For the resulting sum rules, we have included the terms up to order of m_A/m_b in the light-cone expansion. The numerical impact of $1/m_b$ corrections is under control. As discussed in Sec. III, it should be stressed that the expansion parameter in the light-cone sum rules is m_A/m_b, instead of twist. In the present study, because the decay constants $f_{^3P_1}$ and $f^\perp_{^1P_1}$, which are defined in Eqs. (2.1) and (2.2), are of the same sign, the form factors for $B \to 1^1P_1$ and $B \to 1^3P_1$ transitions have opposite signs. The sum rule results could be improved in the future by including $\mathcal{O}(\alpha_s)$ corrections to the sum rules and by improving the input parameters describing the light-meson distribution amplitudes, for instance from lattice calculations. We have presented the results for the semileptonic decay rates of $B_{u,d,s} \to Ae\bar{\nu}_e$. Theses will allow further tests of our form factor results and of the mixing angles in the future.

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APPENDIX A: TWO-PARTON DISTRIBUTION AMPLITUDES

The chiral-even LCDAs are given by

$$\langle A(P,\lambda)|\bar{q}_1(x)\gamma_\mu\gamma_5 q_2(0)|0\rangle = if_A m_A \int_0^1 du e^{iupx}\Big\{p_\mu \frac{\epsilon^{*(\lambda)}x}{px}\Phi_\parallel(u) + \varepsilon^{*(\lambda)}_{\perp\mu} g_\perp^{(a)}(u) - \frac{1}{2}x_\mu \frac{\epsilon^{*(\lambda)}x}{(px)^2} m_A^2 g_3(u) + \mathcal{O}(x^2)\Big\}, \quad \text{(A1)}$$

$$\langle A(P,\lambda)|\bar{q}_1(x)\gamma_\mu q_2(0)|0\rangle = -if_A m_A \epsilon_{\mu\nu\rho\sigma}\epsilon^{*\nu}_{(\lambda)} p^\rho x^\sigma \times \int_0^1 du e^{iupx}\Big(\frac{g_\perp^{(v)}(u)}{4} + \mathcal{O}(x^2)\Big), \quad \text{(A2)}$$

where u and $\bar{u} \equiv 1-u$ are the momentum fractions carried by the q_1 and $\bar{q}_2$ quarks, respectively, in the axial-vector meson. The chiral-odd LCDAs are defined by

$$\langle A(P,\lambda)|\bar{q}_1(x)\sigma_{\mu\nu}\gamma_5 q_2(0)|0\rangle = f_A^\perp \int_0^1 du e^{iupx}\Big\{(\varepsilon^{*(\lambda)}_{\perp\mu}p_\nu - \varepsilon^{*(\lambda)}_{\perp\nu}p_\mu)\Phi_\perp(u) + \frac{m_A^2\epsilon^{*(\lambda)}x}{(px)^2}(p_\mu x_\nu - p_\nu x_\mu)h_\parallel^{(t)}(u) + \frac{1}{2}(\varepsilon^{*(\lambda)}_{\perp\mu}x_\nu - \varepsilon^{*(\lambda)}_{\perp\nu}x_\mu)\frac{m_A^2}{px}h_3(u) + \mathcal{O}(x^2)\Big\}, \quad \text{(A3)}$$

$$\langle A(P,\lambda)|\bar{q}_1(x)\gamma_5 q_2(0)|0\rangle = f_A^\perp m_A^2(\epsilon^{*(\lambda)}x)\int_0^1 du \times e^{iupx}\Big(\frac{h_\parallel^{(p)}(u)}{2} + \mathcal{O}(x^2)\Big), \quad \text{(A4)}$$

where $p_\mu = P_\mu - m_A^2 x_\mu/(2Px)$. Here, $\Phi_\parallel$, $\Phi_\perp$ are of twist 2, $g_\perp^{(a)}$, $g_\perp^{(v)}$, $h_\parallel^{(t)}$, $h_\parallel^{(p)}$ of twist 3, and g_3, h_3 of twist 4. Note that in the definitions of LCDAs, the longitudinal and transverse *projections* of polarization vectors $\epsilon_\mu^{*(\lambda)}$ along the x direction for the axial-vector meson are given by

$$\varepsilon^{*(\lambda)}_{\parallel\mu} \equiv \frac{\epsilon^{*(\lambda)}x}{px}\Big(p_\mu - \frac{m_A^2}{2px}x_\mu\Big), \qquad \varepsilon^{*(\lambda)}_{\perp\mu} = \epsilon^{*(\lambda)}_\mu - \varepsilon^{*(\lambda)}_{\parallel\mu}. \quad \text{(A5)}$$

One should distinguish the above projections from the *exactly* longitudinal ($\epsilon^{*(0)\mu}$) and transverse ($\epsilon_\perp^{*(\lambda)\mu}$) polarization vectors of the axial-vector meson, given in Eq. (3.11), where the results are independent of the coordinate variable x.

In SU(3) limit, due to G parity, $\Phi_\parallel$, $g_\perp^{(a)}$, $g_\perp^{(v)}$, and g_3 are symmetric (antisymmetric) under the replacement $u \to 1-u$ for the 1^3P_1 (1^1P_1) states, whereas $\Phi_\perp$, $h_\parallel^{(t)}$, $h_\parallel^{(p)}$, and h_3 are antisymmetric (symmetric). In other words, in the SU(3) limit it follows that

$$\int_0^1 du\Phi_\perp(u) = \int_0^1 du h_\parallel^{(t)}(u) = \int_0^1 du h_\parallel^{(p)}(u) = \int_0^1 du h_3(u) = 0 \quad \text{(A6)}$$

for 1^3P_1 states, but becomes

$$\int_0^1 du\Phi_\parallel(u) = \int_0^1 du g_\perp^{(a)}(u) = \int_0^1 du g_\perp^{(v)}(u) = \int_0^1 du g_3(u) = 0 \quad \text{(A7)}$$

for 1^1P_1 states. The above integrals are not zero if $m_{q_1} \neq m_{q_2}$, and the detailed discussions can be found in Ref. [11]. For convenience, we therefore normalize the distribution amplitudes of the 1^3P_1 and 1^1P_1 states to be subject to

$$\int_0^1 du\Phi_{\parallel}^{^3P_1}(u) = 1 \quad \text{and} \quad \int_0^1 du\Phi_{\perp}^{^1P_1}(u) = 1. \tag{A8}$$

We set $f^{\perp}_{^3P_1} = f_{^3P_1}$ and $f_{^1P_1} = f^{\perp}_{^1P_1}(\mu = 1\ \text{GeV})$ in the study, such that we have

$$\begin{aligned}&\langle 1^3P_1(P,\lambda)|\bar{q}_1(0)\sigma_{\mu\nu}\gamma_5 q_2(0)|0\rangle \\ &\quad = f_{^3P_1} a_0^{\perp,^3P_1}(\epsilon_\mu^{*(\lambda)}P_\nu - \epsilon_\nu^{*(\lambda)}P_\mu),\end{aligned} \tag{A9}$$

$$\begin{aligned}&\langle 1^1P_1(P,\lambda)|\bar{q}_1(0)\gamma_\mu\gamma_5 q_2(0)|0\rangle \\ &\quad = i f^{\perp}_{^1P_1}(1\ \text{GeV}) a_0^{\parallel,^1P_1} m_{^1P_1}\epsilon_\mu^{*(\lambda)},\end{aligned} \tag{A10}$$

where $a_0^{\perp,^3P_1}$ and $a_0^{\parallel,^1P_1}$ are the Gegenbauer zeroth moments and vanish in the SU(3) limit.

We take into account the approximate forms of twist-2 distributions for the 1^3P_1 mesons to be [11]

$$\Phi_{\parallel}(u) = 6u\bar{u}\left[1 + 3a_1^{\parallel}\xi + a_2^{\parallel}\frac{3}{2}(5\xi^2 - 1)\right], \tag{A11}$$

$$\Phi_{\perp}(u) = 6u\bar{u}\left[a_0^{\perp} + 3a_1^{\perp}\xi + a_2^{\perp}\frac{3}{2}(5\xi^2 - 1)\right], \tag{A12}$$

and for the 1^1P_1 mesons to be

$$\Phi_{\parallel}(u) = 6u\bar{u}\left[a_0^{\parallel} + 3a_1^{\parallel}\xi + a_2^{\parallel}\frac{3}{2}(5\xi^2 - 1)\right], \tag{A13}$$

$$\Phi_{\perp}(u) = 6u\bar{u}\left[1 + 3a_1^{\perp}\xi + a_2^{\perp}\frac{3}{2}(5\xi^2 - 1)\right], \tag{A14}$$

where $\xi = 2u - 1$.

For the relevant two-parton twist-3 chiral-even LCDAs, we take the approximate expressions up to conformal spin 9/2 and, $\mathcal{O}(m_s)$ [11]

$$\begin{aligned}g_\perp^{(a)}(u) &= \frac{3}{4}(1+\xi^2) + \frac{3}{2}a_1^{\parallel}\xi^3 + \left(\frac{3}{7}a_2^{\parallel} + 5\zeta^V_{3,^3P_1}\right)(3\xi^2 - 1) + \left(\frac{9}{112}a_2^{\parallel} + \frac{105}{16}\zeta^A_{3,^3P_1} - \frac{15}{64}\zeta^V_{3,^3P_1}\omega^V_{^3P_1}\right)(35\xi^4 - 30\xi^2 + 3) \\ &\quad + 5\left[\frac{21}{4}\zeta^V_{3,^3P_1}\sigma^V_{^3P_1} + \zeta^A_{3,^3P_1}\left(\lambda^A_{^3P_1} - \frac{3}{16}\sigma^A_{^3P_1}\right)\right]\xi(5\xi^2 - 3) - \frac{9}{2}\bar{a}_1^{\perp}\tilde{\delta}_+\left(\frac{3}{2} + \frac{3}{2}\xi^2 + \ln u + \ln\bar{u}\right) \\ &\quad - \frac{9}{2}\bar{a}_1^{\perp}\tilde{\delta}_-(3\xi + \ln\bar{u} - \ln u),\end{aligned} \tag{A15}$$

$$\begin{aligned}g_\perp^{(v)}(u) &= 6u\bar{u}\Bigg\{1 + \left(a_1^{\parallel} + \frac{20}{3}\zeta^A_{3,^3P_1}\lambda^A_{^3P_1}\right)\xi + \left[\frac{1}{4}a_2^{\parallel} + \frac{5}{3}\zeta^V_{3,^3P_1}\left(1 - \frac{3}{16}\omega^V_{^3P_1}\right) + \frac{35}{4}\zeta^A_{3,^3P_1}\right](5\xi^2 - 1) \\ &\quad + \frac{35}{4}\left(\zeta^V_{3,^3P_1}\sigma^V_{^3P_1} - \frac{1}{28}\zeta^A_{3,^3P_1}\sigma^A_{^3P_1}\right)\xi(7\xi^2 - 3)\Bigg\} - 18a_1^{\perp}\tilde{\delta}_+(3u\bar{u} + \bar{u}\ln\bar{u} + u\ln u) \\ &\quad - 18a_1^{\perp}\tilde{\delta}_-(u\bar{u}\xi + \bar{u}\ln\bar{u} - u\ln u),\end{aligned} \tag{A16}$$

for the 1^3P_1 states, and

$$\begin{aligned}g_\perp^{(a)}(u) &= \frac{3}{4}a_0^{\parallel}(1+\xi^2) + \frac{3}{2}a_1^{\parallel}\xi^3 + 5\left[\frac{21}{4}\zeta^V_{3,^1P_1} + \zeta^A_{3,^1P_1}\left(1 - \frac{3}{16}\omega^A_{^1P_1}\right)\right]\xi(5\xi^2 - 3) + \frac{3}{16}a_2^{\parallel}(15\xi^4 - 6\xi^2 - 1) \\ &\quad + 5\zeta^V_{3,^1P_1}\lambda^V_{^1P_1}(3\xi^2 - 1) + \frac{105}{16}\left(\zeta^A_{3,^1P_1}\sigma^A_{^1P_1} - \frac{1}{28}\zeta^V_{^1P_1}\sigma^V_{^1P_1}\right)(35\xi^4 - 30\xi^2 + 3) - 15\bar{a}_2^{\perp}\left[\tilde{\delta}_+\xi^3 + \frac{1}{2}\tilde{\delta}_-(3\xi^2 - 1)\right] \\ &\quad - \frac{3}{2}[\tilde{\delta}_+(2\xi + \ln\bar{u} - \ln u) + \tilde{\delta}_-(2 + \ln u + \ln\bar{u})](1 + 6a_2^{\perp}),\end{aligned} \tag{A17}$$

$$\begin{aligned}g_\perp^{(v)}(u) &= 6u\bar{u}\Bigg\{a_0^{\parallel} + a_1^{\parallel}\xi + \left[\frac{1}{4}a_2^{\parallel} + \frac{5}{3}\zeta^V_{3,^1P_1}\left(\lambda^V_{^1P_1} - \frac{3}{16}\sigma^V_{^1P_1}\right) + \frac{35}{4}\zeta^A_{3,^1P_1}\sigma^A_{^1P_1}\right](5\xi^2 - 1) \\ &\quad + \frac{20}{3}\xi\left[\zeta^A_{3,^1P_1} + \frac{21}{16}\left(\zeta^V_{3,^1P_1} - \frac{1}{28}\zeta^A_{3,^1P_1}\omega^A_{^1P_1}\right)(7\xi^2 - 3)\right] - 5a_2^{\perp}[2\tilde{\delta}_+\xi + \tilde{\delta}_-(1+\xi^2)]\Bigg\} \\ &\quad - 6[\tilde{\delta}_+(\bar{u}\ln\bar{u} - u\ln u) + \tilde{\delta}_-(2u\bar{u} + \bar{u}\ln\bar{u} + u\ln u)](1 + 6a_2^{\perp}),\end{aligned} \tag{A18}$$

for the 1^1P_1 states, where

$$\tilde{\delta}_{\pm} = \frac{f_A^{\perp}}{f_A}\frac{m_{q_2} \pm m_{q_1}}{m_A}, \qquad \zeta_{3,A}^{V(A)} = \frac{f_A^{V(A)}}{f_A m_A}. \tag{A19}$$

APPENDIX B: AN ALTERNATIVE DEFINITION OF FORM FACTORS

Following Ref. [21], the semileptonic form factors for the $B \to A$ transition are alternatively defined in the following way

$$\langle A(p,\lambda)|A_\mu|\bar{B}_q(p_B)\rangle = i\frac{2}{m_{B_q}-m_A}\varepsilon_{\mu\nu\alpha\beta}\epsilon^{*\nu}_{(\lambda)}p_B^\alpha p^\beta A^{B_qA}(q^2),$$
$$\langle A(p,\lambda)|V_\mu|\bar{B}_q(p_B)\rangle = -\Big\{(m_{B_q}-m_A)\epsilon_\mu^{(\lambda)*}V_1^{B_qA}(q^2) - (\epsilon^{(\lambda)*}\cdot p_B)(p_B+p)_\mu\frac{V_2^{B_qA}(q^2)}{m_{B_q}-m_A} - 2m_A\frac{\epsilon^{(\lambda)*}\cdot p_B}{q^2}q^\mu[V_3^{B_qA}(q^2) - V_0^{B_qA}(q^2)]\Big\}, \tag{B1}$$

where $q = p_B - p$, $V_3^{B_qA}(0) = V_0^{B_qA}(0)$ and

$$V_3^{B_qA}(q^2) = \frac{m_{B_q}-m_A}{2m_A}V_1^{B_qA}(q^2) - \frac{m_{B_q}+m_A}{2m_A}V_2^{B_qA}(q^2),$$
$$\langle A|\partial_\mu V^\mu|B_q\rangle = 2im_A(\epsilon^* p_B)V_0^{B_qA}(q^2). \tag{B2}$$

Following the above parametrization, the numerical results of the light-cone sum rules for the form factors are listed in Table VIII.

APPENDIX C: DECAY AMPLITUDES

The differential decay rates for $\bar{B}_q^0 \to A^+e^-\bar{\nu}_e$ are given by

$$\frac{d\Gamma(\bar{B}_q^0 \to A^+e^-\bar{\nu}_e)}{dE_e dq^2} = \frac{G_F^2}{128\pi^3}|V_{\rm CKM}|^2\frac{q^2}{m_B^2}\times[(1-\cos\theta)^2 \times H_-^2 + (1+\cos\theta)^2H_+^2 + 2(1-\cos^2\theta)H_0^2], \tag{C1}$$

TABLE VIII. Following the parametrization given in Eq. (B1), form factors for $B_{u,d,s} \to A$ transitions obtained in the light-cone sum rule calculation are fitted to the 3-parameter form in Eq. (4.7). Here, because the decay constants $f_{^3P_1}$ and $f^\perp_{^1P_1}$ are of the same sign, the form factors for $B_{(s)} \to 1^1P_1$ and $B_{(s)} \to 1^3P_1$ transitions have opposite signs.

F	$F(0)$	a	b	F	$F(0)$	a	b
$V_1^{Ba_1}$	0.60 ± 0.11	0.645	0.250	$V_1^{Bb_1}$	-0.32 ± 0.06	0.748	0.063
$V_2^{Ba_1}$	0.26 ± 0.05	1.48	1.00	$V_2^{Bb_1}$	-0.06 ± 0.01	0.539	1.76
$V_0^{Ba_1}$	0.30 ± 0.05	1.77	0.926	$V_0^{Bb_1}$	-0.39 ± 0.07	1.22	0.426
A^{Ba_1}	0.30 ± 0.05	1.64	0.986	A^{Bb_1}	-0.16 ± 0.03	1.69	0.910
$V_1^{BK_{1A}}$	0.56 ± 0.11	0.635	0.211	$V_1^{BK_{1B}}$	$-0.48^{+0.13}_{-0.08}$	0.729	0.074
$V_2^{BK_{1A}}$	0.25 ± 0.05	1.51	1.18	$V_2^{BK_{1B}}$	$-0.10^{+0.03}_{-0.02}$	0.919	0.855
$V_0^{BK_{1A}}$	0.22 ± 0.04	2.40	1.78	$V_0^{BK_{1B}}$	$-0.45^{+0.12}_{-0.08}$	1.34	0.690
$A^{BK_{1A}}$	0.27 ± 0.05	1.60	0.974	$A^{BK_{1B}}$	$-0.22^{+0.06}_{-0.04}$	1.72	0.912
$V_1^{Bf_1}$	0.37 ± 0.07	0.640	0.153	$V_1^{Bh_1}$	-0.21 ± 0.04	0.612	0.078
$V_2^{Bf_1}$	0.16 ± 0.03	1.47	0.956	$V_2^{Bh_1}$	-0.04 ± 0.01	0.500	1.63
$V_0^{Bf_1}$	0.18 ± 0.03	1.81	0.880	$V_0^{Bh_1}$	-0.24 ± 0.04	1.16	0.294
A^{Bf_1}	0.18 ± 0.03	1.63	0.900	A^{Bh_1}	-0.10 ± 0.02	1.54	0.848
$V_1^{Bf_8}$	0.26 ± 0.05	0.644	0.209	$V_1^{Bh_8}$	-0.18 ± 0.03	0.623	0.094
$V_2^{Bf_8}$	0.11 ± 0.02	1.49	1.09	$V_2^{Bh_8}$	-0.03 ± 0.01	0.529	1.53
$V_0^{Bf_8}$	0.12 ± 0.02	1.84	0.749	$V_0^{Bh_8}$	-0.18 ± 0.03	1.22	0.609
A^{Bf_8}	0.13 ± 0.02	1.64	0.919	A^{Bh_8}	-0.08 ± 0.02	1.56	0.827
$V_1^{B_sK_{1A}}$	0.49 ± 0.10	0.635	0.211	$V_1^{B_sK_{1B}}$	$-0.42^{+0.11}_{-0.07}$	0.729	0.074
$V_2^{B_sK_{1A}}$	0.22 ± 0.04	1.51	1.18	$V_2^{B_sK_{1B}}$	$-0.09^{+0.03}_{-0.02}$	0.919	0.855
$V_0^{B_sK_{1A}}$	0.19 ± 0.04	2.40	1.78	$V_0^{B_sK_{1B}}$	$-0.40^{+0.11}_{-0.07}$	1.34	0.690
$A^{B_sK_{1A}}$	0.24 ± 0.04	1.60	0.974	$A^{B_sK_{1B}}$	$-0.19^{+0.05}_{-0.03}$	1.72	0.912
$V_1^{B_sf_1}$	0.33 ± 0.06	0.640	0.153	$V_1^{B_sh_1}$	-0.18 ± 0.04	0.612	0.078
$V_2^{B_sf_1}$	0.14 ± 0.03	1.47	0.956	$V_2^{B_sh_1}$	-0.04 ± 0.01	0.500	1.63
$V_0^{B_sf_1}$	0.16 ± 0.03	1.81	0.880	$V_0^{B_sh_1}$	-0.21 ± 0.04	1.16	0.294
$A^{B_sf_1}$	0.16 ± 0.03	1.63	0.900	$A^{B_sh_1}$	-0.09 ± 0.02	1.54	0.848
$V_1^{B_sf_8}$	-0.46 ± 0.09	0.644	0.209	$V_1^{B_sh_8}$	0.32 ± 0.05	0.623	0.094
$V_2^{B_sf_8}$	-0.19 ± 0.03	1.49	1.09	$V_2^{B_sh_8}$	0.05 ± 0.02	0.529	1.53
$V_0^{B_sf_8}$	-0.21 ± 0.04	1.84	0.749	$V_0^{B_sh_8}$	0.32 ± 0.05	1.22	0.609
$A^{B_sf_8}$	-0.23 ± 0.04	1.64	0.919	$A^{B_sh_8}$	0.14 ± 0.03	1.56	0.827

with the helicity amplitudes being

$$H_{\pm} = (m_{B_q} + m_A)V_1(q^2) \mp \frac{\tilde{\lambda}^{1/2}}{m_{B_q} + m_A} A(q^2),$$
$$H_0 = \frac{1}{2m_A(q^2)^{1/2}}\Big[(m_{B_q}^2 - m_A^2 - q^2)(m_{B_q} + m_A) \times V_1(q^2) - \frac{\tilde{\lambda}}{m_{B_q} + m_A} V_2(q^2)\Big]. \tag{C2}$$

Here, E_e is the electron energy in the B_q rest system. θ is the polar angle between the A and e^- in the $(e^-, \bar{\nu}_e)$ system, and is given by

$$\cos\theta = \frac{(m_{B_q}^2 - m_A^2 + q^2) - 4m_{B_q}E_e}{\tilde{\lambda}^{1/2}}, \tag{C3}$$

with $\tilde{\lambda} = (m_{B_q}^2 + m_A^2 - q^2)^2 - 4m_{B_q}^2 m_A^2$. For a fixed electron energy, q^2 varies over the region $0 \le q^2 \le q^2_{\max}$, where

$$q^2_{\max} = \frac{2E_e(m_{B_q}^2 - m_A^2 - 2m_{B_q}E_e)}{m_{B_q} - 2E_e}, \tag{C4}$$

and the related range of E_e is $0 \le E_e \le (m_{B_q}^2 - m_A^2)/(2m_{B_q})$.

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